HW Assignment 5

Solution

Question 1

Question 4.4-3 from the Lathi book.

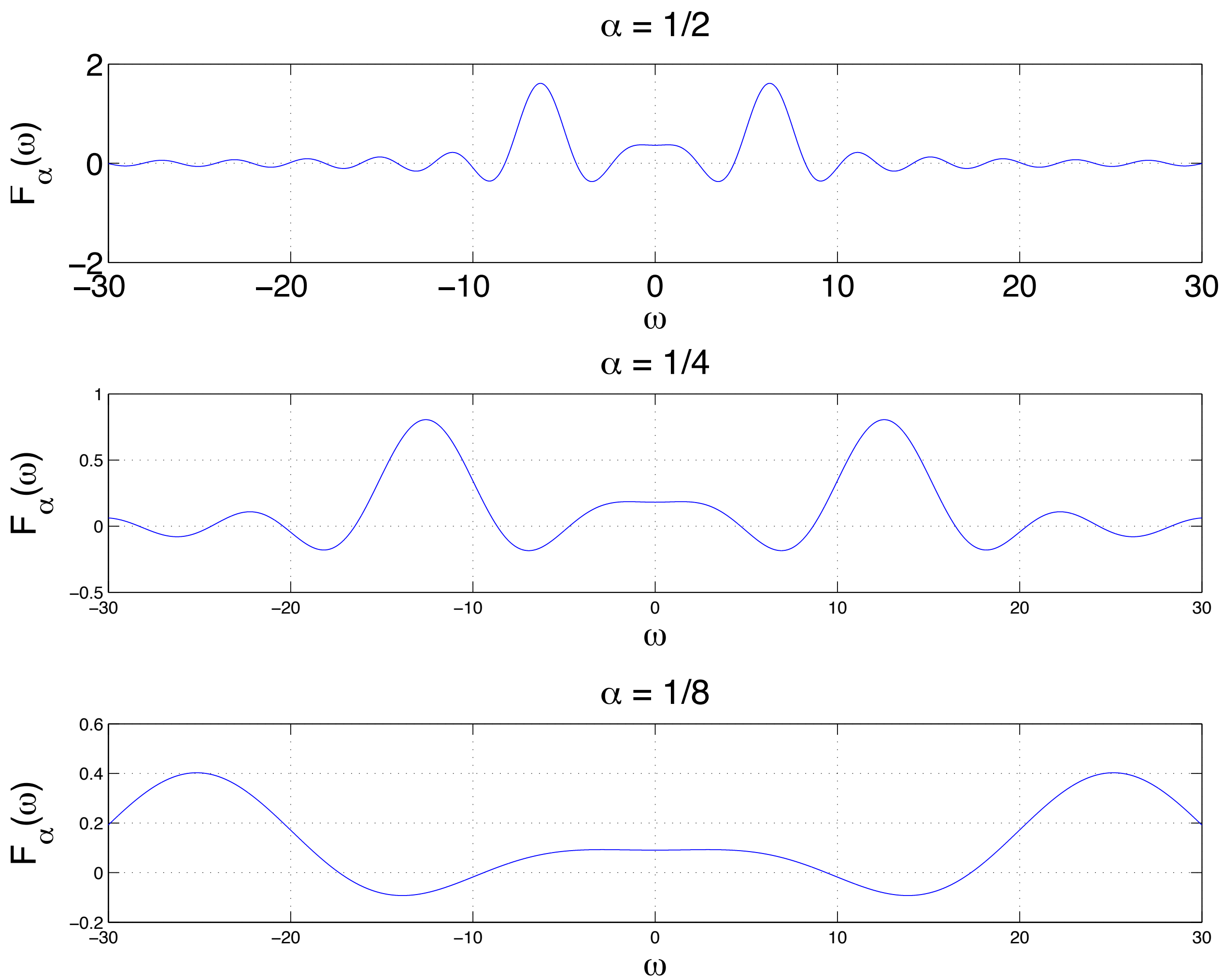
Question 2

1. $f_\alpha = \frac{1}{2}\left(rect\left(\frac{t}{2\alpha}\right) + \frac{1}{2} e^{j\frac{\pi}{\alpha}t} rect\left(\frac{t}{2\alpha}\right) + \frac{1}{2} e^{-j\frac{\pi}{\alpha}t} rect\left(\frac{t}{2\alpha}\right)\right).$

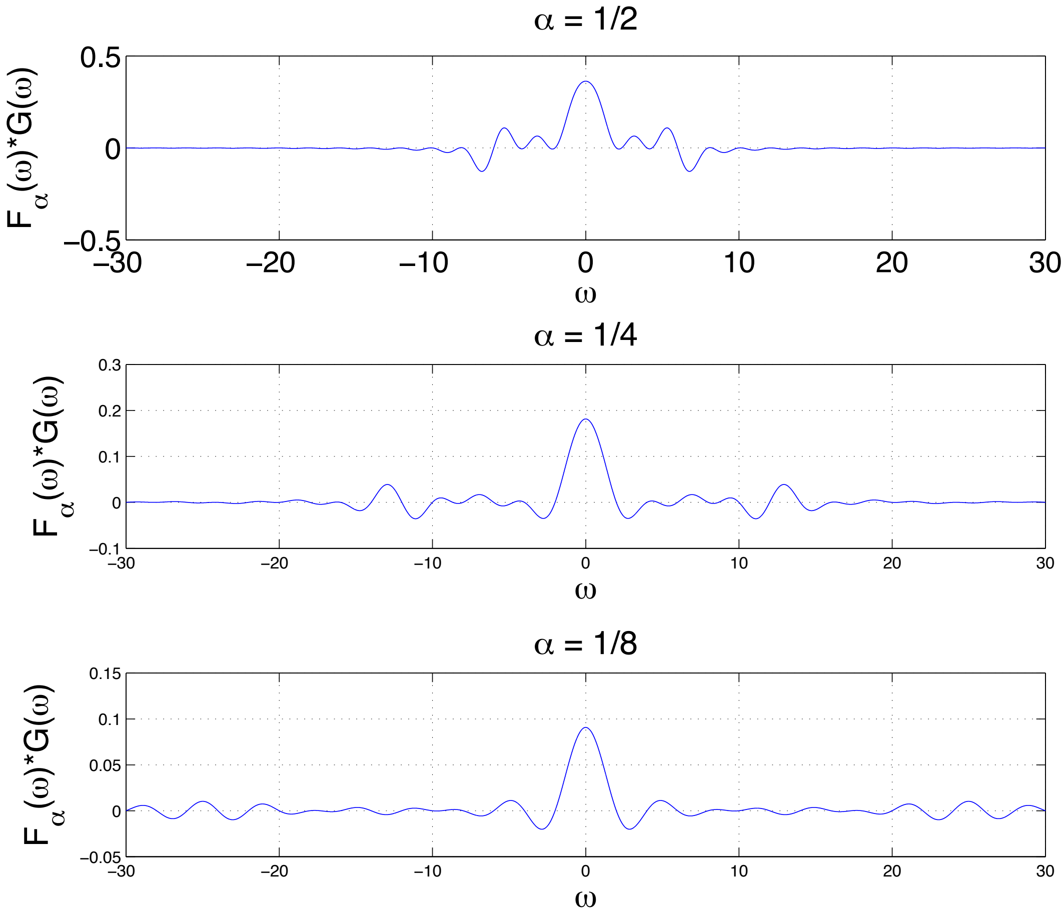
2. $F_\alpha(\omega) = \frac{1}{2}\left(2\alpha \mathrm{sinc}(\alpha\omega) + \pi\delta\left(\omega - \frac{\pi}{\alpha}\right) * 2\alpha \mathrm{sinc}(\alpha\omega) + \pi\delta\left(\omega + \frac{\pi}{\alpha}\right) * 2\alpha \mathrm{sinc}(\alpha\omega)\right) = .$

   $= \left(\alpha \mathrm{sinc}(\alpha\omega) + \alpha\pi \mathrm{sinc}(\alpha\omega - \pi) + \alpha\pi \mathrm{sinc}(\alpha\omega + \pi)\right)$

3.



4. $Fourier\{f_\alpha * g\} = F_\alpha \cdot G = \left(\alpha \mathrm{sinc}(\alpha\omega) + \alpha\pi \mathrm{sinc}(\alpha\omega - \pi) + \alpha\pi \mathrm{sinc}(\alpha\omega + \pi)\right)\mathrm{sinc}\left(\omega/2\right)$

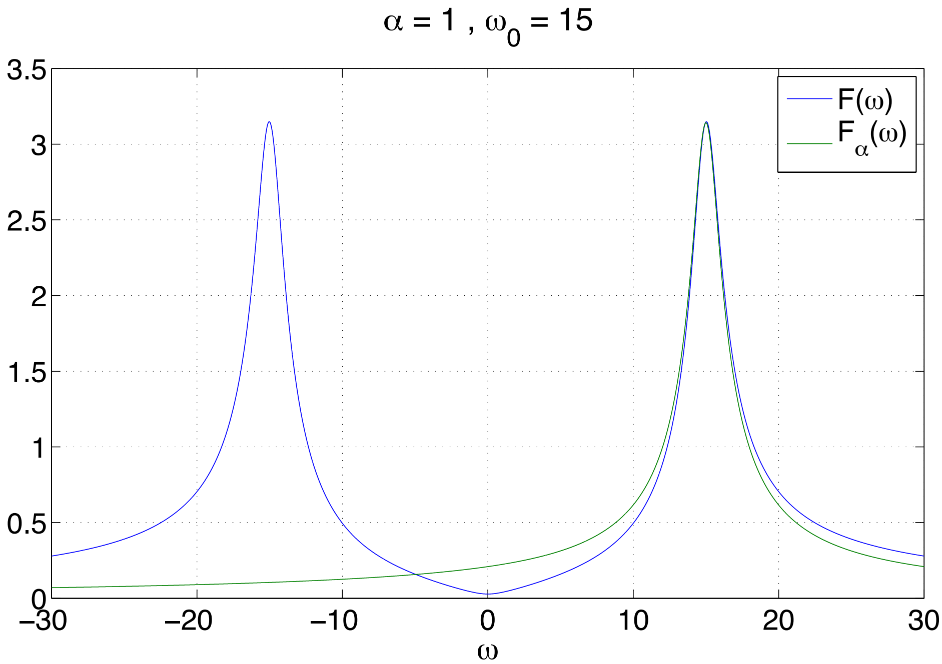


Question 3

1. $$F(\omega)=\frac{1}{\alpha+j\omega}*\pi\delta(\omega-\omega_0)+\frac{1}{\alpha+j\omega}*\pi\delta(\omega+\omega_0)=\pi\left[\frac{1}{\alpha+j(\omega-\omega_0)}+\frac{1}{\alpha+j(\omega+\omega_0)}\right].$$

$$F_a(\omega)=\frac{1}{\alpha+j\omega}*\pi\delta(\omega-\omega_0)=\pi\frac{1}{\alpha+j(\omega-\omega_0)}$$

2.



3. $\left|\frac{1}{F_a(\omega)}\right|^2 = \frac{1}{\pi^2}\left[\alpha + j(\omega-\omega_0)\right]\left[\alpha - j(\omega-\omega_0)\right] = \frac{1}{\pi^2}\left[\alpha^2 + (\omega-\omega_0)^2\right] \rightarrow$

$$\frac{d}{d\omega}\left|\frac{1}{F_a(\omega)}\right|^2 = \frac{2}{\pi^2}(\omega-\omega_0) = 0 \Rightarrow \omega_r = \omega_0$$

The maximum is attained at $\omega_r = \omega_0$ and it's value is $\left|F_a(\omega=\omega_0)\right| = \frac{\pi}{a}$

4. The half-amplitude frequencies are equivalent to finding:

$$\left|\frac{1}{F_a(\omega)}\right|^2 = 2^2 \cdot \left(\frac{\alpha}{\pi}\right)^2$$

$$4\alpha^2 = \alpha^2 + (\omega-\omega_0)^2 \Rightarrow \omega^2 - 2\omega_0\omega + \omega_0{}^2 - 3\alpha^3 = 0$$

$$\omega_{1,2} = \omega_2 \pm \frac{1}{2}\sqrt{4\omega_0{}^2 - 4\left(\omega_0{}^2 - 3\alpha^2\right)} = \omega_0 \pm \sqrt{3}\alpha$$

5. $Q = \frac{\omega_r}{\left|\omega_h - \omega_l\right|} = \frac{\omega_0}{\left|\omega_0 + \sqrt{3}\alpha - \left(\omega_0 - \sqrt{3}\alpha\right)\right|} = \frac{\omega_0}{\sqrt{12}\alpha}$.

Question 4

1. $x(t) = u(t)e^{-4t}$.
2. $x(t) = -u(-t)e^{-4t}$.
3. $x(t) = u(t)\cos(t)$.
4. $x(t) = u(t)e^{2t}$.
5. $x(t) = u(t)e^{2t}\cos(t)$.

Question 5

From Lathi's book, Ques. 6.2-4.

Question 6

1. $X(s) = \int_0^\infty e^{-t}e^{-st}\,dt = \left.\frac{e^{-(1+s)t}}{-(1+s)}\right|_0^\infty = \frac{1}{s+1}$ $\quad \text{ROS}=\left\{s : \text{Re}\{s\} > -1\right\}$.

2. $X(\omega) = \int_0^\infty e^{-t}e^{-j\omega t}\,dt = \left.\frac{e^{-(1+j\omega)t}}{-(1+j\omega)}\right|_0^\infty = \frac{1}{1+j\omega}$. It exists because $s : \text{Re}\{s\} = 0 \in ROC$.

3. $$X(s) = \frac{1}{s+1} = \frac{1}{\underbrace{\sigma + j\omega}_{=s} + 1}$$
$$X(\omega) = \frac{1}{1+j\omega} = X(s)\Big|_{\sigma=0}$$
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