Project 2

Redmond Xia

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Starting the initial value or global functions

Question 1

```
## rho is:  -0.05548391
```

Question 2

```
## The expected value of max(0,(X^3 + sin(Y) + X^2 * Y)) =  1.444804
```

Question 3

3a

Table 1: The expected values by simulation of the functions for each time steps

	t = 1	t = 3	t = 5
A(t)	0.9987437	3.0368393	4.957696
B(t)	1.0000005	0.9791352	1.242632

3b The values seems centered around 1, but variances may have increased.

We check the variances of the first B(t) sequence

Table 2: Adding the variance of B(t)

	t = 1	t = 3	t = 5
A(t)	0.9987437	3.0368393	4.957696
B(t)	1.0000005	0.9791352	1.242632
Variance of B(t)	0.5405122	8.9773760	74.363183

This verify our claim that the variance increses as we increase time.

3c We run the antithetic variates to reduce the variances

Table 3: Antithetic Variates variance reduction of B(t)

	t = 1	t = 3	t = 5
mean	0.9865741	0.9461283	0.8122921
variance	0.5512911	8.9913507	72.6507568

```
## The expected value of B(t) is  0.8122921
```

There does not seem to be a much improvement on the the variance of B(t), if any. Since these are estimates, both values could be within standard error with each other. This means that there may not even be any improvements. This likely due to the asymmetrical function of cosine so that we don't find much improvements.

Question 4

4a

```
## The call price by monte carlo simulation is  18.3616
```

4b The value below is is different than the monte carlo simulated ones

```
## Black Scholes call price:  18.28377
```

4c Implementing the antithetic variates to reduce the variances.

```
## The estimated call price of red 17.91131
```

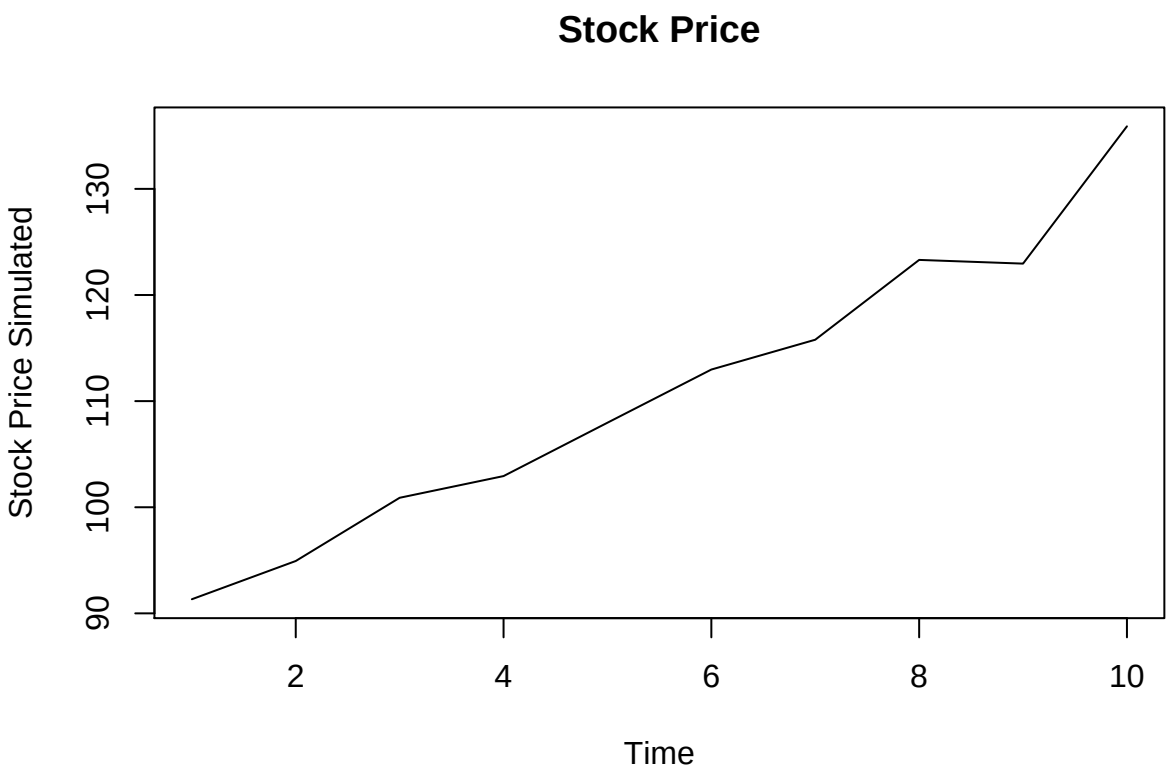
```
## The reduced variance call price  342.5138
```

```
## The original variance of call price  1099.299
```

We see that we significantly reduced the variance. The variance is dropped by roughly a factor of 3. However the price is different. The reduced method is closer to the black scholes price which makes it better -> less biased and less variance.

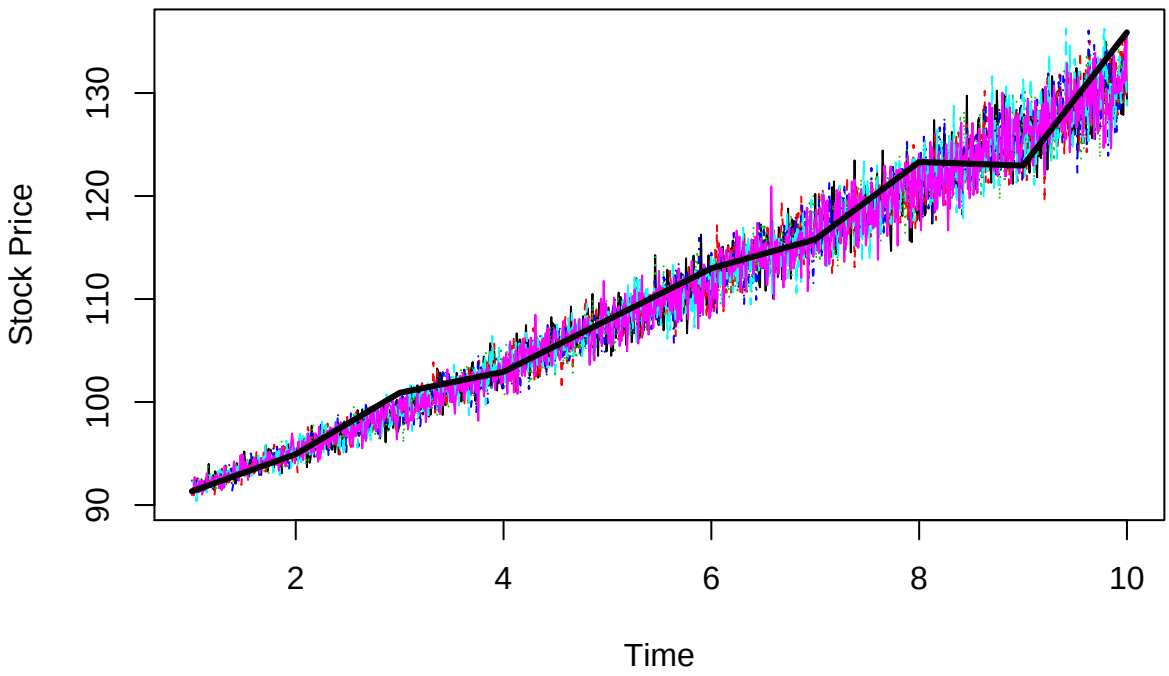
Question 5

5a Simulated stock price for 10 time steps.

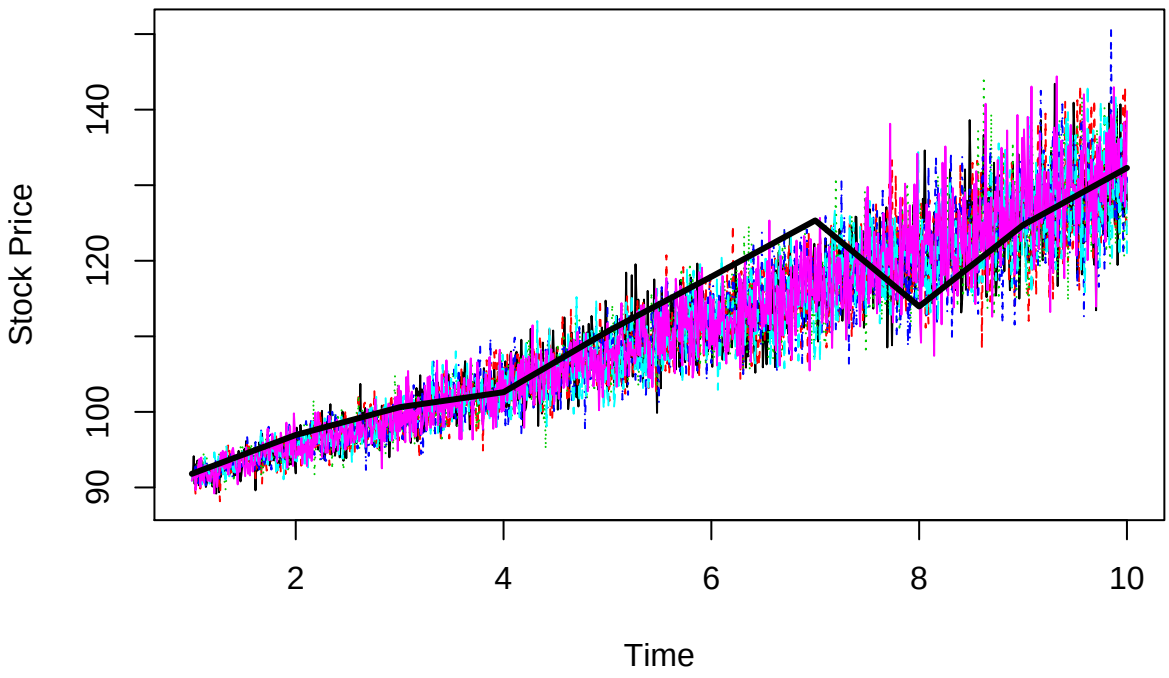


5b Computing the 6 generated random walk paths.

5c The dark black line is the simulated path in part a



5d Again, the dark black line is the simulated path in part a



As we increase ES_n we should see no difference in the average values, but see a huge range of possibilities for the stock price as shown in the graph. The stock ranges widens.

Question 6

6a

```
## The value of the integral is  3.141791
```

6b

```
## The Monte Carlo Estimated Pi:  3.151411
```

6c We use the Wei Cai model for importance sampling

```
## Importance Sampling (IS) Method Estimated Value  3.131786
```

```
## Comparing the variances.
##  Importance sampling standard deviation:  0.3277848
```

```
## MonteCarlo Estimated standard deviation:  0.8836129
```

```
## The absolute difference between actual pi and IS method  0.009807015
```

```
## The absolute difference between actual pi and Monte Carlo method  0.009818256
```

We see that the importance Sampling simulation methods reduces the variance. Again almost by a factor of 3. We also observe that the error for IS method is lower, so we're in favor of that method. However, the error is within the standard error so the expected value may or may not be better.