Fractals - Math 370

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Chapter 1

Introduction

1.1 What are fractals?

In what follows we will be investigating fractals and some of their interesting properties. First we define a fractal.

Definition. A **fractal** is a curve or geometric figure, each part of which has the same statistical character as the whole.

What this means in layterms is that a fractal is a shape or set that has similar properties at all magnifications. A classic example is the *Koch Curve* (Fig 1.1) which looks identical at all levels of magnification. We should note that there are certain degenerate cases which satisfy our definition such as a straight line. The distinction between a true fractal and a degenerate case can be made with the notion of *fractal dimension* which we will touch upon in Chapter 4 which deals with measure theory.

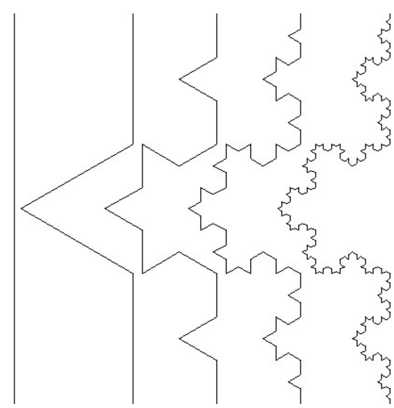

Figure 1.1: The first few iterations in generating the Koch Curve

1.2 Fractal Generation

It is often useful to think of a fractal as a set of points. However it is often difficult to explicitly state which points are in our fractal. A standard technique for fractal generation is to describe the *generation* of a fractal *iteratively* or *corecursively*. Typically we start with a base case, usually a set A_0, and define some sort of operation φ on a set and define $A_{i+1} = \phi(A_i)$ for $i \in \mathbb{Z}^+$. In Fig 1.1 we see that our base case is a straight line and our operation φ replaces the middle third of each line segment in A_i with two line segments forming the top part of a triangle.

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