Introduction to Digital Signal Processing

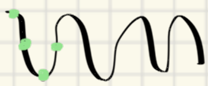
SIGNAL

> physical phenomenon that carries and convey information from one place to another

Signal Processing

— interpretation and manipulation of sound, image, time-varying measurement values.

Types of Signal Processing

- Analog Signal Processing  — limited lang ang makukuhang info since 4 lang ang dots
- Digital signal Processing  — mas madaming information and makukuha

— mas maraming dots smoother or mas maraming information ang makukuha
— mas mataas ang resolution

Digital — operating by the use of discrete signal to represent data

Signal — parameter to convey information

Processing — a series operation performed according to programmed instruction

DSP

> Program Memory — stores the program

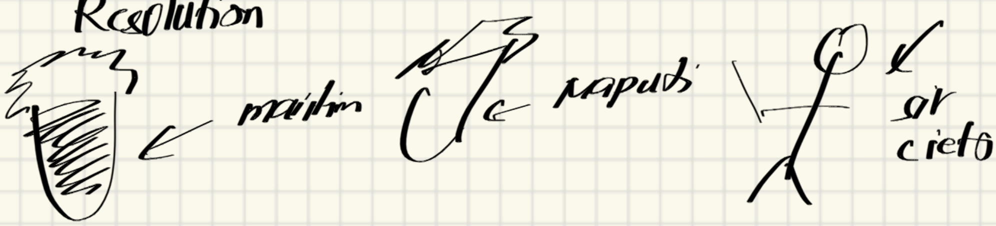
> Data Memory — stores the information

> Compute Engine — perform math

> Input / output

consist of anti-aliasing filter, ADC,

Resolution



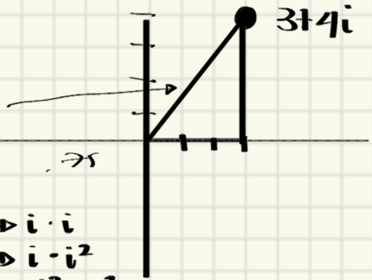
Antialias filter - adlib

Frequency Resolution -

Quantization Error

Jan 28

3+4i



(.25) $i = \sqrt{-1}$

(.50) $i^2 = -1 \rightarrow i \cdot i$

(.75) $i^3 = -i \rightarrow i \cdot i^2$

(divisible) $i^4 = 1 \rightarrow i^2 \cdot i^2$

$i^5 = \sqrt{-1}$

$i^6 = -1 \rightarrow i^4 \cdot i^2 / i^3 \cdot i^3$

$i^7 = -i$

$i^8 = 1$

$i^{29} = \sqrt{1} \rightarrow i^{28} \cdot i^1$

$\rightarrow (i^4)^7 \cdot i^1$

$\rightarrow (1)^7 \cdot \sqrt{-1}$

$= 1 \cdot \sqrt{-1}$

$= \sqrt{-1}$ or i

$$4\overline{)65} = 16, \quad 4, \quad 25, \quad 24, \quad 1$$

$i^{65} = (i)^{4}$

$(4+8i) - (3-5i)$

$= (1+13i)$

$(4+\sqrt{25}) + (3-\sqrt{-81})$

$= (4+5i) + (3-9i)$

$= \boxed{7-4i}$

$(5-3i)^3 = -10-198i$

$(3+4i)(3-4i)$

$9 \quad -12i^2$

$9+16 = 25$

$\frac{4+3i}{5-2i} \times \frac{5+2i}{5+2i}$] conjugate

$23i$

$\frac{20+6i^2+15i+8i}{(5-2i)(5+2i)}$

$\frac{20-6+23i}{25-4i^2} = \boxed{\frac{14+23i}{29}}$

$25+4=29$

simplify

$a+bi$

$\boxed{\frac{14}{29} + \frac{23}{29}i}$

Example

$\frac{5-2i}{i} \cdot \frac{i}{i}$

$\frac{(5-2i)(i)}{i^2} = \frac{5i-2i^2}{i^2} = \left[\frac{5i+2}{-1}\right]^{-1} = \boxed{-5i-2}$

$\frac{7+2i}{3-i} \cdot \frac{3+i}{3+i} = \frac{(7+2i)(3+i)}{(3-i)(3+i)}$

$= \frac{21+7i+6i-2}{9-i^2}$

$= \frac{21+13i-2}{10} = \frac{19+13i}{10} = \boxed{\frac{19}{10} + \frac{13i}{10}}$

$\frac{5-2i}{(2+3i)^2}$

$6i \quad 6i$

$(2+3i)(2+3i)$

$4 \quad 9i^2$

$4-9+12i$

$-5+12i$

$\frac{5-2i}{-5+12i} \quad \frac{-5-12i}{-5-12i}$

$\frac{-25-60i+10i+24i^2(-24)}{25-\cancel{60i}+\cancel{60i}-144i^2}$

$\frac{-49-50i}{169} = \boxed{\begin{array}{c} a+bi \\ \frac{-49}{169} - \frac{50i}{169} \end{array}}$

$x^2 - 36 = 0$ $\quad (x+6)(x-6) = 0$

± 6

$\sqrt{x^2} = \sqrt{36}$

$x = 6$

$x^2 + 36 = 0$ $\quad (x+6i)(x-6i)$

$\sqrt{x^2} = \sqrt{-36}$

$x = 6i$

$3x^2 + 48 = 0$

$\frac{3x^2}{3} = \frac{-48}{3}$

$\sqrt{x^2} = \sqrt{-16}$

$x = \pm 4i$

$\boxed{3(x+4i)(x-4i)}$

$x = \pm 4i$

$x^2 + \frac{1}{9} = 0$

$\sqrt{x^2} = \sqrt{-\frac{1}{9}}$

$x = \pm \frac{1}{3} i$

$\boxed{\left(x + \frac{1}{3}i\right)\left(x - \frac{1}{3}i\right)}$

feb 3 - DSP

Basic Signal

Signal is a pattern of variation of some form
↳ variables that carry information

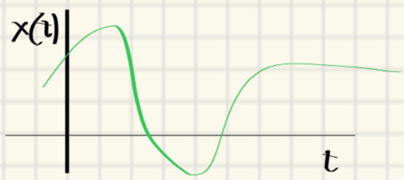
Examples of Signal

- Electrical Signal
  ↳ voltage & currents
- Accoustic Signal
  ↳ sound
- Mechanical
  ↳ velocity
- Video
  ↳ Intensity level of Pixel

Continous Time Signal

↳ most signal in the real world are continous time

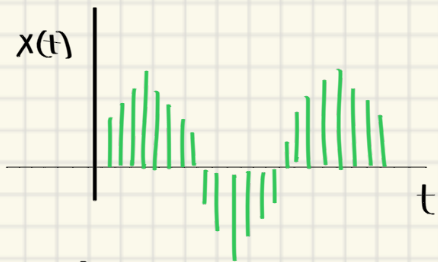
eg. (voltage, velocity)



Discrete Time Signal

↳ some real world and many digital signal are discrete time

eg. (pixels, daily stock price)



System

↳ process input signal to produce output signal

Properties of a System

- Causal
- Linear
- Time Invariance

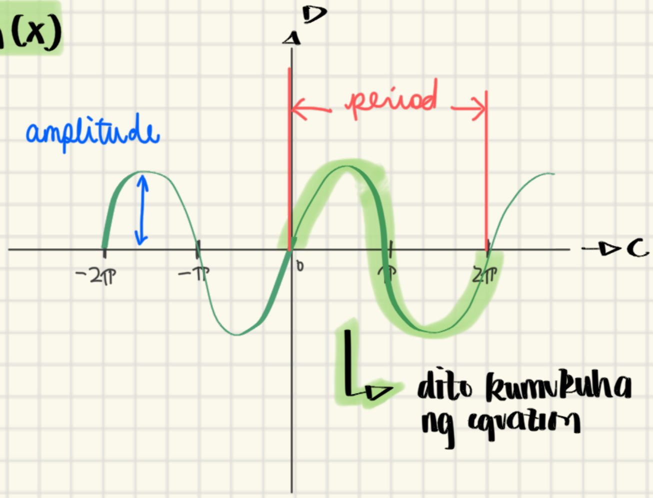
Sinusoidal

↳ sine wave or used to describe a curve

Sinusoidal Graph

↳ based on the sine function $y = \sin(x)$

↳ repeats every 2π / period



Formula of the Sinusoidal

$$y = A\sin(B(x-C)) + D$$

↳ $\frac{2\pi}{B}$ is the period

↳ $|A|$ is the amplitude

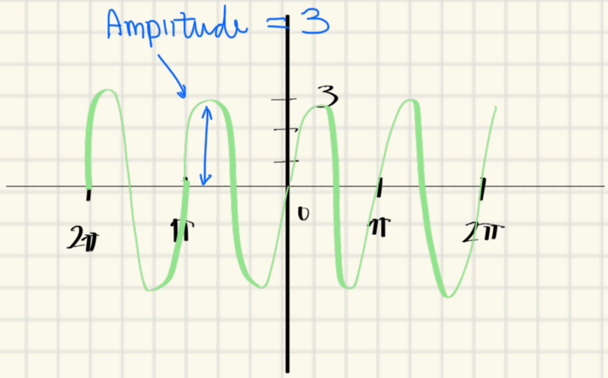
↳ C is the horizontal Shift

↳ D is the vertical Shift

Exmple

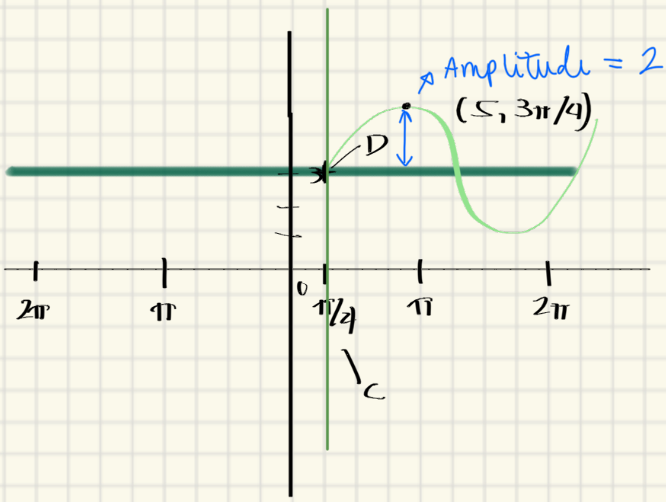
1. Graph $= \overset{A}{3}\sin \overset{B}{2}(x)$

- Period $= \frac{2\pi}{B} = \frac{\cancel{2}\pi}{\cancel{2}} = \pi$
- $|A| = 3$
- $C = 0$, no horizontal shift
- $D = 0$, no vertical shift

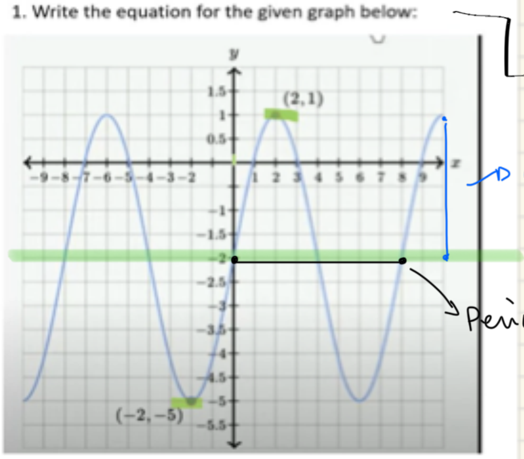


2. $y = \overset{A}{2}\sin\overset{B}{(}x - \overset{C}{\frac{\pi}{4}}) + \overset{D}{3}$

- Period $= \frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$
- $|A| = 2$
- $C = \pi/4$
- $D = 3$



3. 1. Write the equation for the given graph below:



↳ $\boxed{3\sin\left(\frac{\pi}{4}x\right) - 2}$

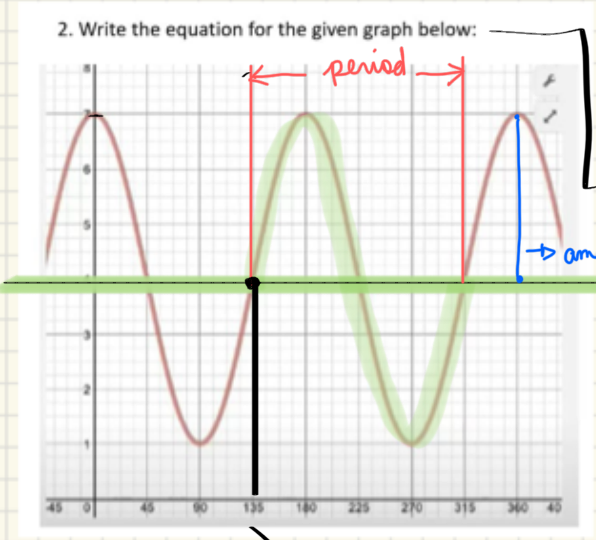
↳ amplitude = 3

Period $= \left[8 = \frac{2\pi}{\cancel{B}}\right]B$

$$\frac{\cancel{8}B = 2\pi}{\cancel{8}} = \frac{\pi}{4}$$

Feb 7

4.



D

C

$A\sin B(x-C)+D$

$D = 4$ $A = 3$ $C = 135 / \frac{3\pi}{4}$

$\boxed{3\sin 2(x-\frac{3\pi}{4})+4}$ $\boxed{B}$ = 180 or π

$\left[\pi = \frac{2\pi}{B}\right]B$

$\frac{\cancel{\pi}B = 2\cancel{\pi}}{\cancel{\pi}}$

$B = 2$

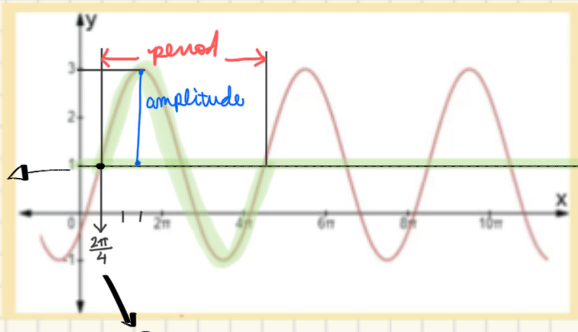
$|A| = \frac{max - min}{2}$

$|A| = \frac{7+1}{2}$

$|A| = 4$ (point sa graph)

Amplitude = 3 (tatlo ang ulo

5.



D

C

$D = 1$ $\boxed{2\sin \frac{1}{2}(x-\frac{\pi}{2})+1}$

$C = \frac{\cancel{2}\pi}{\cancel{4}} = \frac{\pi}{2}$

$B = 4\pi = \left[4\pi = \frac{2\pi}{B}\right]B$

$A = 2$

$\frac{4\pi B = 2\pi}{4\pi} = B = \frac{2\pi}{4\pi} = \frac{1}{2}$

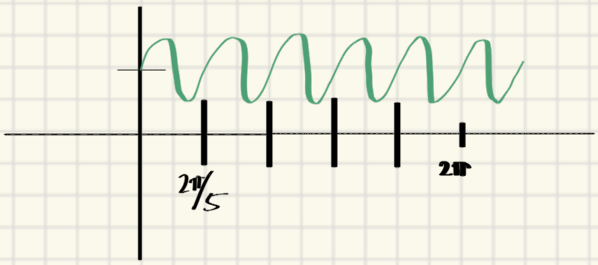
Note : $y = \frac{3+(-1)}{2} = 1 = D$

Practice Problem from the internet

1. $f(x) = \overset{A}{} \sin(\overset{B}{5}x) + \overset{D}{2}$

$y = A\sin(B(x-C)) + D$

$A = 1$
$B = 5$
$C = 0$, no hori shift
$D = 2$

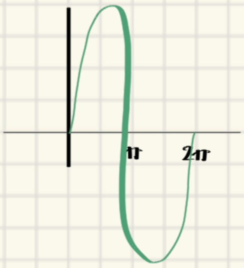


$$\text{Period} = \frac{2\pi}{B}$$

$$= \frac{2\pi}{5} \quad \text{or} \quad \frac{360}{5} = 72°$$

2. $4\sin x$

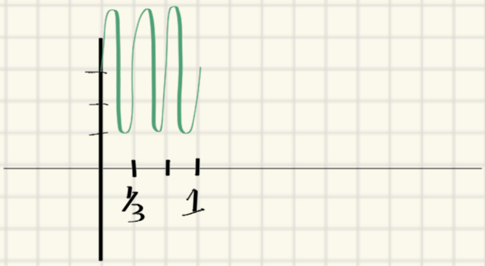
$|A| = 4$
$B = 1 \quad = \frac{2\pi}{1} = 2\pi$



3. $2\sin(6\pi x) + 3$

$|A| - 2$
$B = 6\pi = \frac{2\pi}{6\pi} = 1/3$

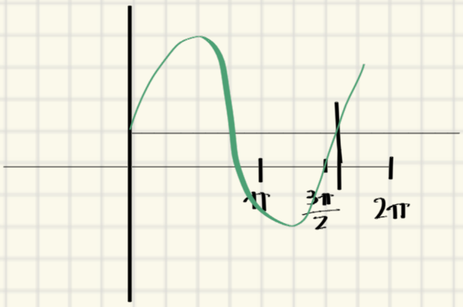
$D = 3$



4) $3\sin\left(\frac{5}{4}x\right) + 1$

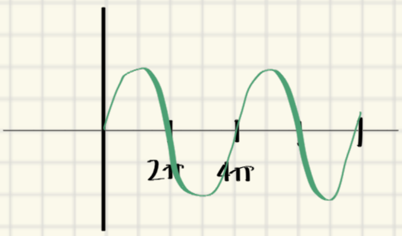
$|A| = 3$

$B = \frac{5}{4} = \frac{2\pi}{5/4} = 2\pi(4/5) = \frac{8\pi}{5} = \frac{8(180)}{5} = 288$

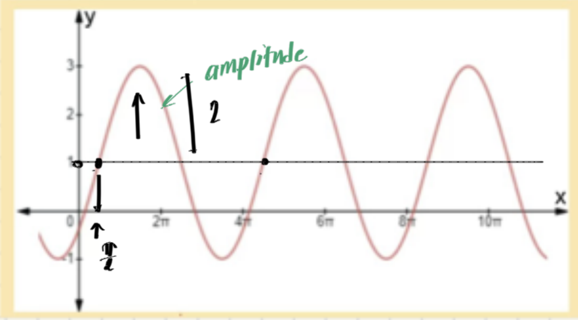


5) $2\sin(1/2 x)$

$A = 2$
$B = 1/2 = \frac{2\pi}{1/2} = 4\pi$



$A \sin B(x - C) + D$



midline = D = $\frac{\text{maximum} + \text{minimum}}{2} = \frac{3 + (-1)}{2} = \frac{2}{2} = 1$

amplitude = maximum − midline
$3 - 1$
$= 2$

Period = 4π

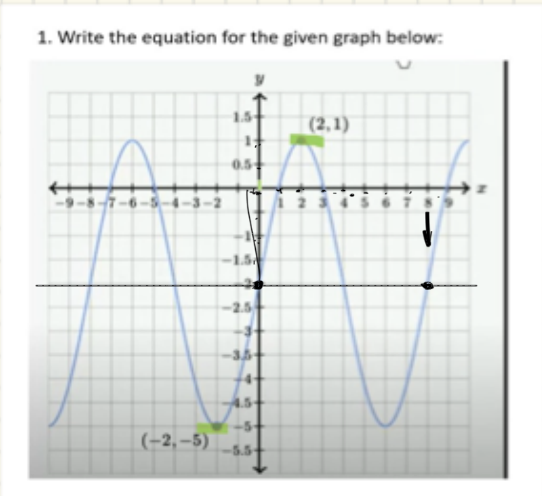
$4\pi = \frac{2\pi}{B}$

$\frac{\cancel{4\pi} B}{\cancel{4\pi}} = \frac{2\pi}{4\pi} = \frac{1}{2}$ B

$C = \frac{\pi}{2}$

$A \sin B(x - C) + D$

$2 \sin \frac{1}{2}\left(x - \frac{\pi}{2}\right) + 1$



midline = $\frac{\text{maximum} + \text{minimum}}{2}$

$= \frac{1 + (-5)}{2}$

$= \frac{-4}{2} = -2$

amplitude = maximum − midline
$= 1 + (-2)$ [sic]
$= 3$

Period = 8

$8 = \frac{2\pi}{B}$

$\frac{\cancel{8}B}{\cancel{8}} = \frac{2\pi}{8}$

$B = \frac{2\pi}{8} = \frac{\pi}{4}$

$C = 0$

$A \sin B(x - C) + D$

$3 \sin \frac{\pi}{4}(x) + (-2)$

$3 \sin \frac{\pi}{4}(x) - 2$

Linear Time Invariant

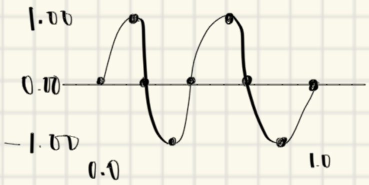
$x[n] \to$ Digital
$x(n) \to$ Analog

A ADC D DAC A

SAMPLING

Hz = cycles/sec.
$T_s = \frac{1}{f_s}$

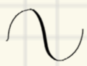
ex. if $f_s = 5$ samples/second, then there are
$T_s = \frac{1}{5} = 0.2$ seconds/sample

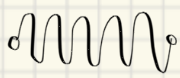


2Hz sine

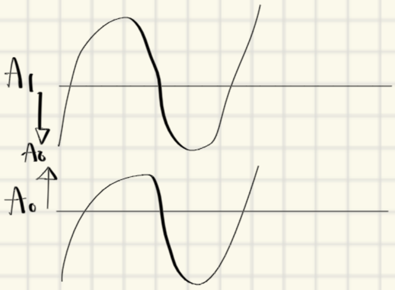
$f = 2$ cycle/sec.
$T = 1/2 = 0.5$ sec. per cycle
$f_s = 8$ samples/sec.
$T_s = 1/8$ sec. per sample

$B = \int$

$\sin x$ 

$\sin 5x$ 

$A \sin B(x - C) + D$



$$\overset{D}{A_0} + \overset{A}{A_1} \cos(2\pi \overset{B}{f} t + \overset{C}{\phi})$$

$x(t) = A_0 + A_1 \cos(2\pi f t + \phi)$

same

$A \sin B(x - C) + D$

Example

$5x_1 n - 3x_2 n + 2x_3 n$

$x = 0$. $x = [2]$
$x = 1$ $x = [-1]$

[0, 1, 0, -1]
5
-3 5
2 -2 5
0 2 -3 5

until margin 0 2

Aliasing

↳ is a problem because we wish to reconstruct the original signal

Anti-Aliasing

↳ performed using a low pass filter.

Question

Suppose that we have a signal bandlimited to 5Hz. What is the minimum Fs necessary to avoid aliasing.

Ans. 10Hz

$X(t) = -2 + \sin(40\pi t)$. What is the lowest possible sampling frequency that would avoid aliasing?

Ans. $F_s > 2f$ 40

March 10

What is a filter?

↳ any medium through which a signal propagate, which modifies the signal

For each of the following filters, state the order of the filter and identify the values of its coefficients.

a) $y_n = 2x_n - x_{n-1}$ ⟶ Order 1: $a_0 = 2$, $a_1 = 1$

b) $y_n = x_{n-2}$ ⟶ Order 2:

Continuous Time	Discrete Time
Integration	Summation
Differential Eqn	Difference Eqn

Non recursive $y[n] = a_1 x[n] + a_2 x[n-1] + a_3 x[n-2] \dots$

recursive $y[n] = b_1 y[n] + a_1 x[n]$

$X = X + 1$ $\qquad$ $x[n] \to \text{System} \to y[n]$

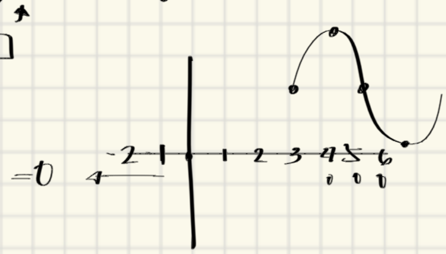
Ex. $y[n] = 2x[n] - 3x[n-1] + x[n-2]$

What is the output when $x[n] = \{\overset{0}{3}, \overset{1}{0}, \overset{2}{1}, \overset{3}{-1}\}$?

$y[0] = 2x[0] - 3x[0-1] + x[0-2]$

$= 2(3) - 3(0) + 1(0)$

$y[0] = 6$



$y[1] = 2x[1] - 3x[1-1] + x[1-2]$

$= 2(0) - 3(3) + 1(0)$

$= -9$

$y[2] = 2x[2] - 3x[2-1] + x[2-2]$

$= 2(1) - 3(0) + 1(3)$

$= 2 + 3$

$= 5$

$y[3] = 2x[3] - 3x[3-1] + x[3-2]$

$= 2[-1] - 3[1] + 1[0]$

$= -2 - 3$

$= -5$

$y[4] = 2x[4] - 3x[4-1] + x[4-2]$

$= 2[0] - 3[-1] + 1[1]$

$= 3 + 1 = 4$

$y[5] = 2x[5] - 3x[5-1] + x[5-2]$

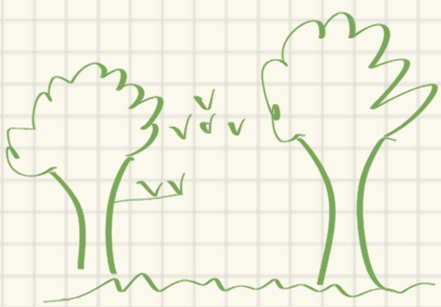
$= 2[0] - 3[0] + 1[-1]$

$= -1$

$\boxed{y[n] = \{6, -9, 5, -5, 4, -1\}}$ //

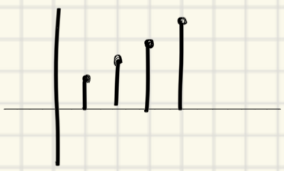
Checking / Easy Way

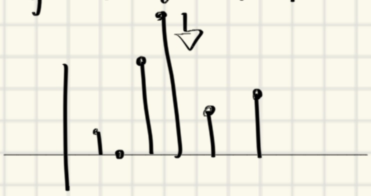
2	-3	1	
3	0	0	→ = 6
0	3	0	→ = -9
1	0	3	→ = 5
-1	1	0	→ = -5
0	-1	1	→ 4
0	0	-1	→ -1



Ex. Time Domain Analysis

$x[n] = \{1, 2, 3, 4\}$ $\quad\quad$ $y[n] = \{1, 0, 13, 62, 2, 7\}$





Impulse function (signal)

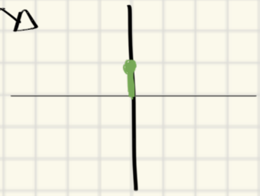
$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

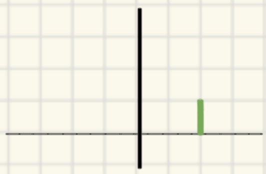
$\downarrow$

delta

n	$\delta[n]$
-2	0
-1	0
0	1
1	0
2	0

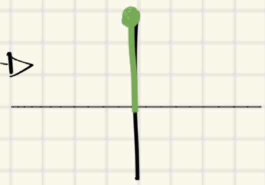
$\delta[n] \rightarrow$





n	$\delta[n-2]$	
-1	-3	0
0	-2	0
1	-1	0
2	0	1
3	1	0

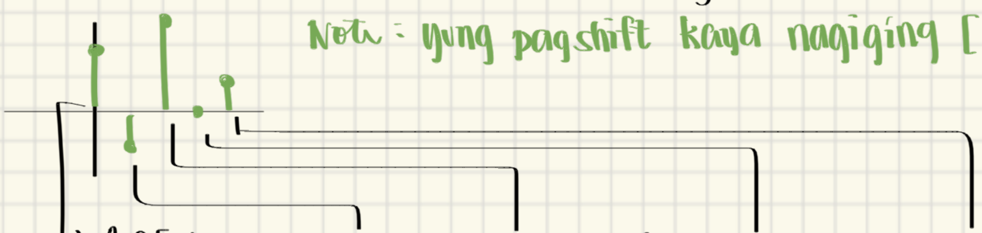
$3\delta[n] \rightarrow$



Ex #2

$x[n] = \{2, -1, 3, 0, 1\}$ $\quad\quad$ $y[n] = a_1 x[n] + a_2 x[n-1] + a_3 x[n-2]$

Note: yung pagshift kaya nagiging $[n-1]$



$\triangleright\ 2\delta[n] \triangleright -\delta[n-1] + 3\delta[n-2] + 0\delta[n-3] + 1\delta[n-4]$

When we have an LTI System
its response to the unit impulse function
its called unit impulse response

Ex #3

$$y[n] = \overset{1}{x[n]} \overset{-2}{-2x[n-1]} + \overset{4}{4x[n-2]}$$

$x[n] = \{\overset{0}{-1}, \overset{1}{3}\}$

$y[0] = 1[0] - 2[0-1] + 4[0-2]$

$= -1 - \cancel{2[0] + 4[0]}$

$y[1] = 1[1] - 2[1-1] + 4[1-2]$

$= 3 - 2[-1] \cancel{+ 4[0]}$

$= 5$

$\triangleright -x[n] + 5x[n-1] - 10x[n-2] + 12x[n-3]$

$y[n] = \{-1, 5, -10, 12\}$

$y = [2] = 1[2] - 2[2-1] + 4[2-2]$

$= 0 - 2[3] + 4[-1]$

$= -6 - 4$

$= -10$

$y = [3] = 1[3] - 2[3-1] + 4[3-2]$

$= 0 - 2[0] + 4[3]$

$= 12$

when $\delta[n]$ is input then $h[n]$ will be the output

$h[n] = \delta[n] - 2\delta[n-1] + 4\delta[n-2]$

$\delta[n]$ $1 \rightarrow n=0$, $0 \rightarrow n \neq 0$

$h[0] = \underset{1}{\delta[0]} - 2\underset{0}{\delta[0-1]} + 4\delta[0-2] \rightarrow 1$

$h[1] = \underset{0}{\delta[1]} - \underset{-2}{2\delta[1-1]} + 4\underset{0}{\delta[1-2]} \rightarrow -2$

$h[2] = \quad 0 \quad\quad 0 \quad\quad 4 \quad \rightarrow 4$

$\Big\}$ filter weights

FIR vs IIR

↳ Finite Impulse Response

↳ Infinite Impulse Response

$y[n] = x[n] - 2x[n-1]$

FIR

$h[n] = \delta[n] - 2\delta[n-1]$

$h[0] = \underset{1}{\delta[0]} - \underset{0}{2\delta[0-1]} = 1$

$h[1] = 0 - 2 = -2$

$h[2] = 0 - 0 = 0$

IIR

$y[n] = y[n-1] + x[n]$

$h[n] = h[n-1] + \delta[n]$

$h[0] = 0 + 1 \rightarrow 1$

$h[1] = 1 + 0 \rightarrow 1$

$h[2] = \underset{1}{h[1]} + \underset{0}{\delta[n]} \rightarrow 1$

March 11

Digital Convolution

↳ obtaining weighted average of a function of a signal

$x[n] \rightarrow \text{System} \rightarrow y[n]$

$y[n] = x[n] * h[n]$

impulse response

$x[n] \rightarrow y[n]$

$\delta[n] \rightarrow h[n]$

LTI System $h[n] = \{-1, 4, 2\}$

$x[n] = \{\overset{0}{1}, \overset{1}{0}, \overset{2}{-2}\}$

$y[n] = -x[n] + 4x[n-1] + 2x[n-2]$

$y[0] = -x[0] + 4x[0-1] + 2x[0-2]$

$= -1 + 0 + 0$

$= -1$

$y[1] = -x[1] + 4x[1-1] + 2x[1-2]$
$= 0 + 4 + 0$
$= 4$

$y[2] = -x[2] + 4x[2-1] + 2x[2-2]$
$= 2 \quad + 0 + 2$
$= 4$

$y[3] = -x[3] + 4x[2] + 2x[1]$
$0 \quad + 4(-2) + 0$
$= -8$

$y[4] = -x[4] + 4x[3] + 2x[2]$
$= 0 + 0 + 2(-2)$
$= -4$

$y[n] = x[n] * h[n]$

$x[n]$	1	0	-2		
$h[n]$	-1	4	2		
	-1	0	2		
		4	0	-8	
			2	0	-4
	-1	4	4	-8	-4

Properties of Convolution

1. Commutative $\quad a + b = b + a$
2. Associative $\quad a+b+c = (a+b)+c = a+(b+c)$
3. Distributive $\quad a(b+c) = ab + ac$

Associativity

$x[n] \longrightarrow \underset{h_1[n]}{\boxed{\text{LTI } 1}} \longrightarrow \underset{h_2[n]}{\boxed{\text{LTI } 2}} \longrightarrow y[n]$

$x[n] = \{4, 1, 3\}$
$h_1[n] = \{1, 1, 2\}$
$h_2[n] = \{-1, 2\}$

$x[n] * [h_1 n] =$
$x[n] * [h_2 n] =$

$x[n] \to$ 4 1 3
$h_1[n] \to$ 1 1 2

4 1 3
4 1 3
8 2 6

4 5 12 5 6

$h_2[n] \to$ -1 2

-4 -5 -12 -5 -6
8 10 24 10 12

$y_1[n]$ = -4 3 -2 19 4 12

Note : $x[n] * h_1[n] = ?$
$? * h_2[n] = y_1[n]$

$x[n] \to$ 4 1 3
$h_2[n] \to$ -1 2

-4 -1 -3
8 2 6

-4 7 -1 6

$h_1[n] \to$ 1 1 2

-4 7 -1 6
-4 7 -1 6
-8 14 -2 12

$y_2[n]$ = -4 3 -2 19 4 12

Note : $x[n] * h_2[n] = ?$
$? * h_1[n] = y_2[n]$

they have the same output

$x[n] = \{-1, 2, 1\}$
$h_1[n] = \{3, 4\}$
$h_2[n] = \{1, 2, 0, 3\}$

$h_1[n]$ 3 4
$h_2[n]$ 1 2 0 3
$h_{12}[n]$ 4 6 0 3

$x[n] * h_{12}[n]$

-1 2 1
3 4

-3 6 3
-4 8 4

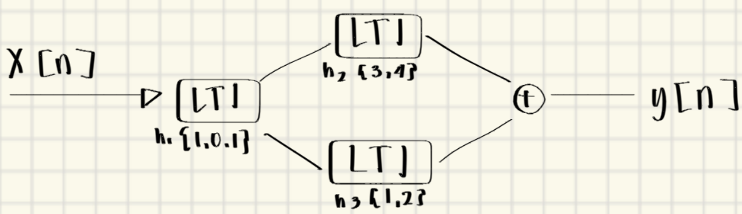
-3 2 11 4

1 2 0 3
-1 2 1

-1 -2 0 -3
2 4 0 6
1 2 0 3

-1 0 5 -1 6 3
+ -3 2 11 4

$y[n]$ = -4 2 16 3 6 3



$x[n] = \{-2, 0, 1\}$

$x[n] \to$ -2 0 1
$h_1 \to$ 1 0 1

-2 0 1
0 0 0
-2 0 1

$xh_1[n] \to$ -2 0 -1 0 1

$xh_1 \rightarrow$	-2	0	-1	0	1		
$h_2 \rightarrow$	3	4					
	-6	0	-3	0	3		
		-8	0	-4	0	4	
$xh_{12} \rightarrow$	-6	-8	-3	-4	3	4	
$xh_{13} \rightarrow$	-2	-4	-1	-2	1	2	
$y[n] \rightarrow$	-8	-12	-4	-6	4	6	: note this is the long process

$xh_1 \rightarrow$	-2	0	-1	0	1		
	1	2					
	-2	0	-1	0	1		
		-4	0	-2	0	2	
$xh_{13} \rightarrow$	-2	-4	-1	-2	1	2	

Short Process

$x[n] \rightarrow$	-2	0	1			
$h_1 \rightarrow$	1	0	1			
	-2	0	1			
		0	0	0		
			-2	0	1	
$xh_1 \rightarrow$	-2	0	-1	0	1	

$h_2 \rightarrow$	3	4
$h_3 \rightarrow$	1	2
$h_{23} \rightarrow$	4	6

$xh_1 \rightarrow$	-2	0	-1	0	1	
$h_{23} \rightarrow$	4	6				
	-8	0	-4	0	4	
		-12	0	-6	0	6
$y[n] \rightarrow$	-8	-12	-4	-6	4	6

Process in Sir

1	0	1			
4	6				
4	0	4			
	6	0	6		
4	6	4	6		
-2	0	1			
-8	-12	-8	-12		
	0	0	0	0	
		4	6	4	6
-8	-12	-4	-6	4	6