Empirical Methods Homework 8

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Given that the the sample mean excess return on the market is 0:05 and the sample standard deviation of excess market returns is 15%, and also based on the given historical regression of excess asset returns on the excess market return and the sample variance-covariance matrix of residual returns, we can compute the sample mean, standard deviation, and Sharpe ratio of the excess returns of the three assets.

```
# 1
mean_rm = 0.05
sd_rm = 0.15

mean_r1 = 0.01 + 0.9 * mean_rm
mean_r2 = -0.015 + 1.2 * mean_rm
mean_r3 = 0.005 + 1.0 * mean_rm

var_r1 = (0.9^2) * (sd_rm^2) + (0.1^2)
var_r2 = (1.2^2) * (sd_rm^2) + (0.15^2)
var_r3 = (1.0^2) * (sd_rm^2) + (0.05^2)
sd_r1 = sqrt(var_r1)
sd_r2 = sqrt(var_r2)
sd_r3 = sqrt(var_r3)

sr_r1 = mean_r1 / sd_r1
sr_r2 = mean_r2 / sd_r2
sr_r3 = mean_r3 / sd_r3

result1 <- data.frame(sample_mean=c(mean_r1,mean_r2,mean_r3),
                      standard_deviation=c(sd_r1,sd_r2,sd_r3),
                      sharpe_ratio=c(sr_r1,sr_r2,sr_r3))
rownames(result1) <- c('asset 1', 'asset 2', 'asset 3')
kable(result1, caption='Sample Data Summary')
```

Table 1: Sample Data Summary

	sample_mean	standard_deviation	sharpe_ratio
asset 1	0.055	0.1680030	0.3273752
asset 2	0.045	0.2343075	0.1920553
asset 3	0.055	0.1581139	0.3478505

The summary statistics is provided above.

If we construct the market-neutral versions of the three assets by hedging out the market risk, the sample average return will simply become the α in the regression model, and the sample standard deviation of asset return will become the standard deviation of ϵ.

```
# 2
result2 <- data.frame(sample_mean=c(0.01,-0.015,0.005),
                      standard_deviation=c(0.1,0.15,0.05),
                      sharpe_ratio=c(0.01/0.1, 0.015/-0.15, 0.005/0.05))
rownames(result2) <- c('asset 1', 'asset 2', 'asset 3')
kable(result2, caption='Market Neutral Version Summary')
```

Table 2: Market Neutral Version Summary

	sample_mean	standard_deviation	sharpe_ratio
asset 1	0.010	0.10	0.1
asset 2	-0.015	0.15	-0.1
asset 3	0.005	0.05	0.1

The summary statistics of the market-neutral versions of the three assets are provided above.

We can then try to maximize the Sharpe ratio by optimally combining assets available.

```
# 3
avg_re1 <- result2[,1]
omega1 <- matrix(c(0.1^2,0,0,0,0.15^2,0,0,0,0.05^2), nrow=3, byrow=T)
sr_mve1 <- sqrt(t(avg_re1)%*%solve(omega1)%*%avg_re1)
```

If we only combine the three hedged assets, the maximum Sharpe ratio we can obtain is 0.1732051.

```
# 4
avg_re2 <- c(result2[,1],0.05)
omega2 <- matrix(c(0.1^2,0,0,0, 0,0.15^2,0,0, 0,0,0.05^2,0, 0,0,0,0.15^2),
                 nrow=4, byrow=T)
sr_mve2 <- sqrt(t(avg_re2)%*%solve(omega2)%*%avg_re2)
```

If we only combine the three hedged assets with the market portfolio, the maximum Sharpe ratio we can obtain is 0.3756476.

Lastly, we can construct a porfolio that meets the criteria of providing the maximum Sharpe ratio of returns and having an volatility of 15%.

```
# 5/a)
k <- as.numeric(0.15 / sr_mve2)
wt_mve <- k * solve(omega2)%*%avg_re2
colnames(wt_mve) <- c('weight')
rownames(wt_mve) <- c('asset 1', 'asset 2', 'asset 3', 'market')
kable(wt_mve, caption='MVE Portfolio Weight')
```

Table 3: MVE Portfolio Weight

	weight
asset 1	0.3993104
asset 2	-0.2662070
asset 3	0.7986209
market	0.8873565

The weight of three assets and market porfolio is provided above.

```r
# 5/b)
er_mve <- t(wt_mve)%*%c(result1[,1],0.05)
sd_mve <- k*sr_mve2
sr_mve <- sr_mve2
result3 <- data.frame(sample_average=er_mve,
                      standard_deviation=sd_mve,
                      sharpe_ratio=sr_mve)
kable(result3, caption='MVE Portfolio Summary')
```

Table 4: MVE Portfolio Summary

	sample_average	standard_deviation	sharpe_ratio
weight	0.0982747	0.15	0.3756476

And the summary statistic of the MVE portfolio is also provided.