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FSS:6836

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Fuzzy Sets and Systems ••• (••••) •••–•••

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Fuzzy sets for image processing and understanding

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Received 4 January 2015; accepted 22 June 2015

Abstract

This paper proposes a short overview of models and methods based on fuzzy sets for image processing and image understanding, from low level to higher level interpretation. Recent trends are highlighted.

Keywords: Image processing; Image understanding; Fuzzy sets; Spatial information; Structural information; Spatial reasoning

1. Fuzzy sets for image processing and understanding: introduction

Since the introduction of fuzzy sets [158], this theory was rapidly exploited and further developed for image processing and image understanding problems. One of the reasons is that it is of major interest for dealing with imprecise information and knowledge, and provides a consistent mathematical framework for knowledge representation, information modeling at different levels, fusion of heterogeneous information, reasoning and decision making. In this paper, we will summarize the main research lines and point some new areas that deserve to be investigated. Some references are given as examples, and by far do not aim at being exhaustive.

1.1. Sources of imprecision

Imprecision is often inherent to images, and its causes can be found at several levels: observed phenomenon (imprecise limits between structures or objects), acquisition process (limited resolution, numerical reconstruction methods, potential artifacts), image processing steps (imprecision induced by a filtering for instance). Imprecision also occurs in the available knowledge, about the acquisition process, the properties of the obtained images, the domain, etc.

1.2. Interest of fuzzy sets

Fuzzy sets have several advantages for representing such imprecisions, as in other domains of information processing [71], since they constitute a unified framework for representing and processing both numerical and symbolic

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http://dx.doi.org/10.1016/j.fss.2015.06.017
0165-0114/

Please cite this article in press as: I. Bloch, Fuzzy sets for image processing and understanding, Fuzzy Sets and Systems (2015), http://dx.doi.org/10.1016/j.fss.2015.06.017

information. First, they are able to represent several types of imprecision in images, as for instance imprecision in spatial location of objects, or imprecision in membership of an object to a class. For instance, partial volume effect finds a consistent representation in fuzzy sets (membership degrees of a pixel or voxel to objects directly represent partial membership to the different objects mixed up in this pixel or voxel, leading to a consistent modeling with respect to reality). Secondly, image information can be represented at different levels with fuzzy sets (local, regional, or global), as well as under different forms (numerical, or symbolic). For instance, classification based only on grey levels involves very local information (at the pixel level); introducing spatial coherence in the classification or relations between features involves regional information; and introducing relations between objects or regions for scene interpretation involves more global information and is related to the field of spatial reasoning. Thirdly, the fuzzy set framework allows for the representation of very heterogeneous information, and is able to deal with information extracted directly from the images, as well as with information derived from some external knowledge, such as expert knowledge. This is exploited in particular in model-based pattern recognition, where fuzzy information extracted from the images is compared and matched to a model representing knowledge expressed in fuzzy terms.

Therefore this theory can achieve tasks at several levels, from low level (e.g. grey-level based classification) to high level (e.g. model based structural recognition and scene interpretation). It provides a flexible framework for information fusion as well as powerful tools for reasoning and decision making. From a mathematical point of view, fuzzy sets can be equipped with a complete lattice structure, which is suitable for its association with other theories of information processing based on such structures, such as mathematical morphology or logics. While first applications mainly addressed reasoning at low level for classification, edge detection or filtering, higher level information modeling and processing is now more widely developed and still the topic of current research. This includes dealing with spatial information at intermediate or higher level, via mathematical morphology, spatial reasoning, ontologies, graphs, or knowledge-based systems, as well as advances in machine learning, higher level descriptions of image content, handling different levels of granularity, to name but a few.

1.3. Semantic gap

One important problem when reasoning at higher level, for instance based on symbolic models, is the semantic gap [147]. The question to be addressed is as follows: how to link visual percepts from the images to symbolic descriptions? This is also known as the symbol grounding problem in artificial intelligence [56,82]. The interest of fuzzy sets to answer this question relies in their capability to model linguistic as well as quantitative knowledge and information. A typical example is the notion of linguistic variable [159], where symbolic values have semantics defined by membership functions on a concrete domain.

1.4. A short review of existing textbooks

Let us now summarize the evolution of the domain, based on published textbooks or collections of more or less independent papers. The first works dealt with classification and pattern recognition and textbooks on these topics were published in the late 1990's [17,18,52,119]. Indices of fuzziness and cluster validity were also introduced, with the main drawback of providing better results for crisp classifications, which contradicts the initial claimed need for fuzzy classes. The book by Tizhoosh [149] also mentions some work in geometry and mathematical morphology applied on fuzzy sets. It mainly focuses on low level processing, for edge detection and image quality improvement, using rules applied on local neighborhoods of pixels, or minimizing a fuzziness index. The evolution towards rule-based systems and neuro-fuzzy approaches is acknowledged by the book edited by Bezdek et al. [17]. All these approaches have been rapidly used in different application domains, such as medical imaging [148]. The Ghent team has edited several books, providing an overview of the advances in fuzzy mathematical morphology, filtering using local approaches, control and rule-based methods, with the associated applications in [91], and more specifically in image filtering in [114]. New applications in remote sensing, image retrieval, video analysis, medical imaging are included in [113], as well as some theoretical advances. Let us also mention the review paper by Karmakar et al. [90], covering techniques for fuzzy segmentation, clustering, IF-THEN rules, mostly adapted to bi-modal images (or images with a known number of classes or clusters). Additional applications in compression are included in [120], as well as various soft computing approaches, still mostly for low level processing.

Please cite this article in press as: I. Bloch, Fuzzy sets for image processing and understanding, Fuzzy Sets and Systems (2015), http://dx.doi.org/10.1016/j.fss.2015.06.017

More recently, the book by Chaira et al. [50] deals again with low level processing, with some mentions of image retrieval and applications in remote sensing. Interestingly enough, the book also includes Matlab® examples, thus highlighting the concrete practical use of all these methods.

Higher level methods were then progressively introduced. As an evidence of this evolution, let us mention the book edited by Matsakis et al. [105] and the review paper [25], dealing with spatial relations, such as topological, metric, directional relations. This domain has then evolved towards more complex relations. Note that the interest of fuzzy sets for spatial relations had been recognized already in 1975 [76], but formal models for them appeared much later. These models are now exploited for model-based segmentation and recognition of structures in images. The book on granular computing by Pedrycz et al. [127] deals with fuzzy sets, as well as with other models for imprecision modeling such as interval analysis and rough sets. Some chapters deal with images and spatial reasoning. Noticeably, the preface mentions images as a typical domain where the methods described in the book are useful (for instance for reasoning from pixel level to object level). Advances in databases, data mining, machine learning, case-based reasoning are described in [77]. A few chapters deal with spatial data, as part of the larger domain of fuzzy information processing.

Finally, let us mention the now large number of special sessions in conferences, dedicated to fuzzy approaches for image processing and understanding (e.g. IPMU, WILF, FUZZ-IEEE, ICIP, EUSFLAT, i.e. both conferences focusing on fuzzy sets and conferences in image processing and computer vision). Several of these sessions are organized by the Working Group on Soft Computing in Image Processing.[1] This group is a follow-up of the SCIP group (Soft Computing in Image Processing),[2] and is now recognized as a working group of the European Society for Fuzzy Logic and Technology (EUSFLAT).[3]

2. Representations

Fuzzy sets can be used to represent both image information, along with its imprecision, and domain and expert knowledge.

Fuzzy sets representing image information can be considered from two points of view. First, a membership function can be a function from the space on which the image is defined into [0, 1], representing the membership degree of each point to a spatial fuzzy object. Such models may represent different types of imprecision, either on the boundary of the objects (due for instance to partial volume effect, or to the spatial resolution), on the variability of these objects, on the potential ambiguity between classes, etc. Secondly, a membership function can be a function from a space of attributes into [0, 1]. At numerical level, such attributes are typically the grey levels. The membership value then represents the degree to which a grey level supports the membership to an object or a class, described in vague terms, such as bright, dark, etc. At intermediate level, attributes can refer for instance to the shape of image regions. Membership functions then allow determining to which degree an image region is elongated, regular, etc.

As for knowledge representation, fuzzy sets are typically used to model, in a semi-quantitative way, symbolic or qualitative knowledge describing the expected content of the images (appearance and shape of the objects, spatial relations, as described in Section 5, type of objects, etc.). The concept of linguistic variable is then often used. These representations contribute to reduce the semantic gap, by associating a symbolic or qualitative value with a representation in a concrete domain (spatial domain or attribute domain). This applies to different types of knowledge useful in image understanding: generic knowledge on the type of observed scene and on the type of image, specific knowledge related to images, used for extracting meaningful information from these images, and knowledge linking image and model.

3. Low level: clustering, enhancement, filtering, edge detection

Low level processing relies on a representation of image information at pixel or voxel level (or in a small neighborhood around pixels). One of the most common tasks addressed at this level is clustering and classification. The idea is to model classes with imprecise boundaries as fuzzy classes, and to find the best partition optimizing some criteria. Among these methods, the most used is the fuzzy C-means algorithm (FCM) [16], often applied to the only grey level

[1] http://graphicwg.irafm.osu.cz/index.php/Main_Page.
[2] http://www.fuzzy.ugent.be/SCIP/index.html.
[3] http://www.eusflat.org/research_wg_scip.php.

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information. The main drawback of this method is that, due to a normalization constraint, the membership functions are not decreasing with respect to the distance to the class centers. An alternative solution to FCM, which avoids the normalization drawbacks, is given by possibilistic C-means (PCM) [93]. More recently, approaches have been proposed by modifying the objective function to increase the robustness of FCM to noise, and to incorporate spatial information, by defining membership functions that depend on a local neighborhood around each point, e.g. [1,74,92,98,101,132,143]. Fuzzy classification has been associated with other formalisms, such as Markov random fields [142] or belief functions [104].

Representations based on fuzzy transforms [129] have also been proved to be useful for low-level image processing, such as image compression or coding [63,130].

Another important task in local processing concerns filtering, enhancement and edge detection. The main approaches can be grouped into two classes: techniques based on functional optimization on the one hand, and rule based techniques on the other hand. These aspects have been largely developed in the literature (see e.g. [6,7,17,19,48,89,96,100,150]). Functional approaches consist in minimizing or maximizing a functional, which can be interpreted as an analytical representation of some objective. For instance, enhancing the contrast of an image according to this technique amounts to reduce the fuzziness of the image. This can be performed by a simple modification of membership functions (for instance using intensification operators), by minimizing a fuzziness index such as entropy, or even by determining an optimal threshold value (for instance optimal in the sense of minimizing a fuzziness index) which provides an extreme enhancement (until binarization) [121,122]. Other methods consist in modifying classical filters (median filter for instance) by incorporating fuzzy weighting functions [97]. Rule based techniques rely on ideal models (of filters, contours, etc.). These ideal cases being rare, variations and differences with respect to these models are permitted through fuzzy representations of the models, as fuzzy rules [41,95,139,140]. Note that rules are sometimes only a different representation of functional approaches. Their main advantage is that they are easy to design (in particular for adaptive operators) and to interpret, and they facilitate the communication with the user.

Although these low level approaches are the most widely developed (and it is not possible to review all of them here), they remain however limited, and the impact of the fuzzy sets is not always the most prominent one. It increases when working at intermediate or higher level, as summarized in the next sections.

4. Intermediate level

Working at intermediate level is performed via geometrical, topological or metrical models and operations. Some geometrical objects such as points, disks, rectangles or lines have been extended to make them fuzzy [45,79,136]. Several operations have been defined in the literature on fuzzy objects, in particular spatial fuzzy objects, since the early works of Zadeh [159] on set operations, and of Rosenfeld on geometrical operations [136]. Typical examples of geometrical operations are area and perimeter of a fuzzy object. They can be defined as crisp numbers, where the computation involves each point up to its degree of membership. But when objects are not well defined, it is convenient to consider that measures performed on them are imprecise too. This point of view leads to definitions as fuzzy numbers [68]. It has been shown in [145,146] that using fuzzy representations of digital objects allows deriving more robust measures than using crisp representations, and in particular dealing properly with the imprecision induced by the digitization process. Such geometrical measures can typically be used in shape recognition, where geometrical attributes of the objects are taken into account (e.g. in mammography images [42,131]), or as descriptors for indexing and data mining applications.

Let us now consider fuzzy connectivity as an example of a topological feature. It was initially defined in [136], and then exploited in fuzzy connectedness notions [151], now widely used for instance in medical image segmentation and incorporated in freely available softwares such as ITK.[4] More general classes of fuzzy connectivity have later been developed, with again applications in medical imaging [44,115,124]. Using both topology and metrics, the notion of skeleton and medial axis was also extended to fuzzy sets [84,102,118,123].

Thanks to the strong algebraic structure of fuzzy sets, extension of mathematical morphology to the fuzzy case was very natural. Initial developments can be found in the definition of fuzzy Minkowski addition [69]. Then this problem has been addressed by several authors independently, e.g. [12,20,37,58,57,60,103,110,133,144]. These works

[4] http://www.itk.org.

can be divided into two main approaches. In the first one [37,58,59], an important property that is put to the fore is the duality between erosion and dilation, the two core operations of mathematical morphology. A second type of approach is based on the notions of adjunction and fuzzy implication, and was formalized in [60]. The links between both approaches have been clarified in [29], with conditions of their equivalence. It was also proved that the definitions of dilation and erosion in these approaches are the most general ones if we want them to share a set of classical properties with standard mathematical morphology. The general setting of complete lattices has been further investigated in [31], with extensions to deal also with bipolar information in [32,106,111,112]. Besides the classical applications of mathematical morphology for filtering, enhancement, segmentation, its fuzzy version was used to model spatial imprecision (e.g. [39]), to define fuzzy spatial relations [25,26], as will be seen in Section 5, to define median fuzzy sets and series of interpolating fuzzy sets [28], etc.

5. Higher level

5.1. Representations of structural information

The main information contained in the images consists of properties of the objects and of relations between objects, both being used for pattern recognition and scene interpretation purposes. Relations between objects are particularly important since they carry structural information about the scene, by specifying the spatial arrangement of objects. These relations highly support structural recognition based on models. These models can be of iconic type, as an atlas, or of symbolic type, as linguistic descriptions, conceptual or semantic graphs, or ontologies.

Spatial relations are strongly involved in linguistic descriptions of visual scenes. They constitute a very important information to guide the recognition of structures embedded in a complex environment, and are more stable and less prone to variability (even in pathological cases) than object characteristics such as shape or size. Mathematical models of several spatial relations (adjacency, distances, directional relations, symmetry, betweenness, parallelism...) have been proposed in the framework of fuzzy sets theory, strongly relying on mathematical morphology operators [22,23,25,35,38,40,54,152]. For instance, the semantic of a relation such as *close to, to the right of* can be modeled as a fuzzy structuring element, and the dilation of a reference object by this structuring element provides the fuzzy region of space where the corresponding relation is satisfied.

These fuzzy representations can enrich ontologies and contribute to reduce the semantic gap between symbolic concepts, as expressed in the ontology, and visual percepts, as extracted from the images. These ideas were used in particular in the segmentation and recognition methods described in [11,34,36,53,85,116,153]: a concept of the ontology is used for guiding the recognition by expressing its semantic as a fuzzy set, for instance in the image domain or in an attribute domain, which can therefore be directly linked to image information.

Similarly, such spatial relations are useful attributes in graphs and fuzzy graphs, and endow recognition and mining methods based on similarity between graphs with structural information [5,15,49,128], benefiting from the huge literature on fuzzy comparison tools (see e.g. [43]). Spatial relations can also be embedded in conceptual graphs and their fuzzy extensions, as in [153].

5.2. Fusion

A lot of approaches for image processing and understanding, whatever their level, involve fusion steps. Information fusion becomes increasingly important due to the multiplication of imaging techniques. The information to be combined can be issued from several images, or from one image only, using for instance combination of several relations between objects or several features of the objects, or from images and a model, like an anatomical atlas or a conceptual graph, or knowledge expressed in linguistic form or as ontologies. The advantages of fuzzy sets and possibilities rely in the variety of combination operators, offering a lot of flexibility in their choice, that can be adapted to any situation at hand, and which may deal with heterogeneous information [70,156]. A classification of these operators was proposed in [21], with respect to their behavior (in terms of conjunctive, disjunctive, compromise [70]), the possible control of this behavior, their properties and their decisiveness, which proved to be useful for several applications in image processing. The combination process can be done at several levels of information representation, from pixel level to higher level. Local fusion is often limited because spatial information is not really taken into account, and

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working at intermediate or higher level (for instance combining several spatial relations to guide the understanding process) is more interesting and powerful. Examples can be found in various domains [34,53,107,116,125,137,153].

5.3. Scene understanding

Scene understanding using fuzzy approaches mostly belongs to the domain of spatial reasoning, which can be defined as the domain of spatial knowledge representation, in particular spatial relations between spatial entities, and of reasoning on these entities and relations. This field has been largely developed in artificial intelligence, in particular using qualitative representations based on logical formalisms [3]. In image interpretation and computer vision it is much less developed and is mainly based on quantitative representations.

A typical example in this domain concerns model-based structure recognition in images, where the model represents spatial entities and relations between them. Two main components of this domain are spatial knowledge representation and reasoning. In particular spatial relations constitute an important part of the knowledge we have to handle. Imprecision is often attached to spatial reasoning in images, and can occur at different levels, from knowledge to the type of question we want to answer. The reasoning component includes fusion of heterogeneous spatial knowledge, decision making, inference, recognition. Two types of questions are raised when dealing with spatial relations:

(1) given two objects (possibly fuzzy), assess the degree to which a relation is satisfied;
(2) given one reference object, define the area of space in which a relation to this reference is satisfied (to some degree).

It has been shown in [26] that the association of three frameworks in a unified way, namely mathematical morphology, fuzzy sets and logics, allows on the one hand matching two important requirements: expressiveness and completeness with respect to the types of spatial information to be represented [2], and on the other hand performing successful reasoning tasks for image understanding.

A common representation of structural information to guide image interpretation consists of a graph, where vertices represent objects or image regions (possibly with attributes such as shape, size, color or grey level), and edges carry the structural information (spatial relations between objects, radiometric contrast between regions...). Although this type of representation has become popular in the last 30 years [55], there are still a number of open problems to efficiently use them for interpretation. One type of approach consists in deriving a graph from the image itself, based on a preliminary segmentation of the image into homogeneous regions, and to express the recognition as a graph matching problem between the image graph and the model graph, which however raises combinatorial problems [46,55]. In [80,99,126,141,157] an initial labeling of the image regions is performed, and spatial relations are used to refine this labeling or to extract the objects of interest.

All these approaches assume a correct initial segmentation of the images. However this is known to be a very difficult problem in image processing, for which no universal solution exists. The segmentation is usually imperfect and no isomorphism exists between the graphs to be matched. An inexact matching should then be found, for instance by allowing several image regions to be assigned to one model vertex, or by relaxing the notion of morphism to the one of fuzzy morphism [49,128]. As an example, in [61,62], an over-segmentation of the image is used, which is easier to obtain. A model structure is then explicitly associated with a set of regions and the recognition is expressed as a constraint satisfaction problem. Still relying on a preliminary segmentation, recent approaches have been proposed, for instance using ontologies [85,117], with fuzzy extensions, besides other types of methods (grammatical or probabilistic ones).

To overcome the difficulty of obtaining a relevant segmentation, the segmentation and the recognition can also be performed simultaneously. For instance, the method proposed in [36,53] consists in sequentially segmenting and recognizing each object of interest, in a pre-calculated order [75]. The objects that are easier to segment are considered first and taken as reference. Spatial relations to these reference objects encoded in the structural model are used as constraints to guide the segmentation and recognition of other objects. However the extraction of the first objects can be difficult if it is not sufficiently constrained, and due to the sequential nature of the process, the errors are potentially propagated. Backtracking may then be needed, as proposed in [75].

To overcome the problems raised by sequential approaches, while avoiding the need of an initial segmentation, another method, still relying on a structural model, but solving the problem in a global way, was proposed in [116]. A solution is the assignment of a region of space to each model object, that satisfies the constraints expressed in the model. A solution is obtained by reducing progressively the solution domain for all objects by excluding assignments that are inconsistent with the structural model. Constraint networks [138] constitute an appropriate framework both for the formalization of the problem and for the optimization. This approach was extended in [153] to fuzzy constraint satisfaction problems (extending [66]) do deal with more complex relations, or involving an undetermined number of objects.

Besides recognition and segmentation, fuzzy spatial relations and more generally fuzzy spatial information have proved useful for other tasks, such as multiple object tracking [154,155], graph kernels for machine learning [5], facial expression understanding [135], navigation in unknown environments in robotics [47,65,78], among others.

6. Ongoing research work and perspectives

6.1. Mining and retrieval

Mining and retrieval are already quite old topics in imaging, justified by the large (and ever increasing) size and number of images. Little has been done in this domain using fuzzy sets, despite their potential [94]. Some works use low level features such as color for image retrieval (e.g. [51,81,88]), and even less use structural information (e.g. [5]). Machine learning approaches have also been extended to the fuzzy case (e.g. [5,108,109]). This is clearly still an open research direction, including the associated questions of symbol grounding and semantic gap.

6.2. Towards bipolarity

A recent trend in contemporary information processing focuses on bipolar information, both from a knowledge representation point of view, and from a processing and reasoning one. Bipolarity is important to distinguish between (i) positive information, which represents what is guaranteed to be possible, for instance because it has already been observed or experienced, and (ii) negative information, which represents what is impossible or forbidden, or surely false [67,72]. This domain has recently motivated work in several directions. In particular, fuzzy and possibilistic formalisms for bipolar information have been proposed [13,14,67,73]. Three types of bipolarity are distinguished in [73]: (i) symmetric univariate, (ii) symmetric bivariate, (iii) asymmetric or heterogeneous, where two types of information are not necessarily linked together and may come from different sources. This last type is particularly interesting in image interpretation and spatial reasoning, and can benefit from mathematical morphology [27,30–33, 106,111,112]. Further work in this direction, in association with reasoning based on different types of logics (see a first step e.g. in [86]), is challenging.

6.3. Towards more interactions between knowledge and image information

Such interactions should be understood in both directions, and are likely to give rise to a lot of research work in the next years. As mentioned in Section 5, knowledge, in particular expressed in a fuzzy form, is useful to guide image understanding and spatial reasoning. However, reasoning aspects deserve to be more developed. For instance image understanding could be expressed as an abduction process [8,10], or information updating based on temporal information as a revision process. Merging temporal and spatial information is also an important direction to pursue, for applications such as video analysis, change detection, etc. Several frameworks could be involved, such as modal logics [24], description logics [10,64,87], formal concept analysis [4,9,10], etc. Conversely, results obtained from images could be further exploited to provide a linguistic description of the observed scene, e.g. in the expert domain language, considering the process of image understanding as the generation of a verbal description of the image content [134]. Recent work on fusion of multi-granularity linguistic terms could be helpful [83].

Acknowledgements

I would like to thank the editors of this issue, H. Maître, D. Dubois, H. Prade and A. Ralescu for recurrent useful and enlightening discussions, L. Zadeh and M. Anvari for the sabbatical stay I did at Berkeley almost twenty years ago and which was a source of inspiration for several of my contributions, J. Atif for his useful suggestions on this paper, and all the colleagues and students who participated in the cited works.

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