Empirical Methods in Finance
Homework 2

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Problem 1: AR(p) Processes

3. Consider an AR(2) process with $\phi_1 = 1.1$ and $\phi_2 = -0.25$ (following the notation in Lecture 5).

   (a) Plot the autocorrelation function for this process for lags 0 through 20.

   (b) Is the process stationary? Explain why or why not.

   (c) Give the dynamic multiplier for a shock that occurred 6 periods ago. That is, calculate $\frac{\partial[r_{t+6}-\mu]}{\partial\varepsilon_t}$ (following the notation in Lecture 5). This requires some algebra.

   (d) Now, instead assume $\phi_1 = 0.9$ and $\phi_2 = 0.8$. Give the dynamic multiplier for a shock that occurred 6 periods ago. Is the process stationary? Why/why not?

   (e) Instead of analytically solving for dynamic multipliers, we can easily simulate a full impulse response (that is, dynamic multipliers at all horizons). In particular, consider a positive ε_t shock with magnitude one standard deviation. Assume the standard deviation is 1 for simplicity. Define $x_t \equiv r_t - \mu$ as in class. Thus:

   $$x_t = 1.1x_{t-1} - 0.25x_{t-2} + \varepsilon_t.$$

   Set the initial values equal to the unconditional mean: $x_{t-1} = x_{t-2} = 0$. Set all future shock equal to their expectations, $\varepsilon_{t+j} = 0$ for all $j > 0$. As stated earlier, let $\varepsilon_t = 1$. Simulate x_{t+j} for $j = 0, ..., 60$ given the above initial values and sequence of shocks. Plot the resulting series from x_{t-1} through x_{t+60}. This is the Impulse-Response plot for a one standard deviation positive shock to ε_t.

Problem 2: Applying the Box-Jenkins methodology[1]

In PPIFGS.xls you will find quarterly data for the Producer Price Index. Our goal is to develop a quarterly model for the PPI, so we can come up with forecasts. Our boss needs forecasts of inflation, because she wants to hedge inflation exposure. There is not a single 'correct' answer to this problem. Well-trained econometricians can end up choosing different specifications even though they are confronted with the same sample. However, there definitely are some wrong answers.

1. We look for a covariance-stationary version of this series. Using the entire sample, make a graph with four subplots:
   (a) Plot the PPI in levels.
   (b) Plot ΔPPI
   (c) Plot $\log PPI$
   (d) Plot $\Delta \log PPI$.
2. Which version of the series looks covariance-stationary to you and why? Let's call the covariance stationary version $y_t = f(PPI_t)$.
3. Plot the ACF of y_t for 12 quarters. What do you conclude? If the ACF converges very slowly, re-think whether y_t really is covariance stationary.
4. Plot the PACF of y_t for 12 quarters. What do you conclude?
5. On the basis of the ACF and PACF, select two different AR model specifications that seem the most reasonable to you. Explain why you chose these.
   (a) Using the entire sample, estimate each one of these. Report the coefficient estimates and standard errors. Check for stationarity of the parameter estimates.
   (b) Plot the residuals. (Note: the residuals will have conditional heteroskedasticity or 'GARCH effects'. We will talk about this later. However, in well-specified models, the residuals should not be autocorrelated.)

[1] In Matlab, there is an **Econometrics Toolbox** and a series of functions : 'arima, estimate, forecast, infer, simulate, lbqtest' that can help you solve this problem. Alternatively, you can download Kevin Sheppard's **MFE toolbox**, which is freely available. You can just Google this and find it. In *R* there is a package called 'MTS' for *Multivariate Time Series*, by Ruey Tsay. This is a very useful package, that we will also use when estimating time-varying volatility models.

(c) Report the Q-statistic for the residuals for 8 and 12 quarters, as well as the AIC and BIC. Select a preferred model on the basis of these diagnostics. Explain your choice.

6. Re-estimate the two models using only data up to the end of 2005 and compute the MSPE (mean squared prediction error) on the remainder of the sample for one-quarter ahead forecasts:

$$\frac{1}{H}\sum_{t=1}^{H} v_t^2$$

where H is the length of the hold-out sample, and v_i is the one-step ahead prediction error. Note: the one-step ahead prediction error means that while you fix the parameters to those estimated up until 2005, you will use new data. So, for the and of Q2 forecast for Q3 2006 you use data up to end of Q2 2006. If you like, you can re-estimate the model sequentially to have parameters estimated, continuing with this example, up until end of Q2 2006. This mimicks more what you would do in a real-world situation.

Also, for comparison, report the MSPE assuming there is no predictability in y_t, i.e. assuming y_t follows a random walk with drift. What do you conclude?