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Homework 2
MTH 361 - Introduction to Probability
Section 2.7: 7, 25, 33, 41, 53—Section 3.5: 7, 24, 29, 35, 37

2.7 #7

Exercise

A commercial for Glade Plug-ins says that by inserting 2 of a choice of 11 scents into the device, you can make more than 50 combinations. If we exclude the boring choice of two of the same scent, how many possibilities are there?

Solution

Ways to pick the first.

There are 11 ways to pick the first item.

Ways to pick the second.

There are then 10 ways to pick the second item.

Total ordered pickings

There $11 * 10$ ways to pick two different scents; however since the order of picking is not relavent, this duplicates the number of combinations by 2.
Thus there are 55 ways to combine 2 different scents from 11.

2.7 #25

Exercise

How many ways can 5 history books, 3 math books, and 4 novels be arranged on a shelf if the books of each type must be together.

Solution

Frame the problem.

Because the books have to be together, this is how many ways can the books be arranged within their categories times how many ways can the categories be arranged?

Category Arrangement

When viewing the problem as how to place three items, placing each item reduces the number of placement choices by one.

In placing the first book, there are three choices, then two for the second, and only one remaining for the final item.

As such, there are 3! or $3 * 2 * 1 = 6$ ways to arrange the items.

Individual Book Arrangements

This is the number of arrangements within each category times themselves.

$$\text{Arrangements Within Categories} = 3!4!5!$$
$$= 17,280$$

Total Arrangements

$$\text{Total Arrangements } = 17,280 * 6$$
$$= 103,680$$

2.7 #33

Exercise

David claims to be able to distinguish brand B beer from brand H, but Alice claims that he just guesses. They set up a taste test with 10 small glasses of beer. David wins if he gets 8 or more right. WHat is the probability that he will win (a) if he's just guessing? (b) If he gets the right answer with probability 0.9?

Solution

When David is Just Guessing

Assuming when David is just guessing, he guesses correct with $P(\text{Correct Beer}) = 0.5$.

In order to win he needs to get 8 or more correct. Thus, this is $P(X \geq 8)$.

The probability of $P(X = x)$ is simply the binomial distribution.

This can be solved using $P(X \geq 8) = P(X = 8) + P(X = 9)) + P(X = 10)$

$$P(X = k) = \binom{10}{k} p^k q^{n-k}$$

$$P(X \geq 8) = \binom{10}{8}0.5^{10} + \binom{10}{9}0.5^{10} + \binom{10}{10}0.5^{10}$$

$$= 9.77 * 10^{-4}\left(\binom{10}{8} + \binom{10}{9} + \binom{10}{10}\right)$$

$$= 0.054688$$

When David Has $P(\textbf{Correct Beer}) = 0.9$.

This is the same problem, but with a much greater probability of success.
Again. . .

$$P(X = k) = \binom{10}{k}p^k q^{n-k}$$

and here, $q = 1 - p$.
At this point I simply used python to compute

$$P(X \geq 8) = 1 - F_x(7)$$

$$= 0.929809$$

2.7 #41

Exercise

In one of the New York state lottery games, a number is chosen at random between 0 and 999. Suppose you play this game 250 times. Use the Poisson approximation to estimate the probabiltiy that you will never win and compare with the exact answer.

Solution

Exact

The exact solution is the binomial with $P(X = 0)$.

$$P(X = k) = \binom{250}{0}\frac{1}{1000}^0 \frac{999}{1000}^{250}$$

$$= 0.77870338$$

Approximation

Poisson Approximation: $\lambda = pn = \frac{250}{1000} = 0.25$

$$P(X = 0) = \frac{(\lambda t)^0}{0!}e^{\lambda t}$$

$$= \frac{1}{1}e^{0.25t}$$

$$= 0.0.7788008$$

In comparison the two answers are very close. It seems the larger the n, the closer the two outcomes become.

2.7 #53

Exercise

Four people are chosen at random from 5 couples. What is the probability that two men and two women are selected?

Solution

Frame the Problem

This is the number of ways to choose 2 men from five, then 2 women from 5 divided by the total number of ways to choose 4 people from 10.

$$P(2 \text{ and } 2) = \frac{\binom{5}{2}^2}{\binom{10}{4}}$$
$$= \frac{100}{210} = \frac{10}{21}$$

3.5 #7

Exercise

Suppose 60% of the people subscribe to newspaper A, 40% to newspaper B, and 30% to both. If a person is picked at random, what is the probability that she subscribes to newspaper A?

Solution

The solution is in wording the problem as 'What is the probability of A given A union B?'

$$P(A|(A \cup B)) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$$

Using the mutual exclusion principle...

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.6 + 0.4 - 0.3$$
$$= 0.7$$

And noting that . . .

$$P(A \cap (A \cup B)) = P(A)$$

Finally

$$P(A|(A \cup B)) = \frac{P(A)}{P(A \cup B)}$$
$$= \frac{0.6}{0.7}$$

3.5 #24

Exercise

John takes the bus with probability 0.3 and the subway with probabilty 0.7. He is late 40% of the time he takes the bus, but only 20% of the time when he takes the subway. What is the probability that he is late for work?

Solution

This is just a sort of weighted probability sum.

$$P(\text{late}) = P(\text{Sub})P(\text{Late}|\text{SUB}) + P(\text{Bus})P(\text{Late}|\text{Bus})$$
$$= 0.3 * 0.4 + 0.7 * 0.2$$
$$= 0.12 + 0.14$$
$$= 0.26$$

A point of note here is that $P(\text{Sub})$ and $P(\text{Bus})$ are probably better thought of as $P(S)$ and $P(\neg S)$.
Then the total probability of being late L becomes. . .

$$P(L) = P(\neg S)P(L|\neg S) + P(S)P(L|S)$$

3.5 #29

Exercise

A student is taking a multiple-choice test in which each question has four possible answers. She knows the answers to 50% of the questions, can narrow the choices down to two 30% of the time, and does not know anything about 20% of the questions. What is the probability that she will correctly answer a question chosen at random from the test?

Solution

$$P(C) = P(C|X)P(X) + P(C|Y)P(Y) + P(C|Z)P(Z)$$
$$= 1.0(0.50) + 0.50(0.30) + 0.25(0.20)$$
$$= 0.50 + 0.15 + 0.05$$
$$= 0.70$$

3.5 #35

Exercise

The alpha fetal protein test is meant to detect spina bifida in unborn babies, a condition that affects 1 out of 1,000 children who are born. The literature on the test indicates that 5% of the time a healty baby will cause a positive reaction. We will assume that the test is positive 100% of the time when spina bifida is present. Your doctor has just told you that your alpha fetal protein test was positive. What's the chance that your baby actually has spina bifida?

Solution

$$P(\text{Pos}|\text{TestPos}) = \frac{P(\text{TestPos}|\text{Pos})P(\text{Pos})}{P(\text{TestPos})}$$
$$= \frac{(1)P(\text{Pos})}{P(\text{Pos}) + P(\text{TestPos}|\text{Neg})}$$
$$= \frac{0.001}{0.001 + 0.05}$$
$$= 0.0197$$

So, less than 2% chance that your baby actually has spina bifida.

3.5 #37

Exercise

To improve the reliability of the channel described in the last example, we repeat each digit in the message three times. What is the probability that 111 was sent given that (a) we recieved 101, (b) We received 000?
As per the previous problem, p=0.9, q=0.1. Ones are as likely as zeroes.

Solution

Received a 101

$$P(111|101) = \frac{P(101|111)P(111)}{P(101)}$$

Because the sent digits have to be either 111 or 000, the P(111)=0.5, and P(000)=0.5. From here, the probability that 111 was sent given 101 was received is. . .

$$
\begin{aligned}
P(111|101) &= \frac{P(101|111)}{P(101|000) + P(101|111)} \\
&= \frac{(0.1^1 * 0.9^2)}{0.1^3 + (0.1^1 * 0.9^2)} \\
&= \frac{0.081}{0.001 + 0.081} \\
&= 0.987805
\end{aligned}
$$