Inference and Hypothesis Testing

Review: one sample t-test

Goal: determine whether an unknown population mean is different from a specific value.

Example1:

- Eric has dark hair
- Evidence (observation): *The person has blond hair*

⇒ This person is *not* Eric

Example2

Test statistic: weight

- Adult chickens weigh 6.2 ± 0.8 lb ⟵ Null distribution

Null hypothesis: this is a chicken

- Evidence/observation: *This weighs 10 lb* ⟵ Observation

⇒ This is probably not a chicken

p-value

Null distributions

- The null hypothesis is the statement of the quantity we want to test for Denoted H_0

- Alternate hypothesis is everything else

Denoted H_1 or H_a

Example1: H_0 :This person is Eric

H_1 :This person is NOT Eric

Example2: H_0 :This animal is a chicken

H_1 :This animal is NOT a chicken

Test statistics

- The test statistics is the quantity we use to determine rejection or acceptance of the null hypothesis
- <u>denoted T</u>

Example1: H_0 :This person is Eric

H_1 :This person is NOT Eric

T : color of the hair

Example2: H_0 :This animal is a chicken

H_1 :This animal is NOT a chicken

T: mean weight

Null distribution

The null distribution is the distribution of the test statistic if the null hypothesis is true

- For test statistic T: $P(T = t \mid H_0)$

Example2: H0 :This animal is a chicken

H1 :This animal is NOT a chicken

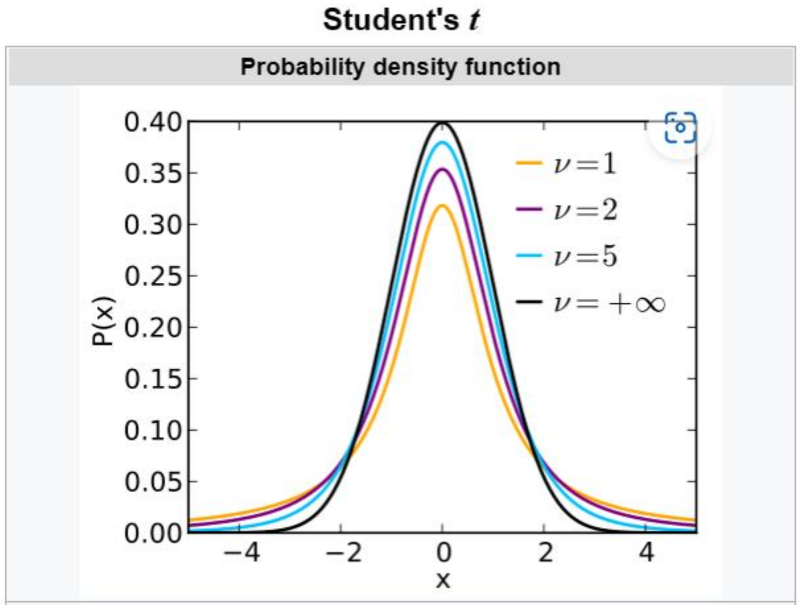
T: mean weight

Null distribution: 6.2 ± 0.8 lb

A more general way to find the test statistics – t test

- T = $\dfrac{Observed - Hypothesized}{SE(Observed)}$

For this test statistics T, we can easily find its null-distribution, which is a t-distribution

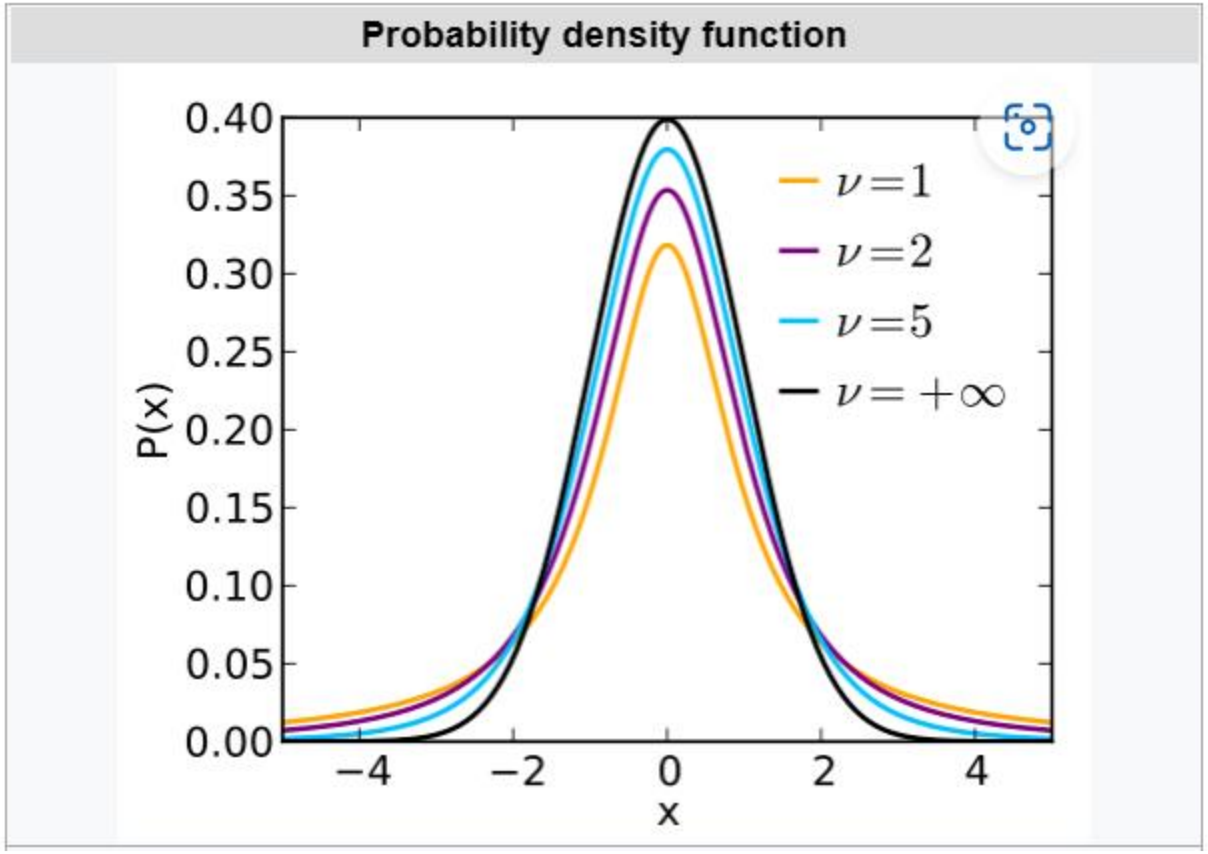


- Chicken example:
  - Adult chickens weigh 6.2 ± 0.8 lb
  - Observed mass is 10 lb
  - T = (10 – 6.2) / 0.8 = 4.75
  - P(|T| ≥ 4.75 | This is a chicken) = 2.03×10^{-6}

A more general way to find the test statistics – t test

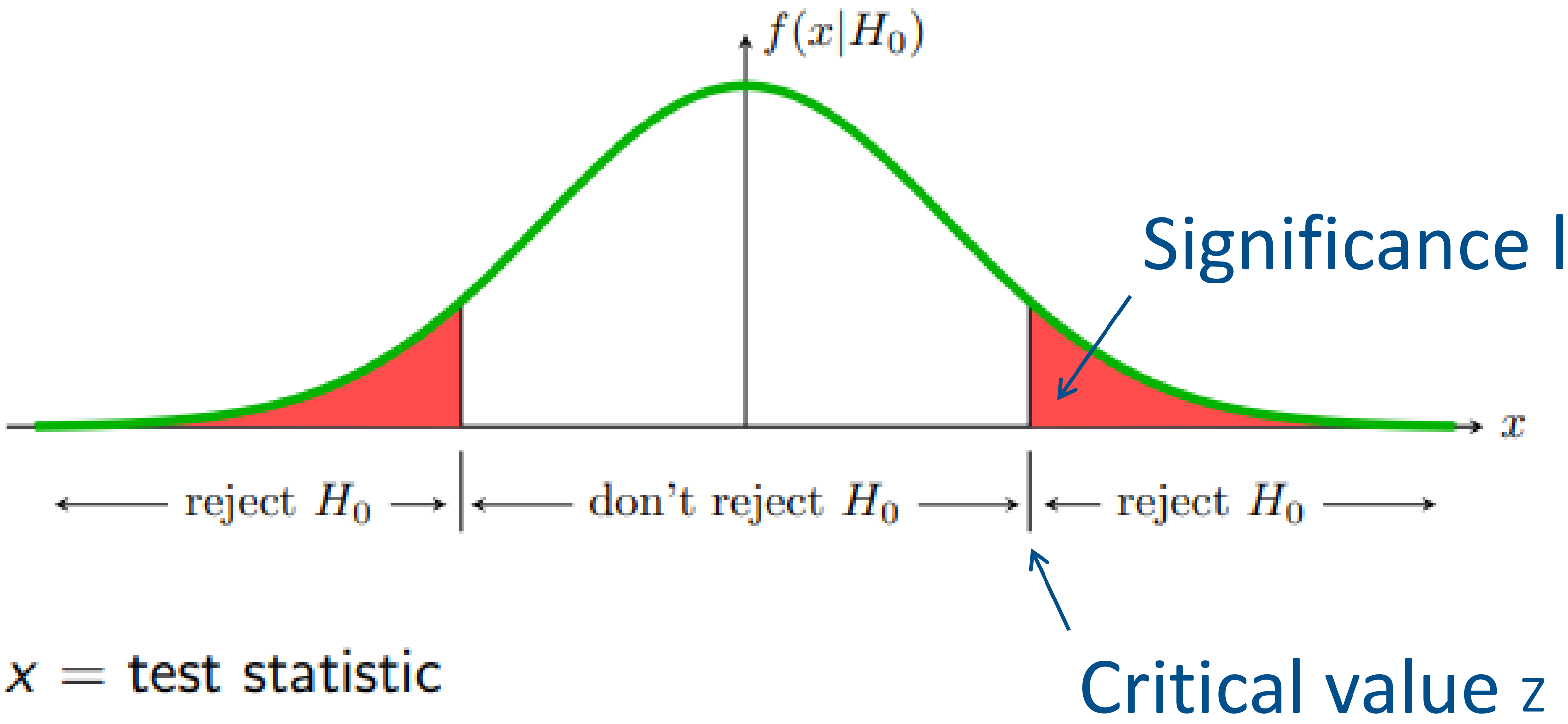
- T = $\dfrac{Observed - Hypothesized}{SE(Observed)}$

Student's *t*



- Sample mean example:
  - Training data x_1 x_2 …x_n
  - $H_0 : \mu = 0$ $H_1 : \mu \neq 0$ ·
  - $T = \frac{\bar{x}-0}{SE(\bar{x})}$ $\sim t_{n-1}$
- Where $SE(\bar{x}) = \frac{\sigma}{n}$, $\sigma = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}}$
  - Example: $n = 3, x_1 = 2, x_2 = 3, x_3 = 1, \bar{x} = \frac{2+3+1}{3} = 2, SE(\bar{x}) = \frac{1}{3}$, t = 6
  - $P(|T| \geq 6 \mid H_0) = 1.01 \times 10^{-5}$

Understand this figure

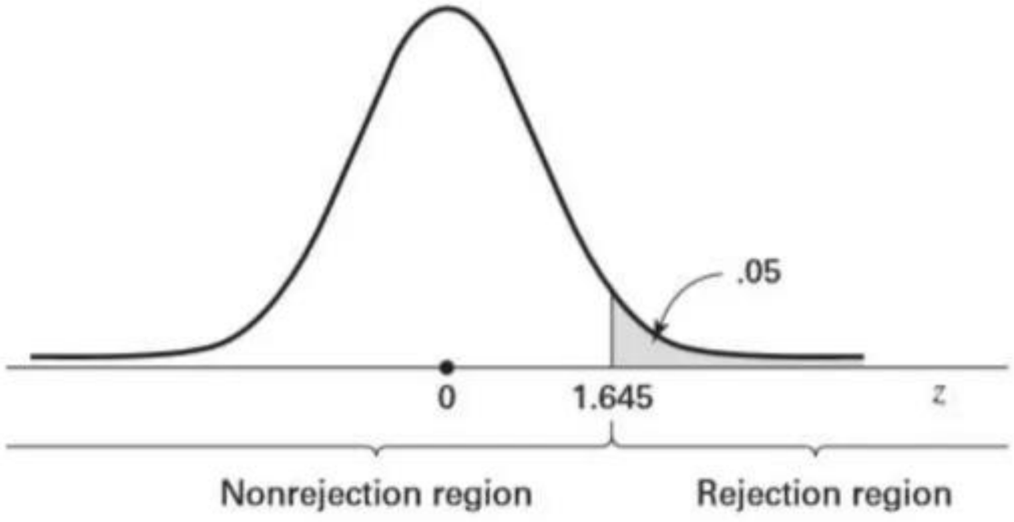


$\alpha = P(|x| \geq z)$

- x = test statistic
- $f(x|H_0)$ = pdf of null distribution = green curve
- Rejection region is a portion of the x-axis.
- Significance = probability over the rejection region = red area.

Lookup table for the critical value z

z	0.00	0.01	0.02	0.03	0.04	$\boxed{0.05}$	0.06	0.07	$\boxed{0.08}$	0.09	z
0.00	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	0.00
0.10	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	0.10
0.20	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	0.20
0.30	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	0.30
0.40	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	0.40
0.50	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	0.50
0.60	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	0.60
0.70	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852	0.70
0.80	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	0.80
0.90	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	0.90
1.00	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	1.00
1.10	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	1.10
1.20	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	1.20
1.30	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	1.30
1.40	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1.40
1.50	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1.50
1.60	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1.60
1.70	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1.70
1.80	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1.80
1.90	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1.90
2.00	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	2.00
2.10	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	2.10
2.20	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	2.20
2.30	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	2.30
2.40	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	2.40
2.50	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	2.50
2.60	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	2.60
2.70	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	2.70
2.80	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	2.80



p-values

- The <u>p-value</u> is the probability of observing an *equal or more extreme* value of the test statistic than observed t, assuming the null hypothesis
    - For test statistic T, this is $P(T \geq t \mid H_0)$
    - Lower -> observation is more unlikely -> more surprised

- p-values < α are considered "significant"
    - reject the null hypothesis
    - t is more extreme than z

EXAMPLE

- The goal of a study by Klingler et al. was to determine how symptom recognition and perception influence clinical presentation as a function of race. They characterized symptoms and care-seeking behavior in African-American patients with chest pain seen in the emergency department. One of the presenting vital signs was systolic blood pressure. Among 157 African-American men, the mean systolic blood pressure was 146 mm Hg with a standard deviation of 27. We wish to know if, on the basis of these data, is there any evidence to support the claim at $\boldsymbol{\alpha = 0.05}$ that the mean systolic blood pressure for a population of African-American men is greater than 140.

Solution

- Hypotheses

$$H_0 : \mu \le 140$$
$$H_A : \mu > 140$$

We want to make a decision of rejection or non-rejection with 95% confidence

Step 1 Set $\alpha = 0.05$

Step 2 Use the look up table to find z= 1.65

Step 3 Compute t, the test statistics using the given dataset

$$t = \frac{146 - 140}{27/\sqrt{157}} = \frac{6}{2.1548} = 2.78$$

Step 4 Reject H_0 since 2.78>1.65 (t>z)

Group work

- Find whether we can reject the null hypothesis with 99% confidence? What about 99.9% confidence?