2.3 In a restaurant known for its unusual service, the time X, in minutes, that a customer has to wait before he captures the attention of a waiter is specified by the following CDF:

$$F_X(x) = \begin{cases} \left(\dfrac{x}{2}\right)^2, & \text{for } 0 \le x \le 1, \\ \dfrac{x}{4}, & \text{for } 1 \le x \le 2, \\ \dfrac{1}{2}, & \text{for } 2 \le x \le 10, \\ \dfrac{x}{20}, & \text{for } 10 \le x \le 20, \\ 1, & \text{for } x \ge 20. \end{cases}$$

(a) Sketch $F_X(x)$. (b) Compute and sketch the pdf $f_X(x)$. Verify that the area under the pdf is indeed unity. (c) What is the probability that the customer will have to wait (1) at least 10 minutes, (2) less than 5 minutes, (3) between 5 and 10 minutes, (4) exactly 1 minute?

2.7 A noisy resistor produces a voltage $v_n(t)$. At $t = t_1$, the noise level $X \triangleq v_n(t_1)$ is known to be a Gaussian RV with pdf

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right].$$

Compute and plot the probability that $|X| > k\sigma$ for $k = 1, 2, \ldots$.

2.9 Write the *probability density functions* (using delta functions) for the Bernoulli, binomial, and Poisson PMF's.

2.10 The pdf of a RV X is shown in Figure P2.10. The numbers in parentheses indicate area. (a) Compute the value of A; (b) sketch the CDF; (c) compute $P[2 \le X < 3]$;

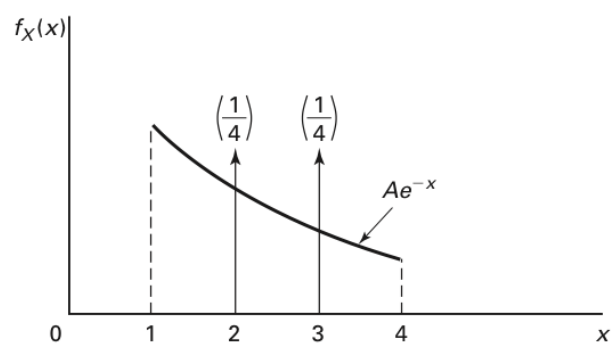

Figure P2.10 pdf of a Mixed RV.

(d) compute $P[2 < X \le 3]$; (e) compute $F_X(3)$.

2.19 It has been found that the number of people Y waiting in a queue in the bank on payday obeys the Poisson law as

$$P[Y = k|X = x] = e^{-x}\frac{x^k}{k!}, \quad k \geq 0, x > 0$$

given that the normalized serving time of the teller x (i.e., the time it takes the teller to deal with a customer) is constant. However, the serving time is more accurately modeled as an RV X. For simplicity let X be a uniform RV with

$$f_X(x) = \tfrac{1}{5}[u(x) - u(x-5)].$$

Then $P[Y = k|X = x]$ is still Poisson but $P[Y = k]$ is something else. Compute $P[Y = k]$ for $k = 0, 1$, and 2. The answer for general k may be difficult.

2.22 Consider the joint pdf of X and Y:

$$f_{XY}(x, y) = \frac{1}{3\pi}e^{-\frac{1}{2}[(x/3)^2+(y/2)^2]}u(x)u(y).$$

Are X and Y independent RVs? Compute the probability of $\{0 < X \leq 3, 0 < Y \leq 2\}$.

2.28 Let X be a random variable with *pdf*

$$f_X(x) = \begin{cases} 0, & x < 0, \\ ce^{-2x}, & x \geq 0, \end{cases} \qquad (c > 0).$$

(a) Find c;
(b) Let $a > 0$, $x > 0$, find $P[X \geq x + a]$;
(c) Let $a > 0$, $x > 0$, find $P[X \geq x + a|X \geq a]$.

2.30 A U.S. defense radar scans the skies for unidentified flying objects (UFOs). Let M be the event that a UFO is present and M^c the event that a UFO is absent. Let $f_{X/M}(x|M) = \frac{1}{\sqrt{2\pi}}\exp(-0.5[x-r]^2)$ be the conditional pdf of the radar return signal X when a UFO is actually there, and let $f_{X/M}(x/M^c) = \frac{1}{\sqrt{2\pi}}\exp(-0.5[x]^2)$ be the conditional pdf of the radar return signal X when there is no UFO. To be specific, let $r = 1$ and let the *alert level* be $x_A = 0.5$. Let A denote the event of an alert, that is, $\{X > x_A\}$. Compute $P[A|M]$, $P[A^c|M]$, $P[A|M^c]$, $P[A^c|M^c]$.

2.31 In the previous problem assume that $P[M] = 10^{-3}$. Compute

$$P[M|A], P[M|A^c], P[M^c|A], P[M^c|A^c]. \text{ Repeat for } P[M] = 10^{-6}.$$

Note: By assigning drastically different numbers to $P[M]$, this problem attempts to illustrate the difficulty of using probability in some types of problems. Because a UFO appearance is so rare (except in Roswell, New Mexico), it may be considered a *one-time event* for which accurate knowledge of the prior probability $P[M]$ is near impossible. Thus, in the surprise attack by the Japanese on Pearl Harbor in 1941, while the radar clearly indicated a massive cloud of incoming objects, the signals were ignored by the commanding officer (CO). Possibly the CO assumed that the prior probability of an attack was so small that a radar failure was more likely.

2.38 A laser used to scan the bar code on supermarket items is assumed to have a constant conditional failure rate $\lambda\,(>0)$. What is the maximum value of λ that will yield a probability of a first breakdown in 100 hours of operation less than or equal to 0.05?