Session 3 : Circles

Precalculus: A Problem-Solving Approach

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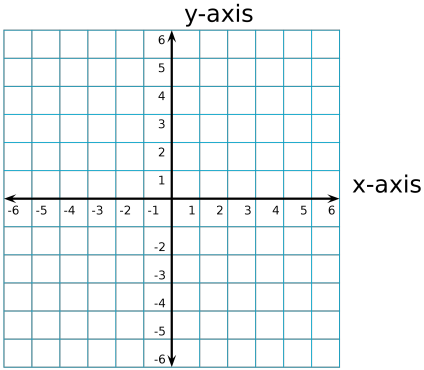
Session Outline

1. The Cartesian Plane
2. Definition of a Point
3. Distance between Two Points
4. The Circle
5. References

The Cartesian Plane

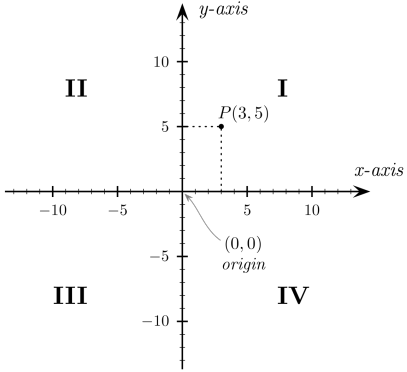


The Rectangular Coordinate System

Parts of a Plane

- $x - axis$, $x - values$
- $x - values$ moves right to $+\infty$
- $x - values$ moves left to $-\infty$
- $y - axis$, $y - values$
- $y - values$ shifts up to $+\infty$
- $y - values$ shifts down to $+\infty$
- $x - axis \perp y - axis$ at $O(0, 0)$
- Units n and Gridlines

The Four Quadrants



The Quadrants

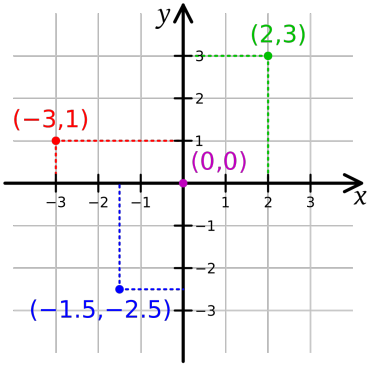
Values in the Quadrants

- First Quadrant $I(+x, +y)$
- Second Quadrant $II(-x, +y)$
- Third Quadrant $III(-x, -y)$
- Fourth Quadrant $IV(+x, -y)$

Realizations

An infinite plane can be divided into smaller but infinite planes.

The Definition of a Point



Some Points on the Plane

Points

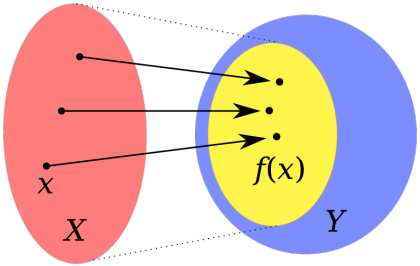
- Point $P(x, y)$ is an exact two-dimensional location
- P, Point Name
- x, $x - value$ or abscissa
- y, $y - value$ or ordinate
- (x, y), *ordered pair* or coordinates
- • Point, Close Point
- ◦ Hollow Point, Open Point
- Locations can be estimated

Problems 3.1 - 3.8

Estimate the Location of the following Points

1. $A(1, 2)$
2. $B(5, 0)$
3. $C(-3, 4)$
4. $D(0, 4)$
5. $E(7, -3)$
6. $F(-4, 0)$
7. $G(-\pi, -\sqrt{2})$
8. $H(0, -\pi)$

Domain & Range



$f : X \to Y$

Mapping Diagram

Domain x and Range y

- Two Points make a line (Euclid)
- Domain, set of all $x - values$
- Range, set of all $y - values$

Interval Notation

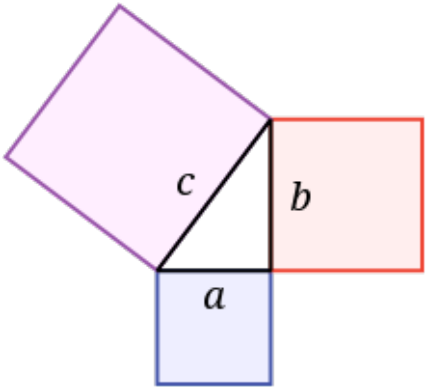
- Exclusivity $(x), >, <, \neq, \circ$
- Inclusivity $[x], \geq, \leq, =, \bullet$
- Infinity $(\pm\infty)$ is always exclusive

Problems 3.9 - 3.16

Illustrate reference lines using the following notations

1. $x(-\infty, +\infty)$
2. $x(-\infty, 8)$
3. $x(-\infty, 6]$
4. $x(11, +\infty)$
5. $x[-2, +\infty)$
6. $x[4, 10] \cup (11, 18)$
7. $x(6, 12] \cup [18, 22)$
8. $x[-1, 9] \cup (15, 27]$

Distance Between Two Points



The Pythagorean Theorem

Increment Δ

$$\Delta P = P_1 - P_2$$

Also, *discriminant* $\Delta = b^2 - 4ac$

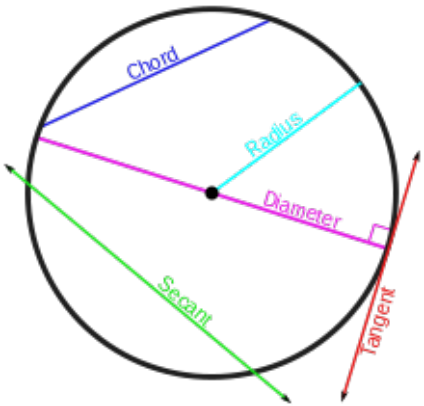
Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Distance d

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

The Circle



Circle & Its Parts

Definition of a Circle

- A conic section produced when a double-napped cone is sliced by a plane parallel to the base
- A circle is the set of all points P_n having the same distance from a center point $C(h, k)$

The Radius

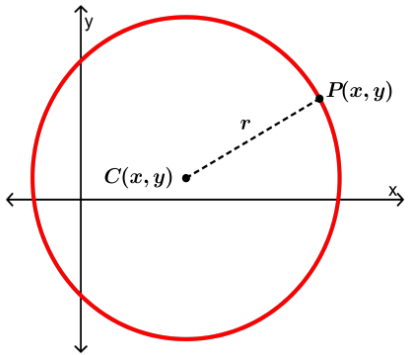
The Radius r is the distance from the center of a circle to a point on the circle.

Problems 3.17 - 3.24

Find the Standard and General Equations, and estimate the graphs of the following circles.

1. Center at the $origin$, radius 4
2. Center $(-4, 3)$, radius $\sqrt{7}$
3. $A(3, 1),\ r = 5$
4. $B(-2, -1),\ r = 4$
5. $C(3, 2),\ r = 3$
6. Center $(5, -6)$, tangent to the $y - axis$
7. Center $(5, -6)$, tangent to the $x - axis$
8. Has a diameter with endpoints $A(-1, 4), B(4, 2)$

The Equation of a Circle



The Radius

The Radius

$$d \to r = \sqrt{\Delta x^2 + \Delta y^2}$$

General Equation of a Circle

$$Ax^2 + By^2 + Cx + Dy + E = 0$$

Condition: $A, B > 0$

Standard Equation of a Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

h, k are vertices of circle C

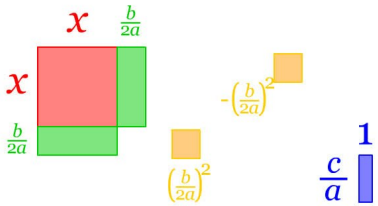
Problems 3.25 - 3.32

Find the radius r

1. $A(0,0),\ S(0,3)$
2. $B(1,-7),\ T(4,-7)$
3. $C(-2,-6),\ U(-8,11)$
4. $D(-3,5),\ V(-12,-15)$
5. $E(-4,4),\ W(-14,-13)$
6. $F(-5,-3),\ X(-10,9)$
7. $G(6,-2),\ Y(6,-5)$
8. $H(7,1),\ Z(2,1)$

The Perfect Square

$$a\left(x^2 + \frac{b}{a}x + \quad + \frac{c}{a}\right)$$



Completing Squares

The Perfect Square

$$x \times x = x^2$$
$$(x \pm y)^2 = x^2 \pm 2xy + y^2$$
$$PST : ax^2 + bx + c$$

Missing Middle Term

Given $ax^2 + c$ then $bx = 2\sqrt{ax^2}\sqrt{c}$

Missing Final Term

Given $ax^2 + bx$ then $c = \left(\frac{bx}{2\sqrt{ax^2}}\right)$

Problems 3.33 - 3.40

Complete the following squared equations, and find the values of x

1. $x^2 - 1 = 0$
2. $y^2 + 4 = 0$
3. $x^2 - 6x - 4 = 0$
4. $y^2 - 14x + 1 = 0$
5. $16x^2 + 96x = 45$
6. $4y^2 - 32y = 12$
7. $x^2 + 2x = -1$
8. $2x^2 + 3x = 7$

Problems 3.41 - 3.48

Transform the following General Equations into Standard Equations, and find the radius r

1. $x^2 + y^2 - 6x = 7$
2. $x^2 + y^2 - 14x + 2y = -14$
3. $16x^2 + 16y^2 + 96x - 40y = 315$
4. $x^2 + y^2 - 5x + 4y = 96$
5. $4x^2 + 4y^2 + 40x - 32y = 5$
6. $x^2 + y^2 - 4x + 6y + 4 = 0$
7. $x^2 + y^2 - 8x - 16y - 89 = 0$
8. $x^2 + y^2 + 1 = 0$

Seatwork

GENERAL DIRECTIONS:

- Only write with a black-inked ballpoint pen
- Read the instructions and problems carefully
- Legibly write your solutions, and box your final answers
- Avoid cheating, using devices, and making erasures
- Physical Scientific Calculators are allowed

IT'S YOUR TURN

Answer *pg. 17* Nos. 1 - 2

IT'S YOUR TURN

Answer *pg. 19* Nos. 1 - 3

References

DOCUMENTATION

This slide presentation is made with LaTeX. The source code is available at: `https://github.com/redundies/ueshsprecal`

REFERENCES

- Garces, Ian June et. al (2016) *Precalculus: Specialized Subject*
- Tamayo, Joycelyn et. al (2018) *Precalculus for SHS Students*
- De Guzman, Danilo et. al (2019) *Precalculus: A Worktext*
- Most Images from (`https://wikimedia.com`), PD-CC0L