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MTH 361 Introduction to Probability

Homework 6

1.

Let X be a continuous random variable with probability density function given by

$$f_X(x) = \begin{cases} c(1-x^2) & -1 < x < 1 \\ 0 & others \end{cases}$$

(a) Find the value of c that makes $f_X(x)$ a pdf. Here, $\int_{-\infty}^{\infty} f_X(x)dx$ must be 1.

$$\begin{aligned} 1 &= \int_{-\infty}^{\infty} f_X(x)dx \\ &= \int_{-\infty}^{\infty} c(1-x^2)[u(x+1) - u(x-1)]dx \\ &= \int_{-1}^{1} c(1-x^2)dx \\ &= c[\int_{-1}^{1} 1dx - \int_{-1}^{1} x^2dx] \\ &= c[x\Big|_{-1}^{1} - \frac{x^3}{3}\Big|_{-1}^{1}] \\ &= c[2 - (\frac{(1)^3}{3} - \frac{(-1)^3}{3})] \\ &= c\frac{4}{3} \\ c &= \frac{3}{4} \end{aligned}$$

(b) What is the CDF of X?

$$
\begin{aligned}
F_X(x) &= \int_{-\infty}^{x} (3/4)(1-x^2)[u(x+1)-u(x-1)]dx \\
&= \int_{-1}^{x} (3/4)(1-x^2)dx, \quad \text{for } x \leq 1 \\
&= \frac{3}{4}[x - \frac{x^3}{3}]\big|_{-1}^{x}, \quad \text{for } x \leq 1 \\
&= \frac{3}{4}[x - \frac{x^3}{3} - [-1 - \frac{-1^3}{3}]] \\
&= \frac{3}{4}[x - \frac{x^3}{3} - [-\frac{2}{3}]] \\
&= \frac{3}{4}[x - \frac{x^3}{3} + -\frac{2}{3}]
\end{aligned}
$$

(c) $P[X \geq \frac{1}{2}]$

$$
\begin{aligned}
F_X(1/2) &= \frac{3}{4}[\frac{1}{2} - \frac{(\frac{1}{2})^3}{3} + -\frac{2}{3}] \\
&= \frac{3}{4}[\frac{1}{2} - \frac{\frac{1}{8}}{3} + \frac{2}{3}] \\
&= \frac{3}{4}[\frac{1}{2} - \frac{1}{24} + \frac{2}{3}] \\
&= 0.84375
\end{aligned}
$$

2.