Empirical Methods Homework 5

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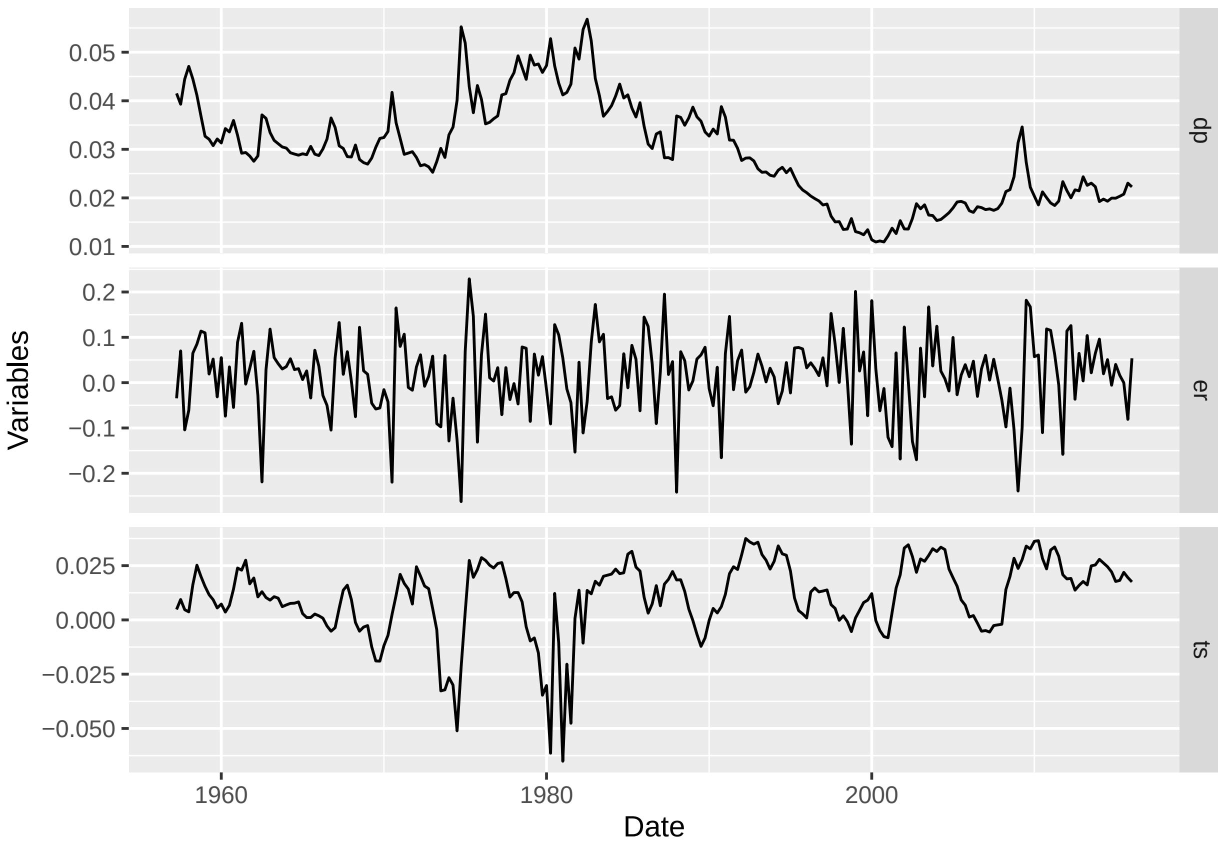
Question 1

```r
# import dataset
df <- read_xlsx('MktRet_DP_TermSpread.xlsx') %>%
  rename(er = MktExRet) %>%
  rename(dp = Mkt_DP) %>%
  rename(ts = y10minFedFunds)

# plot each series
gather(df, er:ts, key='series', value='Variables') %>%
  ggplot() +
  geom_line(aes(x=Date, y=Variables)) +
  facet_grid(series~., scale='free')
```




```r
m1 <- mean(df$er)
sd1 <- sd(df$er)
fac1 <- as.numeric(lm(df$er ~ lag(df$er))[[1]][2])
```

```r
hl1 <- log(0.5) / log(fac1)

m2 <- mean(df$dp)
sd2 <- sd(df$dp)
fac2 <- as.numeric(lm(df$dp ~ lag(df$dp))[[1]][2])
hl2 <- log(0.5) / log(fac2)

m3 <- mean(df$ts)
sd3<- sd(df$ts)
fac3 <- as.numeric(lm(df$ts ~ lag(df$ts))[[1]][2])
hl3 <- log(0.5) / log(fac3)

result1 <- data.frame(mean = c(m1,m2,m3),
                      standard_deviation = c(sd1,sd2,sd3),
                      first_order_ac = c(fac1,fac2,fac3),
                      half_life = c(hl1,hl2,hl3))
rownames(result1) <- c('excess_return', 'dividend_yield', 'term_spread')
kable(result1, caption='Summary of Series')
```

Table 1: Summary of Series

	mean	standard_deviation	first_order_ac	half_life
excess_return	0.0160553	0.0843499	0.0696720	0.2601945
dividend_yield	0.0293646	0.0103388	0.9600270	16.9914604
term_spread	0.0106280	0.0170230	0.8040117	3.1775116

Question 2

```r
phi0 <- matrix(0, 3, 1)
phi1 <- matrix(0, 3, 3)
lm1 <- lm(data=df, er ~ lag(er) + lag(dp) + lag(ts))
phi0[1] <- lm1$coefficients[1]
phi1[1,] <- lm1$coefficients[2:4]
wse1 <- sqrt(diag(vcovHC(lm1, type='HC0')))
rs1 <- summary(lm1)$r.squared

lm2 <- lm(data=df, dp ~ lag(er) + lag(dp) + lag(ts))
phi0[2] <- lm2$coefficients[1]
phi1[2,] <- lm2$coefficients[2:4]
wse2 <- sqrt(diag(vcovHC(lm2, type='HC0')))
rs2 <- summary(lm2)$r.squared

lm3 <- lm(data=df, ts ~ lag(er) + lag(dp) + lag(ts))
phi0[3] <- lm3$coefficients[1]
phi1[3,] <- lm3$coefficients[2:4]
wse3 <- sqrt(diag(vcovHC(lm3, type='HC0')))
rs3 <- summary(lm3)$r.squared

phi0_r = phi0
rownames(phi0_r) <- c('Excess Return', 'DP Ratio', 'Term Spread')
colnames(phi0_r) <- c('Phi0')
kable(phi0_r, caption='Coefficients of Phi0')
```

Table 2: Coefficients of Phi0

	Phi0
Excess Return	-0.0382747
DP Ratio	0.0021214
Term Spread	0.0018723

```r
phi1_r = phi1
rownames(phi1_r) <- c('Excess Return', 'DP Ratio', 'Term Spread')
colnames(phi1_r) <- c('Excess Return', 'DP Ratio', 'Term Spread')
kable(phi1_r, caption='Coefficients of Phi1')
```

Table 3: Coefficients of Phi1

	Excess Return	DP Ratio	Term Spread
Excess Return	0.0485191	1.4509224	1.0495809
DP Ratio	-0.0022822	0.9402772	-0.0388263
Term Spread	-0.0148361	0.0104774	0.8216413

```r
ws <- rbind(wse1,wse2,wse3)
rownames(ws) <- c('Excess Return', 'DP Ratio', 'Term Spread')
kable(ws, caption='White Standard Error')
```

Table 4: White Standard Error

	(Intercept)	lag(er)	lag(dp)	lag(ts)
Excess Return	0.0194643	0.0725420	0.5742116	0.3894893
DP Ratio	0.0005825	0.0023082	0.0193400	0.0168374
Term Spread	0.0018678	0.0077732	0.0734799	0.0863311

```r
rs <- rbind(rs1,rs2,rs3)
rownames(rs) <- c('Excess Return', 'DP Ratio', 'Term Spread')
colnames(rs) <- c('r_sq')
kable(rs, caption='R-squared')
```

Table 5: R-squared

	r_sq
Excess Return	0.0607173
DP Ratio	0.9298475
Term Spread	0.6515704

Question 3

```r
eigen(phi1)$values
```

```
## [1] 0.94071594 0.79530199 0.07441967
```

Because all the eigenvalues are less than one, the VAR model is stationary.

Question 4

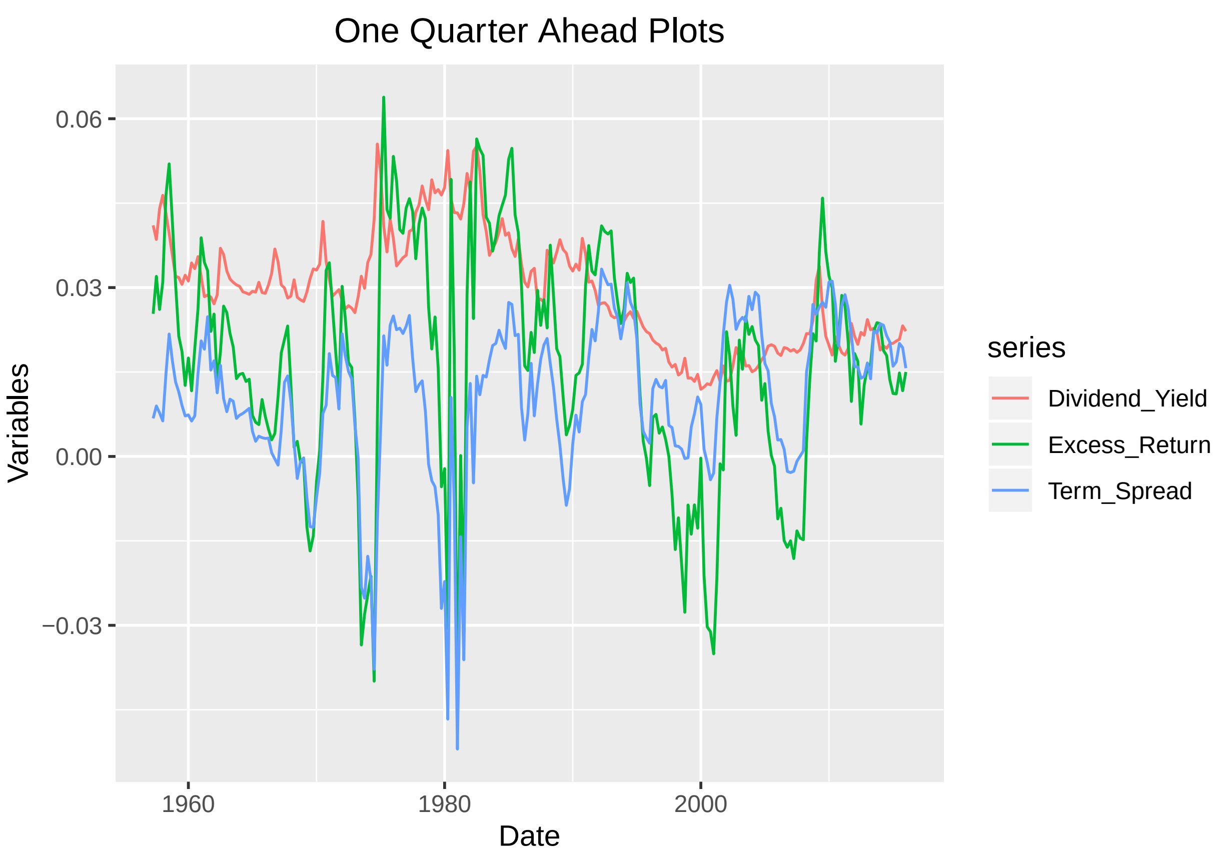
```
sd_et <- sd(lm1$residuals)
```

The volatility of quarterly expected returns given the return forecasting regression is 0.0818607.

Question 5

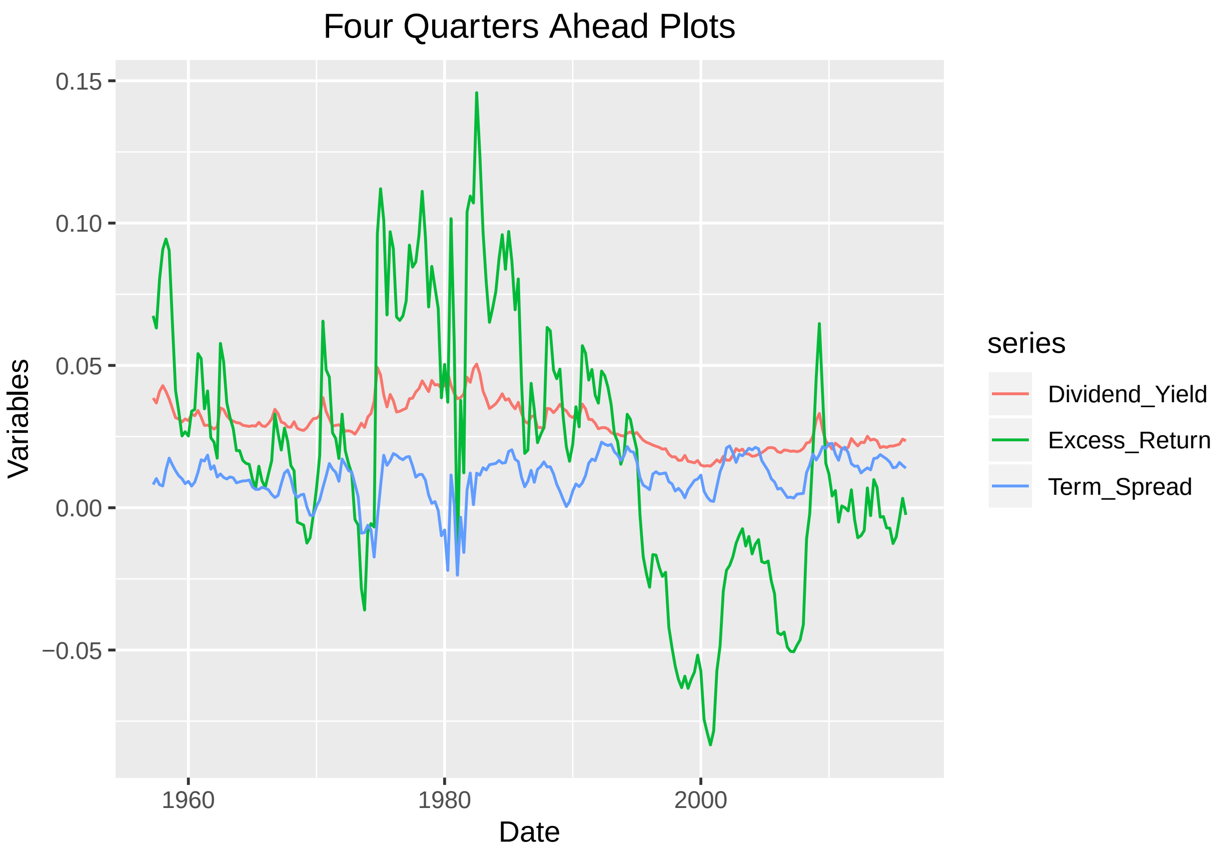
```
one <- diag(3)
mu <- solve(one-phi1) %*% phi0
t <- nrow(df)
mu_m <- matrix(rep(mu,3), nrow=t, ncol=3, byrow=T)
zt <- df[,2:4] - mu_m
et1 <- t(phi1 %*% t(zt)) + mu_m
et1_df <- as.data.frame(et1) %>%
  mutate(Date = df$Date) %>%
  rename(Excess_Return = V1) %>%
  rename(Dividend_Yield = V2) %>%
  rename(Term_Spread = V3)

gather(et1_df, Excess_Return:Term_Spread, key='series', value='V') %>%
  ggplot(aes(x=Date, y=V, col=series)) +
  geom_line() +
  labs(x = 'Date', y= 'Variables',
       title = 'One Quarter Ahead Plots') +
  theme(plot.title = element_text(hjust = 0.5))
```



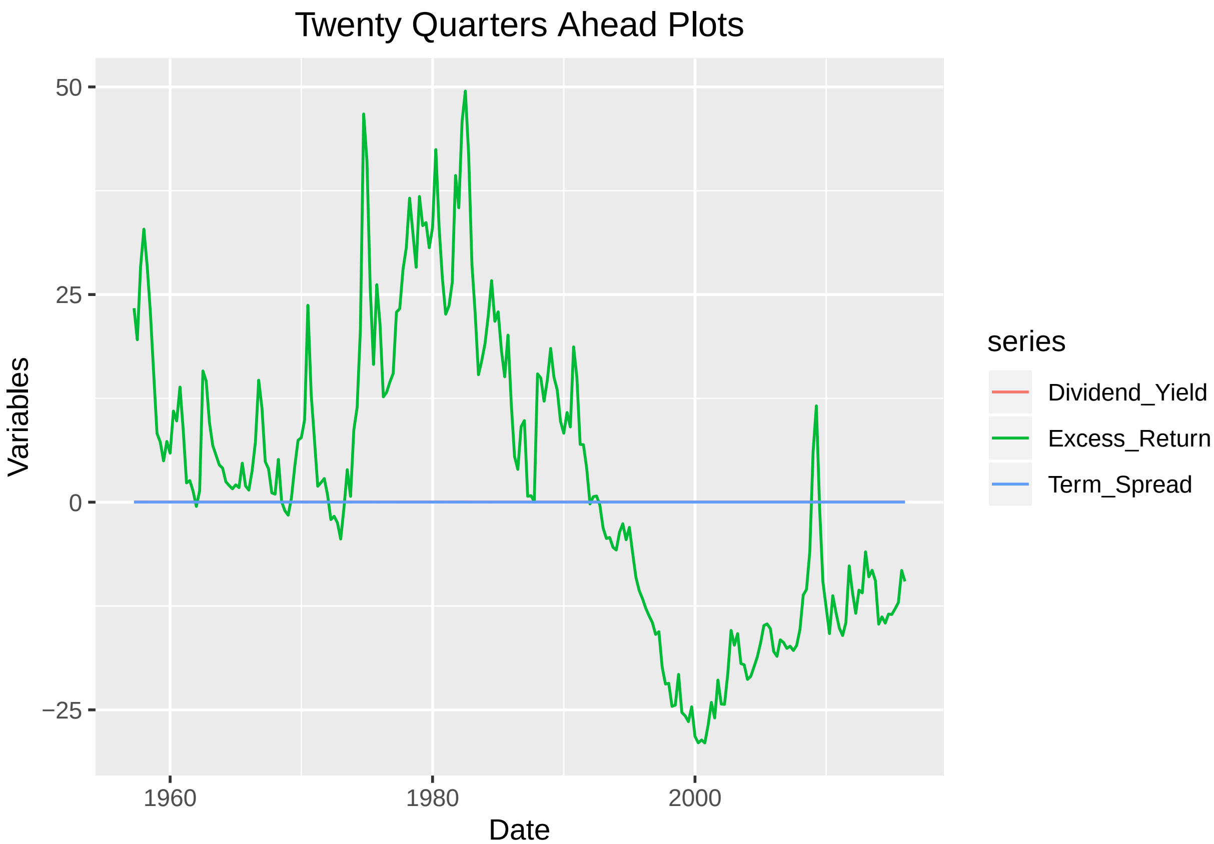
```r
et4 <- t(phi1^4 %*% t(zt)) + mu_m
et4_df <- as.data.frame(et4) %>%
  mutate(Date = df$Date) %>%
  rename(Excess_Return = V1) %>%
  rename(Dividend_Yield = V2) %>%
  rename(Term_Spread = V3)

gather(et4_df, Excess_Return:Term_Spread, key='series', value='V') %>%
  ggplot(aes(x=Date, y=V, col=series)) +
  geom_line() +
  labs(x = 'Date', y= 'Variables',
       title = 'Four Quarters Ahead Plots') +
  theme(plot.title = element_text(hjust = 0.5))
```




```r
et20 <- t(phi1^20 %*% t(zt)) + mu_m
et20_df <- as.data.frame(et20) %>%
  mutate(Date = df$Date) %>%
  rename(Excess_Return = V1) %>%
  rename(Dividend_Yield = V2) %>%
  rename(Term_Spread = V3)

gather(et20_df, Excess_Return:Term_Spread, key='series', value='V') %>%
  ggplot(aes(x=Date, y=V, col=series)) +
  geom_line() +
  labs(x = 'Date', y= 'Variables',
       title = 'Twenty Quarters Ahead Plots') +
  theme(plot.title = element_text(hjust = 0.5))
```



From the 1 quarter ahead plot, we can observe that excess return is negatively correlated with DP ratio and positively correlated with term spread. However, the 4 quarters ahead plot shows that although expected returns still vary a lot due to effect of persistent DP ratio, forecasting power of term spread on excess return gradually damped out. Moreover, the 20 quarters ahead plot suggests that expected returns vary a lot again due to effect of highly persistent DP ratio, and shape of expected return is very similar to that of DP ratio, but forecasting power of term spread on excess return died out and become unconditional average.

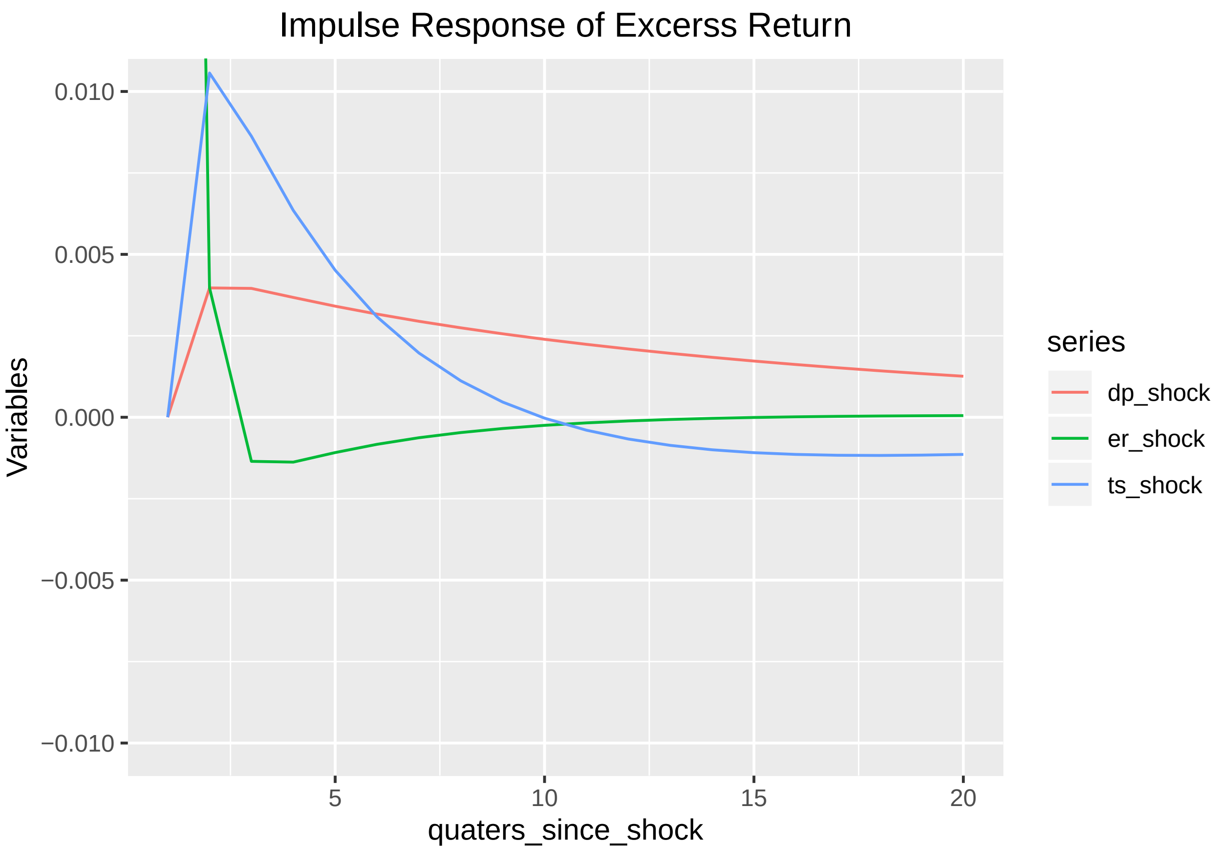
Question 6

```
sigma <- cov(cbind(lm1$residuals,lm2$residuals,lm3$residuals))
stdev <- sqrt(diag(sigma))
phi1_series <- array(rep(0, 3*3*20), dim=c(3, 3, 20))
for (i in 1:20){
  phi1_series[,,i] <- matrix.power(phi1,i-1)
}

ir_er <- data.frame(x = seq(1,20,1)) %>%
  mutate(er_shock = stdev[1] * phi1_series[1,1,]) %>%
  mutate(dp_shock = stdev[2] * phi1_series[1,2,]) %>%
  mutate(ts_shock = stdev[3] * phi1_series[1,3,])

gather(ir_er, er_shock:ts_shock, key='series', value='V') %>%
  ggplot(aes(x=x, y=V, col=series)) +
  geom_line() +
  coord_cartesian(ylim = c(-0.01,0.01)) +
  labs(x = 'quaters_since_shock', y= 'Variables',
       title = 'Impulse Response of Excerss Return') +
```

```
  theme(plot.title = element_text(hjust = 0.5))
```



From the impulse response plot, we can observe that a shock in excess return will have large influence on excess return in the nearest term, since people tend to over-react to sudden news, but the influence damps out very rapidly and will even cause pull back on stock price. Regarding shock of term spread, it did have some influence on excess return in the near term, but the influence die out after 10 quarters. It's worth noticing that shock of DP ratio has least influence on excess return in the near term, but the influence persist for a long time and damp out very slowly.

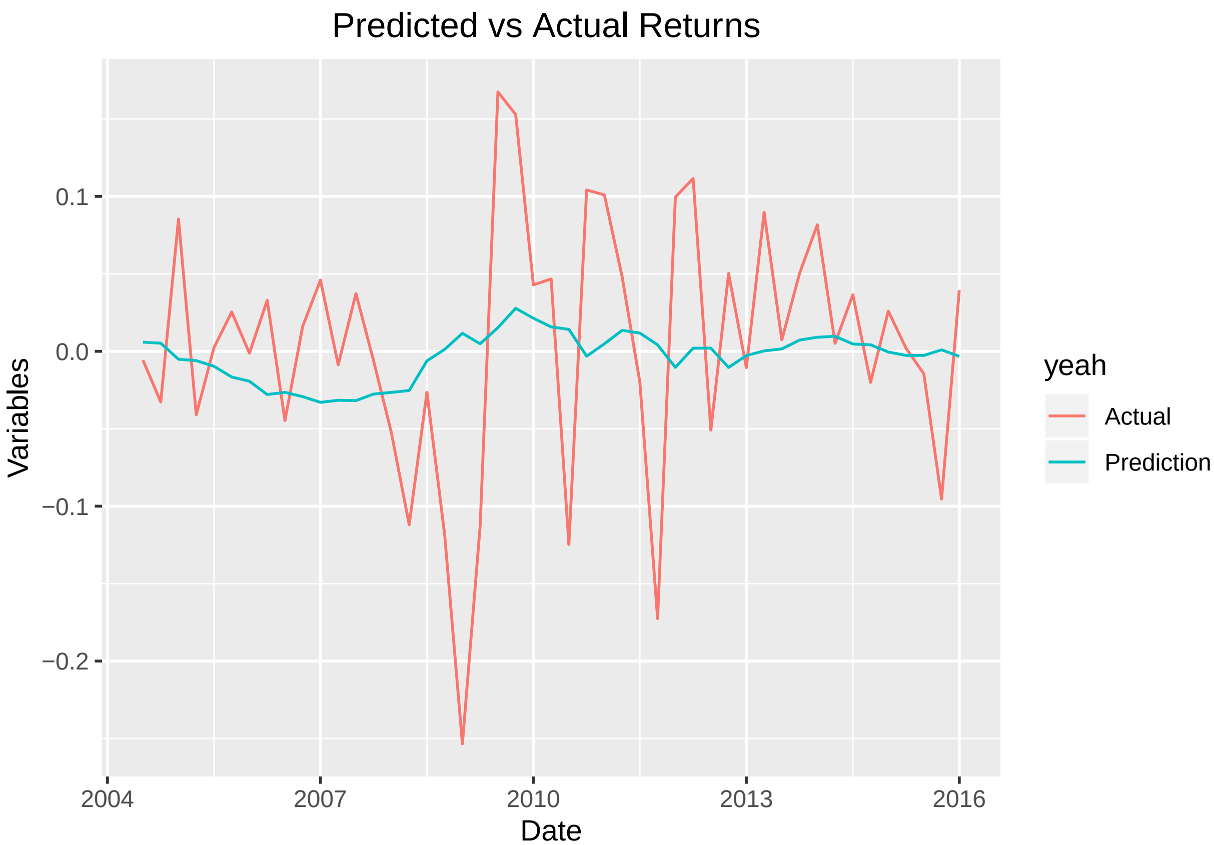
Question 7

```
train <- round(t*4/5)
pred <- double(t-train)
actual <- zt[(train+1):t, 1]

for (i in train:(t-1)){
  out = lm(zt[2:i,1] ~ zt[1:i-1,1] + zt[1:i-1,2] + zt[1:i-1,3])
  pred[1+i-train] <- as.numeric(out$coefficients) %*% t(cbind(1,zt[i,]))
}

oos <- data.frame(Actual = actual, Prediction = pred) %>%
  mutate(Date = df$Date[(train+1):t])
gather(oos, Actual:Prediction, key='yeah', value='V') %>%
  ggplot(aes(x=Date, y=V, col=yeah)) +
  geom_line() +
  labs(x = 'Date', y= 'Variables',
```

```r
      title = 'Predicted vs Actual Returns ') +
  theme(plot.title = element_text(hjust = 0.5))
```




```r
mse = mean((oos$Prediction - oos$Actual) ^ 2)
rmse = sqrt(mse)
r2_oos = 1 - mse / mean((oos$Actual - mean(oos$Actual)) ^ 2)
```

The plot of Predicted vs Actual Returns is showed above. As we can observe, the predicted excess return curve is more smooth than the actual one. The model has a MSE of 0.006628, RMSE of 0.0814128 and out of sample r-squared of -0.0154544.