

Probability, Statistics, and Random Processes for Engineers — Reading Notes

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1 Introduction to Probability

2 Random Variables

2.1 Introduction

2.2 Definition of a RV

2.3 Cumulative Distribution Function

2.4 Probability Density Function

2.5 Continuous, Discrete, and Mixed Random Variables

2.6 Conditional and Joint Distributions and Densities

$$F_X(x|B) = \frac{P[X \leq x] \cap P[B]}{P[B]}$$
$$f_X(x|B) \triangleq \frac{P[X \leq x] \cap P[B]}{P[B]}$$

2.6.1 EX. 2.6–1 Evaluating Conditional CDFs

For the event $B = \{X \leq 10\}$, the set Ω is split above and below $x = 10$. As such, this should be solved piecewise.

For $x \geq 10$, $P[X \leq x, X \leq 10] = P[X \leq 10]$.

$$\begin{aligned} F_X(x|\{X \leq 10\}) &= \frac{P[X \leq x, X \leq 10]}{P[X \leq 10]} \\ &= \frac{P[X \leq 10]}{P[X \leq 10]} \\ &= 1 \end{aligned}$$

And for $x \leq 10$, $P[X \leq x, X \leq 10] = P[X \leq x]$.

$$\begin{aligned} F_X(x|\{X \leq 10\}) &= \frac{P[X \leq x, X \leq 10]}{P[X \leq 10]} \\ &= \frac{P[X \leq x]}{P[X \leq 10]} \end{aligned}$$

EX. 2.6–2 Poisson conditioned on even

Weighted sum of conditionals

Distribution as a weighted sum of conditional distribution functions.

$$F_X(x) = \sum_{i=1}^n F_X(x|A_i)P[A_i]$$

EX. 2.6–3 Defective Chips

One bad chip for every five. Defective Chips (DC) have ttf X which obeys the CDF

$$F_X(x|DC) = (1 - e^{-\frac{x}{2}})u(x)$$

And good chips (GC) have ttf

$$F_X(x|GC) = (1 - e^{-\frac{x}{10}})u(x)$$

What is the probability the chip will fail before six months?

The unconditional CDF as from the above equation is,

$$\begin{aligned} F_X(x) &= F_X(x|DC)(P[DC]) + F_X(x|GC)P[GC] \\ &= (1 - e^{-\frac{x}{2}}u(x))\left(\frac{1}{6}\right) + (1 - e^{-\frac{x}{10}}u(x))\left(\frac{5}{6}\right) \\ F_X(6) &= (1 - e^{-\frac{6}{2}}u(6))\left(\frac{1}{6}\right) + (1 - e^{-\frac{6}{10}}u(6))\left(\frac{5}{6}\right) \end{aligned}$$

Bayes' formula for probability density functions

$$P[B] = \int_{-\infty}^{\infty} P[B|X = x]f_X(x)dx$$

EX. 2.6–4 Detecting a closed switch

This uses the error function in a confusing manner. Still it is a good example of conditional probability and Bayes'.