

9.11)

Similarly to the previous problem, if this problem is going to be NP-complete, we can assume that it is a clique and reduce it to a no-clique-clique problem. If the graph contains a clique (as explained in the previous example), then all that is needed is an additional set of k vertices which are not part of the clique. If the combined graph contains the unconnected k vertices and the connected clique, then the graph satisfies the problem. This results in the problem being NP-complete.

9.12)

To prove that the Eulerian cycle problem is NP-hard we must reduce from another known problem. The Hamiltonian circuit problem is NP-hard and is a good place to start. In Eulerian cycle, you can only visit an edge once, therefore you can only use even numbers of edges from a vertex. Similarly, in a Hamiltonian circuit, you can only visit each vertex once. In an Eulerian cycle, one can visit a vertex multiple times if more edges are required to be visited. Therefore, if the problem is a Hamiltonian problem, then it is also an Eulerian cycle if each vertex has a maximum of 2 edges. A maximal Hamiltonian cycle (maximum vertices) can reduce to a maximal Eulerian cycle (maximum edges) which means that this problem is NP-complete and that finding the number of edges is NP-hard.