HW Assignment 4

Due date: Thursday 31/3/2016

Question 1

The signal $x(t)$ is a rectangle of size T_1:

$$X(t)=\begin{cases} 1 & |t|<\frac{T_1}{2} \\ 0 & else \end{cases}$$

and $\tilde{x}(t)$ is it's periodic repetition with period $T_0=\frac{3}{2}T_1$.

1. Sketch $x(t)$ and $\tilde{x}(t)$.
2. Calculate $X(\omega)$ and sketch $|X(\omega)|$ in the range $|\omega|\le\frac{6\pi}{T_1}$.
3. Calculate the Fourier series coefficients d_n of $\tilde{x}(t)$, and sketch then for $-3\le n\le 3$.
4. How can the Fourier series for a periodic function be obtained by using the Fourier transform of one period of the same function?

Question 2

For the following signals, calculate the Fourier transform and sketch the amplitude and phase:

1. $x(t)=\delta(t-5)$.
2. $x(t)=e^{(-1+2j)t}u(t)$.
3.
   - $x_1(t)=rect(t)$.
   - $x_2(t)=rect\left(\frac{t}{2\pi}\right)$
   - Can you make any observation regarding $X_1(\omega),X_2(\omega)$ band widths (defined as the first frequency ω_0 where $X(\omega_0)=0$)?

Question 3

1. Prove the following Fourier transform integration formula - for $x(t)$ such that $Fourier\{x(t)\}=X(\omega)$:

$$\int_{-\infty}^{t}x(\tau)d\tau \xrightarrow{Fourier} \frac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$$

<u>Hints:</u>

- $\int_{-\infty}^{t}x(\tau)d\tau=\int_{-\infty}^{\infty}x(\tau)\underbrace{u(t-\tau)}_{\text{step function}}.$

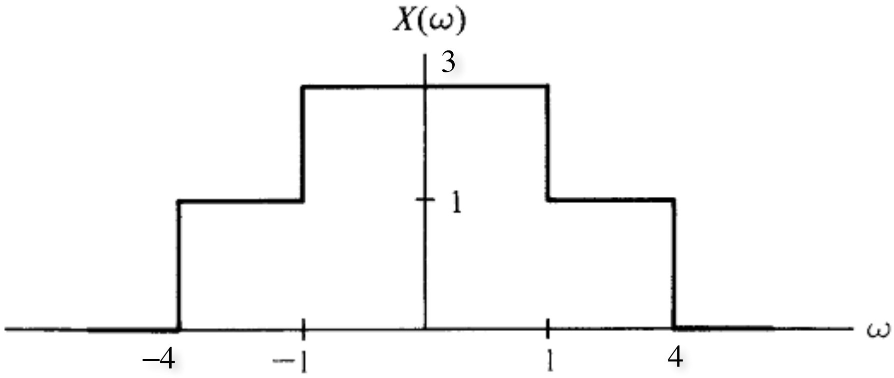
- $fourier\{u(t)\} = \lim_{a\to 0} fourier\{e^{-at}u(t)\}$.
- $\lim_{a\to 0}\left(\frac{a}{a^2+\omega^2}\right) \to \pi\delta(\omega)$

2. Prove the uncertainty property: $f(at) \to \frac{1}{|a|}F\left(\frac{\omega}{a}\right)$.
3. Prove the complex conjugate property: $f^*(t) \to F^*(-\omega)$

   Hints:
   - $f(t) = \mathrm{Re}\{f(t)\} + i\,\mathrm{Im}\{f(t)\}$.
   - $e^{-j\omega t} = \cos(\omega t) - i\sin(\omega t)$.

Question 4

What is the time-domain representation $x(t)$ of the function whose Fourier representation $X(\omega)$ is:



Plot the result (either by hand or using Matlab).

Question 5

The raised-cosine function is defined as

$$x(t) = \begin{cases} 1+\cos(at) & |t| < \pi/a \\ 0 & else \end{cases}$$

1. Calculate it's Fourier transform $X(\omega)$.
2. What is the Fourier transform of it's ramped version $tx(t)$?
3. Calculate $\frac{d}{dt}x(t)$ and it's Fourier transform.

Question 6

Consider a function $f(t)$ whose Fourier transform is given by $F(\omega)$.

1. What is the inverse Fourier transform of $f(t)$?
2. What is the result of applying 3 consecutive Fourier transforms to $f(t)$?
3. What is the result of applying 4 consecutive Fourier transform to $f(t)$?