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* Transfer function of $\Theta(s) / E_a(s)$?

Electrical domain: $e_a - R_a i_a - L_a \frac{di_a}{dt} - e_b = 0$

$$\xrightarrow{\mathcal{L}} \quad E_a(s) - I_a(s) R_a - I_a(s) L_a s - K_b \Theta(s) s$$

$$\Rightarrow \quad I_a(s) = \frac{E_a(s) - K_b \Theta(s) s}{R_a + L_a s} \qquad (1)$$

Mechanical domain: $T - b\dot{\theta} = J\ddot{\theta}$

$$\xrightarrow{\mathcal{L}} \quad T(s) - b s \Theta(s) = J s^2 \Theta(s)$$

$$\Rightarrow \quad K_a I_a(s) - b s \Theta(s) = J s^2 \Theta(s)$$

$$(1) \Rightarrow \quad K_a \frac{E_a(s) - K_b \Theta(s) s}{R_a + L_a s} - b s \Theta(s) = J s^2 \Theta(s)$$

$$\Rightarrow \quad \frac{K_a E_a(s)}{R_a + L_a s} = \left(\frac{K_a K_b s}{R_a + L_a s} + bs + Js^2 \right) \Theta(s)$$

$$\Rightarrow \quad \frac{\Theta(s)}{E_a(s)} = \frac{K_a}{(R_a + L_a s)\left(\frac{K_a K_b s}{R_a + L_a s} + bs + Js^2 \right)}$$

* Linearize $f(v) = av^2$ about a point $v = \bar{v}$

1^{st}-order Taylor series expansion of $f(v)$:

$$f(v) \approx f(\bar{v}) + \left.\frac{df}{dv}\right|_{v=\bar{v}} \cdot (v - \bar{v})$$

$$\Rightarrow f(v) \approx a\bar{v}^2 + 2a\bar{v}(v - \bar{v})$$

* Linearize $f(\theta) = mgl\sin\theta$ about $\theta = \bar{\theta}$

1^{st}-order Taylor series expansion of $f(\theta)$:

$$f(\theta) \approx f(\bar{\theta}) + \left.\frac{df}{d\theta}\right|_{\theta=\bar{\theta}} \cdot (\theta - \bar{\theta})$$

$$\Rightarrow f(\theta) \approx mgl\sin\bar{\theta} + mgl\cos\bar{\theta}\,(\theta - \bar{\theta})$$

* Find the equilibrium solution $(\bar{v}, \bar{F})$ of $F - av^2 = m\dot{v}$

In an equilibrium state, the velocity should not change over time $\Rightarrow \dot{v} = 0$

$$\Rightarrow F - av^2 = m\dot{v} = 0$$

$$\Rightarrow V_{equilibrium} = \pm\sqrt{\frac{F}{a}}$$

and $F_{equilibrium} = F$

* Linearize $\ddot{x} + 2\dot{x} + 2\sqrt{x} = u \quad (1)$ around a nominal input of $u = 6$

- Find operating point $(\bar{x}, \bar{u})$:

we assume the system is at steady state as follows:

$$\begin{cases} \ddot{x} = 0 \\ \dot{x} = 0 \\ u = \bar{u} = 6 \text{ (nominal input)} \end{cases} \Rightarrow 0 + 2 \cdot 0 + 2\sqrt{\bar{x}} = \bar{u} = 6$$

$$\Rightarrow \bar{x} = 9$$

$$\Rightarrow \text{operating point is } \begin{cases} \bar{x} = 9 \\ \bar{u} = 6 \end{cases}$$

- Linearize using Taylor series:

we have
$$\begin{cases} x = \bar{x} + \Delta x \\ \dot{x} = \bar{\dot{x}} + \Delta \dot{x} \\ u = \bar{u} + \Delta u \end{cases}$$

We need to linearize $\sqrt{x}$ around $\bar{x} = 9$. The Taylor series of $\sqrt{x}$:

$$\sqrt{x} = \sqrt{\bar{x}} + \left.\frac{d\sqrt{x}}{dx}\right|_{x=\bar{x}} (x - \bar{x}) = \sqrt{\bar{x}} + \frac{1}{2\sqrt{\bar{x}}} \Delta x$$

$$\bar{x} = 9 \Rightarrow \sqrt{x} = \sqrt{9} + \frac{1}{2\sqrt{9}} \Delta x = 3 + \frac{1}{6} \Delta x$$

- Substitute linearized terms into (1)

$$\ddot{x} + 2\dot{x} + 2\left(3 + \frac{1}{6}\Delta x\right) = \bar{u} + \Delta u = 6 + \Delta u$$

$$\Rightarrow \ddot{x} + 2\dot{x} + \frac{1}{3}\Delta x = \Delta u$$