HW Assignment 5

Due date: Thursday 4/7/2016

Question 1

The signal $f_1(t)=10^4 rect(10^4 t)$ is passed through an ideal low pass filter $H_1(\omega)=rect\left(\frac{\omega}{4\cdot 10^4 \pi}\right)$, to obtain the output signal $y_1(t)$. The signal $f_2(t)=\delta(t)$ is passed through a different low-pass filter $H_2(\omega)=rect\left(\frac{\omega}{2\cdot 10^4 \pi}\right)$ to obtain $y_2(t)$. the two outputs are multiplied $y(t)=y_1(t)\cdot y_2(t)$.

1. Sketch $F_1(\omega)$ and $F_2(\omega)$.
2. Sketch $H_1(\omega)$ and $H_2(\omega)$.
3. Sketch $Y_1(\omega)$ and $Y_2(\omega)$.
4. What is the bandwidth of the signals y_1, y_2 and y?

Note: we define $rect(x)=\begin{cases} 1 & |x|<1/2 \\ 0 & else \end{cases}$

Question 2

consider the functions

$$f_\alpha(t)=\begin{cases} \frac{1}{2}+\frac{1}{2}\cos\left(\frac{\pi}{\alpha}t\right) & |t|<\alpha \\ 0 & else \end{cases} \qquad g(t)=\begin{cases} 1 & |t|<1/2 \\ 0 & else \end{cases}$$

where $0<\alpha\leq 1/2$.

1. Write $f_\alpha(t)$ as a sum of three rect functions (with modulations = complex exponents).
2. Calculate $F_\alpha(\omega)$ - the Fourier transform of $f_\alpha(t)$.
3. Using any plotting software (Matlab is recommended), plot[1] $|F_\alpha(\omega)|$ for 3 different values of α.
4. plot the function $f_\alpha * g$ in the frequency domain for 3 different values of α.

Question 3

Consider the fluctuating solution to a second order ODE and it's analytical version:

$$f(t)=u(t)e^{-\alpha t}\cos(\omega_0 t)$$

$$f_a(t)=u(t)e^{-\alpha t}\frac{1}{2}e^{j\omega_0 t}$$

[1] Note that you are not supposed to calculate the Fourier transform using a computer - you need to calculate the correct formula analytically and then plot it.

1. Calculate the Fourier transforms $F(\omega)$ and $F_a(\omega)$.
2. Sketch (manually or using plotting software) the amplitudes $|F(\omega)|$ and $|F_a(\omega)|$.

Since in the positive range $|F_a(\omega)| \approx |F(\omega)|$, we shall use the analytical signal ($f_a(t)$) from now on, since the calculations are simpler.

3. What is the maximum value of $|F_a(\omega)|$? at what value of ω is the maximum attained? (<u>hint:</u> try to find the maximum value of $\left|\frac{1}{F_\alpha(\omega)}\right|^2$).
4. At what frequencies is the value of $|F_a(\omega)|$ equal to half of the maximum?

Denote by ω_r the frequency of the maximum and by ω_l and ω_h the half-maximum points.

5. Write the quality factor, defined $Q = \frac{\omega_r}{|\omega_h - \omega_l|}$, as a function of α and ω_0. What does it measure?
6. In you own words, describe the connection between the quality factor and attributes of the time-domain function $f_a(t)$. What does it imply about keeping a constant Q for different resonance frequencies ω_0?

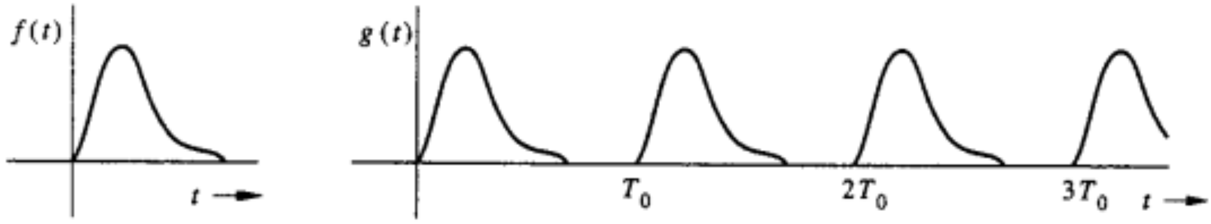
Question 4

Calculate the inverse Laplace transforms for the following functions and ROCs:

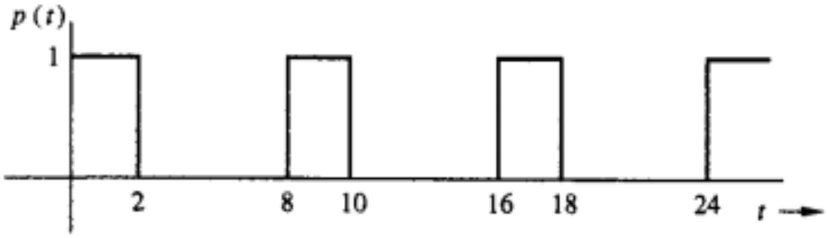
1. $X(s) = \frac{1}{s+4} \quad \mathrm{Re}\{s\} > -4$.
2. $X(s) = \frac{1}{s+4} \quad \mathrm{Re}\{s\} < -4$.
3. $X(s) = \frac{s}{s^2+1} \quad \mathrm{Re}\{s\} > 0$.
4. $X(s) = \frac{1}{s-2} \quad \mathrm{Re}\{s\} > 2$.
5. $X(s) = \frac{1}{(s-2)+\frac{1}{s-2}} \quad \mathrm{Re}\{s\} > 2$.

Question 5

The function $g(t)$ is a periodic continuation of function $f(t)$ with a cycle T_0.



1. Show that $G(s)=\dfrac{F(s)}{1-e^{-sT_0}}$.
2. Using this result, calculate the Laplace transform of the signal $p(t)$



Question 6

consider the function $x(t)=u(t)e^{-t}$

1. Calculate $F(s)$.
2. Calculate $F(\omega)$, how come it exists?
3. What's the connection between $F(s)$ and $F(\omega)$?