Consider

$$x' = \overbrace{\begin{pmatrix} -1 & 5 \\ 5 & -1 \end{pmatrix}}^{A} x$$

$\Rightarrow$ Diagonalize

$$\begin{pmatrix} -1 & 5 \\ 5 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & -6 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1}$$

$\Rightarrow$ General Solution is

$$x(t) = C_1 e^{4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-6t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

If $x(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \Rightarrow c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$\Rightarrow C_1 = C_2 = 1$

Then specific Soln. is

$$x(t) = 1 e^{4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 1 e^{-6t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Then $\lim_{t \to \infty} |x(t)|$

If $x(t)$ = position of a particle (w/mass)

Then ~~Can't Actually work~~

$$\boxed{\lim_{t \to \infty} |x'(t)| = \infty}$$

eventually speed > speed of Light

If $C_1 \neq 0$, Let $x(t) = C_1 e^{4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{-6t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Then $|x(t)| \to \infty$ as $t \to \infty$

And $|x'(t)| \to \infty$ as $t \to \infty$

If
$x(0) = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

then $x(t) = 2e^{-6t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\lim_{t\to\infty} |x(t)| = 0$

$\lim_{t\to\infty} |x'(t)| = 0$

If $x(0) = \begin{pmatrix} 2.0001 \\ -2 \end{pmatrix}$

$c_2 \approx 2$

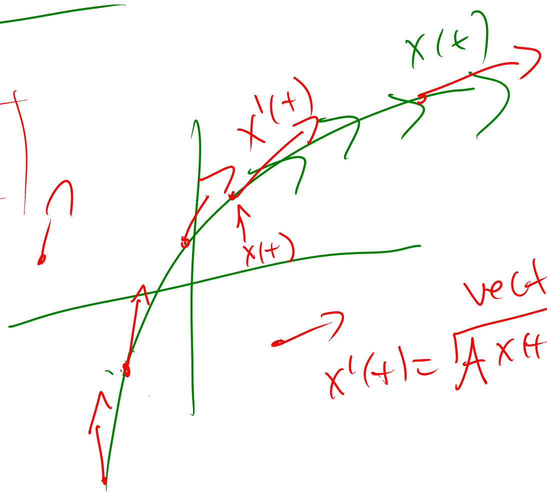
$c_1 \approx 0$

But $c_1 \neq 0$

So lims $\to \infty$

Goal: understand ↑↑ visually.

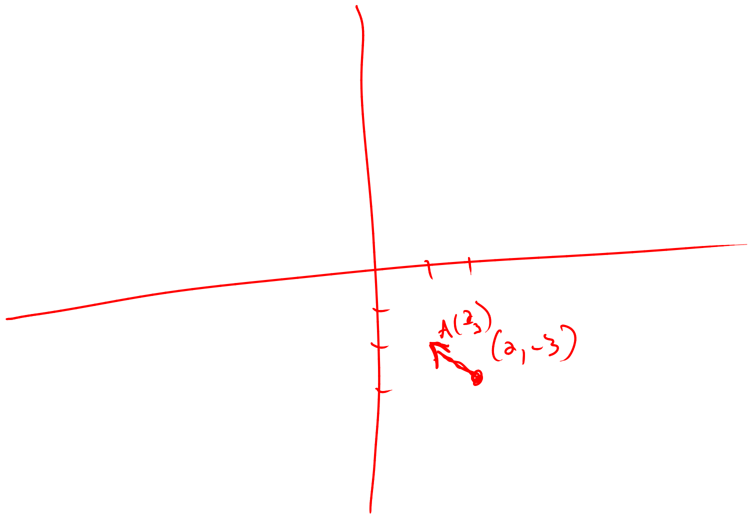
$\boxed{x' = Ax}$



$x'(t) = \boxed{Ax(t)}$ vector obtained by applying "A" to the position $x(t)$.

If tangent to $x(t)$ = A applied to position of $x(t)$ then curve for a solution.

IDEA: Draw Ax for a bunch of "x" And solutions "follow the Arrows"



$$A\begin{pmatrix}2\\-3\end{pmatrix} = \begin{pmatrix}-1 & 5\\ 5 & -1\end{pmatrix}\begin{pmatrix}2\\-3\end{pmatrix} = \begin{pmatrix}-17\\13\end{pmatrix}$$

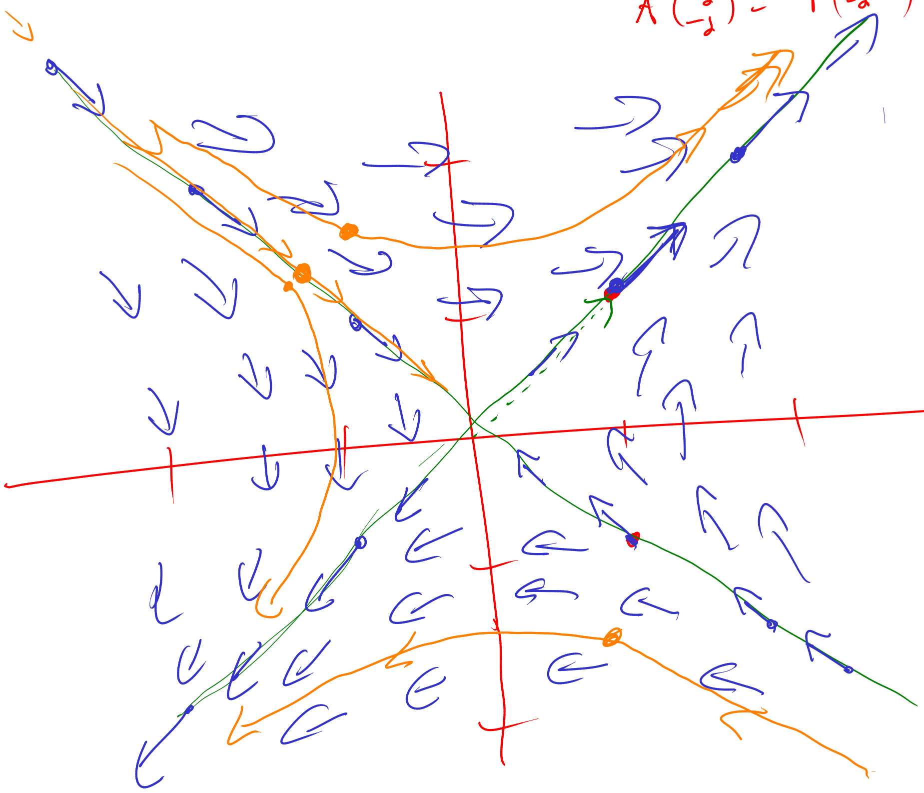
Do this at a bunch of points = "Direction field"

WANT TO Be Systematic [Not just picking Random Points].

$$A = \begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}4 & 0\\ 0 & -6\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}^{-1}$$

$$A\begin{pmatrix}1\\1\end{pmatrix} = 4\begin{pmatrix}1\\1\end{pmatrix}$$
$$A\begin{pmatrix}2\\2\end{pmatrix} = 4\begin{pmatrix}2\\2\end{pmatrix}$$
$$A\begin{pmatrix}-2\\-2\end{pmatrix} = 4\begin{pmatrix}-2\\-2\end{pmatrix}$$

$$A\begin{pmatrix}1\\-1\end{pmatrix} = -6\begin{pmatrix}1\\-1\end{pmatrix}$$



Blue = Dir. field
orange = Solutions
Green = lines w/ e-vects.

To Draw A Dir. Field

1. Diagonalize A

2. Draw lines containing e-vects

3. Depending on <u>sign</u> of eigenvect (i.e. $\pm$?) vects go in/out on lines

4. Interpolate