

Prime factorization

- get the prime factors of a number
main idea
- every composite number(not-prime) have at least 1 prime below \sqrt{n} so we check until \sqrt{n}
- if the remaining is not 1 then the number is prime

time complexity -- $\rightarrow \sqrt{n}$

code

```
vector<int> prime_factorization(int num) {  
    vector<int> factors;  
    if (num < 2) return factors;  
    while (!(num % 2)) {  
        factors.push_back(2);  
        num /= 2;  
    }  
    for (ll i = 3; i * i <= num; i += 2) {  
        while (!(num % i)) {  
            factors.push_back(i);  
            num /= i;  
        }  
    }  
    if (num > 1) factors.push_back(num);  
    return factors;  
}
```

Count_divisors

- count the number of divisors
we just iterate until \sqrt{n}
time complexity -- $\rightarrow \sqrt{n}$

code

```
int count_divisors(int num) {  
    int i, counter = 0;  
    for (i = 1; i * i < num; i++) {  
        if (!(num % i)) counter += 2;  
    }  
    if ((i * i) == num) counter++;  
    return counter;  
}
```

by prime fact

- number of divisors = pow_1+1 *pow₂+1* ..
ex : $12 = 2^2 \cdot 3^1 \rightarrow \text{no div} = 2+1 \cdot 1+1 = 6$

get_divisors

- get the divisors of a number
- we just iterate until \sqrt{n}
time complexity -- $\rightarrow \sqrt{n}$

code

```
vector<int> get_divisors(int num) {
    vector<int> divisors;
    ll i;
    for (i = 1; i * i < num; i++) {
        if (!(num % i)) {
            divisors.push_back(i);
            divisors.push_back(num / i);
        }
    }
    if((i * i) == num) divisors.push_back(i);
    return divisors;
}
```

bonus

- number of divisors = $\text{pow}_1 + 1$ *pow₂ + 1* ..
ex : 12 = 2^2 *3¹* -> *no div = 2 + 1* $1 + 1 = 6$

multiples loop - get divisors

```
const int N = 1000005;
vector<int> dv[N];
void get_divisors() {
    for(int i=1;i<N;++i)
    {
        for(int j=i;j<N;j+=i)
        {
            dv[j].push_back(i);
        }
    }
}
```