

const double PI = 3.14159265358979323846;

1. Distance Between Two Points (Centers of Circles):

The distance d between two points $(x1, y1)$ and $(x2, y2)$ is calculated using the **Euclidean distance formula**:

$$d = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$

This is used to determine the relative positions of the two circles.

2. Area of a Circle:

The area (A) of a circle with radius r is given by:

$$A = \pi r^2$$

This is used to calculate the area of each circle.

3. Conditions for Circle Intersection:

- No Intersection:**
If the distance d between the centers is greater than or equal to the sum of the radii ($d \geq r1 + r2$), the circles **do not intersect**.
 - One Circle Inside the Other:**
If the distance d is less than the absolute difference of the radii ($d < |r1 - r2|$), one circle is completely inside the other without intersecting.
 - Tangent Circles:**
If the distance d equals the sum or difference of the radii ($d = r1 + r2$) or ($d = |r1 - r2|$), the circles touch at exactly one point (tangent).
 - Partial Intersection:**
If the distance d is between the sum and difference of the radii ($|r1 - r2| < d < r1 + r2$), the circles intersect at two points.
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4. Area of Intersection of Two Circles:

When two circles intersect partially, the area of intersection can be calculated using the following steps:

- Step 1:** Calculate the distance d between the centers.
 - Step 2:** Use the **Law of Cosines** to find the angles subtended by the chord at the centers of the circles:
$$\text{angle1} = \cos^{-1} \left(\frac{r1^2 + d^2 - r2^2}{2 \cdot r1 \cdot d} \right)$$
$$\text{angle2} = \cos^{-1} \left(\frac{r2^2 + d^2 - r1^2}{2 \cdot r2 \cdot d} \right)$$
 - Step 3:** Calculate the area of the sectors of the circles:
$$\text{sectorArea1} = r1^2 \cdot \text{angle1}$$
$$\text{sectorArea2} = r2^2 \cdot \text{angle2}$$
 - Step 4:** Calculate the area of the triangles formed by the radii and the chord
$$\text{triangleArea1} = \frac{1}{2} \cdot r1^2 \cdot \sin(2 \cdot \text{angle1})$$
$$\text{triangleArea2} = \frac{1}{2} \cdot r2^2 \cdot \sin(2 \cdot \text{angle2})$$
 - Step 5:** The area of intersection is the sum of the sector areas minus the triangle areas:
$$\text{intersectionArea} = (\text{sectorArea1} - \text{triangleArea1}) + (\text{sectorArea2} - \text{triangleArea2})$$
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5. Area of the Crescent (Helal):

The area of the largest crescent (*Helal*) is the area of the larger circle minus the area of intersection:

$$\text{HelalArea} = \max(\text{area1}, \text{area2}) - \text{intersectionArea}$$

If the circles do not intersect or one is completely inside the other without touching, the *Helal* area is **undefined** (output `-1`).

6. Concentric Circles:

If the circles are concentric (same center) and have the same radius, they do not form a Helal. The output should be `-1`.

```
double d = sqrt((x2 - x1) * (x2 - x1) + (y2 - y1) * (y2 - y1)); // distance
if(d >= r1 + r2){ No Intersection or Externally Tangent Circles
return -1;
}
else if(d < abs(r1 - r2)){ One Circle Completely Inside the Other Without Intersection
return -1;
}
else if (d == 0 and r1 == r2) { Concentric Circles with Same Radius
return -1;
}
else{ Internally Tangent Circles or Partial Intersection – > Calculate the intersection area
double angle1 = acos((r1 * r1 + d * d - r2 * r2) / (2 * r1 * d));
double angle2 = acos((r2 * r2 + d * d - r1 * r1) / (2 * r2 * d));
double area1 = r1 * r1 * angle1 - 0.5 * r1 * r1 * sin(2 * angle1);
double area2 = r2 * r2 * angle2 - 0.5 * r2 * r2 * sin(2 * angle2);
return area1 + area2;
}
```