HW Assignment 5

Solution

Question 1

1. $X(s)=\frac{s+9}{s^2+3s-4}=\frac{s+9}{(s+4)(s-1)}=\frac{a}{s+4}+\frac{b}{s-1}$.

   Common denominator:

$$s+7=a(s-1)+b(s+4)$$

$$\Rightarrow \begin{array}{ll} s: & 1=a+b\rightarrow a=1-b \\ 1: & 9=-a+4b=5b-1\rightarrow\begin{cases}b=2\\a=-1\end{cases}\end{array}$$

   and we got:

$$X(s)=-\frac{1}{s+4}+\frac{2}{s-1}$$

   the inverse Laplace transform for $ROC=\{s:\mathrm{Re}\{s\}>1\}$ will be: $x(t)=u(t)\left[-e^{-4t}+2e^{t}\right]$ (causal but not stable).

2. $X(s)=-\frac{1}{s+4}+\frac{2}{s-1}\rightarrow$ the inverse Laplace transform for $ROC=\{s:-4<\mathrm{Re}\{s\}<1\}$ will be: $x(t)=4u(t)e^{-4t}-2u(-t)2e^{t}$ (stable but not casual).

3. $X(s)=-\frac{1}{s+4}+\frac{2}{s-1}\rightarrow$ the inverse Laplace transform for $ROC=\{s:\mathrm{Re}\{s\}<-4\}$ will be: $x(t)=-4u(-t)e^{-4t}-2u(-t)2e^{t}$ (neither stable nor casual).

4. $X(s)=\frac{s^2-s+1}{s^2(s-1)}=\frac{a}{s}+\frac{b}{s^2}+\frac{c}{s-1}$.

   Common denominator:

$$A(s^2-s)+B(s-1)+Cs^2=(A+C)s^2+(B-A)s-B=s^2-s+1$$

$$\rightarrow \begin{array}{ll} s^2 & b=-1 \\ s: & b-a=-1\rightarrow a=0 \\ 1: & c=1\end{array}$$

   We got:

$$X(s)=-\frac{1}{s^2}+\frac{1}{s-1}$$

   the inverse Laplace transform for $ROC=\{s:0<\mathrm{Re}\{s\}<1\}$ will be: $x(t)=-u(-t)e^{t}-tu(t)$.

5. $X(s)=\frac{s^2+1}{(s+1)^2}=\frac{s^2+2s+1}{s^2+2s+1}+\frac{-2s}{s^2+2s+1}=1-\frac{2s}{(s+1)^2}=1+\frac{a}{(s+1)}+\frac{b}{(s+1)^2}$.

   Common denominator:

$$2s=a(s+1)+b$$

$$\rightarrow \begin{array}{ll} s: & a=-2 \\ 1: & a+b=0\rightarrow b=2\end{array}.$$

   We got:

$$X(s)=1-\frac{2}{(s+1)}+\frac{2}{(s+1)^2}$$

the inverse Laplace transform for $ROC=\{s:\mathrm{Re}\{s\}>-1\}$ will be: $x(t)=\delta(t)+2u(t)e^{-t}(t-1)$.

6. $X(s)=\dfrac{s+1}{(s+1)^2+4}=\dfrac{s+1}{(s+1+2i)(s+1-2i)}=\dfrac{a}{s+1+2i}+\dfrac{b}{s+1-2i}$.

Common denominator:

$$s+1=a(s+1-2i)+b(s+1+2i)$$

$$\rightarrow\begin{cases} s: \quad 1=a+b\rightarrow a=1-b \\ 1: \quad 1=a(1-2i)+b(1+2i)=(1-2i)+4ib\rightarrow\begin{cases} b=\frac{1}{2} \\ a=\frac{1}{2}\end{cases}\end{cases}$$

We got:

$$X(s)=\tfrac{1}{2}\left(\frac{1}{s+1+2i}+\frac{1}{s+1-2i}\right)$$

the inverse Laplace transform for $ROC=\{s:\mathrm{Re}\{s\}>-1\}$ will be:

$x(t)=\dfrac{1}{2}u(t)e^{-t}\left[e^{2it}+e^{-2it}\right]=u(t)e^{-t}\cos(2t)$.

Question 2

1. $X(s)=\int_{-\infty}^{\infty}u(t)e^{-t}e^{-st}\,dt=\int_0^{\infty}e^{-(1+s)t}\,dt=-\left.\dfrac{e^{-(1+s)t}}{s+1}\right|_0^{\infty}=\dfrac{1}{s+1}$, $\mathrm{Re}\{s\}>-1$.

$H(s)=\int_{-\infty}^{\infty}u(t)e^{-3t}e^{-st}\,dt=\int_0^{\infty}e^{-(3+s)t}\,dt=-\left.\dfrac{e^{-(3+s)t}}{s+3}\right|_0^{\infty}=\dfrac{1}{s+3}$, $\mathrm{Re}\{s\}>-3$

2. $Y(s)=X(s)\cdot H(s)=\dfrac{1}{(s+1)(s+3)}\quad \mathrm{Re}\{s\}>-1$

3. $Y(s)=\dfrac{1}{(s+1)(s+3)}=\dfrac{a}{(s+1)}+\dfrac{b}{(s+3)}$. common denominator: $1=a(s+3)+b(s+1)$

$$\begin{matrix} s: & a=-b \\ \rightarrow \quad 1: & 1=3a+b=-2b\Rightarrow\begin{cases} b=-\frac{1}{2} \\ a=\frac{1}{2}\end{cases}\end{matrix}$$ and we got: $Y(s)=\dfrac{1}{2}\left(\dfrac{1}{(s+1)}-\dfrac{1}{(s+3)}\right)$.

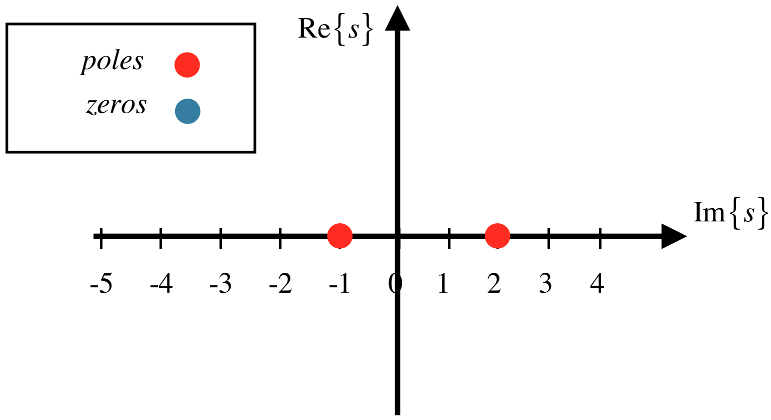
the inverse Laplace transform for $ROC=\{s:\mathrm{Re}\{s\}>-1\}$ will be: $\dfrac{1}{2}u(t)\left[e^{-t}-e^{-3t}\right]$

Question 3

1. $\ddot{y}(t)-\dot{y}(t)-2y(t)=x(t)\ \xrightarrow{Laplace}\ s^2Y(s)-sY(s)-2Y(s)=X(s)$

$\rightarrow(s^2-s-2)Y(s)=X(s)\rightarrow Y(s)=\underbrace{\dfrac{1}{s^2-s-2}}_{H(s)}X(s)=Y(s)=\underbrace{\dfrac{1}{(s-2)(s+1)}}_{H(s)}X(s)$.

We got 2 poles at $s=2,-1$.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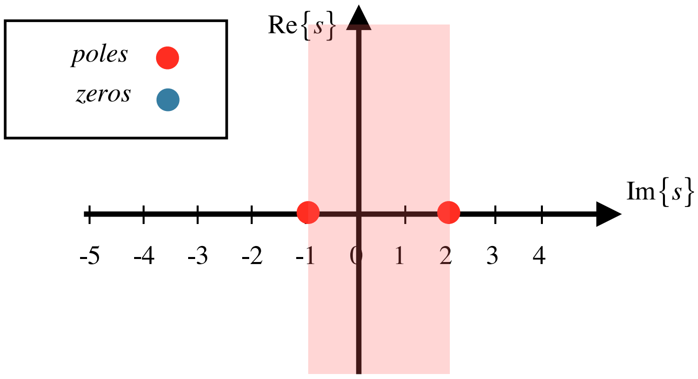
2.

2.1.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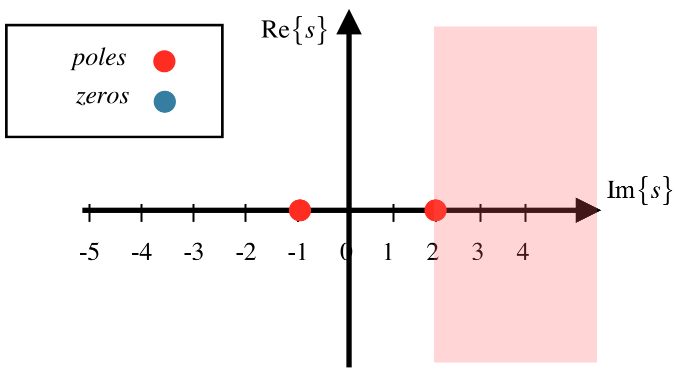
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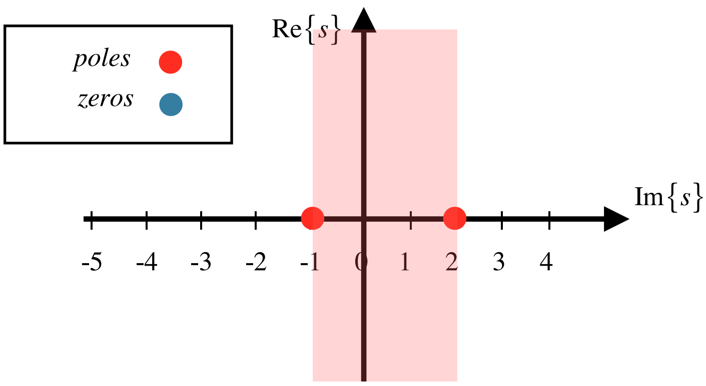
2.3.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3. $H(s)=\frac{1}{(s-2)(s+1)}=\frac{a}{(s-2)}+\frac{b}{(s+1)}\Rightarrow 1=a(s+1)+b(s-2)$.

$$s: \quad 0=a+b\rightarrow a=-b$$

$$1: \quad 1=a-2b\rightarrow\begin{cases}b=-1/3\\ a=1/3\end{cases}$$. and so $H(s)=\frac{1}{3}\left[\frac{1}{s-2}-\frac{1}{s+1}\right]$

For causal system: $ROC=\{s:\mathrm{Re}\{s\}>2\}$ we get: $h(t)=-\frac{1}{3}u(t)\left[e^{-t}+e^{2t}\right]$.

Question 4

$$\dddot{y}(t)+5\ddot{y}(t)+7\dot{y}(t)+3y(t)=2\ddot{x}(t)+\dot{x}(t)\Rightarrow s^3Y(s)+5s^2Y(s)+7sY(s)+3Y(s)=2s^2X(s)+sX(s)$$

$$\downarrow$$

$$\left(s^3+5s^2+7s\right)Y(s)=\left(2s^2+s\right)X(s)\Rightarrow H(s)=\frac{2s^2+s}{s^3+5s^2+7s}=\frac{s(2+s)}{(s+1)^2(s+3)}=\frac{a}{(s+1)}+\frac{b}{(s+1)^2}+\frac{c}{(s+3)}$$

$$\downarrow$$

$$2s^2+s=a\left(s^2+4s+3\right)+b(s+3)+c\left(s^2+2s+1\right)$$

$$\downarrow$$

$$\begin{aligned} s^2: &\quad 2=a+c\rightarrow a=2-c\\ s: &\quad 1=4a+b+2c\\ 1: &\quad 0=3a+3b+c \end{aligned}$$

$$3s-1: \quad 3=9a+5c=18-4c\rightarrow\begin{cases}c=15/4\\ a=-7/4\\ b=1/2\end{cases}$$

$$\left(s^3+5s^2+7s\right)Y(s)=\left(2s^2+s\right)X(s)\Rightarrow H(s)=-\frac{7/4}{(s+1)}+\frac{1/2}{(s+1)^2}+\frac{15/4}{(s+3)}$$

the casual and stable solution:

$$h(t)=u(t)\left[\left(\frac{1}{2}t-\frac{7}{4}\right)e^{-t}+\frac{15}{4}e^{-3t}\right]$$