# Empirical Methods HW3

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## Problem 2.1

$e_t = e_{t-1} + x_t \implies x_t = e_t - e_{t-1}$ $x_t = \phi x_{t-1} + \epsilon_t$ $y_t \equiv e_t - e_{t-4}$

$$\begin{aligned} x_t = e_t - e_{t-1} = \phi x_{t-1} + \epsilon_t = \epsilon_t,\ for\ \phi = 0 \\ x_t = \epsilon_t = e_t - e_{t-1} \\ \epsilon_{t-1} = e_{t-1} - e_{t-2} \\ \epsilon_{t-2} = e_{t-2} - e_{t-3} \\ \epsilon_{t-3} = e_{t-3} - e_{t-4} \\ y_t = \epsilon_t + \epsilon_{t-1} + \epsilon_{t-2} + \epsilon_{t-3} = e_t - e_{t-4} \end{aligned}$$

$$\begin{aligned} E[y_t] = 0 + 0 + 0 + 0 = 0,\ Find\ Cov(y_t, y_{t-j})\ for\ j = 0, 1, 2, 3, 4, 5 \\ j = 0, Cov(y_t, y_t) = Var(y_t) = \epsilon_t^2 + \epsilon_{t-1}^2 + \epsilon_{t-2}^2 + \epsilon_{t-3}^2 = 1 + 1 + 1 + 1 = 4 \\ \\ j = 1, Cov(y_t, y_{t-1}) = E[y_t y_{t-1}] - E[y_t]E[y_{t-1}] = E[y_t y_{t-1}] - 0 = \\ E[(\epsilon_t + \epsilon_{t-1} + \epsilon_{t-2} + \epsilon_{t-3})(\epsilon_{t-1} + \epsilon_{t-2} + \epsilon_{t-3} + \epsilon_{t-4})],\ E[\epsilon_i \epsilon_j] = 0\ \forall i \neq j \\ \implies E[\epsilon_{t-1}^2 + \epsilon_{t-2}^2 + \epsilon_{t-3}^2] = 1 + 1 + 1 = 3 \\ \\ Similarly,\ j = 2, Cov(y_t, y_{t-2}) = E[(\epsilon_t + \epsilon_{t-1} + \epsilon_{t-2} + \epsilon_{t-3})(\epsilon_{t-2} + \epsilon_{t-3} + \epsilon_{t-4} + \epsilon_{t-5})] \\ = E[\epsilon_{t-2}^2 + \epsilon_{t-3}^2] = 1 + 1 = 2 \\ \\ Again,\ j = 3,\ Cov(y_t, y_{t-3}) = E[\epsilon_{t-3}^2] = 1 \\ \\ j = 4, Cov(y_t, y_{t-4}) = 0,\ j = 5, Cov(y_t, y_{t-5}) = 0 \end{aligned}$$

## Problem 2.2

The autocovariance drops to 0 after 3 lags. So the for every value after 3 it isn't correlated to $y_t$. With $\phi = 0$ We get the AR(0), this is a MA(3) then. So we have a ARMA(0,3).

## Problem 3

1.

$$Var(R^e_{t+1}) = \beta^2 Var(x_t) + Var(\epsilon_{t+1}) = 1 * (0.05)^2 + 0.15^2 = 0.025 = \sqrt{0.025} = .15811$$

2.

$$R^2 = \rho^2 = (\frac{Cov(R^e_{t+1}, x_t)}{\sigma_{R^e_{t+1}}\sigma_{x_t}})^2 = (\frac{Cov(\beta x_t + \epsilon_{t-1}, x_t)}{\sigma_{R^e_{t+1}}\sigma_{x_t}})^2 = (\frac{Var(x_t)}{\sigma_{R^e_{t+1}}\sigma_{x_t}})^2 = \frac{Var(x_t)}{Var(R^e_{t+1})} = \frac{0.05^2}{0.025} = 0.1$$

3. Sharpe Ratio $= \frac{E[R^e_{t+1}]}{\sigma_{mkt}} = \frac{0.05}{\sqrt{0.025}} = 0.31622$

4.

$$\gamma = \frac{40}{9}$$
$$\alpha_t = \frac{E[x_t]}{\gamma\sigma^2_t[R^e_{t+1}]} = \frac{E[x_t]}{\gamma(\sigma^2_t[x_t] + \sigma^2_t[\epsilon_{t+1}])}$$
$$So,\ if\ x = 0,\ then\ \alpha_t = \frac{0}{positive} = 0 \implies Sharpe\ Ratio = 0$$
$$if\ x = .1, \alpha_t = \frac{.1}{\frac{40}{9}(0.15^2)} = 1 \implies Sharpe\ Ratio = \frac{.1}{.15} = \frac{2}{3}$$

5.

a) Below is the Expected value of the return

```
gamma <- 40/9
xt <- c(0,0.1)
cond_var <- .15^2
E_x2 <- var(xt)/2 + 0.05^2
uncond_ret <- 0.5*xt[1] + 0.5*xt[2]
alpha = (uncond_ret / (gamma * (cond_var + var(xt)/2)))
E_alpR <- alpha * uncond_ret
E_alpR
```

```
## [1] 0.0225
```

b) The output below is the unconditional standard deviation

```
var_alpR <- (E_x2 + cond_var - uncond_ret^2) * alpha^2
sqrt(var_alpR)
```

```
## [1] 0.07115125
```

c)Below is the sharpe value

```
E_alpR / sqrt(var_alpR)
```

```
## [1] 0.3162278
```

d) i.Below is the implied R^2

```
x <- c(-0.05,.15)
E_x <- mean(x)
var_x <- var(x) / 2
var_R <- var_x + cond_var
E_xSq <- var_x + E_x^2
R_Sq <- var_x / var_R
R_Sq
```

```
## [1] 0.3076923
```

ii.

```r
alpha1 = (E_x / (gamma * (cond_var + var_x)))
var_alpR1 <- (E_xSq + cond_var - E_x^2) * alpha1^2
(alpha1 * E_x)  / sqrt(var_alpR1)
```

```
## [1] 0.2773501
```