Gravitational Wave Data Analysis

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24 Sep 2021: Conditional Probabilities, Bayes Theorem, and Stationary Gaussian Noise

Speaker: Reed Essick

Learning Objectives

- Construct a joint distribution over signal parameters, noise realizations, and observed data
- Marginalize over (unobserved) noise to obtain the likelihood
- Use Bayes Theorem to construct a posterior from the likelihood, prior, and evidence
- Define stationary Gaussian noise
  - Define the Power Spectral Density in terms of the noise autocorrelation function
- Write the standard GW likelihood for an arbitrary detector network and arbitrary signal model

Lecture Notes

Further Reading

- Gaussian Processes
  - Rasmussen and Williams, Gaussian Processes for Machine Learning, MIT Press (2006)
- Frequency-domain Likelihoods
  - Veitch *et al*, Parameter Estimation for Compact Binaries with Ground-Based Gravitational-Wave Observations Using the LALInference Software Library, PRD **91**, 042003 (2015)
- Time-domain Likelihoods
  - Essick *et al*. iDQ: Statistical Inference of Non-Gaussian Noise with Auxiliary Degrees of Freedom in Gravitational-Wave Detectors, Mach. Learn.: Sci. Technol. **2**, 015004 (2020)
  - Isi *et al*, Testing the Black-Hole Area Law with GW150914, PRL **127**, 011103 (2021)
- Detector Response
  - Essick *et al*, Frequency-dependent Responses in Third Generation Gravitational-Wave Detectors, PRD **96**, 084004 (2017)

1 Oct 2021: Parameter Estimation and Information Theory

Speaker: Reed Essick

Learning Objectives

- Review how the posterior is constructed from Bayes Theorem
- Interpret the posterior
  - median vs. mean vs. mode
  - Symmetric vs. HPD credible regions
  - credible region coverage
- Approximations to the posterior
  - Fisher Information Matrix
- Sampling techniques
  - Rejection sampling
  - Inverse transform sampling
  - Gibbs sampler
  - Metropolis-Hastings algorithm
  - Monte Carlo summation and Importance Sampling

Lecture Notes

Further Reading

- Feldman and Cousins, Unified Approach to the Classical Statistical Analysis of Small Signals, PRD **57**, 3873 (1998)
- Cramer-Rao Bound
- Fisher Efficient Estimators (frequentist point-estimators)
- emcee documentation (open-source MCMC ensemble sampler)
- ...

Possible Projects

- Skymap phenomenology
  - determining the component masses from only the posterior over the sky position
- Analytic marginalization over calibration uncertainty
  - Show what types of calibration uncertainty impact different GR parameters
  - Explicitly show how calibration uncertainty limits our inference of parameters in the high SNR limit (very relevant for 3G detectors)

8 Oct 2021: Searches and Likelihood Ratio Tests

Speaker: Reed Essick

Learning Objectives

Basics of decision theory → how to build an optimal search for any type of signal model
- Short-duration signals (templated and unmodeled)
- Continuous signals (templated)
- Stochastic signals (assume Gaussian signal vs popcorn noise)

Neyman-Pearson lemma
Sufficient Statistics
Marginalization vs Maximization

Bayes factors vs Odds ratios → further reading about Ocaam factors
Computational methods (Savage-Dickey Density Ratios, thermodynamic integration (more general power sampling), nested sampling)

Lecture Notes

Further Reading

- Neyman-Pearson Lemma
- Discriminating between NSs and BHs with Imperfect Knowledge of the Maximum Neutron Star Mass
    - Discussion of relative merits of Bayes Factors vs Odds Ratios (impact of prior odds)
- Bayes Factors as detection statistics
    - Leveraging waveform complexity for confident detection of GWs
    - An Information-theoretic approach to the GW burst detection problem
    - Enhancing confidence in the detection of gravitational waves from compact binaries using signal coherence
- Bayes Factor Computation
    - Kass+Raftery
    - Lartillot+Philippe (thermodynamic integration)

Possible Projects

- Probability distributions of bayes factors under signal and noise hypotheses
- Probabilistic Data Quality in a (simplified?) BBH search and/or one way to search for signals without perfect templates
- Incoherent coincidence of poisson distributed events

15 Oct 2021: Bayesian Model Selection between Signal Models

Speaker: Reed Essick

Learning Objectives

How to test for the presence/absence of effects
Hierarchical models for population inference

- formation channels and population models (Aman requested this)

~~Scaling of combined results with number of detections, etc~~

Lecture Notes

Further Reading

- Kass and Raftery, Bayes Factors
- Wagenmakers et al, Bayesian Hypothesis Testing for Psychologists: A Tutorial on the Savage Dickey Method
- Isi et al, Hierarchical Tests of General Relativity with Gravitational Waves
- Zimmerman et al, On Combining Information from Multiple Gravitational Wave Sources
- LVK O3a Tests of GR paper
- LVK O3a Rates & Populations paper
- Reviews on hierarchical bayesian inference
    - Mandel+ (2016)
    - Thrane+Talbot (2019)
    - Vitale+ (2020)
- ...

Possible Projects

- Monte Carlo measures of search sensitivity
- “Theory informed priors” for deviations from GR

Bibliography

Textbooks

- Creighton and Anderson, *Gravitational-Wave Physics and Astronomy: An Introduction to Theory, Experiment and Data Analysis* (2011) (Free Download)
- Saulson, *Interferometric Gravitational Wave Detectors* (2017)
- Maggiore, *Gravitational Wave: Volume 1: Theory and Experiment*
- Maggiore, *Gravitational Waves: Volume 2: Astrophysics and Cosmology*

Gravitational Wave Sources

- …

Gravitational Wave Searches

- Allen *et al*, FINDCHIRP: an algorithm for detection of gravitational waves from inspiraling compact binaries, PRD **85**, 122006 (2012)
- PSD Estimation
  - Littenberg and Cornish, Bayesian inference for spectral estimation of gravitational wave detector noise, PRD **91**, 084034 (2015)
  - Chatziioannou *et al*, Noise spectral estimation methods and their impact on gravitational wave measurement of compact binary mergers, PRD **100**, 104004 (2019)
- Frequency-domain Likelihoods
  - Veitch *et al*, Parameter Estimation for Compact Binaries with Ground-Based Gravitational-Wave Observations Using the LALInference Software Library, PRD **91**, 042003 (2015)
- Time-domain Likelihoods
  - Essick *et al*. iDQ: Statistical Inference of Non-Gaussian Noise with Auxiliary Degrees of Freedom in Gravitational-Wave Detectors, Mach. Learn.: Sci. Technol. **2**, 015004 (2020)
  - Isi *et al*, Testing the Black-Hole Area Law with GW150914, PRL **127**, 011103 (2021)
- ...

Gravitational Wave Detectors

- Essick *et al*, Frequency-dependent Responses in Third Generation Gravitational-Wave Detectors, PRD **96**, 084004 (2017)
- ...

Probability/Information Theory

- Jaynes, Probability Theory: The Logic of Science (1995)
- Feldman and Cousins, Unified Approach to the Classical Statistical Analysis of Small Signals, PRD **57**, 3873 (1998)
- Gaussian Processes (common noise model)
  - Rasmussen and Williams, Gaussian Processes for Machine Learning, MIT Press (2006)
- Samplers
  - emcee documentation
- …

Tutorials / useful code

- undergrad reading series at UChicago: github
- GW Open Science Center: https://www.gw-openscience.org/path/ (click on "Researchers")
- MIT Open Course Ware
  - Signals, Systems, and Inference
  - Introduction to Communication, Control, and Signal Processing
  - Identification, Estimation, and Learning
  - Information Theory