Breaking Fire: An Analysis of Wildfires with Firebreaks

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December 19, 2021

Abstract

In this paper, we use a finite difference method to analyze a wildfire system. We perform Dimensional Analysis to simplify the system and numerically approximate solutions to the equations. We examine the width required for a firebreak to effectively stop the spread of a fire under different conditions, including the presence and absence of wind.

All code and animations for this project can be found at https://bitbucket.org/reecejr/volume4project/src/master/.

1 Background

Wildfires are natural disasters that take human life and cause significant property damage year after year. Several firefighting and fire prevention methods are effective at stopping the spread of wildfires, including the introduction of firebreaks into potential burn areas. A firebreak is a strip of land that has been cleared of foliage to prevent the spread of a fire across it. A firebreak can be effective at containing fires, provided that the firebreak is large enough to work properly. We model the spread of wild fires and investigate the size of firebreaks that would be needed to contain a wildfire using a system of two partial differential equations, one for the heat of the fire and another for the foliage the fire is consuming. We based our implementation on the paper "A Wildland Fire Model with Data Assimilation" [1]. We ran several simulations which employed their equations, and then added an advection term to model the wind and ran several more simulations. Although the paper discusses data assimilation into the model to estimate their coefficients, we did not introduce data into our model, though we did use their derived coefficients. We observed that a firebreak of up to $100m$ in width is necessary to prevent the spread of fires using this model, particularly when there are adverse wind conditions. With favorable wind conditions a firebreak of $36.125m$ or less is often sufficient to hem a fire in. This provides a useful benchmark for physical applications. However, we recommend constructing more specific models before these results are put into practice in a physical area.

2 Modeling

As stated above, our equations and coefficients come from [1]. They, and we, use two equations, one to model the heat of the fire and one to model the foliage consumption. In this paper, a thorough derivation of these equations is made, and the constant terms are carefully computed using real wildfire data. The equations are

$$\frac{dT}{dt} = \nabla \cdot (k\nabla T) - \vec{v} \cdot \nabla T + A\left(Se^{-B/(T-T_a)} - C(T-T_a)\right),$$

and

$$\frac{dS}{dt} = -C_S S e^{-B/(T-T_a)},$$

where,

T (K) is the temperature of the fire layer.

$S \in [0,1]$ is the fuel supply mass fraction (the relative amount of fuel remaining).

k (m^2s^{-1}) is the thermal diffusivity. We use $k = 2.13608 * 10^{-1}$

A (Ks^{-1}) is the temperature rise per second. We use $A = 1.8793 * 10^2$

B (K) is the proportionality coefficient in the modified Arrhenius law. We use $B = 5.5849 * 10^2$
C (K^{-1}) is scaled coefficient of the heat transfer to the environment. We use $C = 4.837 * 10^{-5}$
C_S (s^{-1}) is the fuel relative disappearance rate. We use $C_S = 1.6250 * 10^{-1}$
T_a (K) is the ambient temperature. We generally initialize this to be $300K$
$\vec{v}$ (ms^{-1}) is the wind speed given by atmospheric data or model.

We later add an advection term to these equations to model wind in the environment (replacing the $\vec{v} \cdot \nabla T$ term from the original equations).

2.1 Origin

In this exploration, we relied a lot on the cited research [1]. In it, they explored the given system of PDEs and used data assimilation to derive the constants used, in order to better approximate the behavior of real wildfires. We used these constant values in our experimentation, since we don't have data currently available with which to derive our own constants.

The authors of [1] derive these PDEs principally through physical laws, especially that of diffusion, advection, and combustion. Therefore, the equations don't rely so much on the specific physical and material properties of a specific region and can more easily be generalized to many types of regions.

The first Laplacian term in the Temperature function is based on the diffusion of heat to the surrounding area, and is integrated into the equation by the Divergence Theorem. The wind term is used since the wind generates a direct propagation and transfer of heat. The next term is based on the fact that the heat generated is directly proportional to the amount of fuel lost. Finally, the last term is Newton's law of cooling which represents the loss of heat by convection into the atmosphere. These aspects are combined into the Temperature and constitute a conservation of heat energy equation (equation (1)).

Similarly, we note that the amount of fuel lost is proportional to the rate of burning, as well as the amount of fuel left, which yields our equation for S (equation (2)).

2.2 Solution

To solve this system of PDEs, we used an explicit (forward) Euler temporal approximation with a centered finite difference spatial approximation. To generate the "no wind" equations, these methods gave the following approximations:

$$S^{n+1} = S^n - \Delta t \left(C_S S^n e^{-B/(T^n - T_a)} \right),$$

and

$$T^{n+1}_{i,j} = T^n_{i,j} + \Delta t \left(k \left(\frac{T^n_{i+1,j} - 2T^n_{i,j} + T^n_{i-1,j}}{\Delta x^2} + \frac{T^n_{i,j+1} - 2T^n_{i,j} + T^n_{i,j-1}}{\Delta y^2} \right) + AS^n_{i,j} e^{-B/(T^n_{i,j} - T_a)} - C(T^n_{i,j} - T_a) \right).$$

With the addition of an advection term to model the wind we added this term into the above method after the Laplacian approximation term:

$$\alpha \frac{T^n_{i+1,j} - T^n_{i-1,j}}{2\Delta x} + \beta \frac{T^n_{i,j+1} - T^n_{i,j-1}}{2\Delta y}$$

Here $S^n_{i,j}, T^n_{i,j}$ are Fuel and Temperature (respectively) at timestep n, x index i, and y index j. Note that the centered finite difference term in our approximation approximates the Laplacian $\nabla \cdot (k \nabla T)$ term of our original equation. The magnitude of the α and β coefficients in the wind term determines the strength of the wind and their sign determines the direction of the wind.

For this approximation, we used values of $\Delta t = 1s$ and $\Delta x, \Delta y = 2m$ to generate our numerical approximations and animations. These equations we approximated on our $500m \times 500m$ region (250×250 mesh grid), using Neumann boundary conditions in the spatial dimensions and the same constants as used in [1]. We initialized our region by establishing a firebreak (which we varied in testing, as discussed later) and an initial ignition region (normally $50m \times 50m$). We also padded the T matrix on all edges to ensure that our Centered Finite Difference method would not cause `index out of range` errors. Note that in using the wind term, we could not derive a specific constant for that term (due to lack of data to utilize), so instead we used constants to establish the effect of wind in the x and y directions. Analysis of, and comparison to data, would allow us to determine constants which would allow us to assimilate wind speed into the model in meters/second.

2.3 Stability

In our modeling, we noticed a few issues of stability. We noticed that due to slight perturbations, when the $(T^n_{i,j} - T_a)$ term was slightly negative, it would cause overflow errors and send our exponential term to infinity. To remedy this, we implemented the Arrhenius equation (equation (14) in [1]):

$$r(T) = \begin{cases} e^{-B/(T^n - T_a)}, & T > T_a \\ 0, & T \leq T_a \end{cases}$$

All of our stability testing and correction was dedicated to ensuring that edge cases of this issue worked correctly according to our model. We also artificially limited the temperature to the range of $[T_a, 1200]$ to help correct this instability around the edges, as well as ensure that the burn temperature did not increase exponentially.

3 Results

We ran two separate models, one with the wind term and one without the wind term.

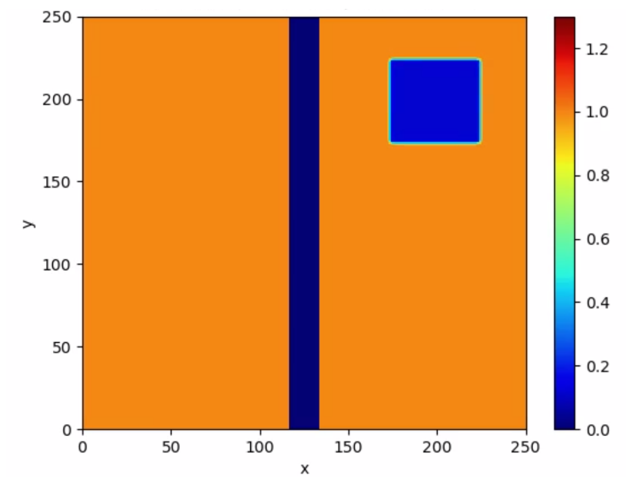


Figure 1: A plot of the density of foliage in the forest fire model. Orange represents 100% foliage, dark blue represent 0% foliage. The vertical strip is the firebreak and the square box is the fire.

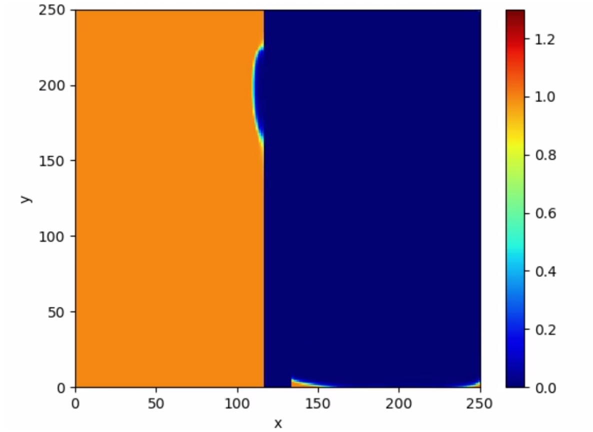
3.1 Without Wind

We began by simulating the spread of a fire through a $500m \times 500m$ (meter) forest of constant foliage density, except that the forest includes a firebreak that runs the length of the forest horizontal. Within the firebreak, there is no foliage. At the start of the model, the ambient temperature across the forest is also constant at 300° K, except within a $50m \times 50m$ section where we increase the temperature to 1200° K. The initial setup is given in Figure 1.

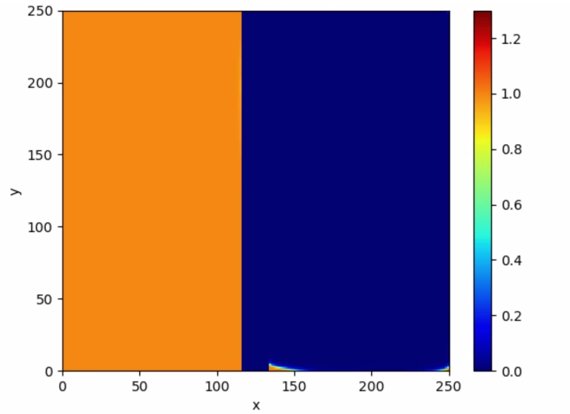
We evolved the system over time using our approximation of the PDE system. We artificially capped the heat of the fire at 1200° K to prevent the temperature values from overflowing (see 2.3 for a discussion of changes we made to the model). Our first model used a firebreak of $35m$. While that seemed promising and held the fire off over 3 minutes, eventually the fire broke through (Figure 2a). However, a firebreak of $36.125m$ was just enough to ward off the fire completely (Figure 2b).

After running these simulations, we increased the density of the forest by 30%. Here our 36.125 firebreak was no longer sufficient to stop the fire (Figure 3a); rather a $43m$ firebreak was required (Figure 3b). In addition, we observed that the fire would spread much more rapidly through the dense forest.

Finally, in an attempt to make a more accurate model, we initialized a forest with random permutations of starting foliage density. The density ranged from 70%-130% of the original constant density. Again, an $36.125m$ firebreak is sufficient to stop the fire (Figure 4).

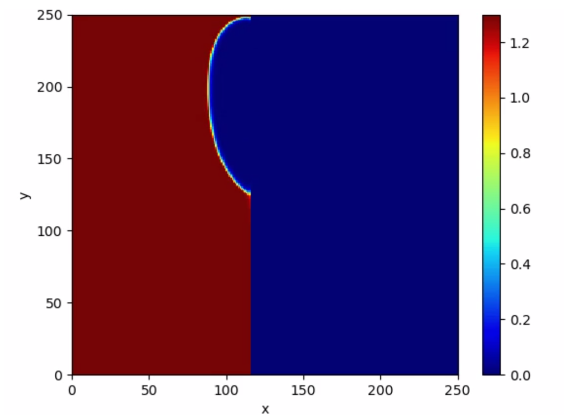


(a) 3 min 15 sec with a 35m firebreak.

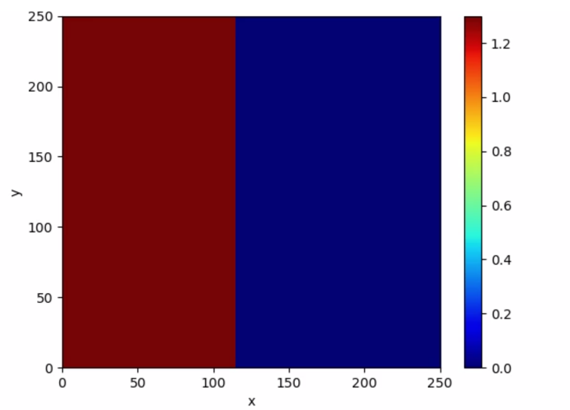


(b) 3 min 15 sec with a 36.125m firebreak.

Figure 2: Progression of the fire in a forest of 100% foliage density.



(a) 3 min 15 sec with a 36.125m firebreak.



(b) 3 min 15 sec with a 43m firebreak.

Figure 3: Progression of the fire in a forest of 130% foliage density.

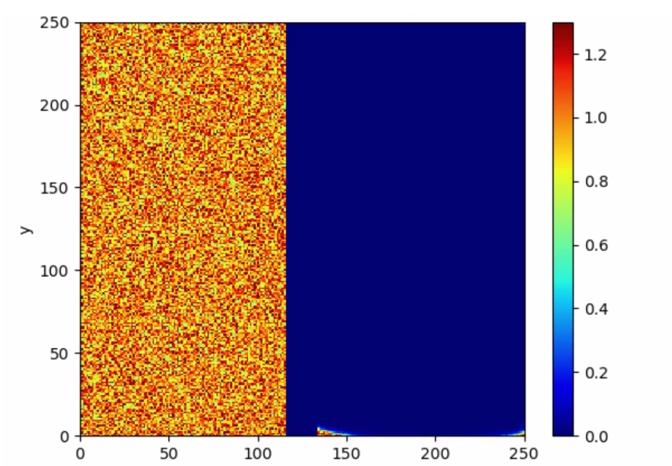


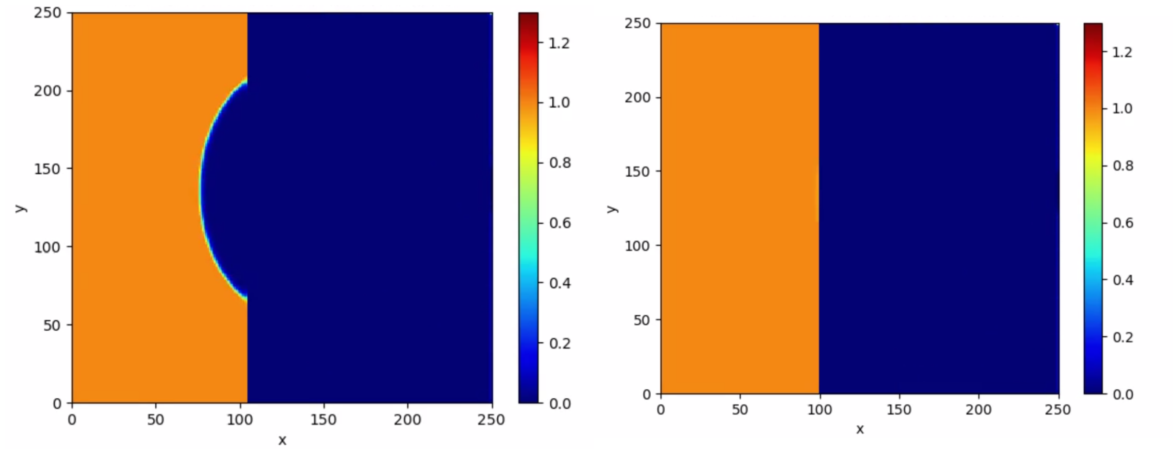
Figure 4: Progression of a fire after 3 min 15 sec with a 36.125m firebreak in a forest of random initial foliage density.

3.1.1 Temperature Variation

We briefly explored the effect of increasing the ambient temperature to 315° K to simulate a hotter condition as in California. Though the fire spread a little faster than before, the temperature variation was not enough to noticeably change the required width of firebreak to stop the fire's progression.

3.2 With Wind

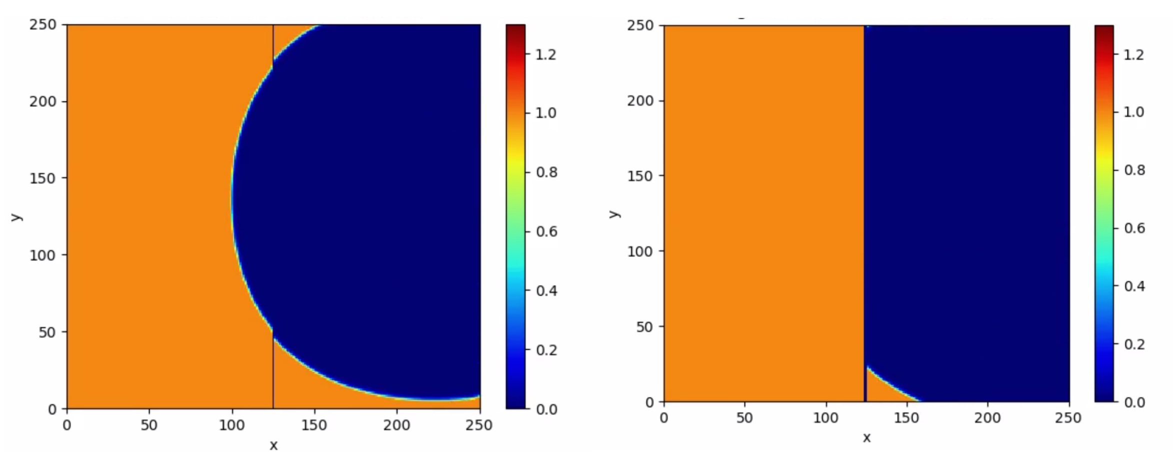
After finishing our first construction of the model, we added an additional advection term. This advection term models the effect of wind on the spread of the fire. We tested the model with a wind blowing from various directions, in each case with a speed of 0.5 (see 2.2 for discussion about wind measurement units). For a wind blowing west towards the firebreak, a firebreak of $80m$ failed to stop the fire (Figure 5a), however, a 100 meter firebreak did stop it (Figure 5b). For a wind blowing to the east, away from the firebreak, a firebreak of $2m$ failed (Figure 6a) while a firebreak of only $4m$ succeeded (Figure 6b). Lastly, when the wind was blowing parallel to the firebreak, the results were the same as in the case with no wind, i.e. a 35 meter firebreak failed to stop the fire (Figure 7a) while a firebreak of $36.125m$ did stop it (Figure 7b).



(a) $80m$ firebreak.

(b) $100m$ firebreak.

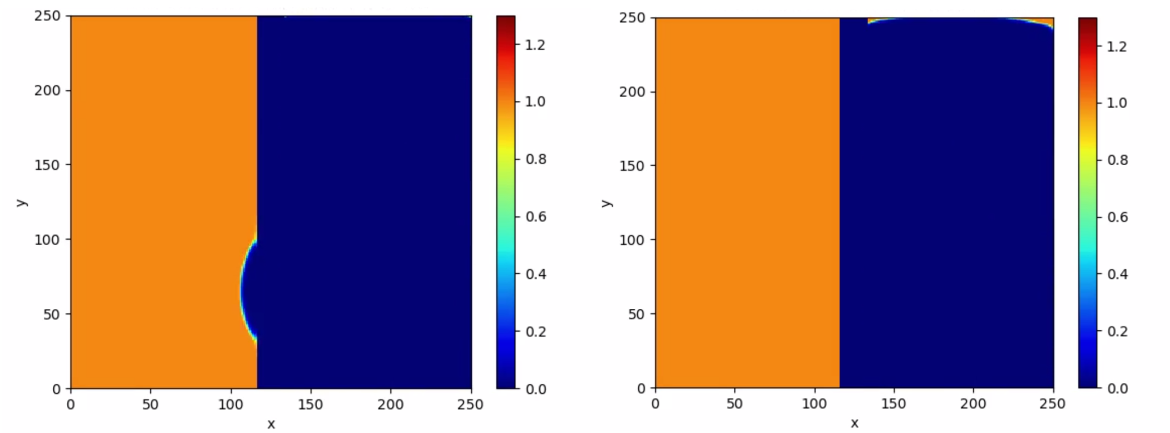
Figure 5: Progression of the fire in a forest of 100% foliage density with a west wind.



(a) $2m$ firebreak.

(b) $4m$ firebreak.

Figure 6: Progression of the fire in a forest of 100% foliage density with a east wind.



(a) $35m$ firebreak.

(b) $36.125m$ firebreak.

Figure 7: Progression of the fire in a forest of 100% foliage density with a south wind.

4 Analysis and Conclusions

While our model has shortcomings, the results from this experiment could be extremely helpful in fighting fires. This model can predict the exact size of firebreaks that will be needed to stop the spread of a forest fire no matter what the density of the fuel or the direction of the wind. Using this data, firefighters could minimize the amount of land needed to be burned to create a firebreak. If these firebreaks are successful then lives, property, and forests could be saved.

Another conclusion from our model was that the initial size of the fire did not affect the overall spread. For example, whether the fire started with a $1m \times 1m$ area or a $25m \times 25m$ area, the spread of the fire and the length of firebreak needed to stop it were the same. These findings underscore the significance of firebreaks in prevention of fire spread. This is important to recognize because some devastating forest fires were started by very small things. For example, the Mendacino fire in 2018 which burned over 450,000 acres was said to be started by a spark from a man using a hammer [2].

4.1 Future Work

In our work with the model, one of the most significant aspects that stood out to us was the lack of a term in the PDEs that corresponds to a decrease in fire temperature as the fuel runs out. In a few of our trials of firebreak width, we noticed that the fire would be halted by the firebreak for an extended duration of time, and only jump across after a long period of time has elapsed. We believe the likelihood of this phenomenon is much smaller in reality, since a longer duration of time would correspond to a lower temperature as the fire burns through the available fuel. As the model currently stands, the fire burns perpetually at temperatures $\geq 1200°$ K, even when all available fuel is consumed.

Another potential area of improvement is more experimentation with fuel density. Depending on the desired region to model, fuel density can be modified to better represent the vegetation distribution native to the region, as well as to better represent the dryness and ease of combustion of said vegetation.

Another factor to take into account is the slope of the terrain over which a wildfire travels. It is true that a fire travels faster uphill, and to account for this, [1] writes that this effect can be incorporated into the wind term of the Temperature PDE since the effect of terrain gradient is similar to a wind blowing in that direction. That, combined with a vector field of air currents and wind velocities for the region, would better simulate the native nuances to a region and would better model a fire's progression in that region.

Ultimately, the most useful and applicable further work we could do on this model is to simulate the effect of fire-fighting measures on the wildfire. This would provide insights into the efficacy of different techniques, as well as provide guidance as to the most effective locations to implement said measures.

References

[1] Jan Mandel, Lynn S. Bennethum, Jonathan D. Beezley, Janice L. Coen, Craig C. Douglas, Minjeong Kim, Anthony Vodacek, "A Wildland Fire Model with Data Assimilation," *Mathematics and Computers in Simulation, Volume 79, Issue 3,* 2008, Pages 584-606, ISSN 0378-4754, https://doi.org/10.1016/j.matcom.2008.03.015.

[2] Tim Arango, "Behind Most Wildfires, a Person and a Spark: 'We Bring Fire With Us'" *The New York Times*, August 20, 2018. https://www.nytimes.com/2018/08/20/us/california-wildfires-human-causes-arson.html