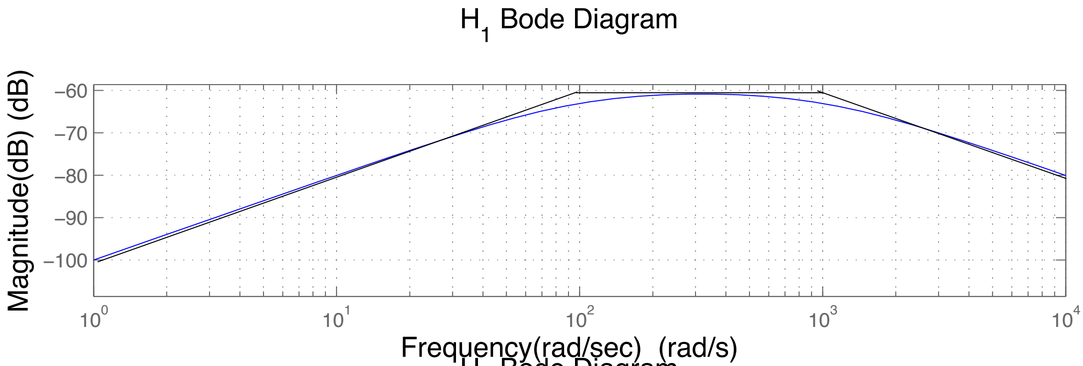
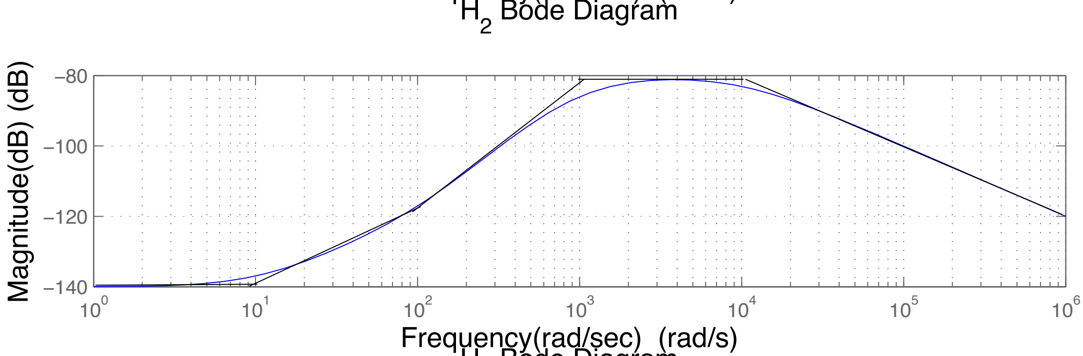
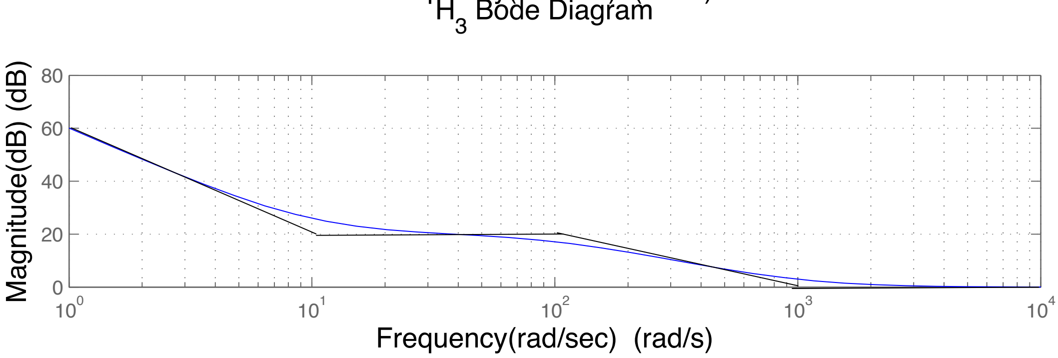
HW Assignment 8

Solution

Question 1







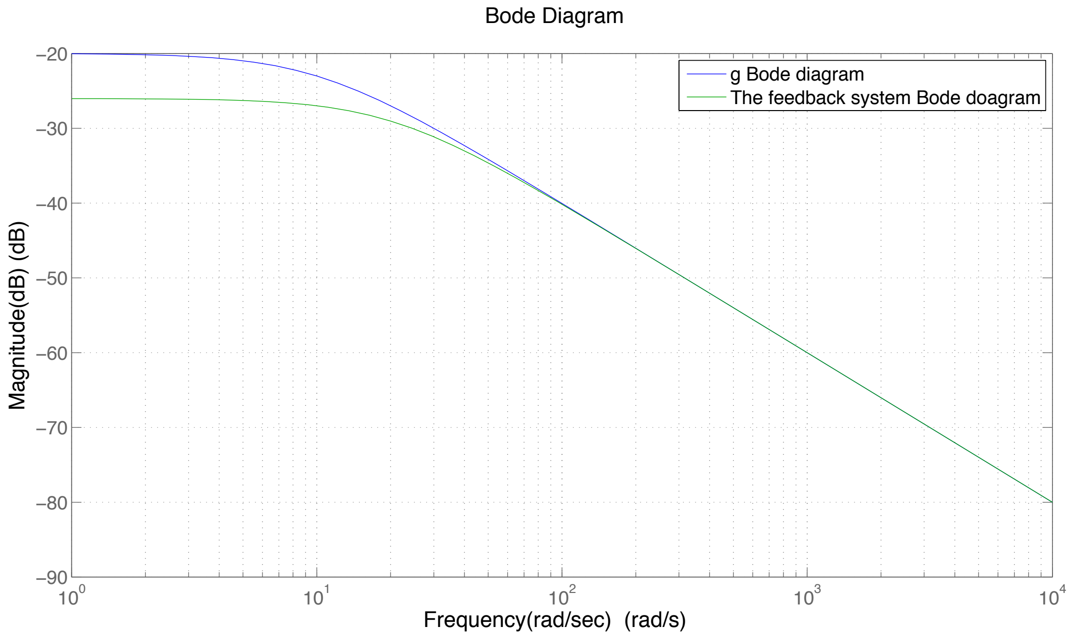
Question 2

1. The causal solution $Laplace^{-1}\left\{\frac{1}{s+\omega_c}\right\} = e^{-\omega_c t}u(t)$. In order for the system to be stable we have to demand $\omega_c > 0$.

2. & 3. The transfer function of the feedback system:

$$\left[X(s)-10Y(s)\right]G(s)=Y(s)\Rightarrow Y(s)\left[1+10G(s)\right]=G(s)X(s)\Rightarrow Y(s)=\frac{G(s)}{1+10G(s)}X(s)$$

$$H(s)=\frac{G(s)}{1+10G(s)}=\frac{\frac{1}{s+\omega_c}}{1+10\frac{1}{s+\omega_c}}=\frac{\frac{1}{s+10}}{1+\frac{10}{s+10}}=\frac{1}{s+20}$$



4. The feedback system's transfer function:

$$H(s)=\frac{G(s)}{1+kG(s)}=\frac{\frac{1}{s-10}}{1+k\frac{1}{s-10}}=\frac{1}{s-10+k}$$

In order for the system to be stable we want the pole to be negative, so we'll demand:

$$k-10>0\Rightarrow \boxed{k>10}$$

Question 3

1. $W(s)=\left[X(s)-G(s)W(s)\right]H_1(s)\rightarrow W\left[1+GH_1\right]=H_1X\rightarrow W=\frac{H_1}{1+GH_1}X$.

$$Y(s)=\left[W(s)+H_0(s)\right]X(s)=\underbrace{\left[\frac{H_1(s)}{1+G(s)H_1(s)}+H_0(s)\right]}_{\text{The transfer function}}X(s)$$

2. $$\left.\begin{aligned}W(s)&=\left[X(s)-Y(s)G_2(s)\right]H_2(s)\\ Y(s)&=\left[W(s)-G_1(s)Y(s)\right]H_1(s)\end{aligned}\right\}\rightarrow Y=\left\{\left[X-G_2Y\right]H_2-G_1Y\right\}H_1 .$$

$$\rightarrow Y\left[1+G_2H_2H_1+G_1H_1\right]=XH_1H_2\rightarrow Y(s)=\underbrace{\frac{H_1(s)H_2(s)}{1+G_1(s)H_1(s)+G_2(s)H_1(s)H_2(s)}}_{\text{The transfer function}}X(s)$$

Question 4

$$Y(s)=K\left[X(s)-\frac{s+1}{s+100}Y(s)\right]\rightarrow Y(s)\left(1+\frac{Ks+K}{s+100}\right)=Y(s)\left(\frac{s(K+1)+100+K}{s+100}\right)=KX(s)$$

$$\Rightarrow H(s)=\frac{Ks+100K}{s(K+1)+100+K}=\frac{100K}{100+K}\cdot\frac{\frac{s}{100}+1}{\frac{(K+1)}{K+100}s+1}$$

Zeros are always at $s_z = -100$, the poles are at:

1. $s_p = -99.0198$ 2. $s_p = -50.5$ 3. $s_p = -10$ 4. $s_p = -1.9802$

Question 5

1. The impulse response is $h(t) = ae^{-at}u(t)$. The step response is:

$$y_u(t) = u(t) * h(t) = \int_{-\infty}^{\infty} u(t-\tau)u(\tau)ae^{-a\tau}\,d\tau = a\int_0^t e^{-a\tau}\,d\tau = \left(1-e^{-at}\right)u(t)$$

2. $Y(s) = \dfrac{a}{s+a}\left(k_2F(s) + k_1Y(s)\right) \to Y(s)\left[1 - \dfrac{k_1a}{s+a}\right] = Y(s)\left[\dfrac{s+a-k_1a}{s+a}\right] = \dfrac{k_2a}{s+a}F(s)$.

   $\to Y(s) = \dfrac{k_2a}{s+a(1-k_1)}F(s)$

   The system is stable for any $k_1 < 1$.

3. In order for the gain to be corrected we must have $k_2 = 1 - k_1$ and so we get:

   $Y(s) = \dfrac{k_2a}{s+a(1-k_1)}F(s)$ define $\tilde{a} = a(1-k_1)$ (the effective mass of the liquid) and we get:

   $\to Y(s) = \dfrac{\tilde{a}}{s+\tilde{a}}F(s)$ that is now stable.

4. The more k_1 approaches to 1, the effective mass of the liquid decreases, and the step response becomes closer to a step function: $\dfrac{\tilde{a}}{s+\tilde{a}} \underset{k_1\to 1}{\to} \dfrac{\tilde{a}}{s}$.