Internship Report on

Numerical Simulation of holographic endoscopic multimode Fibers

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Abstract

The optical fibers, especially, multimode fibers have various applications in communication and medicine. In order to harness their power experimentally, a detailed theoretical analysis is a necessity for which this project presents some simulation results to analyse how light propagate through the fiber material using weakly guiding approximation taking into consideration some perturbations and their effects. The work simulates the influence of bending curvature in the output of a single focal points and analyses the optical abberations from a single point pixel.

1 Introduction

Multimode Fibers(MMFs) have a myriad of potential applications in the field of telecommunications, medicine, optical metrology, etc. The theoretical model behind the light propagation through an optical fiber has been there for years, however, one of the most innovative ideas of Fiber Technology in the field of medicine is their use as holographic endoscopes for invivo imaging to study the deep neural activities. This technique has advantages over the traditional probes: low cost, minimized probe size, better resolution, minimal intrusion with the living cells, better sampling rate. [2]

Currently, the scientists are working on not only optimizing the image resolution but also on minimizing the fiber's size to ensure the minimal damage inside the living body. Although the MMFs generate some speckled patterns at the end of fiber tip when light passes through it, studies shows that it is possible to predict their behaviour through the theoretical model to tens to hundreds of millimeters.
In the experimental part, some holographic techniques have been implemented to determine the output modes and Transmission Matrices have been measured which aids in analysing the light propagation over distances. However, several external factors like temperature, bending, etc. play major role in distortion that one obtain at the fiber end. The process to analyse the process can be rather cumbersome, which can be overcome by different approach: finding a transmission matrix through numerical modelling.

2 Theoretical Framework

2.1 Electromagnetic Fiber modes

Consider a cylindrical step-index (SI) multi-mode fiber with the radius and refractive index of core to be a and n_c respectively and the refractive index of the cladding is n. The wave number is given by $k = \frac{2*\pi}{\lambda}$ and the propagation constant as β which is limited by $n_c * k >= \beta >= n * k$. In order to understand the fiber modes, which are the solution to the electromagnetic fields of Maxwell Equations, we need to solve the Bessel differential equation. [1]

$$E_y = E_l * cos(l\phi) \begin{Bmatrix} J_l(ur/a)/J_l(u) \\ K_l(ur/a)/K_l(u) \end{Bmatrix} \tag{1}$$

The solution to light propagation through core is given by Bessel's first kind, J_l, while through the cladding is given by modified Bessel's second kind, K_l. Here u and w are the parameters defined as,

$$u = a(k^2 n_c^2 - \beta^2)^{\frac{1}{2}} \tag{2}$$

$$w = a(\beta^2 - k^2 n^2)^{\frac{1}{2}} \tag{3}$$

Solving the equations for LP modes at the tangential interface for all the components, we get characteristics equations for (l-1) ϕ and (l+1) ϕ,

$$\frac{u}{n_c} \frac{J_{l\pm1}(u)}{J_l(u)} = \pm \frac{w}{n} \frac{K_{l\pm1}(w)}{K_l(w)} \tag{4}$$

The vectorial model for the Fiber modes simulation consists of electric and magnetic field in all the three direction, however, the scalar field requires only E_x or E_y leading to

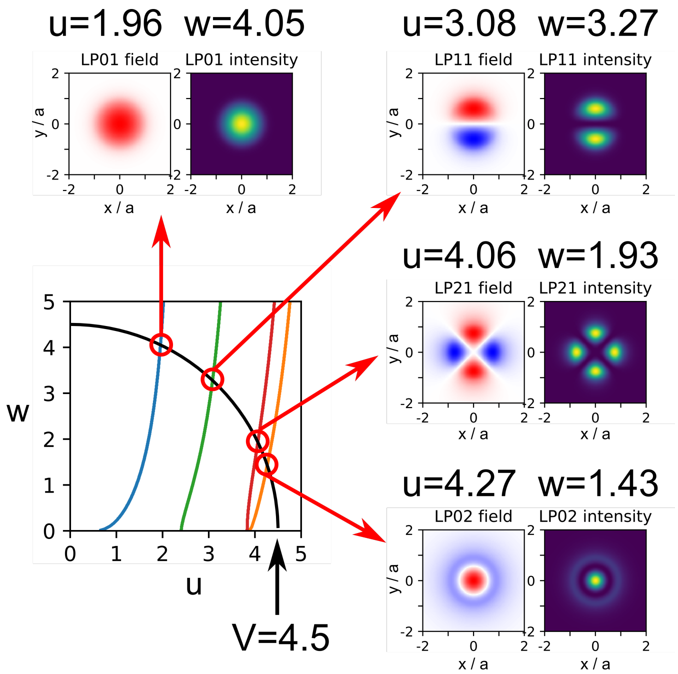


Figure 1: u and w branch point intersection for finding solutions of modes [3]

a much simpler approach. Nonetheless, it restricts us from getting information about the polarization direction.
Following the derivation, which is excluded in this report, we get another parameter,

$$V = a * k * (n_c^2 - n^2)^{\frac{1}{2}} \tag{5}$$

where $(n_c^2 - n^2)^{\frac{1}{2}}$ is Numerical Apperture (NA) and V is the V-parameter or normalised frequency. For a step-index multimode fibers, the number of modes is approximated by $V^2/2$. The simulation follows weakly guided approximation where,

$$\Delta = \frac{n_c - n}{n} << 1 \tag{6}$$

2.2 Propagation Invariant Modes

The theoretical prediction of light propagation through an optical fiber can be done formulating a transmission matrix which acts as an optical system. Following the equation below, the information on optical amplitudes and phases at the output focal point can be extracted. Based on the weakly guiding and paraxial approximation, the azimuthal intensities should remain unchanged, only with the shift in phase, known as Propagation Invariant Modes (PIM). This is mathematically shown through the equation,

$$\bar{M} = TMT^{\dagger} \tag{7}$$

where T is the conversion matrix and M is the transmission matrix. The conversion matrix is basically a superposition of all the input modes where each row represents a 2D eigen modes.Transmission matrix is given by the evolution operator of the state along the fiber $\exp(i\boldsymbol{B}L)$ where L is the length of the fiber and $\bar{M}$ is a diagonal matrix which retains the conservation of the input modes, however, a non-diagonal matrix means that there is some interaction with the coupled modes.

2.3 Optimization of Propagation constant

Propagation constant (β) is a measure to estimate how much the amplitude and phases changes in a particular direction. In a weakly guiding approximation for LP modes, the value of β can be approximated using the formula,

$$\beta = nk[1 + \Delta - \Delta\frac{u^2}{v^2}] \tag{8}$$

The value of propagation constants need to be optimised based on the deviation of refractive index profile solving the Helmholtz equation and a reminiscent part of perturbation theory,

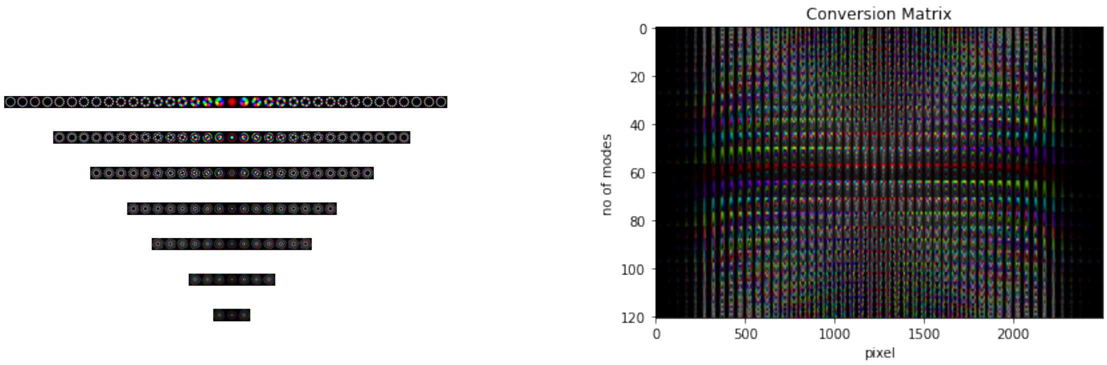
$$[\frac{\partial}{\partial z} + k_0^2 n^2 - \beta'^2(1 + \frac{x}{\rho})^2]\psi(r, \phi) = 0 \tag{9}$$

where z is the optical axis direction and $\beta^{'2}$ is the perturbed propagation constant. The derivation leads to the solution,

$$\beta_{ij} = \beta_i \delta_{ij} - (\frac{n_c k_0}{\rho})(< \psi_i|x|\psi_j > + < \psi_i|y|\psi_j >) \tag{10}$$

where ρ is the curvature(bending) and $< \psi_i|x|\psi_j > + < \psi_i|y|\psi_j >$ determines how the curvature affects in the x and y coordinates. To take the effect of bending of the fiber into account, the radius of the curvature is considered to be comparatively larger than the radius of the core. For a straight fiber, the radius of the curvature is considered to be infinitely large. For a straight fiber, $\rho \to \infty$ which means the perturbed propagation constant is the same as unperturbed.

3 Simulation and Results

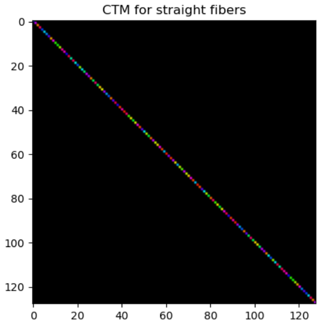


(a) (b)

Figure 2: (a) Fiber Modes with azimuthal index on x-axis and orbital index on y-axis (b) Conversion Matrix

(a) (b)

Figure 3: Transmission Matrix for (a) straight SI MMFs and (b) bent SI MMFs with the curvature bent of $25m^{-}1$ on both x and y axis and with a fiber length of 100mm



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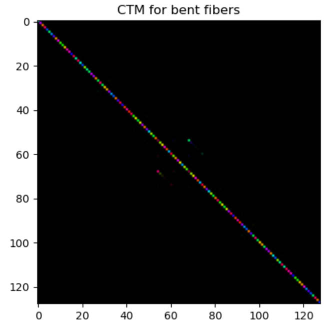

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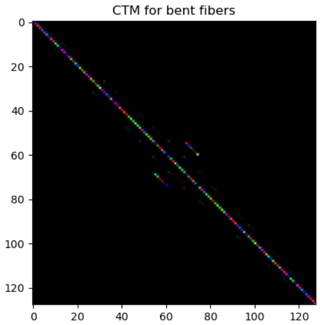

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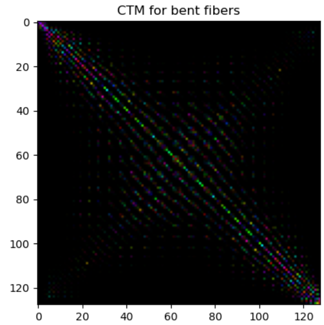

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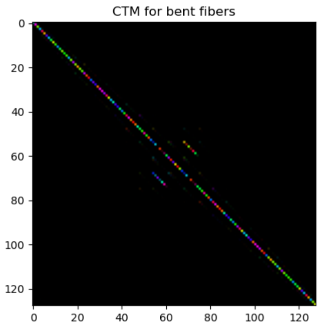

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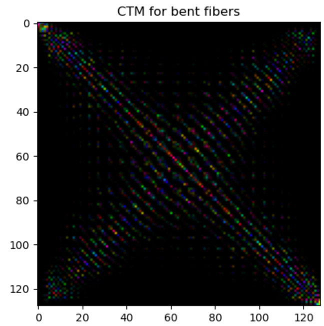

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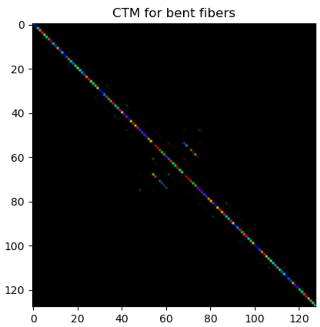

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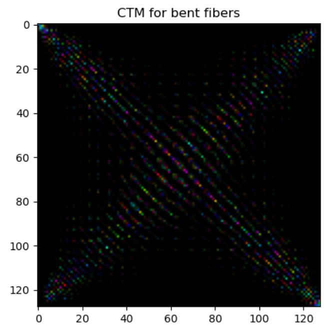
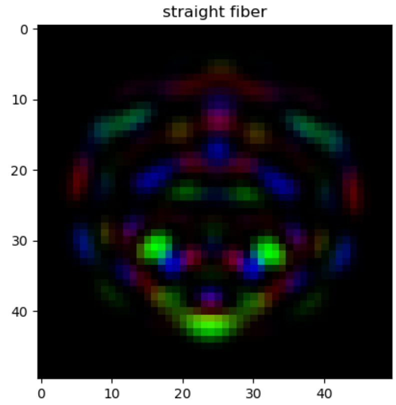


(h)

Figure 4: Converted Transmission Matrix for (a) straight SI-MMFs and bent SI-MMFs with the curvature bent of (b) $2m^{-1}$ (c) $5m^{-1}$ (d) $70m^{-1}$on both x and y axis and a fiber length of 100mm (e)$5m^{-1}$ and (f) $70m^{-1}$on both x and y axis and a fiber length of 500mm. (g)$5m^{-1}$ and (h) $70m^{-1}$on both x and y axis and a fiber length of 1m.



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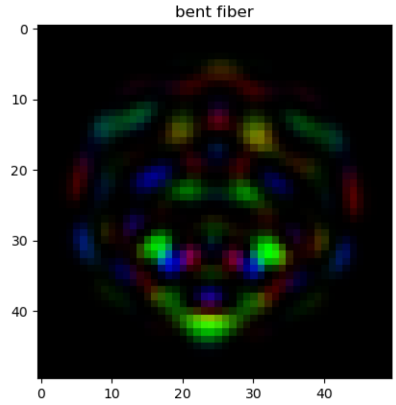

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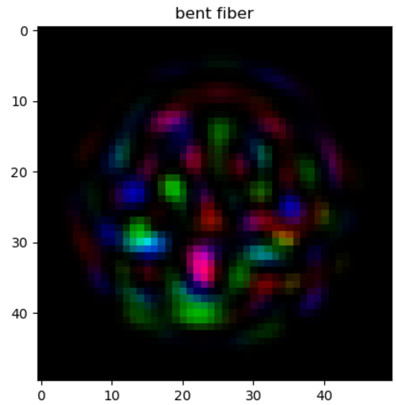

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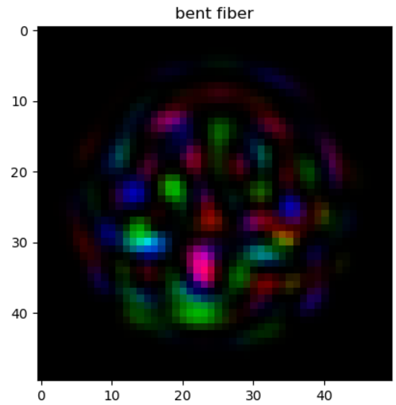

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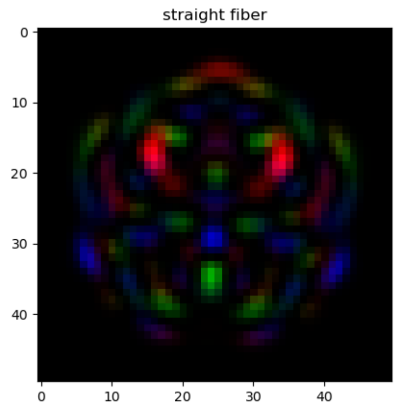

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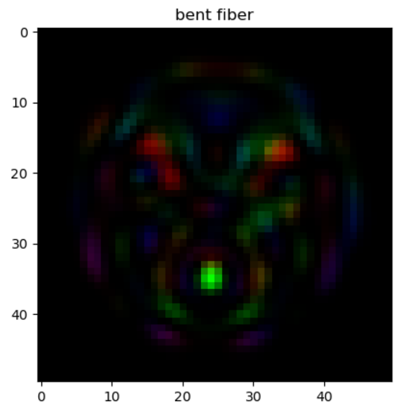

(f)

Figure 5: Propagation of a single test pixel point for (a) straight SI-MMFs and bent SI-MMFs with the curvature bent of (b) $2m^{-1}$ (c) $25m^{-1}$ (d) $70m^{-1}$on both x and y axis and a fiber length of 100mm (e)straight SI-MMFs and (f) $2m^{-1}$on both x and y axis and a fiber length of 500mm.

Around 252 theoretically predicted modes is calculated for a Numerical aperture of 0.095 and radius core of 40 μm. Not all the modes are shown in the above image (2a), however, the modes for both the polarization in positive an negative directions are shown in (2b).

The conversion matrix is just a representation of superposition of all the 2D modes into a matrix of 1D. The complete picture of TM in the fig(3a)represents polarization for both the orthogonal states which is given by positive and negative orbital angular direction indexed by l. The Converted Matrices shown in fig (4) for both straight and bent fibers show clearly a diagonal line which represents the conservation of polarization of a given mode, while the off diagonal components indicate the mutual coupling between the modes.From the results, it is observed that the mutual coupling between the modes is more strong with the increase in bending of the curvature. The simulation results above has been in good agreement with the experimental results [4]. Experimentally, the the mutual coupling of the modes could be influenced not just by the bending curvature, also other factors like misalignment of the light into the fiber, temperature, pressure, etc. This shows that the truly considered LP PIM are no more invariant. This interference in mode coupling is also seen to be slightly distinguishable with the increase in the propagating length of the fiber. Fig (5) shows the transformation of a single pixel light into a fiber for different bending an length. The focal point at the output generates a speckle pattern for both straight and curved fibers which shows the optical aberrations. Similar repeated simulation for other pixels and averaging can help us with subtract the aberration for the optical component.

4 Conclusion

In this project, I have numerically simulated and analysed theoretical model to understand different possible modes in a step-index multimode fibers both in a straight and a bending fibers for different length scales which is in agreement with [5]. The results showed that the optical fibers are more sensitive to the bending than the propagation length. This could be a crucial information in the field of Deep Imaging where the fibers have to undergo deeper along with some deformations. Nevertheless, the fiber could be used under threshold bending (for instance, $5^{-1}m$ upto a meter long scale. The aim of the MMFs should be to avoid any deformation possible, however, the reality is different than the simulation, thus, this could aid the process of designing experimental setups in future. Further theoretical investigations could be done to simulate the influence of other perturbations on the light propagation and the PIMs. With the advancement in engineering side, the optical fibers would certainly open doors to many other applications.

References

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