HW Assignment 11

Solution

Question 1

1. The given system:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} f$$

so the transition matrix A is:

$$\boxed{A = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}}$$

2. The new state vector ω

$$\begin{pmatrix} \omega_1(t) \\ \omega_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

so the base transfer matrix:

$$P = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$$

$$\hat{A} = PAP^{-1} = P = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -3 & 1 \end{pmatrix}$$

$$\hat{B} = PB = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

so the state equation using the new state variables:

$$\boxed{\begin{pmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} f(t)}$$

3. The eigenvalues of A:

$$|\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ 1 & \lambda+1 \end{vmatrix} = \lambda(\lambda+1)+1 = \lambda^2+\lambda+1 \Rightarrow \lambda_{1,2} = \frac{-1 \pm j\sqrt{3}}{2}$$

The eigenvalues of $\hat{A}$:

$$|\lambda I - \hat{A}| = \begin{vmatrix} \lambda+2 & -1 \\ 3 & \lambda-1 \end{vmatrix} = (\lambda+2)(\lambda-1)+3 = \lambda^2+\lambda-2+3 = \lambda^2+\lambda+1 \Rightarrow \lambda_{1,2} = \frac{-1 \pm j\sqrt{3}}{2}$$

same eigenvalues.

Question 2

Question 1 from moed A 2015.

Question 3

1. The system is controllable since $\hat{b}_i \neq 0$ for $i = 1,2$.

2. First will find $\begin{pmatrix} x_1(0^+) \\ x_2(0^+) \end{pmatrix}$ in the diagonal basis:

$$\begin{pmatrix} \omega_1(0^+) \\ \omega_2(0^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_1(0^+) \\ x_2(0^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

Now we'll "guess" the solution $f(t) = c\delta(t) + d\dot{\delta}(t)$ and find c,d:

$$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} b_1 & b_1 a_{11} \\ b_2 & b_2 a_{22} \end{pmatrix}^{-1} \begin{pmatrix} \omega_1(0^+) \\ \omega_1(0^+) \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & -3 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 3 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

so we got:

$$f(t) = \delta(t) - \dot{\delta}(t)$$