

## Having a point, p(a, b)

To get the *angle* between this vector, We can use *arg* or *atan2* functions.

Those two function output the angle with radian, if we need to convert it into degrees, just *(Val 180 / M\_PI)\**

```
#define ld long double
typedef ld T;
typedef complex<T> pt;

void RIM() {
    ld a, b; cin >> a >> b;
    pt p = {a, b};
    cout << arg(p) * 180 / M_PI << '\n';
    cout << atan2(b, a) << '\n';
}
```

Some Helpful functions: sq, abs, hypot, sgn

1. `T sq(pt p){`  
    `return p.x * p.x + p.y * p.y;`  
    `}`
2. `abs(p)` with `complex`  
    If `p = {x, y}` is a `complex<T>` point, then:  
    `abs(p) = sqrt(x*x + y*y)`  
    *It gives the distance from (0, 0) to (x, y) — the length of the vector.*  
    `abs` here equivalent to `hypot(a, b)`
3. `int sgn(T val){ --> return if the Val Pos, Neg, or Zero. (With doubles)`  
    `if(val > EPS) return 1;`  
    `if(val < -EPS) return -1;`  
    `else return 0;`  
    `}`

## Basic Trigonometry in Triangles (for CP)

### Right Triangle with sides:

- `a` – adjacent (base)
- `b` – opposite (height)
- `r` – hypotenuse (i.e., `r = sqrt(a2 + b2)` )

### Trig Ratios:

```
sin(θ) = b / r
cos(θ) = a / r
tan(θ) = b / a
```

### Law of Sines (for general triangles):

$$a / \sin(\alpha) = b / \sin(\beta) = c / \sin(\gamma) = 2R$$

Where `R` is the *circumradius* of the triangle. "نصف قطر المثلث في دائرة"

## Law of Cosines

- In any triangle with sides `a`, `b`, and `c`, and angle `θ` between `a` and `b`:  
$$\cos(\theta) = \frac{a^2 + b^2 - c^2}{2 * a * b}$$
- Use this to compute *angles* when all three sides are known.
- Rearranged to find side `c` if angle `θ` is known:  
$$c^2 = a^2 + b^2 - 2ab \cdot \cos(\theta)$$

## Tips:

- Use `atan2(b, a)` for angle from x-axis.
- Use `hypot(a, b)` instead of `sqrt(a*a + b*b)` for numerical stability.
- Memorize  $\sin^2\theta + \cos^2\theta = 1$  — useful for checks.

## Transformations

1. Translation
2. Scaling
3. Rotating

```
pt translate(pt p, pt v){ // translate point p with vector v
return p+v;
}
```

```
pt scale(pt c, ld factor, pt p){ // scale point p with specific factor around point c
return c + (p-c) * factor;
}
```

```
pt rotate(pt p, pt c, ld a){ // To rotate point p around point c with a specific angle with radian
pt v = p - c; // vector between two points
pt rotate = {cos(a), sin(a)};
return c + rotate * v;
}
```

```
pt linearTransformation(pt p, pt q, pt r, pt fp, pt fq){
return fp + (r-p) * (fq-fp) / (q-p);
}
```

---

## Dot product and cross product + angles

### Dot Product

```
T dot(pt v, pt w){
return v.x * w.x + v.y * w.y;
}
```

### Cross product

```
T cross(pt v, pt w){
return v.x * w.y - v.y * w.x;
}
```

### to get angle between two vectors out from the origin point (0, 0)

```
T angle(pt v, pt w){
return acos(clamp(dot(v, w) / abs(v) / abs(w), (T)-1.0, (T)1.0));
}
```

### to get an angle between a vector and x-axis (Polar angle)

```
const ld EPS = 1e-9;
const ld PI = 3.141592653589793238L;
```

```
int sgn(T val){
if(val > EPS) return 1;
if(val < -EPS) return -1;
else return 0;
}
```

```
void RIM() {
ld x, y;
cin >> x >> y;
ld c = atan2(y, x);
if(sgn(c) == -1)
```

```
c += 2 * PI;
cout << fixed << setprecision(10) << c;
}
```

To check if an angle between two vectors is perpendicular

```
bool isPerp(pt v, pt w){ return fabs(dot(v, w)) < EPS; }
```

if we have a vector (a, b), vector(-b, a) will be perpendicular on it on it's left side.

```
pt perp(pt v){
return (-v.y, v.x);
}
```