HW Assignment 1

Due date: Thursday 10/3/2016

Question 1

1. Plot the signal

$$u(t)=\begin{cases} -1 & t\in[0,1] \\ 2-t & t\in[1,2] \\ 0 & else \end{cases}$$

2. Given the system $T[x](t)=x\left(\frac{3}{2}t+1\right)$, plot the effect of T on $u(t)$.
3. Is T linear? Is T time-invariant?
4. Given the system $R[x](t)=x^2(t)$, plot the effect of $R[T[x]](t)$ on $u(t)$.
5. Is R linear? Is R time-invariant?
6. Is $R\circ T$ linear? Is $R\circ T$ time-invariant?

Question 2

Find the energy of the signal $u(t),v(t),u(t)+v(t)$ and $u(t)-v(t)$ for the following pairs of signals:

1. $u(t)=\begin{cases} 1 & t\in[0,2] \\ 0 & else \end{cases}$ $\quad v(t)=\begin{cases} 1 & t\in[0,1] \\ -1 & t\in[1,2] \\ 0 & else \end{cases}$.

2. $u(t)=\begin{cases} 1 & t\in\left[0,\pi/4\right] \\ -1 & t\in\left[\pi/4,\pi\right] \\ 0 & else \end{cases}$ $\quad v(t)=\begin{cases} 1 & t\in[0,\pi] \\ 0 & else \end{cases}$.

Can you make any observation from these results?

Question 3

1. Show that the derivative operator $D[x](t)=\frac{d}{dt}x(t)$ is LTI.
2. Show that the second derivative operator $D^2[x](t)=\frac{d^2}{dt^2}x(t)$ is LTI. Use two methods - direct and by using the result of the previous question.
3. Prove that the nth derivative operator $D^n[x](t)=\frac{d^n}{dt^n}x(t)$ is LTI (i.e linear <u>and</u> time-invariant), <u>hint</u>: prove by induction.

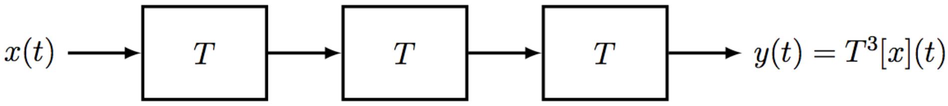
Question 4

Let T_a, T_b be two LTI systems.

1. Show that $T_{sum} = T_a + T_b$ is LTI (i.e linear <u>and</u> time-invariant).
2. Show that $T_{series} = T_b\left[T_a[x]\right](t)$ is time-invariant (we showed linearity in class).

Question 5 - generalization of question 3 and 4

Given a system T, we denote by T^n the application of T to the signal n times in series (see example in the diagram).



1. Show that if T is LTI than T^n is LTI.
2. Show that if T is LTI than the polynomial system $\sum_n \alpha_n T^n$ is LTI.