# Homework 3 Solutions

## 1 Complex Diagonalization

Diagonalize each of the following matrices

A. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

- $\begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix} \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix}^{-1}$

B. $\begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix}$

- $\begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 3+2i & 0 \\ 0 & 3-2i \end{pmatrix} \begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix}^{-1}$

C. $\begin{pmatrix} 7 & -5 \\ 13 & -9 \end{pmatrix}$

- $\begin{pmatrix} 8+i & 8-i \\ 13 & 13 \end{pmatrix} \begin{pmatrix} -1+i & 0 \\ 0 & -1-i \end{pmatrix} \begin{pmatrix} 8+i & 8-i \\ 13 & 13 \end{pmatrix}^{-1}$

## 2 Complex General Solutions

For each of the following ODEs, find the (complex) general solution.

A. $x' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$

- $c_1 e^{it} \begin{pmatrix} i \\ 1 \end{pmatrix} + c_2 e^{-it} \begin{pmatrix} -i \\ 1 \end{pmatrix}$

B. $x' = \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} x$

- $c_1 e^{(3+2i)t} \begin{pmatrix} i \\ 1 \end{pmatrix} + c_2 e^{(3-2i)t} \begin{pmatrix} -i \\ 1 \end{pmatrix}$

C. $x' = \begin{pmatrix} 7 & -5 \\ 13 & -9 \end{pmatrix} x$

- $c_1 e^{(-1+i)t} \begin{pmatrix} 8+i \\ 13 \end{pmatrix} + c_2 e^{(-1-i)t} \begin{pmatrix} 8-i \\ 13 \end{pmatrix}$

## 3 Real General Solutions

For each of the following ODEs, find the real general solution.

A. $x' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$

- $c_1 \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} + c_2 \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

B. $x' = \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} x$

- $c_1 e^{3t} \begin{pmatrix} -\sin 2t \\ \cos 2t \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix}$

C. $x' = \begin{pmatrix} 7 & -5 \\ 13 & -9 \end{pmatrix} x$

- $c_1 e^{-t} \begin{pmatrix} 8\cos t - \sin t \\ 13\cos t \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} \cos t + 8\sin t \\ 13\sin t \end{pmatrix}$

## 4 Initial Value Problems

Find the solution to each of the following IVPs

A. $x' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$, $x(0) = (1,2)$

- Same as question 3 but $c_1 = 2$ and $c_2 = 1$.

B. $x' = \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} x$, $x(0) = (1,-1)$

- Same as question 3 but $c_1 = -1$ and $c_2 = 1$

C. $x' = \begin{pmatrix} 7 & -5 \\ 13 & -9 \end{pmatrix} x$, $x(0) = (3, 4)$

- Same as question 3 but $c_1 = 4/13$ and $c_2 = 7/13$.

# 5 Limiting Behavior

For each of the following IVPs, compute the limit $\lim_{t \to \infty} |x(t)|$, for the solution $x$.

A. $x' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$, $x(0) = (1, 2)$

- $\sqrt{5}$

B. $x' = \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} x$, $x(0) = (1, -1)$

- $\infty$

C. $x' = \begin{pmatrix} 7 & -5 \\ 13 & -9 \end{pmatrix} x$, $x(0) = (3, 4)$

- $0$

# 6 Direction Fields

Draw the direciton field (approximately) for each of the following ODEs

A. $x' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$

- counterclockwise circles (periodic)

B. $x' = \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} x$

- counterclockwise outward spirals

C. $x' = \begin{pmatrix} 7 & -5 \\ 13 & -9 \end{pmatrix} x$

- counterclockwise inward spirals