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Fall, 2017

Homework 5

EE 520 - Random Processes

Problems: 8.1, 8.22, 8.40, 8.44, and 8.51

8.21

Exercise

Given a random sequence $X[n]$ for $n \geq 0$ with conditional pdfs

$$f_X(x_n|x_{n-1}) = \alpha e^{-\alpha(x_n - X_{n-1})} u(x_n - x_{n-1}), \text{ for } n \geq 1,$$

with $u(x)$ the unit-step function and the intitial pdf $f_X(x_0) = \delta(x_0)$. Take $\alpha > 0$.

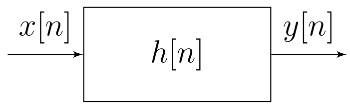
(a) Find the first-order pdf $f_X(x_n)$ for $n = 2$.

(b) Find the first-order pdf $f_X(x_n)$ for arbitrary $n > 1$ using mathematical induction.

8.22

Exercise

Let x[n] be a deterministic input to the LSI discrete-time system H shown in the figure below.



(a) Use linearity and shift-invariance properties to show that

$$y[n] = x[n] * h[n] \triangleq \sum_{k=-\infty}^{\infty} x[k|h[n-k] = h[n] * x[n].$$

(b) Define the Fourier transform of a sequence a[n] as

$$A(\omega) \triangleq \sum_{n=-\infty}^{\infty} a[n]e^{-j\omega n}, \; -\pi \leq \omega \leq \pi$$

and show that the inverse Fourier transform is

$$a[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} A(\omega)e^{+jwn} d\omega, \; -\infty < n < \infty$$

(c) Using the results in (a) and (b), show that

$$Y(\omega) = H(\omega)X(\omega)text, -\pi \leq \omega \leq \pi$$

for an LSI discrete time system.

8.40

Exercise

Consider using a first-order Markov sequence to model a random sequence $X[n]$ as

$$X[n] = rX[n-1] + Z[n],$$

where $Z[n]$ is white noise of variance σ_Z^2. Thus, we can look at X[n] as the output of passign $Z[n]$ through a lineqr system. Take $|r| < 1$ and assume the system has been running for a long time, that is, $-\infty < n < \infty$

(a) Find the psd of $X[n]$, that is, $-\infty < n < \infty$.

(b) Find the correlation function $R_{XX}[m]$.

8.44

Exercise

Given a Markov chain X[n] on $n \geq 1$, with the transistion probabilities given as $P\left[x[n]|x[n-1]\right]$, find an expression for the two-step transition probabilities $P\left[x[n]|x[n-2]\right]$. Also show that

$$P\left[x[n+1]x[n-1], x[n-2], \ldots, x[1]\right] = P\left[x[n+1]x[n-1]\right], \text{ for } n \geq 1.$$

8.51

Exercise

Let $X[n]$ be a real-valued random sequence on $n \geq 0$, made up from a stationary and *independent increments*, that is, $X[n] - X[n-1] = W[n]$, "the increment" with $W[n]$ being a stationary and independent random sequence. The random sequence always starts with $X[0] = 0$. We also know that at time $n = 1$, $E[X[1]] = \nu$ and $Var[X[1]] = \sigma^2$.

(a) Fin $\mu_X[n]$ and $\sigma_X^2[n]$, the mean and variance functions of the random sequence X at time n for any time $n > 1$.

(b) Prove that $\frac{X[n]}{n}$ converges in probability to ν as the time n approaches infinity.