Joshua Reed

Fall, 2017

Homework 1

EE 520 - Random Processes

Problems: 2.3, 2.7, 2.9, 2.10, 2.19, 2.22, 2.28, 2.30, 2.31, and 2.38

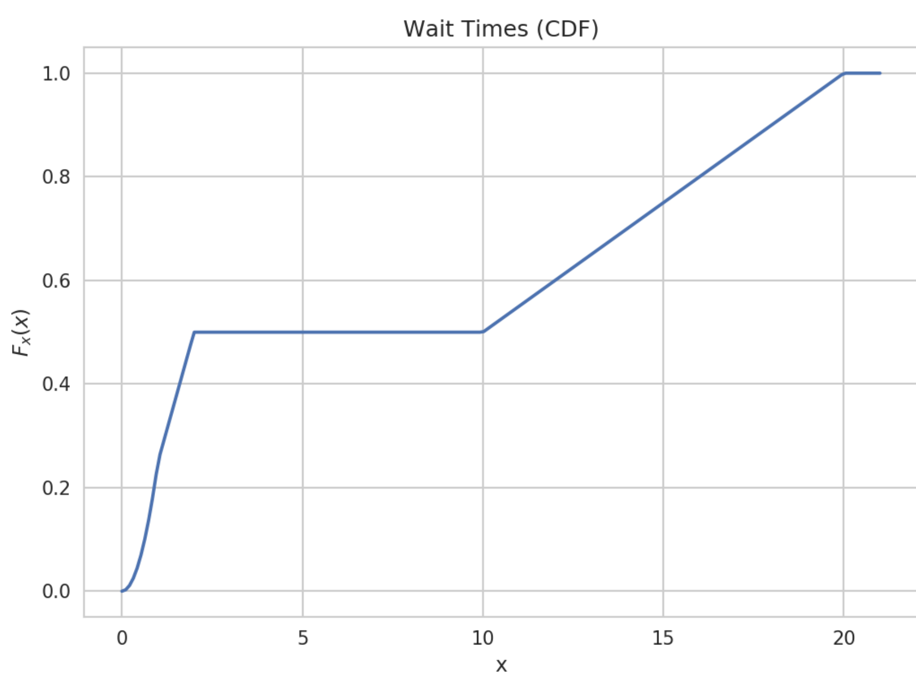
2.3

Exercise

In a restaurant known for its unusual service, the time X, in minutes, that a customer has to wait before he captures the attention of a waiter is specified by the following.

$$F_X(x) = \begin{cases} (\frac{x}{2})^2, & \text{for } 0 \le x \le 1, \\ \frac{x}{4}, & \text{for } 1 \le x \le 2, \\ \frac{1}{2}, & \text{for } 2 \le x \le 10, \\ \frac{x}{20}, & \text{for } 10 \le x \le 20, \\ 1, & \text{for } x \ge 20, \end{cases}$$

Part A



Sketch $F_x(x)$.

Part B
Compute and sketch the PDF $f_x(x)$

The PDF is the differential of the CDF, so the derivative of $F_x(X)$ will be the PDF $f_x(x)$.

For this problem, I am assuming that $F_x(x)$ is piecewise differentiable.

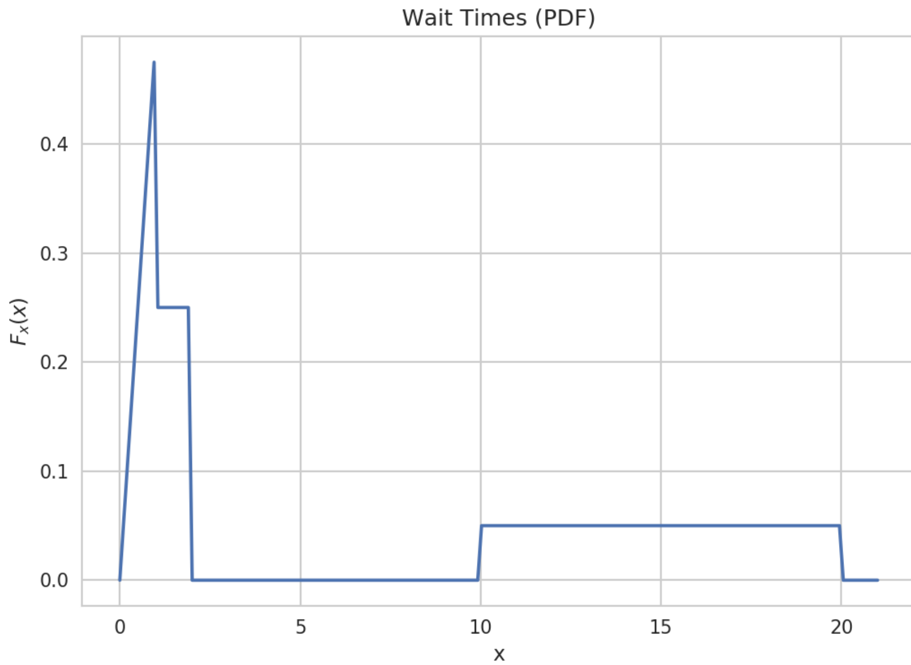
$$\begin{aligned}\frac{d}{dx}((\frac{x}{2})^2) &= \frac{d}{dx}(\frac{x^2}{4}) \\ &= 2\frac{x}{4} \\ &= \frac{x}{2}\end{aligned}$$

$$\frac{d}{dx}(\frac{x}{4}) = \frac{1}{4}$$

$$\frac{d}{dx}(\frac{1}{2}) = 0$$

$$\frac{d}{dx}(\frac{x}{20}) = \frac{1}{20}$$

$$F_X(x) = \begin{cases} \frac{x}{2}, & \text{for } 0 \leq x \leq 1, \\ \frac{1}{4}, & \text{for } 1 \leq x \leq 2, \\ 0, & \text{for } 2 \leq x \leq 10, \\ \frac{1}{20}, & \text{for } 10 \leq x \leq 20, \\ 0, & \text{for } x \geq 20, \end{cases}$$



Part C
What is the probability that the customer will have to wait (1) at least 10 minutes, (2) less than 5 minutes, (3) between 5 and 10 minutes, (4) exactly 1 minute?

1. $P(X \geq 10) = P(x \leq \infty) - P(x \leq 10) = F_x(20) - F_x(10) = 1 - 1/2 = 1/2$
2. $P(X \leq 5) = F_x(5) = 1/2$
3. $P(5 \leq x \leq 10) = P(x \leq 10) - P(x \leq 5) = 1/2 - 1/2 = 0$
4. $P(x = 1) = f_x(1) =$ either 1/2, 1/4, or more plausably 0.

Because the likelihood of being served at **exactly** 1 minute is zero.

2.7

Exercise

A noisy resistor produces a voltage $v_n(t)$. At $t = t_1$, the noise level $X \triangleq v_n(t_1)$ is known to be a Gaussian RV with PDF

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(\frac{x}{\sigma})^2}$$

Compute and plot the probability that $|X| > k\sigma$ for $k = 1, 2, \ldots$

2.9

Exercise

Write PDFs using the δ functions for the Bernoulli binomial, and Poisson PMF's.

Solution

Bernoulli PMF

With probability of success p.

$$f_x(x) = \begin{cases} p^x(1-p)^{1-x}, & \text{for } x = 0, 1 \\ 0, & \text{for all others} \end{cases}$$

2.10

Exercise

T pdf of a RV X is shown in figure P2.10. The numbers in the parenthesis indicate area.

Part A

Compute the value of A.

Part B

Sketch the CDF.

Part C

Compute $P[2 \leq X < 3]$.

Part D

Compute $P[2 < X \leq 3]$

Part E

Compute $F_x(3)$.

2.19

Exercise

It has been found that the number of people Y waiting in a queue in the bank on payday obeys the Poisson law as

$$P[Y = k|X = x] = e^{-x}\frac{x^k}{k!}, k \geq 0, x > 0$$

given that the normalized serviing time of the teller x is constant. However, the serving time is more accurately modeled as an RV X. For simplicity let X be a uniform RV with

$$f_X(x) = \frac{1}{5}[u(x) - u(x-5)].$$

Then $P[Y = k|X = x]$ is still Poisson but $P[Y = k]$ is something else. Compute $P[Y = k]$ for $k = 1, 2$, and 2. The answer for general k may be difficult.

2.22

Exercise

Consider the joint PDF of X and Y:

$$f_{xy}(x, y) = \frac{1}{3\pi} e^{-\frac{1}{2}[(x/3)^2+(y/2)^2]} u(x)u(y)$$

Are X and Y independent RVs? Compute the probability of $\{0 < X \leq 3, 0 < Y \leq 2\}$.