Experiment No.: 06
Fuzzy sets and relations

Aim: To implement fuzzy set operations and relations in SCILAB / Python.

Apparatus: SCILAB / Python

Theory: Fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth- truth values between "completely true" and "completely false". As its name suggests, it is the logic underlying modes of reasoning which are approximate rather than exact. The importance of fuzzy logic derives from the fact that most modes of human reasoning and especially common sense reasoning are approximate in nature. The essential characteristics of fuzzy logic as founded by Zader Lotfi are as follows:

- In fuzzy logic, exact reasoning is viewed as a limiting case of approximate reasoning.
- In fuzzy logic everything is a matter of degree.
- Any logical system can be fuzzified
- In fuzzy logic, knowledge is interpreted as a collection of elastic or, equivalently, fuzzy constraint on a collection of variables
- Inference is viewed as a process of propagation of elastic constraints.

Fuzzy definitions :

- ***Universe of Discourse***

  The Universe of Discourse is the range of all possible values for an input to a fuzzy system.

- ***Fuzzy Set***

  A Fuzzy Set is any set that allows its members to have different grades of membership (membership function) in the interval [0,1].

- ***Support of a fuzzy set***

  The Support of a fuzzy set F is the crisp set of all points in the Universe of Discourse U such that the membership function of F is non-zero.

- ***Crossover point***

  The Crossover point of a fuzzy set is the element in U at which its membership function is 0.5.

- ***Fuzzy Singleton***

  A Fuzzy singleton is a fuzzy set whose support is a single point in U with a membership function of one.

Fuzzy set operations:

Union:

The membership function of the Union of two fuzzy sets A and B with membership functions μ_A and μ_B respectively is defined as the maximum of the two individual membership functions. This is called the *maximum* criterion.

$$\mu_{A \cup B} = \max(\mu_A, \mu_B)$$

Intersection:

The membership function of the Intersection of two fuzzy sets A and B with membership functions μ_A and μ_B respectively is defined as the minimum of the two individual membership functions. This is called the *minimum* criterion.

$$\mu_{A \cap B} = \min(\mu_A, \mu_B)$$

Complement:

The membership function of the Complement of a Fuzzy set A with membership function μ_A is defined as the negation of the specified membership function. This is caleed the *negation* criterion.

$$\mu_{\overline{A}} = 1 - \mu_A$$

Fuzzy Relations

A **crisp relation** represents the presence or absence of association, interaction, or interconnectedness between the elements of two or more sets. This concept can be generalized to allow for various degrees or strengths of relation or interaction between elements. Degrees of association can be represented by membership grades in a **fuzzy relation** in the same way as degrees of set membership are represented in the fuzzy set.

In fact, just as the crisp set can be viewed as a restricted case of the more general fuzzy set concept, the crisp relation can be considered to be a restricted case of the fuzzy relations.

A *fuzzy relation* is a fuzzy set defined on the Cartesian product of crisp sets $X_1, X_2, \ldots, X_n$, where tuples $(x_1, x_2, \ldots, x_n)$, may have varying degrees of membership within the relation. The membership grade is usually represented by a real number in the closed interval $[0,1]$ and indicates the strength of the relation present between the elements of the tuple.

A fuzzy relation can also conveniently be represented by an *n*-dimensional membership array whose entries correspond to *n*-tuples in the universal set. These entries take values representing the membership grades of the corresponding *n*-tuples.

Problem:

1. Consider two fuzzy sets A and B as, $A = \left\{ \frac{0.4}{x_1} + \frac{0.6}{x_2} + \frac{0.1}{x_3} + \frac{0.8}{x_4} \right\}$

$$B = \left\{ \frac{0.5}{x_1} + \frac{0.3}{x_2} + \frac{0.7}{x_3} + \frac{0.2}{x_4} \right\}$$

Find: Union, Intersection and complement of the two fuzzy sets.

2. Verify the De Morgan's Law

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

3. Find the max-min and max product composition for the fuzzy sets given as,

$$A = \left\{ \frac{1}{x_1} + \frac{0.5}{x_2} + \frac{0.2}{x_3} \right\} \qquad B = \left\{ \frac{1}{w_1} + \frac{0.5}{w_2} + \frac{0.3}{w_3} \right\} \qquad C = \left\{ \frac{0.1}{s_1} + \frac{0.6}{s_2} + \frac{0.4}{s_3} \right\}$$

Find, R = A X B
S = C X B
Max-min composition, R ○ S
Max-product composition, R ● S

Results:

1. **Solve the problem step by step and derive the output for the given fuzzy equations. Attach the images for the solved problem in the space below (write SAP ID and name on each page).**

01/10/2021 Reeha Parkar 60001180046

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Fuzzy sets and Relations

1. consider two fuzzy set A and B as:

$$A = \left\{ \frac{0.4}{x_1} + \frac{0.6}{x_2} + \frac{0.1}{x_3} + \frac{0.8}{x_4} \right\}$$

$$B = \left\{ \frac{0.5}{x_1} + \frac{0.3}{x_2} + \frac{0.7}{x_3} + \frac{0.2}{x_4} \right\}$$

Find: union, Intersection and complement of the two fuzzy sets.

$$\underset{(max)}{A \cup B} = \left\{ \frac{0.5}{x_1} + \frac{0.6}{x_2} + \frac{0.7}{x_3} + \frac{0.8}{x_4} \right\}$$

$$\underset{(min)}{A \cap B} = \left\{ \frac{0.4}{x_1} + \frac{0.3}{x_2} + \frac{0.1}{x_3} + \frac{0.2}{x_4} \right\}$$

$$\underset{(1-\mu_A(x))}{\bar{A}} = \left\{ \frac{0.6}{x_1} + \frac{0.4}{x_2} + \frac{0.9}{x_3} + \frac{0.2}{x_4} \right\}$$

$$\underset{(1-\mu_B(x))}{\bar{B}} = \left\{ \frac{0.5}{x_1} + \frac{0.7}{x_2} + \frac{0.3}{x_3} + \frac{0.8}{x_4} \right\}$$

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2. Verify the DeMorgan's law

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$
$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

(a) $\overline{A \cup B} = \bar{A} \cap \bar{B}$

Data taken from previous question:

$$L \cdot H \cdot S = \left\{ \frac{0.5}{x_1} + \frac{0.4}{x_2} + \frac{0.3}{x_3} + \frac{0.2}{x_4} \right\}$$

$$R \cdot H \cdot S = \left\{ \frac{0.5}{x_1} + \frac{0.4}{x_2} + \frac{0.3}{x_3} + \frac{0.3}{x_4} \right\}$$

$L \cdot H \cdot S = R \cdot H \cdot S$ Hence proved.

(b) $\overline{A \cap B} = \bar{A} \cup \bar{B}$

$$L \cdot H \cdot S = \left\{ \frac{0.6}{x_1} + \frac{0.7}{x_2} + \frac{0.9}{x_3} + \frac{0.8}{x_4} \right\}$$

$$R \cdot H \cdot S = \left\{ \frac{0.6}{x_1} + \frac{0.7}{x_2} + \frac{0.9}{x_3} + \frac{0.8}{x_4} \right\}$$

$\therefore L \cdot H \cdot S = R \cdot H \cdot S$

$\therefore$ Hence, proved.

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3. Find the max-min and max product composition for the fuzzy sets given as:

$A = \left\{ \frac{1}{x_1} + \frac{0.5}{x_2} + \frac{0.2}{x_3} \right\}$ $B = \left\{ \frac{1}{\omega_1} + \frac{0.5}{\omega_2} + \frac{0.3}{\omega_3} \right\}$ $C = \left\{ \frac{0.1}{s_1} + \frac{0.6}{s_2} + \frac{0.4}{s_3} \right\}$

Find $R = A \times B$
$S = C \times B$
Max-min composition, $R \circ S$
Max-product composition, $R \bullet S$

$$R = A \times B = \begin{array}{c|ccc} & \omega_1 & \omega_2 & \omega_3 \\ \hline x_1 & 1 & 0.5 & 0.3 \\ x_2 & 0.5 & 0.5 & 0.3 \\ x_3 & 0.2 & 0.2 & 0.2 \end{array}$$

$$S = \cancel{A}\, C \times B = \begin{array}{c|ccc} & \omega_1 & \omega_2 & \omega_3 \\ \hline s_1 & 0.1 & 0.1 & 0.1 \\ s_2 & 0.6 & 0.5 & 0.3 \\ s_3 & 0.4 & 0.4 & 0.3 \end{array}$$

$$R \circ S = \begin{array}{c|ccc} & s_1 & s_2 & s_3 \\ \hline x_1 & 0.1 & 0.6 & 0.4 \\ x_2 & 0.1 & 0.5 & 0.4 \\ x_3 & 0.1 & 0.2 & 0.2 \end{array}$$

$$x_1 S_1 \longrightarrow \begin{array}{c|ccc} & \omega_1 & \omega_2 & \omega_3 \\ \hline x & 1 & 0.5 & 0.2 \\ s_1 & 0.1 & 0.1 & 0.1 \end{array} \text{ min}$$
$$(0.1 \quad 0.1 \quad 0.1) \text{ max} = 0.1$$

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$x_1 S_2 \Rightarrow \begin{array}{c} x_1 \\ s_2 \end{array} \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.6 & 0.5 & 0.3 \end{pmatrix}$ min $= 0.6$

$(0.6 \quad 0.5 \quad 0.3)$ max

$x_1 S_3 \Rightarrow \begin{array}{c} x_1 \\ s_3 \end{array} \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.4 & 0.4 & 0.3 \end{pmatrix}$ min $= 0.4$

$(0.4 \quad 0.4 \quad 0.3)$ max

$x_2 S_1 \Rightarrow \begin{array}{c} x_2 \\ s_1 \end{array} \begin{pmatrix} 0.5 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0.1 \end{pmatrix}$ min $= 0.1$

$(0.1 \quad 0.1 \quad 0.1)$ max

$x_2 S_2 \Rightarrow \begin{array}{c} x_2 \\ s_2 \end{array} \begin{pmatrix} 0.5 & 0.5 & 0.3 \\ 0.6 & 0.5 & 0.3 \end{pmatrix}$ min $= 0.5$

$(0.5 \quad 0.5 \quad 0.3)$ max

$x_2 S_3 \Rightarrow \begin{array}{c} x_2 \\ s_3 \end{array} \begin{pmatrix} 0.5 & 0.5 & 0.3 \\ 0.4 & 0.4 & 0.3 \end{pmatrix}$ min $= 0.4$

$(0.4 \quad 0.4 \quad 0.3)$ max

$x_3 S_1 \Rightarrow \begin{array}{c} x_3 \\ s_1 \end{array} \begin{pmatrix} 0.2 & 0.2 & 0.2 \\ 0.2 & 0.1 & 0.1 \end{pmatrix}$ min $= 0.2$

$(0.2 \quad 0.1 \quad 0.1)$ max

$x_3 S_2 \Rightarrow \begin{array}{c} x_3 \\ s_2 \end{array} \begin{pmatrix} 0.2 & 0.2 & 0.2 \\ 0.6 & 0.5 & 0.3 \end{pmatrix}$ min $= 0.2$

$(0.2 \quad 0.2 \quad 0.2)$ max

$x_3 S_3 = \begin{array}{c} x_3 \\ s_3 \end{array} \begin{pmatrix} 0.2 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.3 \end{pmatrix}$ min $= 0.2$

$(0.2 \quad 0.2 \quad 0.2)$ max

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$$R \circ S = \begin{array}{c|ccc} & s_1 & s_2 & s_3 \\ \hline x_1 & 0.1 & 0.6 & 0.4 \\ x_2 & 0.05 & 0.3 & 0.2 \\ x_3 & 0.02 & 0.12 & 0.08 \end{array}$$

$x_1 s_1 \Rightarrow$ $\begin{array}{c} x_1 \\ s_1 \end{array}\begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0.1 \end{pmatrix}$ pro

$(0.1 \quad 0.05 \quad 0.03)_{max} = 0.1$

$x_1 s_2 \Rightarrow$ $\begin{array}{c} x_1 \\ s_2 \end{array}\begin{pmatrix} \cancel{0.1}\,1 & 0.5 & 0.3 \\ 0.6 & 0.5 & 0.3 \end{pmatrix}$ pro

$(0.6 \quad 0.25 \quad 0.09)_{max} = 0.6$

$x_1 s_3 \Rightarrow$ $\begin{array}{c} x_1 \\ s_3 \end{array}\begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.4 & 0.4 & 0.3 \end{pmatrix}$ pro

$(0.4 \quad 0.2 \quad 0.09)_{max} = 0.4$

$x_2 s_1 \Rightarrow$ $\begin{array}{c} x_2 \\ s_1 \end{array}\begin{pmatrix} 0.5 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0.1 \end{pmatrix}$ pro

$(0.05 \quad 0.05 \quad 0.03)_{max} = 0.05$

$x_2 s_2 \Rightarrow$ $\begin{array}{c} x_2 \\ s_2 \end{array}\begin{pmatrix} 0.5 & 0.5 & 0.3 \\ 0.6 & 0.5 & 0.3 \end{pmatrix}$ pro

$(0.3 \quad 0.25 \quad 0.09)_{max} = 0.3$

$x_2 s_3 \Rightarrow$ $\begin{array}{c} x_2 \\ s_3 \end{array}\begin{pmatrix} 0.5 & 0.5 & 0.3 \\ 0.4 & 0.4 & 0.3 \end{pmatrix}$ pro

$(0.2 \quad 0.2 \quad 0.09)_{max} = 0.2$

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$$x_3 S_1 \Rightarrow \begin{array}{c} x_3 \\ S_1 \end{array} \begin{pmatrix} 0.2 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{pmatrix} \text{pro}$$

$$(0.02 \quad 0.02 \quad 0.02)_{max} = 0.02$$

$$x_3 S_2 \Rightarrow \begin{array}{c} x_3 \\ S_2 \end{array} \begin{pmatrix} 0.2 & 0.2 & 0.2 \\ 0.6 & 0.5 & 0.3 \end{pmatrix} \text{pro}$$

$$(\overset{0.12}{\cancel{0.06}} \quad 0.1 \quad 0.06)_{max} = 0.12$$

$$x_3 S_3 \Rightarrow \begin{array}{c} x_3 \\ S_3 \end{array} \begin{pmatrix} 0.2 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.3 \end{pmatrix} \text{pro}$$

$$(0.08 \quad 0.08 \quad 0.06)_{max} = 0.08.$$

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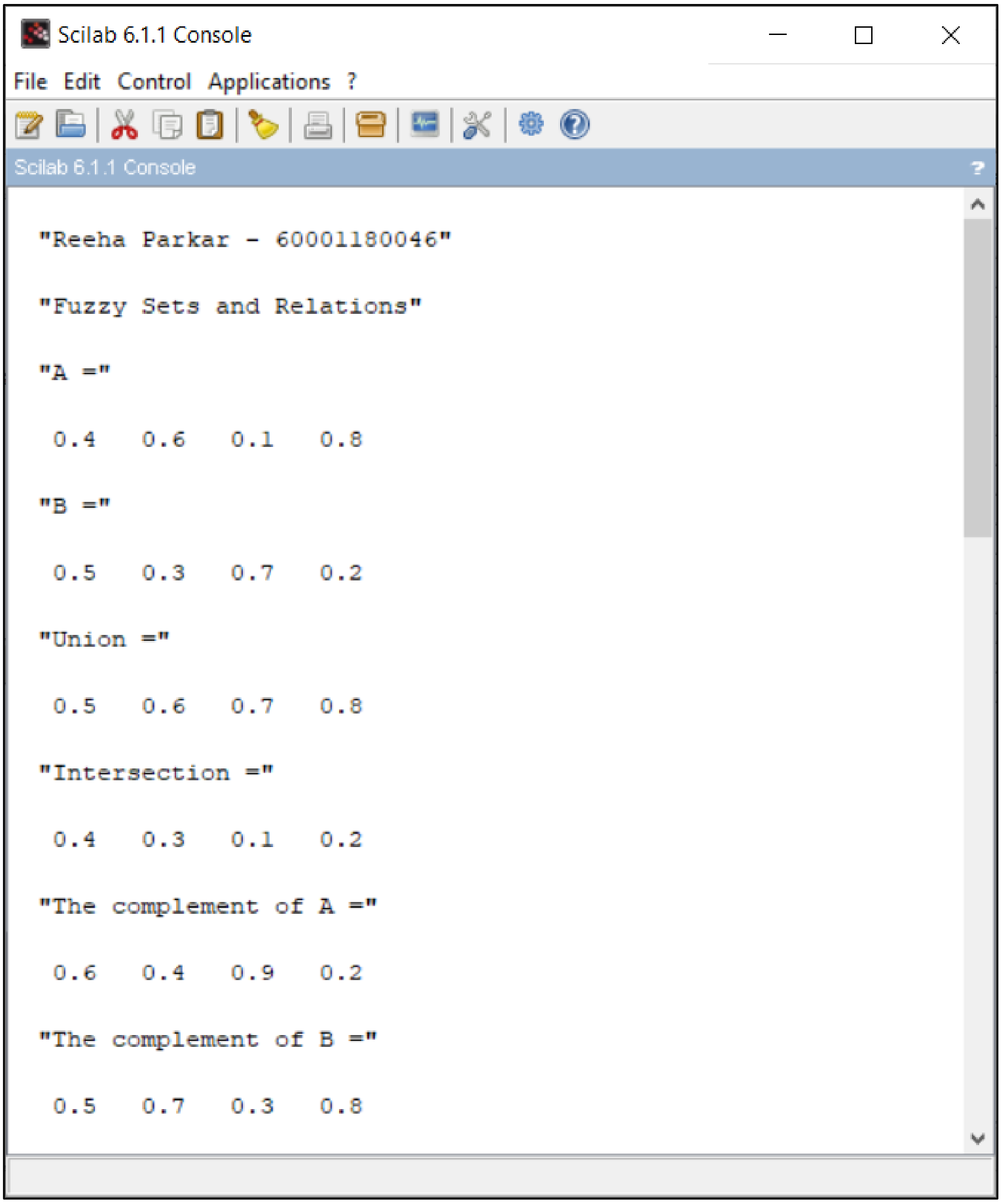
2. Code and outputs for each problem (Mention the title of the experiment, Student name and SAP Id in the code and display it in the output window)

Code:

```
clc;
clear all;
disp("Reeha Parkar - 60001180046");
disp("Fuzzy Sets and Relations");
//input
A = [0.4 0.6 0.1 0.8]
B = [0.5 0.3 0.7 0.2]
disp("A =");
disp(A);
disp("B =");
disp(B);
//Union
AUB = max(A,B);
disp("Union =", AUB);
//Intersection
AIB = min(A,B);
disp("Intersection =", AIB);
//Complement
Abar = 1 - A;
Bbar = 1 - B;
disp("The complement of A =", Abar);
disp("The complement of B =", Bbar);
//De Morgan's Law
disp("Verifying De Morgans Law");
AUBbar = 1 - AUB;
disp("Comp(A union B) =", AUBbar);
AbIBb = min(Abar,Bbar);
disp("Comp(A) intersection Comp(B) =", AbIBb);
if AUBbar == AbIBb then
 disp("LHS is equal to RHS");
 disp("De Morgans law verified");
else
 disp("LHS is not equal to RHS.");
end
AIBbar = 1 - AIB;
disp("Comp(A intersection B) =", AIBbar)
AbUBb = max(Abar,Bbar);
disp("Comp(A) union Comp(B) =", AbUBb)
if AUBbar == AbIBb then
 disp("LHS is equal to RHS");
 disp("De Morgans law verified");
else
 disp("LHS is not equal to RHS.")
end
//Max-Min and max-product Composition
disp("Max-Min and Max-Product Composition");
A = [1 0.5 0.2]
B = [1 0.5 0.3]
C = [0.1 0.6 0.4]
for i = 1:3
 for j = 1:3
 R(i,j) = min(A(i), B(j));
 end
end
disp("R =", R)
for i = 1:3
 for j = 1:3
 S(i,j) = min(C(i), B(j));
 end
end
disp("S =", S)
for i = 1:3
 for j = 1:3
```

```
 X(i,j) = max(min(R(i), S(j)));
 end
end
disp("Max-min composition of R and S =", X);
for i = 1:3
 for j = 1:3
 Y(i,j) = max(R(i) * S(j))
 end
end
disp("Max-product composition of R and S =", Y);
```

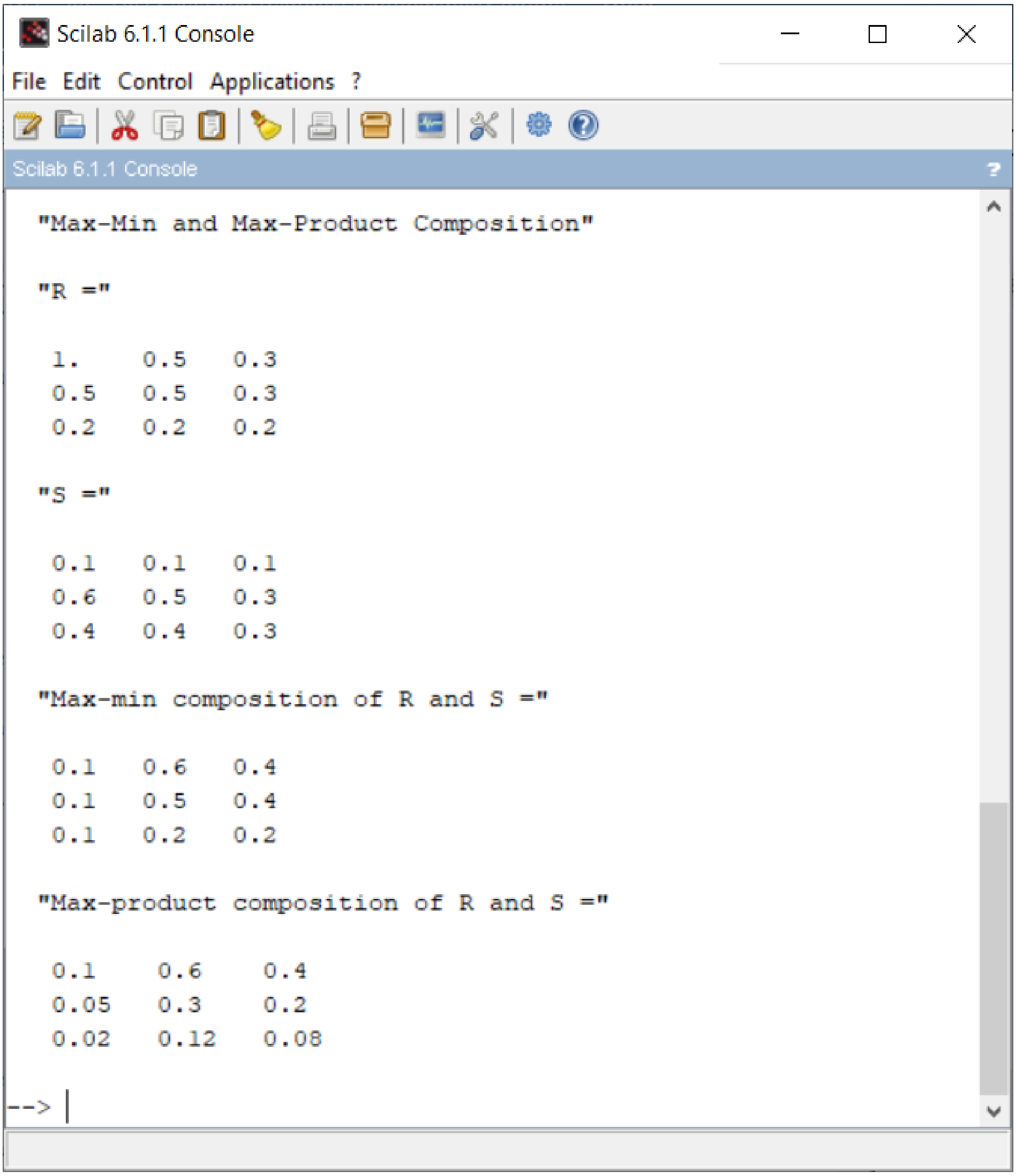
Output:



Scilab 6.1.1 Console

File Edit Control Applications ?

```
"Verifying De Morgans Law"

"Comp(A union B) ="

 0.5   0.4   0.3   0.2

"Comp(A) intersection Comp(B) ="

 0.5   0.4   0.3   0.2

"LHS is equal to RHS"

"De Morgans law verified"

"Comp(A intersection B) ="

 0.6   0.7   0.9   0.8

"Comp(A) union Comp(B) ="

 0.6   0.7   0.9   0.8

"LHS is equal to RHS"

"De Morgans law verified"
```



```
Scilab 6.1.1 Console
File  Edit  Control  Applications  ?
Scilab 6.1.1 Console

 "Max-Min and Max-Product Composition"

 "R ="

   1.     0.5    0.3
   0.5    0.5    0.3
   0.2    0.2    0.2

 "S ="

   0.1    0.1    0.1
   0.6    0.5    0.3
   0.4    0.4    0.3

 "Max-min composition of R and S ="

   0.1    0.6    0.4
   0.1    0.5    0.4
   0.1    0.2    0.2

 "Max-product composition of R and S ="

   0.1     0.6     0.4
   0.05    0.3     0.2
   0.02    0.12    0.08

-->
```

Conclusion:
In this experiment, we implemented Fuzzy Logic in SCILAB. 4 sums were solved using the coded implementation, whose results match accurately with the theoretical values. De Morgan's Law was also verified, including the max-min and max-product composition was coded.