Radiation Pattern Analysis of Linear Antenna Array

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1 Introduction

Linear antenna arrays are fundamental building blocks in phased array systems and are used extensively in radar, wireless communications, and antenna design. By controlling the phase and amplitude of individual array elements, we can steer the main lobe, suppress side lobes, and achieve desired directivity patterns. This report analyzes two key array configurations: broadside and end-fire arrays.

1.1 Array Factor

The radiation pattern of an N-element linear antenna array is given by the array factor (F_{AF}) :

$$F_{AF}(\theta, \phi) = \frac{\sin(N\psi/2)}{N\sin(\psi/2)} \tag{1}$$

where $\psi = 2\pi d/\lambda \cos(\theta) + \Phi$ is the phase difference between adjacent elements, d is the element spacing (in wavelengths), θ is the elevation angle, and Φ is the applied phase shift.

1.2 Broadside Array

A broadside array has:

- Uniform spacing: $d = \lambda/2$
- Uniform phase excitation: $\Phi = 0$
- Maximum radiation perpendicular to the array axis ($\theta = 90$)
- Therefore: $\psi = \pi\cos(\theta)$

The array factor becomes:

$$F_{AFbroadside}(\theta) = \frac{\sin(N\pi\cos(\theta)/2)}{N\sin(\pi\cos(\theta)/2)} \tag{2}$$

1.3 End-Fire Array

An end-fire array has:

- Phase shift: $\Phi = -\pi/2$ (or equivalently $d = \lambda/4$)
- Maximum radiation along the array axis ($\theta = 0$ or 180)
- Progressive phase shift between elements
- Therefore: $\psi = \frac{\pi}{2}(\cos(\theta) - 1)$

The array factor becomes:

$$F_{AFendfire}(\theta) = \frac{\sin(N\pi(\cos(\theta) - 1)/2)}{N\sin(\pi(\cos(\theta) - 1)/2)} \tag{3}$$

2 Methodology

2.1 Simulation Setup

- **Software:** MATLAB
- **Angular resolution:** 1000 points over $[0, 2\pi]$
- **Visualization:** 2D polar plots and 3D Cartesian plots
- **Normalization:** All patterns normalized to maximum value

2.2 Analysis Cases

Case 1: Broadside Array - Variable N

- Fixed spacing: $d = \lambda/2$ (implies $\beta d = \pi$)
- Number of elements: $N = 2, 3, \ldots, 20$
- Phase excitation: $\Phi = 0$

Case 2: Broadside Array - Variable Spacing

- Fixed elements: $N = 8$
- Element spacing: $d/\lambda = 0.1, 0.2, \ldots, 2.0$
- Phase excitation: $\Phi = 0$

Case 3: End-Fire Array - Variable N

- Fixed spacing: $d = \lambda/4$ (implies $\beta d = \pi/2$)
- Number of elements: $N = 2, 3, \ldots, 20$
- Phase shift: $\Phi = \pi/2$

Case 4: End-Fire Array - Variable Spacing

- Fixed elements: $N = 8$
- Element spacing: $d/\lambda = 0.1, 0.2, \ldots, 2.0$
- Phase shift: $\Phi = \pi/2$

3 Results and Discussion

3.1 Broadside Array with Variable N

The broadside array with $d = \lambda/2$ and $\Phi = 0$ creates maximum radiation perpendicular to the array axis. As the number of elements increases from 2 to 20:

- The main lobe narrows significantly, improving directivity
- Side lobes become more pronounced and numerous
- The beamwidth decreases as $BW \approx \lambda/L$ (where L is array length)
- Grating lobes may appear when $d > \lambda/2$

3.1.1 Results

Selected results for $N = 2, 5, 10, 15, 20$ are shown below:

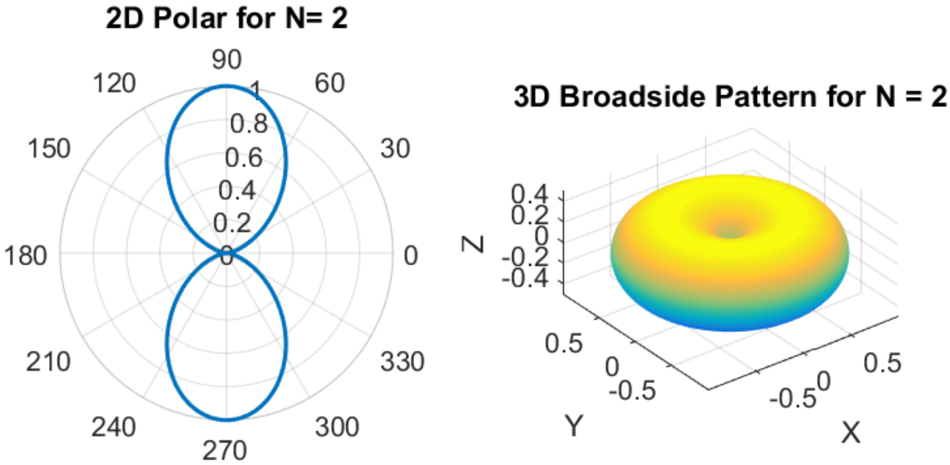

Figure 1: Broadside array radiation pattern for N=2

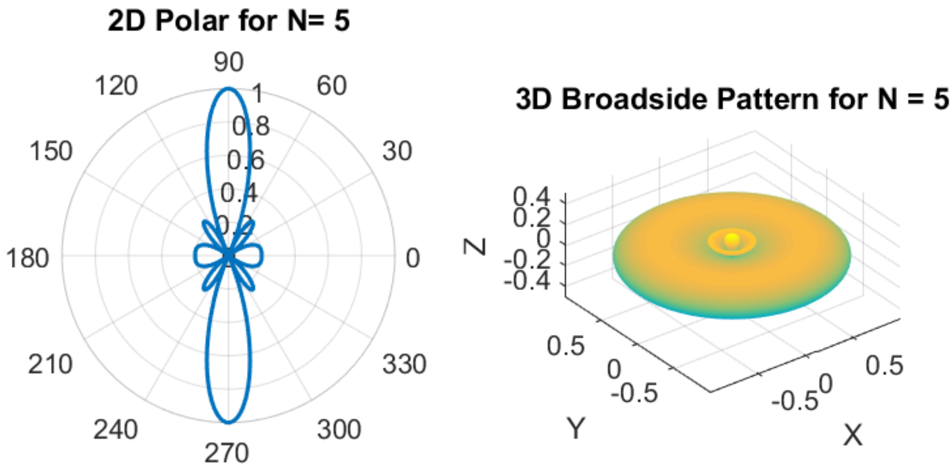

Figure 2: Broadside array radiation pattern for N=5

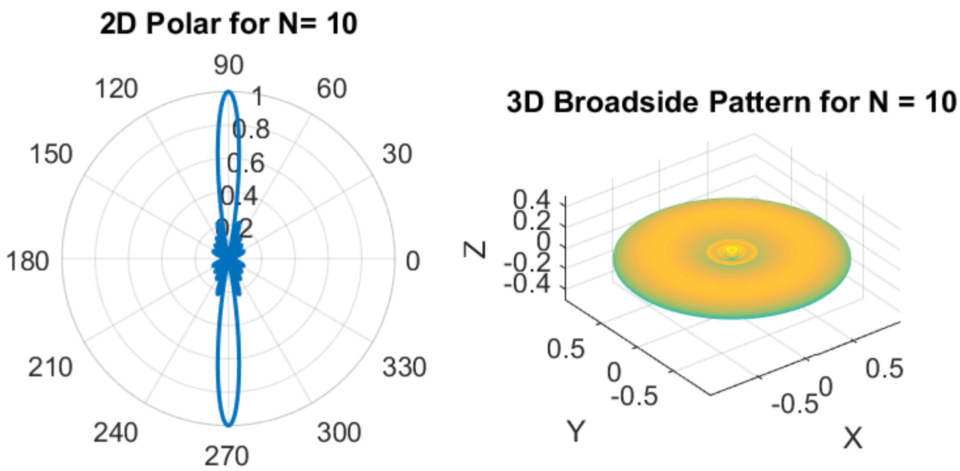

Figure 3: Broadside array radiation pattern for N=10

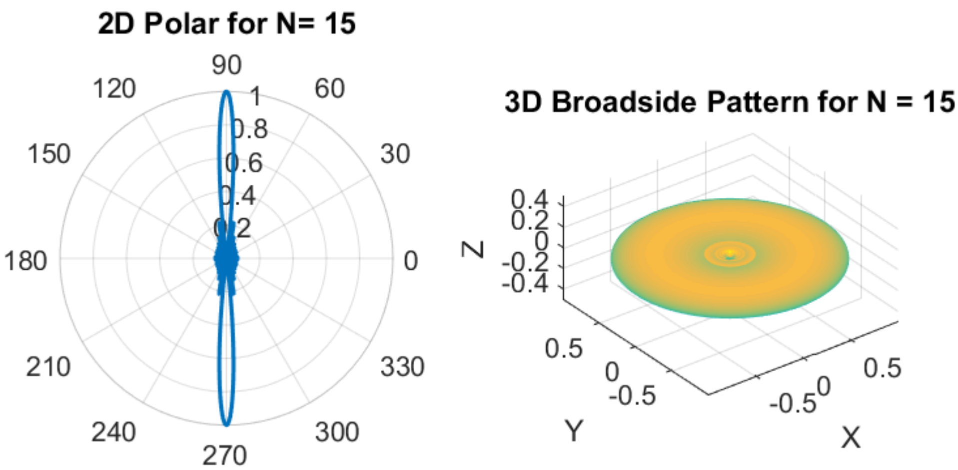

Figure 4: Broadside array radiation pattern for N=15

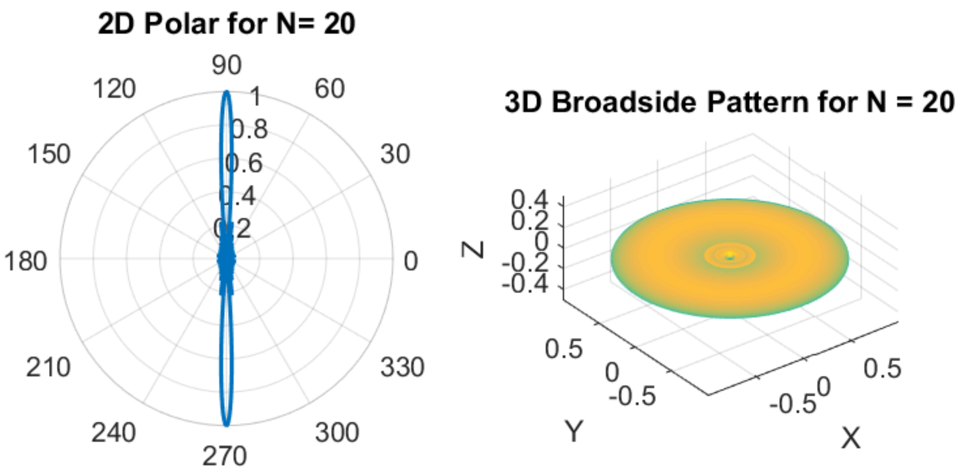

Figure 5: Broadside array radiation pattern for N=20

3.2 Broadside Array with Variable Spacing

With $N = 8$ elements fixed, varying the element spacing from 0.1λ to 2λ shows the effect of interelement spacing on the radiation pattern:

- Small spacing ($d < \lambda/2$): Broader main lobe, few or no side lobes
- Optimal spacing ($d = \lambda/2$): Narrow main lobe with controlled side lobes
- Large spacing ($d > \lambda/2$): Grating lobes appear, pattern becomes complicated

3.2.1 Results

Selected spacing values are shown below:

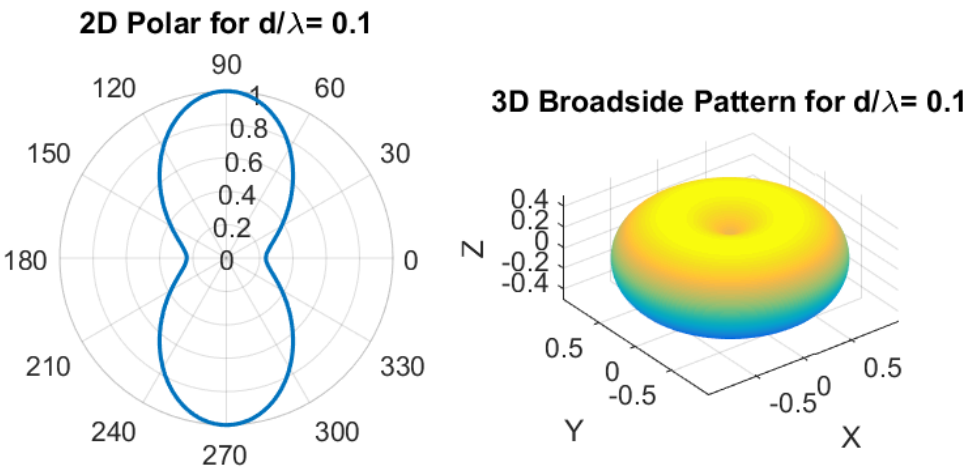

Figure 6: Broadside array (N=8) with d/λ = 0.1

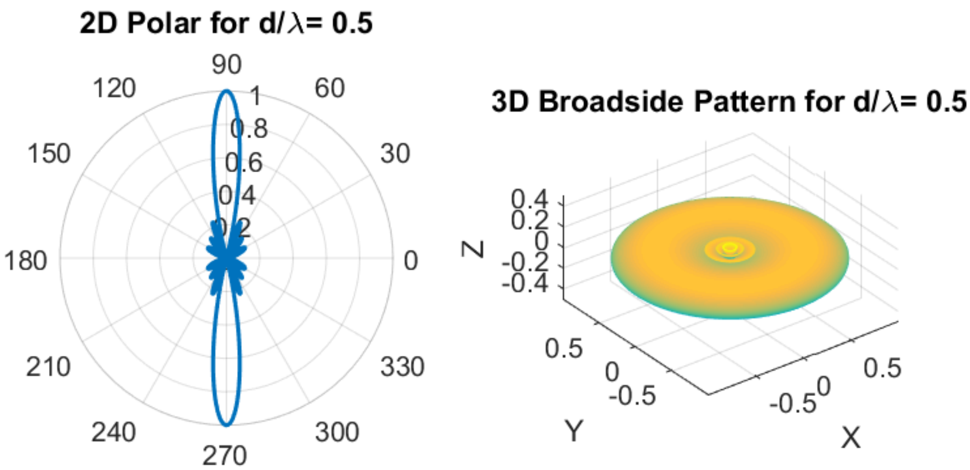

Figure 7: Broadside array (N=8) with d/λ = 0.5

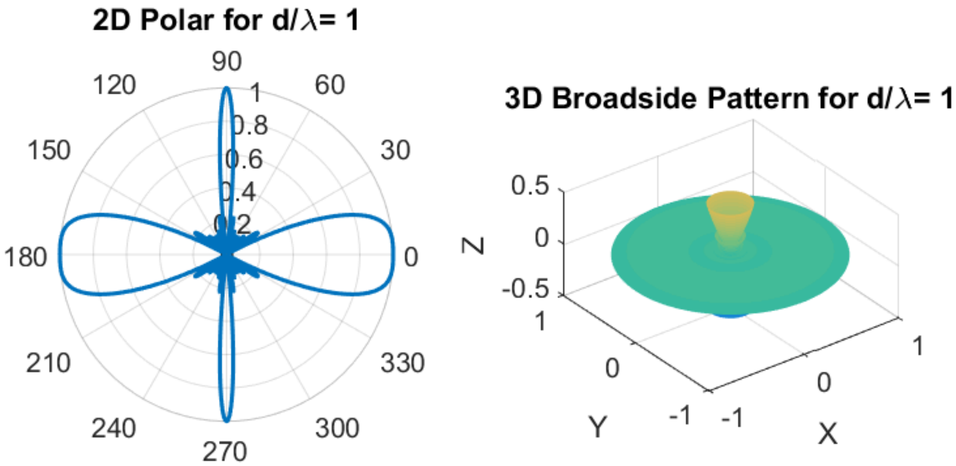

Figure 8: Broadside array (N=8) with d/λ = 1.0

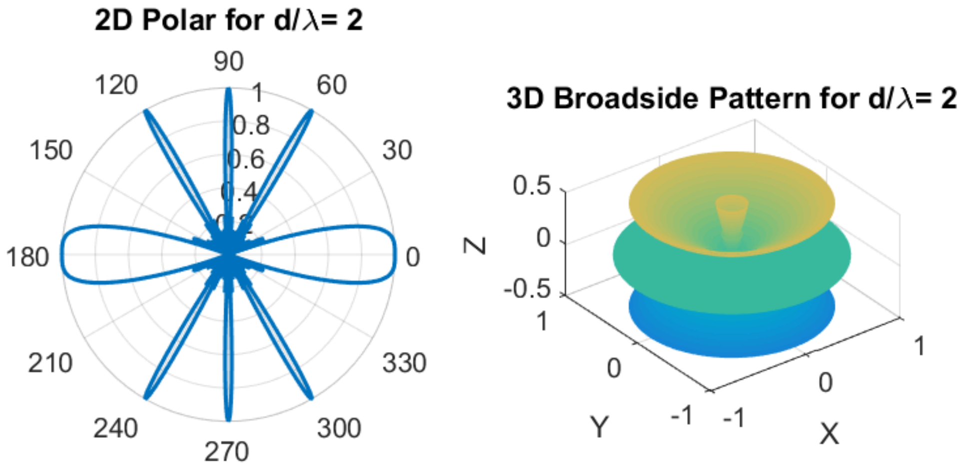

Figure 9: Broadside array (N=8) with d/$\lambda = 2.0$

3.3 End-Fire Array with Variable N

The end-fire configuration with $d = \lambda/4$ and progressive phase shift maximizes radiation along the array axis:

- Main lobe directed along the array axis (end-fire direction)
- Narrower main lobe than broadside for same N
- Lower side lobe levels compared to broadside arrays
- Better directivity in the desired direction

3.3.1 Results

Selected results for $N = 2, 5, 10, 15, 20$ are shown below:

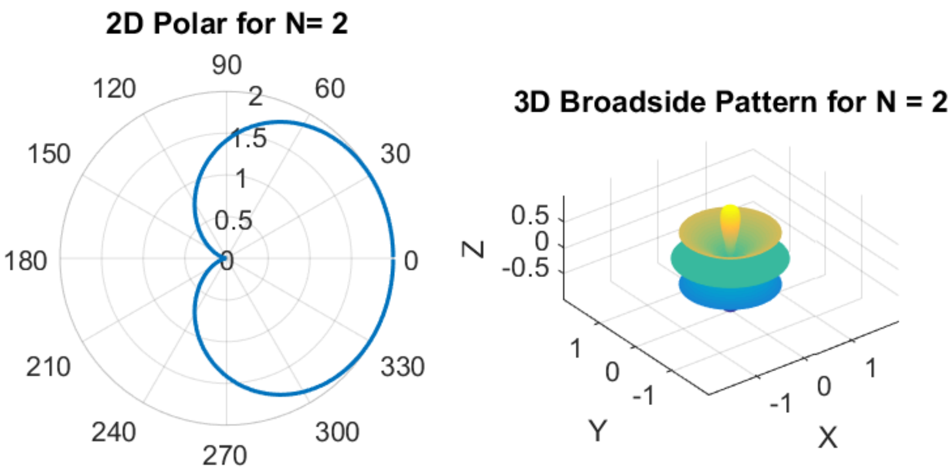

Figure 10: End-fire array radiation pattern for N=2

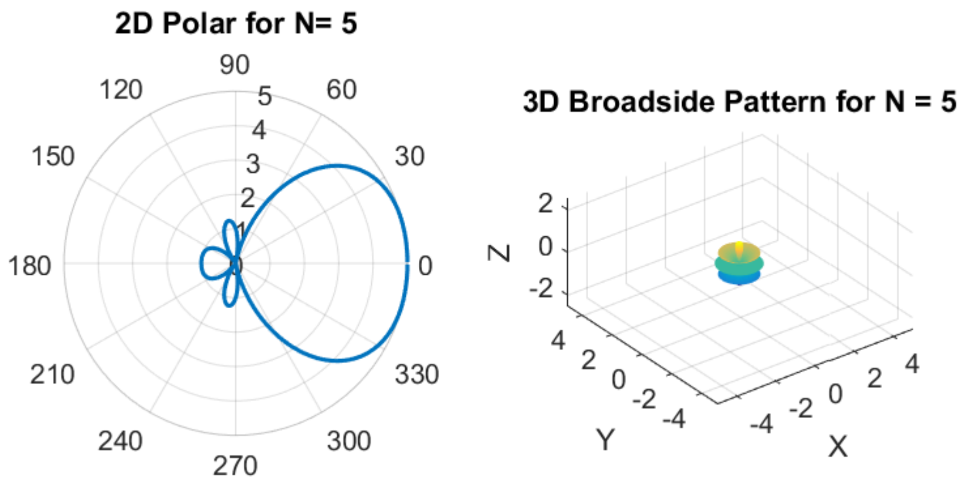

Figure 11: End-fire array radiation pattern for N=5

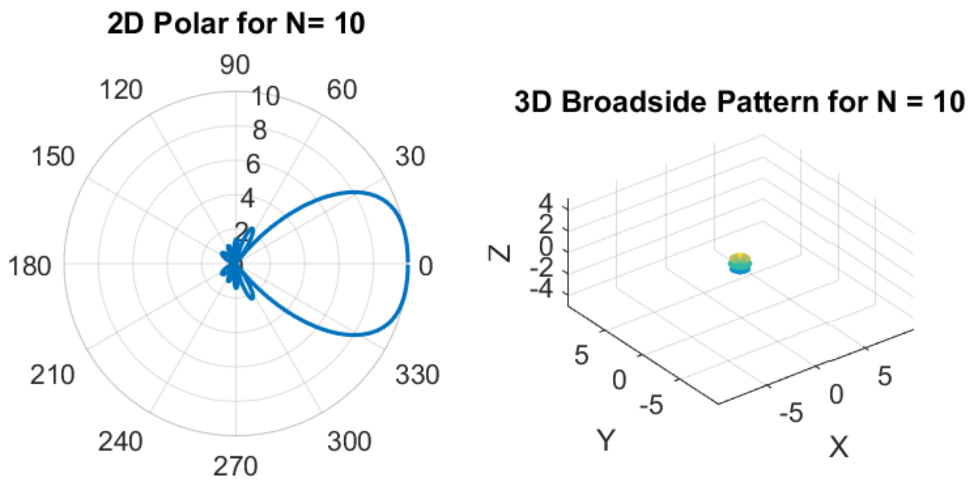

Figure 12: End-fire array radiation pattern for N=10

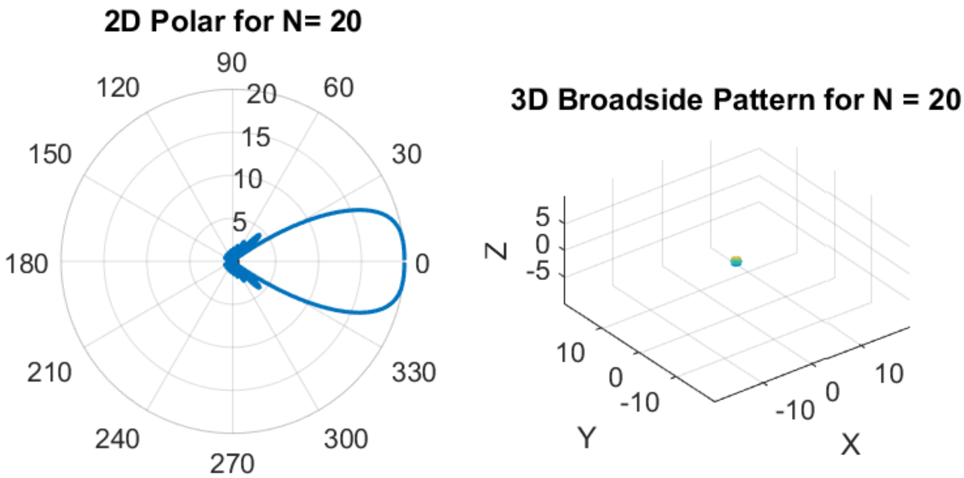

Figure 13: End-fire array radiation pattern for N=20

3.4 End-Fire Array with Variable Spacing

With $N = 8$ elements, varying spacing reveals the trade-offs in end-fire array design:

- The trade-off between beam narrowness and side lobe control
- Optimal performance typically at $d = \lambda/4$
- Larger spacings introduce unwanted grating lobes in the end-fire direction

3.4.1 Results

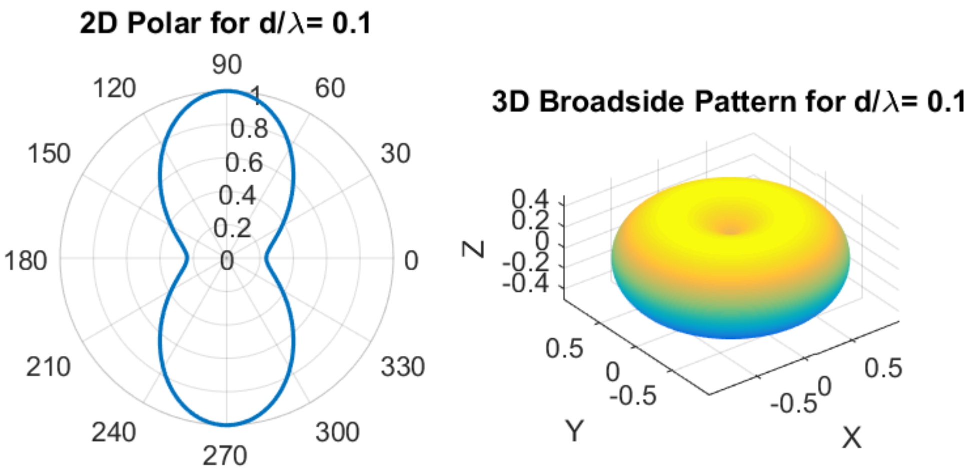

Figure 14: End-fire array (N=8) with d/λ = 0.1

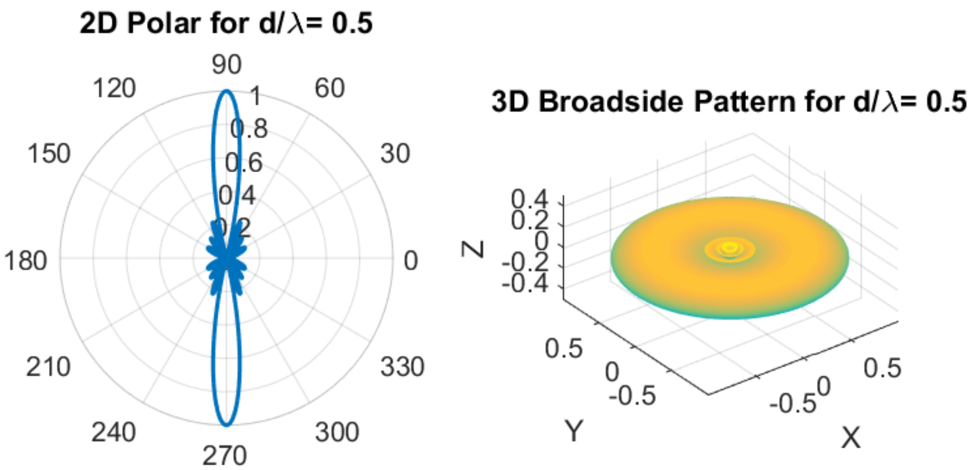

Figure 15: End-fire array (N=8) with d/λ = 0.5

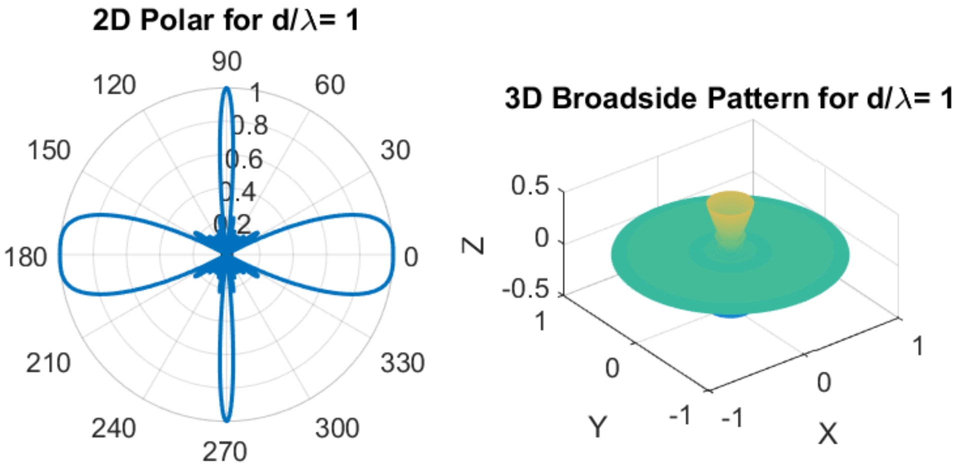

Figure 16: End-fire array (N=8) with d/λ = 1.0

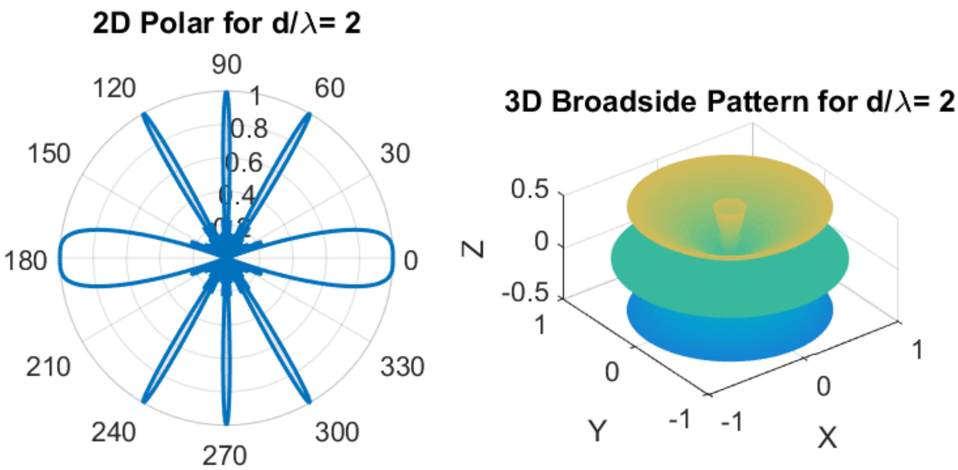

Figure 17: End-fire array (N=8) with d/λ = 2.0

4 Key Observations

1. **Directivity Improvement:** Increasing array elements dramatically improves directivity and reduces beamwidth

2. **Element Spacing:** Optimal spacing depends on array type (broadside vs. end-fire) and application

3. **Side Lobe Suppression:** Careful control of element spacing and phasing is crucial for minimizing side lobes

4. **Grating Lobes:** Must maintain $d < \lambda/2$ to avoid grating lobes in broadside arrays. Must maintain $d < \lambda/4$ to avoid grating lobes in End-Fire arrays

5. **3D Patterns:** 3D visualizations reveal complex lobing behavior not obvious in 2D polar plots

6. **Array Type Comparison:** Broadside arrays provide broad coverage while end-fire arrays provide directional beams

5 Conclusion

This analysis demonstrates the critical role of array geometry and phasing in controlling radiation patterns. Broadside arrays excel at broad-area coverage with perpendicular maximum radiation, while end-fire arrays provide narrow beams along the array axis. The number of elements and spacing must be carefully designed based on specific application requirements for directivity, gain, and side lobe control.

6 Appendix A: Complete MATLAB Code

6.1 Case 1: Broadside Array - Variable N

```
%fix d=lamda/2 and N of broadside antenna 2--20
% Broadside array excited phase =0
% fix d = lamda /2 ,   beta*d=pi
close all

N=2:20;
theta = linspace(0, 2*pi,1000);
phi = linspace(0, 2*pi, 1000);
[Theta, Phi] = meshgrid(theta, phi);
for i = 1:length(N)
    f=figure;
    subplot(1,2,1)
    a= pi*cos(theta);
    F_af_n = (sin(N(i)/2 *a)) ./( N(i)*(sin(a/2)));
    polarplot(theta,abs(F_af_n),'LineWidth', 1.5)
    title(['2D␣Polar␣for␣N=␣',num2str(N(i))]);
    subplot(1,2,2)
    %3d
    a_3= pi*cos(Theta);
    F_af_3 =abs( (sin(N(i)/2 *a_3)) ./( N(i)*(sin(a_3/2))));
    X=F_af_3 .*sin(Theta).*cos(Phi);
    Y=F_af_3 .*sin(Theta).*sin(Phi);
    Z=F_af_3 .*cos(Theta);
    surf(X,Y,Z)
    shading interp;
    title(['3D␣Broadside␣Pattern␣for␣N␣=␣', num2str(N(i))]);
    xlabel('X'); ylabel('Y'); zlabel('Z');
    axis equal;
    axis([-1 1 -1 1 -0.5 0.5] * max(abs(F_af_n(:))));
    scriptFolder = fileparts(mfilename('fullpath'));
    saveas(f, fullfile(scriptFolder, ['Broadside_Rad_N_', num2str
       ↪ (N(i)), '.png']));
end
```

6.2 Case 2: Broadside Array - Variable Spacing

```
d_lamda= [0.1 , 0.2, 0.3 , 0.4 , 0.5 , 0.6, 0.7,0.8, 0.9 , 1,
   ↪ 1.5, 2];
close all
N=8;
theta = linspace(0, 2*pi,1000);
phi = linspace(0, 2*pi, 1000);
[Theta, Phi] = meshgrid(theta, phi);
for i = 1:length(d_lamda)
    f=figure;
    subplot(1,2,1)
```

```
    a= 2*pi*d_lamda(i)*cos(theta);
    F_af_n = (sin(N/2 *a)) ./( N*(sin(a/2)));
    polarplot(theta,abs(F_af_n),'LineWidth', 1.5)
    title(['2D␣Polar␣for␣d/\lambda=␣',num2str(d_lamda(i))]);
    subplot(1,2,2)
    a_3= 2*pi*d_lamda(i)*cos(Theta);
    F_af_3 =abs((sin(N/2 *a_3)) ./( N*(sin(a_3/2))));
    X=F_af_3 .*sin(Theta).*cos(Phi);
    Y=F_af_3 .*sin(Theta).*sin(Phi);
    Z=F_af_3 .*cos(Theta);
    surf(X,Y,Z)
    shading interp;
    title(['3D␣Broadside␣Pattern␣for␣d/\lambda=␣', num2str(
       ↪ d_lamda(i))]);
    xlabel('X'); ylabel('Y'); zlabel('Z');
    axis equal;
    axis([-1 1 -1 1 -0.5 0.5] * max(abs(F_af_n(:))));
    scriptFolder = fileparts(mfilename('fullpath'));
    saveas(f, fullfile(scriptFolder, ['Broadside_Rad_D_', num2str
       ↪ (d_lamda(i)), '.png']));
end
```

6.3 Case 3: End-Fire Array - Variable N

```
close all
N=2:20;
theta = linspace(0, 2*pi,1000);
phi = linspace(0, 2*pi, 1000);
[Theta, Phi] = meshgrid(theta, phi);
for i = 1:length(N)
    f=figure;
    subplot(1,2,1)
    a= (pi/2);
    F_af_n = (sin(N(i)/2 *a*(cos(theta)-1)) ./((sin(a/2*(cos(
       ↪ theta)-1)))));
    polarplot(theta,abs(F_af_n),'LineWidth', 1.5)
    title(['2D␣Polar␣for␣N=␣',num2str(N(i))]);
    subplot(1,2,2)
    F_af_3 = (sin(N(i)/2 *a*(cos(Theta)-1)) ./((sin(a/2*(cos(
       ↪ Theta)-1)))));
    X=F_af_3 .*sin(Theta).*cos(Phi);
    Y=F_af_3 .*sin(Theta).*sin(Phi);
    Z=F_af_3 .*cos(Theta);
    surf(X,Y,Z)
    shading interp;
    title(['3D␣Broadside␣Pattern␣for␣N␣=␣', num2str(N(i))]);
    xlabel('X'); ylabel('Y'); zlabel('Z');
    axis equal;
    axis([-1 1 -1 1 -0.5 0.5] * max(abs(F_af_n(:))));
    scriptFolder = fileparts(mfilename('fullpath'));
```

```
    saveas(f, fullfile(scriptFolder, ['Broadside_Rad_End_N_', num2str(N(i)), '.png']));
end
```

6.4 Case 4: End-Fire Array - Variable Spacing

```
d_lamda= [0.1 , 0.2, 0.3 , 0.4 , 0.5 , 0.6, 0.7,0.8, 0.9 , 1, 1.5, 2];
close all
N=8;
theta = linspace(0, 2*pi,1000);
phi = linspace(0, 2*pi, 1000);
[Theta, Phi] = meshgrid(theta, phi);
for i = 1:length(d_lamda)
    f=figure;
    subplot(1,2,1)
    a= 2*pi*d_lamda(i)*(cos(theta)-1);
    F_af_n = (sin(N/2 *a)) ./( N*(sin(a/2)));
    polarplot(theta,abs(F_af_n),'LineWidth', 1.5)
    title(['2D Polar for d/\lambda= ',num2str(d_lamda(i))]);
    subplot(1,2,2)
    a_3= 2*pi*d_lamda(i)*(cos(Theta)-1);
    F_af_3 =abs((sin(N/2 *a_3)) ./( N*(sin(a_3/2))));
    X=F_af_3 .*sin(Theta).*cos(Phi);
    Y=F_af_3 .*sin(Theta).*sin(Phi);
    Z=F_af_3 .*cos(Theta);
    surf(X,Y,Z)
    shading interp;
    title(['3D Broadside Pattern for d/\lambda= ', num2str(d_lamda(i))]);
    xlabel('X'); ylabel('Y'); zlabel('Z');
    axis equal;
    axis([-1 1 -1 1 -0.5 0.5] * max(abs(F_af_n(:))));
    scriptFolder = fileparts(mfilename('fullpath'));
    saveas(f, fullfile(scriptFolder, ['Broadside_Rad_D_End_', num2str(d_lamda(i)), '.png']));
end
```