Homework 2

Due: Monday, April 19, at 11:00 PM Seattle time on Gradescope

1 Complex Diagonalization

Diagonalize each of the following matrices

A.
$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

B.
$$\begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix}$$

C.
$$\begin{pmatrix} 7 & -5 \\ 13 & -9 \end{pmatrix}$$

2 Complex General Solutions

For each of the following ODEs, find the (complex) general solution.

A.
$$x' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$$

B.
$$x' = \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} x$$

$$C. \ x' = \begin{pmatrix} 7 & -5 \\ 13 & -9 \end{pmatrix} x$$

3 Real General Solutions

For each of the following ODEs, find the real general solution.

A.
$$x' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$$

B.
$$x' = \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} x$$

$$C. \ x' = \begin{pmatrix} 7 & -5 \\ 13 & -9 \end{pmatrix} x$$

4 Initial Value Problems

Find the solution to each of the following IVPs

A.
$$x' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$$
, $x(0) = (1, 2)$

B.
$$x' = \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} x$$
, $x(0) = (1, -1)$

C.
$$x' = \begin{pmatrix} 7 & -5 \\ 13 & -9 \end{pmatrix} x$$
, $x(0) = (3,4)$

5 Limiting Behavior

For each of the following IVPs, compute the limit $\lim_{t\to\infty} |x(t)|$, for the solution x.

A.
$$x' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$$
, $x(0) = (1, 2)$

B.
$$x' = \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} x$$
, $x(0) = (1, -1)$

C.
$$x' = \begin{pmatrix} 7 & -5 \\ 13 & -9 \end{pmatrix} x$$
, $x(0) = (3,4)$

6 Direction Fields

Draw the direction field (approximately) for each of the following ODEs

A.
$$x' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$$

B.
$$x' = \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} x$$

$$C. \ x' = \begin{pmatrix} 7 & -5 \\ 13 & -9 \end{pmatrix} x$$