HW Assignment 2

Solution

Question 1

The characteristic polynomial:

$$\lambda^3+5\lambda^2+7\lambda+3=0$$

We'll use polynomial devision starting with the root -1:

$$\begin{array}{l} \quad\ \ \lambda^2+4\lambda+3 \\ \overline{\lambda^3+5\lambda^2+7\lambda+3}\,\big|\,\lambda+1 \\ -\underline{\lambda^3+\lambda^2} \\ \qquad 4\lambda^2+7\lambda \\ \quad -\underline{4\lambda^2+4\lambda} \\ \qquad\qquad 3\lambda+3 \\ \qquad\quad -\underline{3\lambda+3} \\ \qquad\qquad\quad 0 \end{array}$$

We got: $\lambda^3+5\lambda^2+7\lambda+3=(\lambda^2+4\lambda+3)(\lambda+1)=(\lambda+1)^2(\lambda+3)$,so the solutions of the polynomial are: $-1,-1$ and -3 making y

$$y(t)=(c_1+c_2t)e^{-t}+c_3e^{-3t}$$

by using the initial conditions we get a linear set of equations:

$$y(0)=c_1+c_3=0$$
$$\dot{y}(0)=-c_1+c_2-3c_3=2$$
$$\ddot{y}(0)=c_1-2c_2+9c_3=4$$

making the solution

$$y(t)=(6t-2)e^{-t}+2e^{-3t}$$

Question 2

Writing the above system using the notation we saw in class $Q(D)[y](t)=P(D)[x](t)$ we get:

$$Q(D)=\sum_{i=0}^{n}a_1D^i=D^3+5D^2+7D+3$$
$$P(D)=\sum_{i=0}^{m}b_iD^i=2D^2+D$$

when $n=3,m=2$.

The impulse response in given by $h(t)=b_n\delta(t)+[P(D)y_n]u(t)$ where:

- $n>m$ and therefor $b_n=0$.

- $P(D) = 2D^2 + D$.
- y_n is the solution of $\dddot{y}(t) + 5\ddot{y}(t) + 7\dot{y}(t) + 3y(t) = 0$ (same as Q1)using the initial conditions: $y(0) = \dot{y}(0) = 0, \ddot{y}(0) = 1$:

$$\begin{aligned} y(0) &= c_1 + c_3 = 0 \\ \dot{y}(0) &= -c_1 + c_2 - 3c_3 = 0 \\ \ddot{y}(0) &= c_1 - 2c_2 + 9c_3 = 1 \end{aligned}$$

we get:

$$y_n(t) = \left(\frac{1}{2}t - \frac{1}{4}\right)e^{-t} + \frac{1}{4}e^{-3t}$$

$$\boxed{h(t) = b_n\delta(t) + \left[P(D)y_n\right]u(t) = \left[2\frac{d^2}{dt^2}y_n(t) + \frac{d}{dt}y_n(t)\right]u(t) = \left[\left(\frac{1}{2}t - \frac{7}{4}\right)e^{-t} + \frac{15}{4}e^{-3t}\right]u(t)}$$

Question 3

$$y(t) = x(t) * h(t) = 4e^{-3t}u(t) * \left[\left(\frac{1}{2}te^{-t} - \frac{7}{4}e^{-t} + \frac{15}{4}e^{-3t}\right)u(t)\right] =$$

$$= 2e^{-3t}u(t) * te^{-t}u(t) - 7e^{-3t}u(t) * e^{-t}u(t) + 15e^{-3t}u(t) * e^{-3t}u(t)$$

From the convolution table we get

$$\boxed{y(t) = \left[(-4 + t)e^{-t} + (4 + 15t)e^{-3t}\right]u(t)}$$

Question 4

1. $\frac{d^2}{dt^2}i(t) + \frac{R}{L}\frac{d}{dt}i(t) + \frac{1}{LC}i(t) = 0$.
2. The roots are $\lambda_{1,2} = \frac{-R \pm \sqrt{R^2 - 4\frac{L}{C}}}{2L}$. The three phases depend on the sign go the discriminant $R^2 - 4\frac{L}{C} \gtreqless 0$:
   - If $R^2 - 4\frac{L}{C} > 0$ we get $\lambda_1 \neq \lambda_2 \in \mathbb{R}$. Therefor the solution is: $i(t) = c_1e^{\lambda_1 t} + c_2e^{\lambda_2 t}$.
   - If $R^2 - 4\frac{L}{C} = 0$ we get $\lambda_1 = \lambda_2 = \frac{-R}{2L} \triangleq \lambda \in \mathbb{R}$. Therefor the solution is: $i(t) = (c_1 + tc_2)e^{\lambda t}$.
   - if $R^2 - 4\frac{L}{C} < 0$, define $\alpha = -\frac{R}{L}, \beta = \frac{\sqrt{4\frac{L}{C} - R^2}}{2L}, c_1 = c_2{}^* = \frac{c}{2}e^{i\theta}$ and we get: $i(t) = c_1e^{(\alpha+i\beta)t} + c_2e^{(\alpha-i\beta)t} = ce^{\alpha t}\cos(\beta t + \theta)$

Question 5

In Lathi's book, question 2.4-12.

Question 6

The question is from the MIT Open Courseware (exercise 4.3 in the "downloadable version").

1. $y(t) = \begin{cases} 1-e^{-t} & t>0 \\ 0 & t<0 \end{cases}$.
2. $y(t) = x(t-2)$ - convolution with an impulse is equivalent ti translation.