Assignment 1

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* problem 1.6 : Sample of 10 marbles { red, green

a) pick one sample, compute p(no red marbles)

$$P(\nu=0) = \mu^0 * (1-\mu)^{10}$$

red

$$P(\nu=0) = \begin{cases} 0.95^{10} = 0.5987, & \mu=0.05 \\ 0.5^{10} = 0.0009765, & \mu=0.5 \\ 0.2^{10} = 1024 \times 10^{-10}, & \mu=0.8 \end{cases}$$

b) draw 1 thousand independent samples

$$P(\nu=0)|^{1000} = 1 - P(\nu>0)|^{1000}$$

at least 1 marble out of all samples

$$P(\nu>0)|_{1000} = \prod^{1000} (1 - P(\nu=0)|^{10}) \rightarrow (1-\mu)^{10}$$

↳ independent

$$P(\nu>0)|_{1000} = [1-(1-\mu^{10})]^{1000}$$

$$\therefore P(\nu=0)|_{1000} = 1-[1-(1-\mu^{10})]^{1000}$$

$\mu = 0.05$

$P(\nu=0)|_{1000} \simeq 1$

$= 1 - 3 \times 10^{-397}$

$\mu = 0.5$

$P(\nu=0)|_{1000} = 0.62355$

$\mu = 0.8$

$P(\nu=0)|_{1000}$

$= 0.0001024$

problem 1.6

Repeat b for 1 million independent samples

c) $P(U=0)\big|_{1,000,000} = 1-[1-(1-\mu)^{10}]^{1000000}$

$\mu = 0.05$ | $P(U=0) = 1$

$\mu = 0.5$ | $P(U=0) = 1$

$\mu = 0.8$ | $P(U=0) = 0.9733$

problem 2.5

prove $\sum_{i=0}^{D} \binom{N}{i} \leq N^{D}+1$

hence $m_H(N) \leq N^{d_{vc}(H)}+1$

Soln: 1st way:

From lecture 3 P. 23: $m_H(N) \leq \sum_{i=0}^{d_{vc}(H)} \binom{N}{i}$ ①

and sample complexity P. 27: $N \geq 10\, d_{vc}(H)$

$\because$ N: sample data & D: all dataset

$D \geq N$

$\therefore D \geq 10\, d_{vc}(H) \geq d_{vc}(H)$

$\therefore \sum_{i=0}^{D} \binom{N}{i} \geq \sum_{i=0}^{10 d_{vc}(H)} \binom{N}{i} \geq \sum_{i=0}^{d_{vc}(H)} \binom{N}{i}$ ... ②

"not sure if exists": in case $d_{vc}(H)=0$

$$\therefore \sum_{i=0}^{D} \binom{N}{i} \leq N^{D} + 1 \quad \ldots (3)$$

From (2) & (3)

$$\therefore N^{D}+1 \geq \sum_{i=0}^{duc(H)} \binom{N}{i} \quad \ldots (4)$$

$$\because D \geq duc(H)$$

$$\therefore N^{D}+1 \geq N^{duc(H)}+1$$

$$\therefore \sum_{i=0}^{duc(H)} \binom{N}{i} \leq N^{duc(H)}+1 \quad \ldots (5) \text{ from } 3$$

From (5) & (1)

$$\therefore m_H(N) \leq N^{duc(H)}+1$$

2nd way:

$$\sum_{i=0}^{D} \binom{N+1}{i} = \sum_{i=0}^{D} \binom{N}{i} + \binom{N}{i-1}$$

$$= \sum_{i=0}^{D} \binom{N}{i} + \sum_{j=0}^{D-1} \binom{N}{j}$$

$$\leq N^{D}+1 + N^{D-1}+1 = (N^{D}+N^{D-1}+1)+1$$

$$\leq (N+1)^{D}+1 = \sum_{k=0}^{D} \binom{D}{k} N^{k} + 1$$

$$\therefore M_H(N) \leq \sum_{i=0}^{duc} \binom{N}{i} \leq N^{duc}+1$$