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Lecture 7: given system : $\dddot{x} + 5\ddot{x} + 3\dot{x} + 2x = u$ (1) . State-space ?

- Define $x_1 = x$ , $x_2 = \dot{x}$ , $x_3 = \ddot{x}$ $\Rightarrow$ $\dot{x}_1 = x_2$ , $\dot{x}_2 = x_3$

(1) $\Rightarrow$ $\dot{x}_3 = -5x_3 - 3x_2 - 2x_1 + u$

- Define $y = x_1$
- State-space model is $\left.\begin{aligned} \dot{X} &= Ax + Bu \\ Y &= Cx + Du \end{aligned}\right|$ (2)

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}$$

$$(2) \Rightarrow \begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + Bu \\ Y = C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + Du \end{cases}$$

$$\Rightarrow A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T, \quad D = [0]$$

* Convert given state-space model to a transfer function

- Transfer function has form : $G(s) = C(sI - A)^{-1}B + \overset{[0]}{D}$

$$= C(sI - A)^{-1}B \quad (1)$$

- $(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ -2 & s+3 \end{bmatrix}$

$$(sI - A)^{-1} = \frac{1}{s(s+3) - (-1)(-2)} \begin{bmatrix} s+3 & 1 \\ 2 & s \end{bmatrix}$$

$$(1) \Rightarrow G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{s^2 + 3s - 2} \begin{bmatrix} s+3 & 1 \\ 2 & s \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{s+3}{s^2 + 3s - 2}$$

$$\therefore \; G(s) = \frac{s+3}{s^2 + 3s - 2}$$

Lecture 8 :

$$A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{CR} & -\frac{1}{CR} & 0 \\ -\frac{1}{CL} & \frac{1}{CL} & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} \quad C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T \quad D = [0]$$

Transfer function has form : $G(s) = C(sI - A)^{-1}B + \overset{[0]}{D}$

$$= C(sI - A)^{-1}B \quad (1)$$

$$sI - A = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 \\ 1/CR & -1/CR & 0 \\ -1/CL & 1/CL & 0 \end{bmatrix} = \begin{bmatrix} s & 0 & -1 \\ -1/CR & s+1/CR & 0 \\ 1/CL & -1/CL & s \end{bmatrix}$$

$$\det(sI - A) = \frac{s}{CL}$$

$$(1) \Rightarrow G(s) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (sI - A)^{-1} \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}$$

We focus only element (3,3) of $(sI - A)^{-1}$,

element (3,3) : $\dfrac{1}{s^2 + \frac{s}{CR} + \frac{1}{CL}}$

$$\therefore G(s) = \frac{\frac{1}{L}}{s^2 + \frac{s}{CR} + \frac{1}{CL}}$$