Session 2 : The Conic Sections

Precalculus: A Problem-Solving Approach

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Session Outline

1. History of Analytic Geometry
2. The Cone
3. The Double-Napped Cone
4. The Conic Sections
5. References

A Brief History of Analytic Geometry

Euclid

Pre-Cartesian Era

- Geometry, 2000 B.C.
- No Cartesian Plane
- Plane Geometry
  Two-Dimensional (2D)
  Point, Line, Plane
- Solid Geometry
  Three-Dimensional (3D)
  Cube, Sphere *etc.*

A Brief History of Analytic Geometry

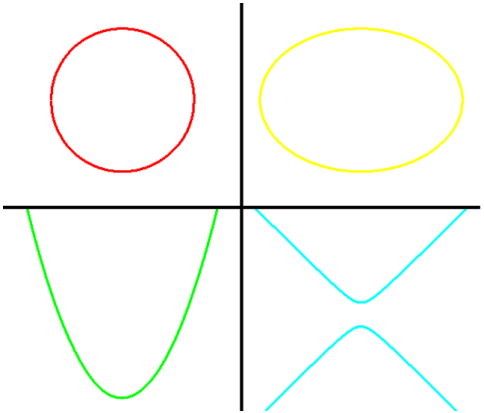
A fountain gets smaller as you move far away from it

Perspective Geometry

- 3D objects can be created from 2D Objects
- 2D objects can be derived from 3D Objects
- 3D objects and 2D objects can be manipulated[1]

[1] *Translated, Rotated, Dilated, Contracted, Sliced, Destroyed, Recreated, Transformed*

A Brief History of Analytic Geometry



The Two-Dimensional Curves

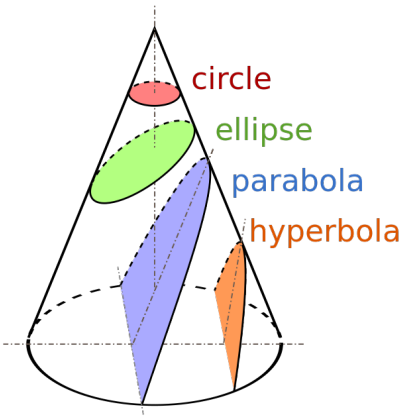
Existence of 2D Curves

- Ellipse
- Circle
- Parabola
- Hyperbola

Geometric Realization

Since 2D objects are derived from 3D objects, what 3D object contains all the curves?

A Brief History of Analytic Geometry

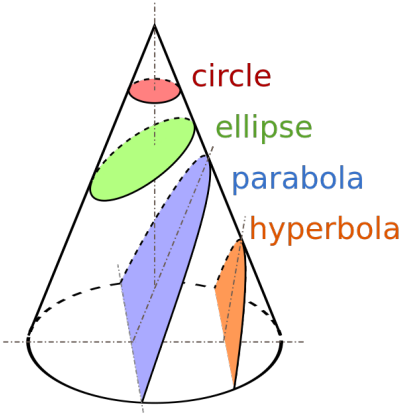


The Cone and the Conic Sections

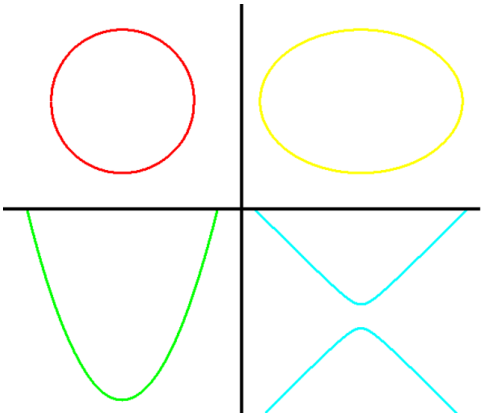
The Cone & The Conic Sections

- Menaechmus (*c. 355 B.C.*)
- A **Cone** is a 3D object that tapers from a circular base to a point
- The curves are derived by slicing a cone
- The curves, being derived parts from a cone, is now called **Conic Sections**

The Cone & The Conic Sections

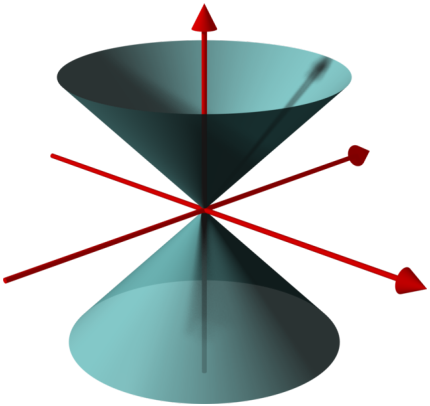


The Cone and the Conic Sections



The Two-Dimensional Curves

The Iconic Mistake

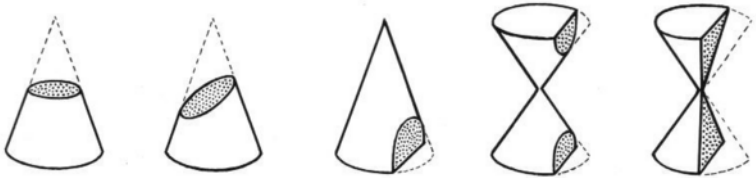


The Double-Napped Cone

The Double-Napped Cone

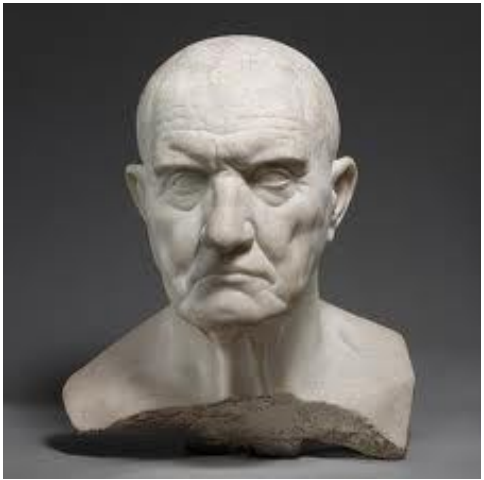
- The Hyperbola has two distinct branches
- A cone has one branch
- In resolution, another cone was put atop of the other
- Apollonius (*c. 240 B.C.*)
- Analytic Geometry was born
- The Conic Equations

The Double-Napped Cone



Conic Sections on the Double-Napped Cone

People in History



Menaechmus

Apollonius of Perga

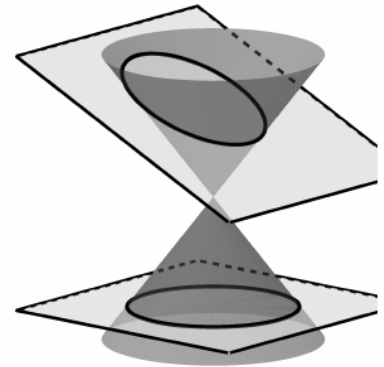
Circle & Ellipse

Definition

A circle is obtained by slicing a double-napped cone with a plane parallel to the base of the cone.

Definition

An ellipse is obtained by slicing a double-napped cone with a tilted plane intersecting only one of the cones.

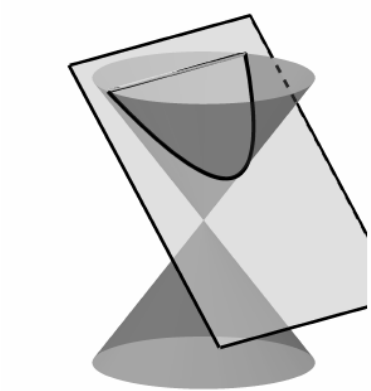


Circle and Ellipse

Parabola

Definition

A parabola is obtained by slicing a double-napped cone with a plane parallel to the edges of the cones.

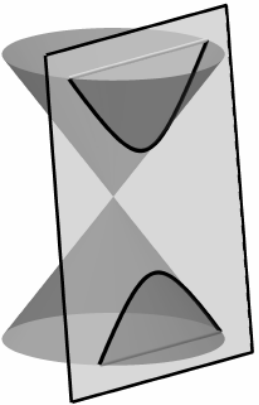


Parabola

Hyperbola

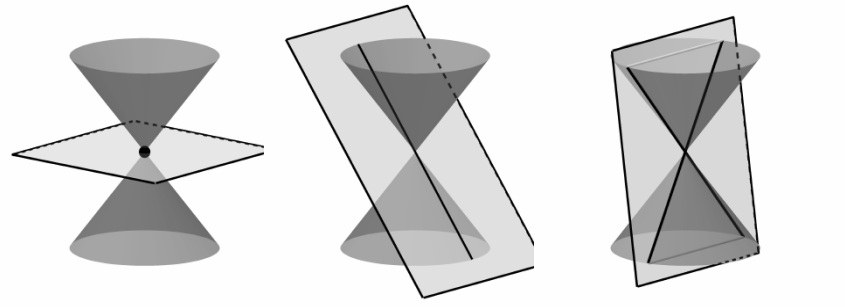
Definition

A hyperbola is obtained by slicing a double-napped cone with a plane intersecting both cones.



Hyperbola

Degenerate Conic Sections



Point, Line, Intersecting Lines

The Conic Equations

Rene Descartes

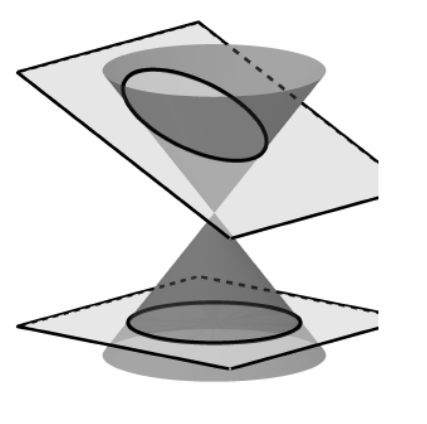
The Cartesian Era

- The Rectangular Coordinate System
- Simplified perspectives of 3D Objects
- Later, The Cartesian Plane

The Conic Equation

$$Ax^2 + By^2 + Cx + Dy + E = 0$$

General Equation of a Circle



Circle and Ellipse

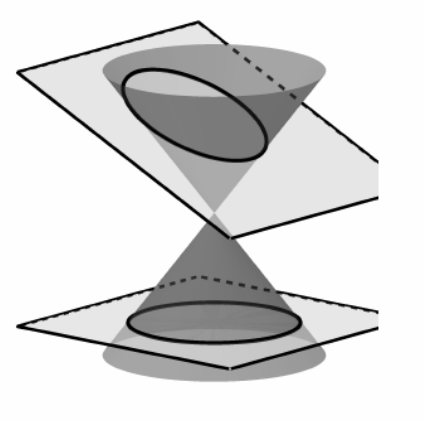
Equation of a Circle

$$A = B, \text{ A,B} > 0$$

Examples

$$4x^2 + 4y^2 + 8x + 20y + 1 = 0$$
$$3x^2 + 3y^2 + 12x - 6y - 11 = 0$$

General Equation of an Ellipse



Circle and Ellipse

Equation of an Ellipse

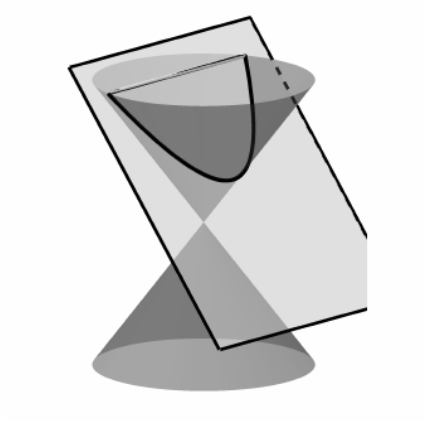
$A \neq B$; liked signed, not 0

Examples

$4x^2 + 5y^2 + 8x + 20y + 1 = 0$

$-x^2 - 2y^2 + 12x - 6y - 11 = 0$

General Equation of a Parabola



Parabola

Equation of a Parabola

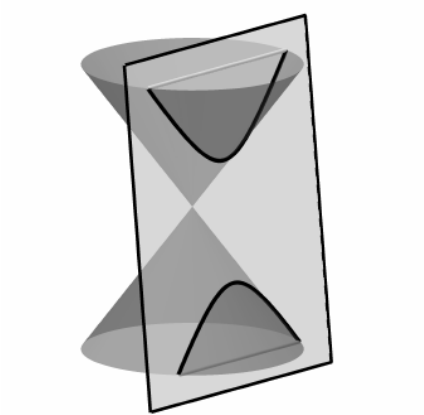
$A, B = 0$, but not both

Examples

$4x^2 + 8x + 20y + 1 = 0$

$-2y^2 + 12x - 6y - 11 = 0$

General Equation of a Hyperbola



Hyperbola

Equation of a Parabola

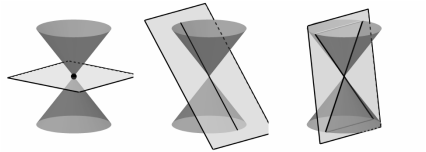
$A, B = 0$, but opposite signs

Examples

$4x^2 - 8y^2 + 8x + 20y + 1 = 0$

$-x^2 + 2y^2 + 12x - 6y - 11 = 0$

General Equations of Degenerate Sections



Point, Line,
Double-Napped Triangle

Examples

Point

$$2x^2 + 5y^2 + 12x - 20y + 38 = 0$$

Double-Napped Triangle

$$2x^2 - 3y^2 - 20x - 18y + 23 = 0$$

Complex Graph

$$x^2 + y^2 + 2x - 6y + 15 = 0$$

Seatwork

GENERAL DIRECTIONS:

- Only write with a black-inked ballpoint pen
- Read the instructions and problems carefully
- Legibly write your solutions, and box your final answers
- Avoid cheating, using devices, and making erasures
- Physical Scientific Calculators are allowed

IT'S YOUR TURN

Answer *pg. 11* Nos. 1 - 3

HOW CLEAR IS THE LESSON TO YOU?

Answer *pg. 12* Nos. 9 - 12 & Nos. 14 - 20

References

DOCUMENTATION

This slide presentation is made with LaTeX. The source code is available at: `https://github.com/redundies/ueshsprecal`

REFERENCES

- Garces, Ian June et. al (2016) *Precalculus: Specialized Subject*
- Tamayo, Joycelyn et. al (2018) *Precalculus for SHS Students*
- De Guzman, Danilo et. al (2019) *Precalculus: A Worktext*
- Most Images from (`https://wikimedia.com`), PD-CC0L