CHUNG QUANG KHANH – 20245360 – Homework 02 – Professor. AHN

Lecture 4

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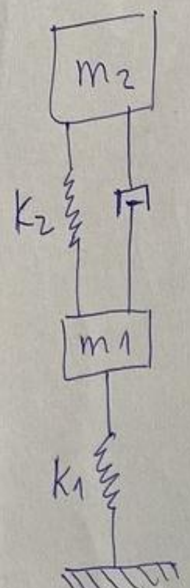

Equations of Motion using Newton's Second Law:

$$m_i \ddot{x}_i = \text{sum of forces on } m_i \quad (1)$$

(+) For m_1: forces acting on m_1

- Spring force from k_1: $-k_1 x_1$
- Spring force from k_2: $k_2(x_2 - x_1)$
- Damping force from b: $b(\dot{x}_2 - \dot{x}_1)$

$$(1) \Rightarrow m_1 \ddot{x}_1 = -k_1 x_1 + k_2(x_2 - x_1) + b(\dot{x}_2 - \dot{x}_1)$$

(+) For m_2:

- Spring force from k_2: $-k_2(x_2 - x_1)$
- Damping force from b: $-b(\dot{x}_2 - \dot{x}_1)$

$$(1) \Rightarrow m_2 \ddot{x}_2 = -k_2(x_2 - x_1) - b(\dot{x}_2 - \dot{x}_1)$$

Lecture 5

Mass-spring-damper example:

$$m\ddot{y} + b\dot{y} + ky = 0$$

$$\Rightarrow \quad \ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = 0 \Rightarrow \ddot{y} + 4\dot{y} + 40y = 0 \quad (1)$$

Assume a solution of the form $y(t) = e^{rt}$

$$(1) \Rightarrow \quad r^2 + 4r + 40 = 0$$

$$\Rightarrow \quad r = \frac{-4 \pm \sqrt{-144}}{2}$$

Since $\sqrt{-144} = 12i$, we have $r = \dfrac{-4 \pm 12i}{2} = -2 \pm 6i$

$\therefore$ General free response $y(t) = e^{-2t}\left(C_1 \cos(6t) + C_2 \sin(6t)\right)$

where $\begin{cases} r = \alpha \pm i\omega \text{ (with } \alpha = -2,\ \omega = 6) \\ C_1, C_2 \text{ are constants determined by initial conditions} \end{cases}$

Lecture 6

$$\text{Let } m = 1,\ b = 4,\ k = 40 \text{ and } u(t) = 1(t),\ \text{find } x(t)$$

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m = 1;
b = 4;
k = 40;

s = tf('s');
H = (b*s + k) / (m*s^2 + b*s + k);
t = linspace(0, 5, 500);
u = ones(size(t));
x = lsim(H, u, t);

figure;
plot(t, x, 'LineWidth', 2);
title('Car Suspension Response using lsim');
xlabel('Time (sec)');
ylabel('x(t)');
grid on;
```