CMSE381 - Practice problems for the final exam

1. (SVM) For the following dataset, if I was given the information that the classification hyperplane has a slope of -1. (a) Could you help to figure out the equation of the MMC?
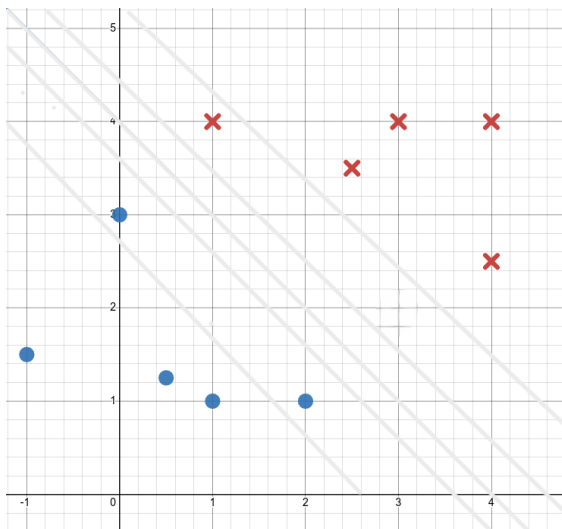
(b) what should the label be for the red class?

(c) Please use the equation to find M, the value of the margin.

(d) What is the main difference between MMC and SVC?

(e) If I want to let $M = \sqrt{2}$. Then how many support vectors would I have?

(f) For each of the support vectors, what is the smallest value for the associated slack variable?

(g) Find the inner product representation of the SVC

(h) Using the kernel $K(x, y) = e^{-\frac{\|x-y\|_2^2}{2}}$ to replace the inner product found in the previous problem, what is prediction of SVM for the new input $x = [1, 5]$?

2. (Decision Tree for classification) (a) which of the following classification tree is preferred during pruning with an $\alpha = 0.1$?

Tree 1: three leaves (L_1, L_2, L_3), training samples classified to L_1 has labels $(-1, 1, 1)$, those classified to L_2 has labels $(-1, 1)$, those classified to L_3 has labels $(1, 1)$

Tree 2: two leaves (L_1, L_2), training samples classified to L_1 has labels $(-1, 1, 1, 1)$, those classified to L_2 has labels $(-1, 1, 1)$.

3. (Neural Network) (a) Draw the diagram for a neural net with input data points with $p = 2$ (i.e., (X_1, X_2)), two units in the hidden layer and two units in the output layer with the following β and $\beta^{(2)}$ matrices as weights for the first and second layer (last columns are the bias, β_{ij} is the weight associated with X_i and A_j),

$$\beta = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 0 \end{pmatrix} \qquad \beta^{(2)} = \begin{pmatrix} 0 & -2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

and using the activation function

$$g(z) = (z)_+ = \begin{cases} 0 & \text{if } z < 0 \\ z & \text{else.} \end{cases}$$

(b) What is the output Y for the new point $(2, 1)$?

(c) Add a softmax layer to the end of the network, what is the predicted Y value for the new point $(2, 1)$ now?

(d) How many parameters are trained by the computer in this setup?

4. (Convolutional Neural Network) (a) Draw a sketch of a CNN which

- takes as input a 36×36 black and white image,
- applies a convolution layer with three filter matrices of size 5×5,
- applies a 2×2 max pool layer,
- flattens the result into a vector.

Be sure to label all sizes of matrices.

5. For the following input "image" matrix X, we convolve with the matrix F resulting in the matrix A.

$$X = \begin{bmatrix} 6 & 5 & -1 & 5 & 4 \\ -6 & 2 & 4 & 0 & 5 \\ 5 & 2 & 1 & 4 & 3 \\ 3 & 2 & -2 & -1 & -6 \\ 5 & 0 & 2 & 2 & 1 \end{bmatrix}, \qquad F = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \qquad A = \begin{bmatrix} 18. & 7. & 9. & 18. \\ 0. & 11. & 8. & 13. \\ 11. & 2. & ??? & 4. \\ 7. & 0. & -2. & -12. \end{bmatrix}$$

(a) What value goes in the missing spot in matrix A?

(b) If we apply a 2x2 max pooling layer to the matrix A, what would the resulting matrix be?

(c) If we apply the ReLU function to every entry in the A matrix, what would the resulting matrix be?