Probabilty

Events

An event is a collection of outcomes of a random experiment

$S = \{$collection of all outcomes of the experiment$\}$

$\phi = \{$empty set$\}$

If $A \cap B = \phi$,
then A and B are mutually exclusive events

DeMorgan's $(A \bar{\cup} B) = (\bar{A} \cap \bar{B})$

Axioms and Properties

Axioms

I. $P(A) \geq 0$

II. $P(S) = 1$

III. If $(A \cap B) = \phi$,
then $P(A \cup B) = P(A) + P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(\bar{A}) = 1 - P(A)$

Independence

If $P\{A \cap B\} = P\{A\}P\{B\}$,
then A and B are independent

If $P(A \cap B|C) = P(A|C)P(B|C)$,
A and B are **conditionally** independent given event C

Mutually Exclusivity

If $P\{A \cap B\} = \phi$,
then A and B are M.E. And, in this case $P(A|B) = P(A)$ and $P(B|A) = P(B)$

Conditional Probability

$P(A|B) = P(A \cap B)/P(B)$ $P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$

Bayes' rule

$P(B|A) = P(A|B)P(B)/P(A)$,

PDF and CDF

PDF

The **P**robability **D**ensity **F**unction is a function that accepts an outcome and returns the probability of that outcome occuring. Written as:

$p(x)$ and $f_x(x)$

PMF and CMF

Are the discrete time versions of the PDF and CDF

CDF

The **C**umulative **D**istribution **F**unction. Commonly written as:

$P(x)$ and $F_x(x)$

Is the integral of the PDF.

$F_x(x) = \int f_x(x)dx$

Distributions

Binomial

General

X = the number of successes in n trials. This is n trials of a Bernoulli random variable.

Probability Mass Function

$P\{X = k\} = \binom{n}{k} p^k q^{n-k}$, for $k = 0, 1, 2, ..., n$, where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Mean

$m_x = np$

Variance

$Var(x) = np(1-p)$

Uniform

General

X has equal likeliness of taking any value in the interval $[a, b]$

Probability Density Function

$f_x(u) = \frac{1}{b-a}$, for $a < u < b$, and is 0 elsewhere

Cumulative Distribution Function

$$F_x(u) = \begin{cases} 0, & u < a \\ \frac{u-a}{b-a}, & a < u < b \\ 1, & b < u \end{cases}$$

Mean

$m_x = (a+b)/2$

Variance

$Var(X) = \frac{(b-a)^2}{12}$

Triangular

General

Upon adding two uniform distributions, we get the triangular density function. The function only has value over $[2a, 2b]$

Density

$$f_x(\alpha) = \begin{cases} \frac{\alpha - 2a}{(b-a)^2}, & 2a < \alpha < (a+b) \\ \frac{2b-\alpha}{(b-a)^2}, & (a+b) < \alpha < (2b) \\ 0, & otherwise \end{cases}$$

Cumulative

This was not listed in the summary, and I need to review to understand why.

Exponential

General

X is the time to arrival or time to failure, where arrival rate is λ

X can also be viewed as departure time with departure rate μ

Probability Density Function

$f_x(t) = \lambda e^{-\lambda t}$, for $t > 0$, and is 0 elsewhere.

Cumulative Distribution Function

$F_x(t) = 1 - e{-}\lambda t$, for $t \geq 0$, and is 0 elsewhere.

Mean and Variance

$m_x = \sigma_x = 1/\lambda$

Gaussian

General

The normal distribution

Probability Density Function

With mean m and standard deviation σ

$f_x(u) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(u-m)^2}{2\sigma^2}}$

Unit Gaussian (normal) $\sigma = 1$, $m = 0$

$f_x(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}$

Unit Gaussian Cumulative Distribution $\phi(u)$

$\phi(u)$ was used to compute the following

$P\{a < X < b\} = \phi(\frac{b-m}{\sigma}) - \phi(\frac{a-m}{\sigma})$

Mean and Variance

m and σ are the mean and standard deviation

σ_x^2 is the variance

Geometric

General

X is the number of trials before the first success

p is the probability of success

Mass Function

$P\{X = k\} = p(1-p)^{k-1}$, for $k = 1, 2, 3, ...$

Mean

$m_x = 1/p$

Variance

$Var(X) = (1-p)/p^2$

Poisson

General

X is the number of arrivals in a time interval t

λ is the arrival rate

Mass Function

$P\{X = k\} = \frac{(\lambda t)^k}{k!} e^{\lambda t}$, for $k = 0, 1, 2, 3, ...$

Mean and Variance

$m_x = Var(X) = \lambda t$

Moments

First

General

The first moment is the mean of the distribution. Sometimes refered to as the center of mass.

Formula

Where $p(x)$ is the probabilty of the outcome x occuring.

$\mu_x = E\{X\} = \int p(x)xdx$

And applies via a sum for the discrete case.

N$^{\text{th}}$ Moment

$E\{X^n\} = \int p(x)x^n dx$

Properties

If $Y = aX + b$,
then $m_y = am_x + b$
and $\sigma_y^2 = a^2\sigma_x^2$

Expectation

General

The expectation E of a function g of a random variable x, $E\{g(X)\}$:

$E\{g(X)\} = \int_{-\infty}^{\infty} g(u)f_x(u)du$

A sum can be substituted for the integral in the discrete case unless using impulse functions

Properties

$E\{C\} = C$

$E\{ag(X) + bh(X)\} = aE\{g(x)\} + bE\{h(X)\}$

If $g(X) \geq 0$, then $E\{g(X)\} \geq 0$

Variance

$Var(X) = E\left[(X - \mu_x)^2\right] = E[X^2] - \mu_x^2$

$Var(X) = \sigma^2 = \int (X - \mu_x)^2 f_x(x)dx$

$Var(X) = \sigma^2 = \sum_{i=1}^{n} p_i(x_i - \mu_x)^2$

Covariance

$Var(X) = Cov(X, X)$

$Cov(X, Y) = \sigma_{XY}$

$\sigma_{XY} = E[(X - \mu_x)(Y - \mu_y)]$

$\sigma_{XY} = E[XY] - E[X]E[Y]$

$cov(X, a) = 0$ $cov(X, Y) = cov(Y, X)$

$cov(X + a, Y + b) = abcov(X, y)$

$cov(X + a, Y + b) = cov(X, Y)$

Covariance for RV Vector

$\sigma_{\vec{X}\vec{Y}} = E[(\vec{X} - E[\vec{X}])(\vec{Y} - E[\vec{Y}])]$

$\sigma_{\vec{X}\vec{Y}} = E[\vec{X}\vec{Y}^T] - E[\vec{X}]E[\vec{Y}]^T$

Correlation

$corr(X, Y) = \rho_{XY}$

$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$

Processes

Poisson Process

Independend Increments
I.I.D.

Basic Maths

Series and Sequences

Geometric Sequence

A series with a constant ration between successive terms.

Ex. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...$

Often defined as using ar

Ex. $a + ar + ar^2 + ar^3 + ...$

For $r \neq 1$, the sum of the first n terms is:

$\sum_{k=0}^{n-1} ar^k = a(\frac{1-r^n}{1-r})$

And for infinite sequences:

$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$, for $|r| < 1$

Arithmetic Series

A series with a constant difference between successive terms.

Ex. $2 + 5 + 8 + 11 + ...$

Sum of an arithmetic series with n terms starting with a_1 and ending with a_2:

$\sum = \frac{n(a_1 + a_2)}{2}$

Power Series

A series of the form:

$\sum_{n=0}^{\infty} = a_n(x-c)^n$

Where often $c = 0$

$\sum_{n=0}^{\infty} = a_n(x)^n$

The power series allows generalization of multiplication, division, subtraction, and addition between like series. It is also possible to integrate or differentiate a power series.

Taylor Series

The Taylor series of $f(x)$ (a function that is infinetely differentiable at a number a) is the power series:

$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a) + ...$

$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

Logarithms

$\log_b c = k$

$b^k = c$

$\ln(xy) = \ln(x) + \ln(y)$

$\ln(x/y) = \ln(x) - \ln(y)$

$\ln(x^y) = yln(x)$

$\ln(e) = 1$

$\ln(1/x) = -\ln(x)$

Integrals

$\int x^n dx = \frac{1}{n+1}x^{n+1}, n \neq -1$

$\int \frac{1}{x}dx = \ln|x|$

$\int udv = uv - \int vdu$

$\int e^{ax}dx = \frac{1}{a}e^{ax}$

$\int xe^{ax}dx = (\frac{x}{a} - \frac{1}{a^2})e^{ax}$

Derivatives

Combinatorics

Permutations

Number of ways to order n distinct elements:

$n!$

k-Permutations of n

Ordered arrangements of a k-element subset of an n-set. $P(n, k) = \frac{n!}{(n-k)!}$

Permutations With Repitition

For a set S of size k, the number of n-tuples over S is. k^n

Combination

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Binomial Theorem

Test

$$\begin{aligned} a &= 5 \\ &= 6 \end{aligned}$$