Lossy Image Compression

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***Abstract*—**

I. Introduction

Image compression is an important area of study due to the ever-increasing consumption of media and explosion of big data image processing. The goal of lossy image compression is to preserve the perceptually relevant portions of an image so that the image appears unaltered while drastically reducing the number of bits required to store the image.

In this paper, I will discuss several methods of image compression and evaluate their performance on metrics of image quality and compression ratio.

II. Background

Humans perceive the different spatial frequencies in images with varying degrees of sensitivity. State of the art lossy image compression systems (e.g. JPEG and JPEG2000 from the Joint Photographic Experts Group) leverage this to dramatically shrink the number of bits required to encode an image while maintaining a strong resemblance of the original image. In order to do this, the original image is transformed into a frequency representation. The two most popular approaches for this transformation are the discrete cosine transform (DCT) and discrete wavelet transform (DWT).

A. Discrete Transforms in 1D and 2D

A frequency transform in general can be viewed as a series of inner products between the input function and a set of basis vectors. Often, these basis vectors are chosen to be orthogonal (such as in the DCT and many variants of the DWT). A one-dimensional transform of an input vector $\mathbf{x}$ can be represented as:

$$\widetilde{\mathbf{x}} = \mathbf{\Omega}\mathbf{x} \tag{1}$$

$$\mathbf{\Omega} = \begin{pmatrix} — & \omega_1^T & — \\ & \vdots & \\ — & \omega_N^T & — \end{pmatrix} \tag{2}$$

Where ω_k are the basis vectors of the transform. For a discrete fourier transform, these are simply complex exponentials, with different frequencies for different basis vectors:

$$\omega_k = \left(1, e^{-\frac{2i\pi k}{N}}, e^{-\frac{4i\pi k}{N}}, \ldots e^{\frac{2(N-1)\pi k}{N}}\right)^T \tag{3}$$

Intuitively, a discrete transform of 2D data (e.g. a matrix) is performed by first taking a discrete transform of each individual column:

$$\widetilde{\mathbf{X}}_c = \begin{pmatrix} — & \omega_1^T & — \\ & \vdots & \\ — & \omega_N^T & — \end{pmatrix} \begin{pmatrix} | & & | \\ \mathbf{x}_1 & \ldots & \mathbf{x}_N \\ | & & | \end{pmatrix} \tag{4}$$

Then taking a discrete transform of each row of the partially transformed matrix:

$$\widetilde{\mathbf{X}}^T = \mathbf{\Omega}\widetilde{\mathbf{X}}_c^T \tag{5}$$

$$\widetilde{\mathbf{X}} = \mathbf{\Omega}\mathbf{X}_c\mathbf{\Omega}^T \tag{6}$$

B. Discrete Cosine Transform

For a discrete cosine transform, the basis vectors are given by

$$\omega_k = \left(\cos\frac{\pi k}{2N}, \cos\frac{3\pi k}{2N}, \ldots \cos\frac{(2N-1)\pi k}{2N}\right)^T$$

C. Discrete Wavelet Transform

D. Singular Value Decomposition

E. Lossless Compression - Entropy Coding

III. Methods

A. Proposed Image Compression Techniques

B. Quantifying Image Quality

C. Estimating Compression Ratio

IV. Results

V. Discussion

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