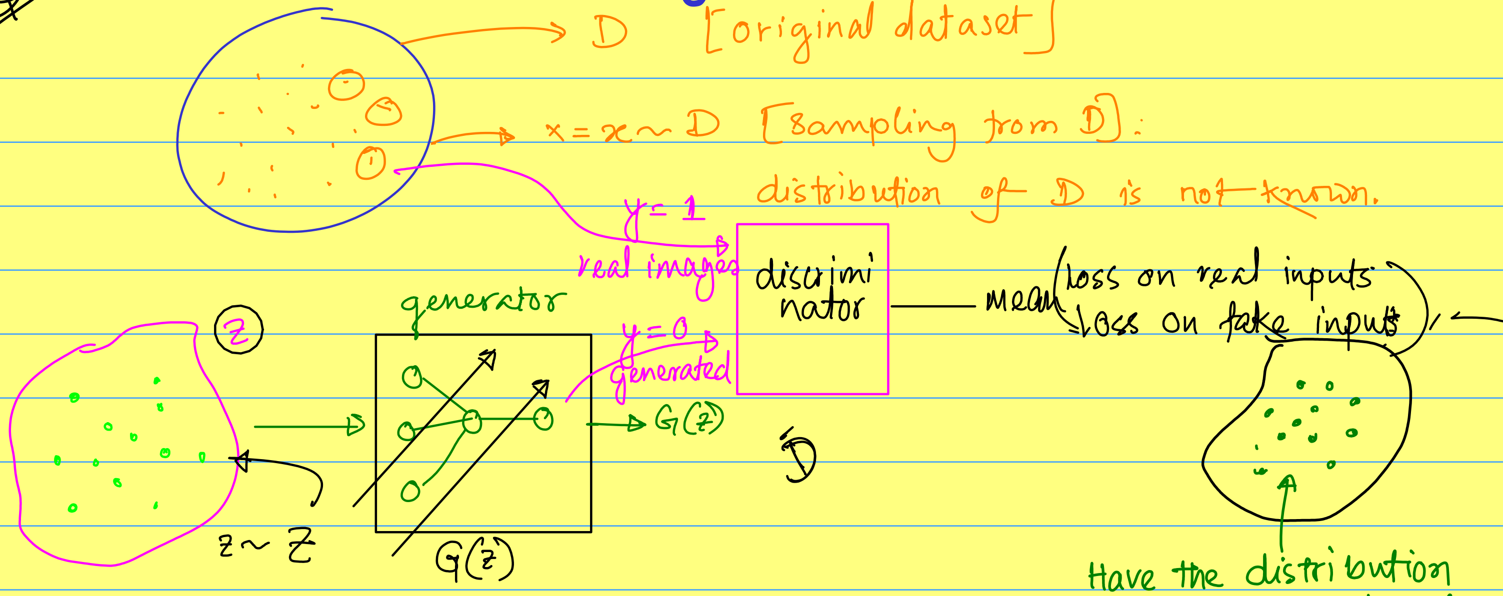
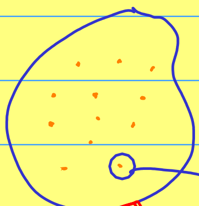
Understanding loss Function of GAN.

Good pen



Have the distribution close to the original distribution D.

Basic Convensions to understand loss function:



$p_{data}(x)$: original data distribution (unknown to us / But we want to know).

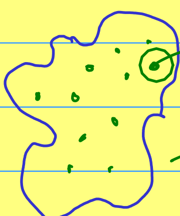
• : X = x [single instantiations of original space]

labels y = 1

$D(x;\theta_2)$ → $D(x)$ transformed | probability that x is coming from original dataset

$D(G(z)) \in \boxed{[0,1]}$ Here $D(x) \in [0,1]$ Basic probability.

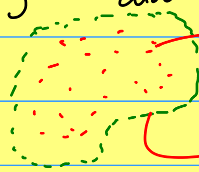
feeding it to discriminator



random noise (z)

z → $G(z;\theta_1)$

$P_Z(z)$



G(z)

$p_g(x)$: we are trying to make $p_{data}(x) \approx p_g(x)$

Here G and D are differentiable function.

Loss function

$$L(\hat{y}, y) = [y \log \hat{y} + (1-y)\log(1-\hat{y})]$$

Loss contribution from real images:

Label for the original data coming from $p_{data}(x)$ will be $y = 1$ and for the output produced by the discriminator D(G(z)) " the labels will be $\hat{y} = D(x)$.

So the amount of loss from the real images will be

$$L(\hat{y}, y) = L(D(x), 1) = [1 \cdot \log D(x) + 0 * \log(1 - D(x))] = \boxed{\log(D(x))} \rightarrow (A)$$

Loss contribution from the generated images:

For data coming from the generator $G(z)$, the ground truth is $y = 0$ and prediction $\hat{y} = D(G(z))$. Therefore

$$L(D(G(z)), 0) = [(1-0)\log(1 - D(G(z)))]$$

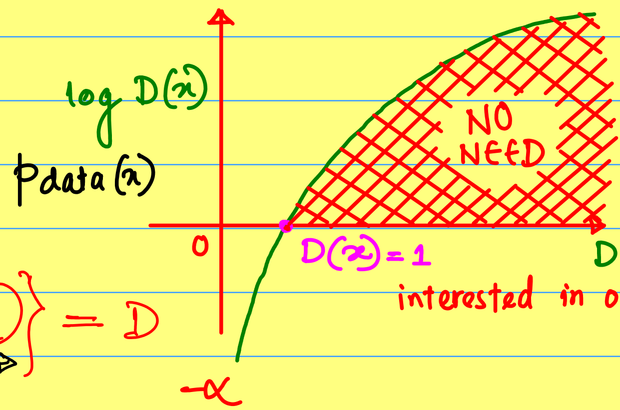
$$= \log(1 - D(G(z))) \quad \text{———} \; (B)$$

objective of the Discriminator: We want to maximize both the A and B

(A) $[\log(D(x))]$

(B) $[\log(1 - D(G(z)))]$

log plot:



force
$D(x) \to 1$

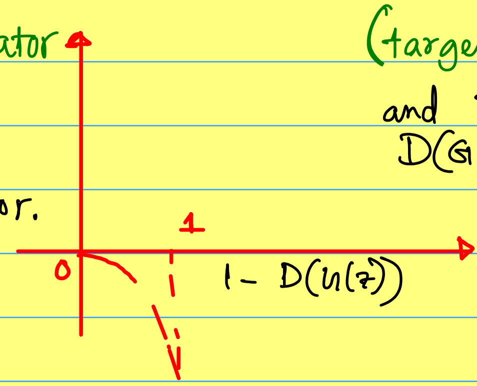
interested in only $D(x) \in [0,1]$

Therfore the objective function

$$\max\left\{\log(D(x)) + \log(1 - D(G(z)))\right\} = D$$

objective of generator: fool the discriminator
$D(G(z)) \to 1$

(target of discriminator)
and force
$D(G(z)) \to 0$

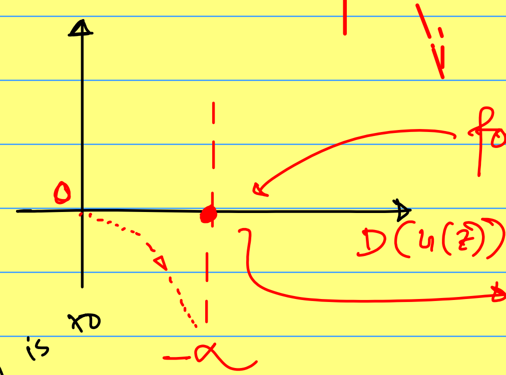


Not at all dependent on the generator.

Look at the second term.

$$\log(1 - D(G(z)))$$

if $D(G(z)) = 1$ then it is fooled.



fooling point

at this point
$\log(1-1) = -\infty$
very small value compared to the other thing.

So here for generator the aim is to

$$\min\left[\log(D(x)) + \log(1 - D(G(z)))\right]$$

there is no gen term here.

produce the data to the original data distribution.

Therefore combining both of the optimization task we get

$$\min_{G} \max_{D} \left\{ \log(D(x)) + \log(1 - D(G(z))) \right\} \text{ --- } (C)$$

Important:

(#) when generator is considered (minimized) only the $\log(1 - D(G(z)))$ term will be considered. i.e only the parameters of $G(z)$ will be updated.

(#) when the discriminator is optimized (maximized), then both of the term will be considered. i.e Both parameters of D and G will be updated.

The above equation (C) is only for a single value or point. But for all the points in the dataset of n points. it will be something like

$$\min_{G} \max_{D} \frac{1}{m} \sum_{i=1}^{m} \log(D(x_i)) + \frac{1}{m} \sum_{i=1}^{m} \log(1 - D(G(z_i)))$$

which can be re written as the expectation value

$$\boxed{\min_{G} \max_{D} V(D, G) = E_{x \sim P_{data}(x)} \log(D(x)) + E_{z \sim P_z(z)} \log(1 - D(G(z)))}$$