San Francisco State University
CSC 665/865: Artificial Intelligence, Spring 2020

Assignment 4

Due Date: April 28, 1:00pm

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Submission via iLearn

Submit your answers online via iLearn as a *single pdf file* with the name **csc665-assign4-lastname-sfsuid**. Write your Name and SFSU ID at the top. Each question's answer should be completed on a separate page or pages. Violation of these rules will lead to the deduction of marks.

Academic Honesty

Plagiarism is a serious offense and will be dealt with harshly. I consider plagiarism to be the unattributed use of an external source (e.g., another student, a web site, a book) in work for which a student takes credit, or the inappropriate use of an external source whether or not attribution is made. The seriousness of the offense depends on the extent to which the student relied upon the external source. All written responses must be your own. All work ideas and quotes that originate from elsewhere must be cited according to standard academic practice. Students caught cheating will automatically fail the course.

1 [10 points] Bayes Rule

After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease, and that the test is 99% accurate (i.e., the probability of testing positive given that you have the disease is 0.99, as is the probability of testing negative given that you don't have the disease). The good news is that this is a rare disease, striking only one in 10,000 people.

- What are the chances that you actually have the disease? (Show your calculations as well as giving the final result.)

Knowns:
E = test positive
H = have disease
-H = not have disease

Bayes Theorem: $P(H \mid E) = \frac{P(E \mid H) * P(H)}{P(H) * P(E \mid H) + P(-H) * P(E \mid -H)}$

$P(E \mid H) = 0.99$
$P(H) = 0.0001$
$P(-H) = 0.9999$
$P(E \mid -H) = 0.01$

$P(H \mid E) = \frac{0.99 * 0.0001}{0.0001 * 0.99 + 0.9999 * 0.01} = \frac{0.000099}{0.000099 + 0.009999} = \frac{0.000099}{0.010098}$

$= 0.009804 = 0.09804$ % chance

2 [10 points] Conditional Independence

Prove that conditional independence is symmetric (i.e. if A is independent of B given C then B is independent of A given C).

Something is conditionally independent when:

P (A, B | C) = P (A | C) * P (B | C)

For A:

$$P(A \mid B, C) = \frac{P(A,B,C)}{P(B,C)} = \frac{P(A,B \mid C) * P(C)}{P(B,C)} = \frac{P(A \mid C) * P(B \mid C) * P(C)}{P(B \mid C) * P(C)} = P(A \mid C)$$

For B:

$$P(B \mid A, C) = \frac{P(A,B,C)}{P(A,C)} = \frac{P(A,B \mid C) * P(C)}{P(A,C)} = \frac{P(A \mid C) * P(B \mid C) * P(C)}{P(A \mid C) * P(C)} = P(B \mid C)$$

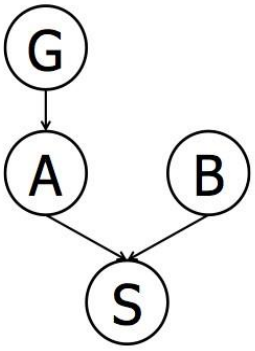
And P (A | C) * P (B | C) = P (A, B | C)

Thus, A & B are independent, and Conditional Independence is Symmetric.

3 [20 points] Bayes Net Inference

$\mathbb{P}(G)$	
$+g$	0.1
$-g$	0.9

$\mathbb{P}(A\|G)$		
$+g$	$+a$	1.0
$+g$	$-a$	0.0
$-g$	$+a$	0.1
$-g$	$-a$	0.9



$\mathbb{P}(B)$	
$+b$	0.4
$-b$	0.6

$\mathbb{P}(S\|A,B)$			
$+a$	$+b$	$+s$	1.0
$+a$	$+b$	$-s$	0.0
$+a$	$-b$	$+s$	0.9
$+a$	$-b$	$-s$	0.1
$-a$	$+b$	$+s$	0.8
$-a$	$+b$	$-s$	0.2
$-a$	$-b$	$+s$	0.1
$-a$	$-b$	$-s$	0.9

Compute the values of the following probabilities:

1. **[3 points]** P (+g, +a, +b, +s)
   = P (+g) * P (+a | +g) * P (+b) * P (+s | +a, +b)

2. **[3 points]** $P(+a)$ $= \sum_g P(g)P(+a \mid g)$
   = P(+g) P(+a | +g) + P(-g) P(+a | -g)
   = 0.1(1.0) + 0.9(0.1)
   = 0.19

3. **[3 points]** $P(+a \mid +b)$ $= \frac{P(+a,+b)}{P(+b)}$
   $= \frac{\sum_g P(g)P(+a \mid g)P(+b)}{P(+b)}$
   $= \frac{0.4 * (0.1(1.0)+0.9(0.1))}{0.4}$
   = 0.19

4. **[5 points]** $P(+a \mid +s,+b)$ $= \frac{P(+s \mid +a,+b) * P(+a \mid +b)}{P(+s \mid b)}$
   $= \frac{0.08819 * 0.19}{0.4(1.0 * 0.8) + 0.6(0.9 * 0.1)}$
   $= \frac{0.0167561}{0.374}$
   = 0.04480 - answer

Separate parts of #4 – NOT FINAL ANSWER

P(-a) = P(+g) P(-a | +g) + P(-g) P(-a | -g) = 0.1(0.0) + 0.9(0.9) = 0.81

P(+s | +a, +b) = P(+a)P(+s | +a) + P(-a)P(+s|-a) * P(+b)P(+s|+b) + P(-b)P(+s | -b)
= (0.19(1.0 * 0.9) + 0.81(0.8 * 0.1)) * 0.4(1.0 * 0.8) + 0.6(0.9 * 0.1)
= 0.08819

5. **[3 points]** $P(+g|+a)$

$$= \sum_{+g} P(a|+g)\,P(+g)$$

$$= \frac{P(+a|+g)P(+g)}{P(+a|+g)P(+g)+P(-a|-g)P(+g)}$$

$$= \frac{1.0(0.1)}{1.0(0.1)+0.9(0.1)}$$

$$= 0.526$$

6. **[3 points]** $P(+g|+b)$

$$= \frac{P(+g)P(+b)}{P(+b)}$$

$$= P(+g)$$

$$= 0.1$$