Heat Equation

→ Solutions will describe possible ways for heat to flow (on a 1-D wire)

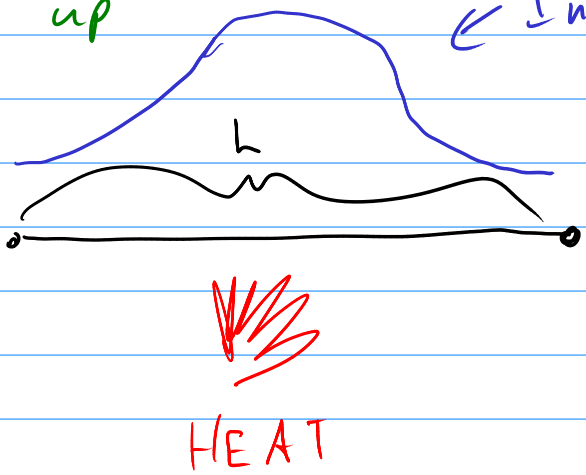
→ Any diffusion Process

→ Scent Diffusion

→ Smoke

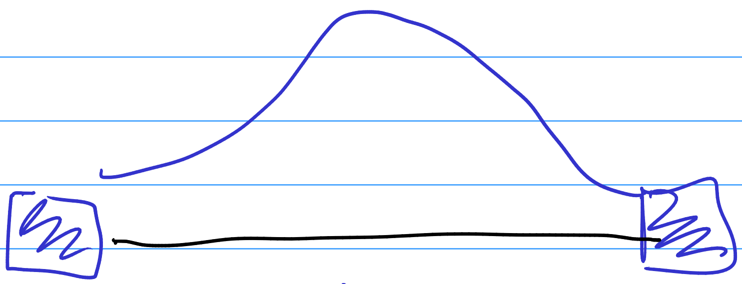
Physical Set up

← Initial temp



→ Start A timer, & Apply ICE TO EDGES & Remove heat Souce

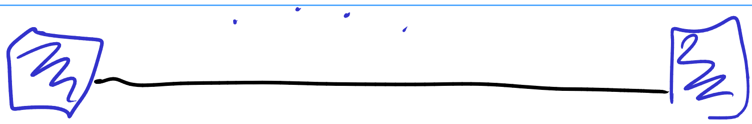
$t=0$



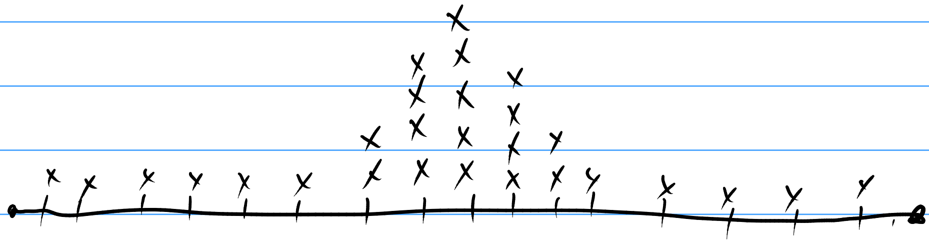
$t=1$

????

what will temp dist be

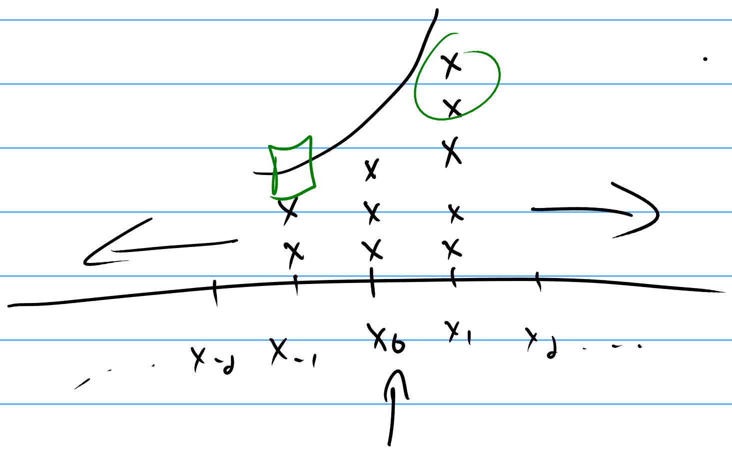


Discrete MODEL:



- $\Delta t \to$ Each heat packet "x"
  Equal probability of moving left, Right or Staying.

SCENARIO 1



After Δt Seconds will heat @ x_0

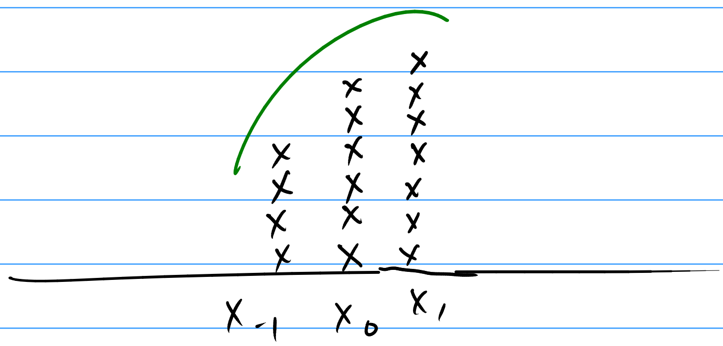
→ Increase ✓

→ Decrease

→ Stay the Same

on average.

SCENARIO 2



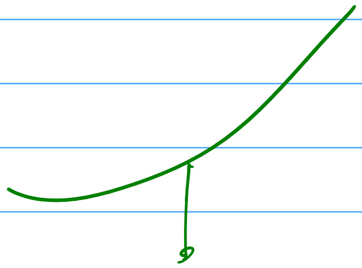
???

→ Increase

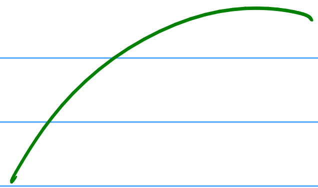
→ Decrease ✓

→ Stay the Same

In Summary



$\Rightarrow$ Increase



$\rightarrow$ Decrease

Concave up $\rightarrow$ Increase

Concave down $\rightarrow$ Decrease

Let $u(x,t)$ = temp @ time t & position x

i.e. $u(x,0) \rightarrow$

Scenarios Indicate

$\partial_t u(x,t)$ proportional to $\partial_x^2 u(x,t)$

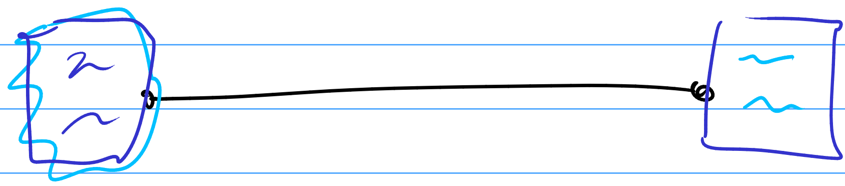
$\boxed{\partial_t u = c^2 \partial_x^2 u}$ ← Heat Equation

(*)

$\partial_t u - c^2 \partial_x^2 u = 0, \quad x \in [0,L], t > 0$ ← Heat eqn

$u(x,0) = f(x), \quad x \in [0,L]$ ← Initial temp.

$\underbrace{u(0,t) = u(L,t) = 0,}_{\text{Ice @ Bndy}} \quad t > 0$ ← Boundary Conditions



$\boxed{u(0,t) = u(L,t) = 0}$ → called "Dirichlet Boundary Conditions"

$u: [0,L] \times [0,\infty) \to \mathbb{R}$

(*) Set of Constraints on u

A "Solution" satisfies all constraints

"x" Notation:

$f: \text{Colors} \times \text{fruits} \to \mathbb{R}$

$f(\text{Red}, \text{banana}) = 7.2$

$f(\text{blue}, \text{apple}) = \pi$

So

$u: [0,L] \times [0,\infty) \to \mathbb{R}$

$u(x,t)$ is a Real #

$x \in [0,L]$ (i.e. position)

$t \in [0,\infty)$ (i.e. time)