PROBLEM 2:

1. GIVEN: $mb_t = \ln(M_t / B_t) = 0.7$

$roe_t = \ln(1 + ROE_t) = \ln(1.2) = 0.1823$

$\varepsilon_t = 0.1$

$$mb_t = \underbrace{E_t \sum_{j=1}^{\infty} \kappa^{j-1} roe_{t+j}}_{CF} - \underbrace{E_t \sum_{j=1}^{\infty} \kappa^{j-1} r_{t+j}}_{DR}$$

$\therefore DR = CF - mb_t$

To calculate $CF = E_t \sum_{j=1}^{\infty} \kappa^{j-1} roe_{t+j}$

$roe_{t+1} = 0.05 + 0.9(roe_t - 0.05) - 0.6\,\varepsilon_t + \varepsilon_{t+1}$ ... given.

$= 0.05 + 0.9(0.1823 - 0.05) - 0.6 \times 0.1 + \varepsilon_{t+1}$

$= 0.10907 + \varepsilon_{t+1}$

$E_t[roe_{t+1}] = 0.10907$

$E_t[roe_{t+2}] = 0.05 + 0.9(roe_{t+1} - 0.05)$

$= 0.005 + 0.9\, E_t[roe_{t+1}]$

$E_t[roe_{t+3}] = 0.005 + 0.9 E_t[roe_{t+2}]$

$= 0.005 + 0.9[0.005 + 0.9 E_t[roe_{t+1}]]$

$= 0.005 + 0.005 \times 0.9 + 0.9^2 E_t[roe_{t+1}]$

$\vdots$

$$E_t[roe_{t+j}] = 0.005[\overbrace{1 + 0.9 + 0.9^2 + \cdots + 0.9^{j-2}}^{\text{Geometric series}}] + 0.9^{j-1} \cdot E_t[roe_{t+1}]$$

$$E_t[roe_{t+j}] = 0.005 \sum_{j=1}^{\infty} 0.9^{j-2} + 0.9^{j-1} E_t[roe_{t+1}]$$

$$= 0.005 \times \left(\frac{1-0.9^{j-1}}{1-0.9}\right) + 0.9^{j-1} E_t[roe_{t+1}]$$

$$CF = \sum_{j=1}^{\infty} k^{j-1} roe_{t+j}$$

$$= \sum_{j=1}^{\infty} 0.97^{j-1} \left[0.005 \left(\frac{1-0.9^{j-2}}{1-0.9}\right) + 0.9^{j-1} \times 0.10907 \right]$$

$$= \sum_{j=1}^{\infty} 0.97^{j-1} \times 0.05 - \sum_{j=1}^{\infty} 0.05 \times 0.97^{j-1} \times 0.9^{j-2} + 0.10907 \sum (0.97 \times 0.9)^{j-1}$$

$$= 0.05 \times \frac{1}{1-0.97} - \frac{0.05 \times 0.97}{0.97 \times 0.9} \times \frac{1}{1-0.873} + 0.10907 \times \frac{1}{1-0.873}$$

$$= 2.08137$$

$$\therefore \boxed{CF_t = 2.08137}$$

$$DR_t = CF_t - mb_t = 2.08137 - 0.7$$

$$\boxed{DR_t = 1.3814}$$

2. GIVEN: $mb_t = 0.2 \rightarrow$ unconditional average

$$roe_{t+1} = 0.05 + 0.9(roe_t - 0.05) - 0.6\,\varepsilon_t + \varepsilon_{t+1}$$

follows an ARMA(1, 1)

Unconditional mean of ARMA(1, 1)

$$E[roe_{t+1}] = \frac{\phi_0}{1-\phi_1} = \frac{0.05 - 0.045}{1-0.9}$$

$$\therefore E[roe_{t+1}] = \frac{0.005}{0.1} = 0.05$$

$$CF = E\left[\sum_{j=1}^{\infty} k^{j-1}\, roe_{t+j}\right]$$

$$\therefore CF = \sum_{j=1}^{\infty} k^{j-1} E\left[roe_{t+j}\right]$$

$$= \frac{1}{1-0.97} \times 0.0\cancel{0}5$$

$$\boxed{\therefore CF = 1.67}$$

$$DR = CF - mb_t$$

$$= 1.67 - 0.2$$

$$\therefore \boxed{DR = 1.47}$$