Optimal Speed Control of DC Motor using Linear Quadratic Regulator and Model Predictive Control

[1]*Satyabrata Sahoo*
Department of Electrical Engineering
C.V. Raman College of Engineering
Bhubaneswar, Odisha, India
Email: jitu_sahoo@yahoo.com

[2]*Bidyadhar Subudhi*
Department of Electrical Engineering
National Institute of Technology
Rourkela, Odisha, India
Email: bidyadhar@nitrkl.ac.in

[3]*Gayadhar Panda*
Dept. of Electrical & Electronics Engg.
National Institute of Technology
Meghalay, India
Email: p_gayadhar@yahoo.com

***Abstract*—The objective of this paper is to control the angular speed in a model of a DC motor using different control strategies like Model Predictive Control and Linear Quadratic Regulator for comparison purpose. Model Predictive Control approach provides online & offline computation of the optimization law by Multi Parametric Quadratic Programming. The controllers are designed based on the optimal control theory. Performance of these controllers has been verified through simulation results using MATLAB/SIMULINK software**

Keywords—model-based control; PID Controller; LQR control method; Model Predictive Control (MPC); state-space models, DC motor; optimal control

I. INTRODUCTION

Control system design refers to the process of selecting feedback gains that meet design specifications in a closed-loop control system. Most design methods are iterative, combining parameter selection with analysis, simulation, and insight into the dynamics of the plant.

Due to the excellent speed control characteristics of a DC motor, it has been widely used in industry (such as cars, trucks and aircraft) even though its maintenance costs are higher than the induction motor. As a result, authors have paid attention to position control of DC motor and prepared several methods to control speed of such motors. Proportional–Integral-Derivative (PID) controllers have been widely used for speed and position control. [1]

They designed a position controller of a DC motor by selection of PID parameters using genetic algorithm and secondly by using Ziegler & Nichols method of tuning the parameters of PID controller. They found that, first method gives better results than the second one.

In [2] they have compared two types of controllers which are PID controller and optimal controller. The PID compensator is designed using (GA), while the other compensator is made optimal and integral state feedback controller with Kalman filter. Computer simulations have been carried out. Finally they found that the second controller gives less settling, less overshoot and better performance encountering with noise and disturbance parameters variations.

[3] Presented a novel PID dual loop controller for a solar photovoltaic (PV) powered industrial permanent magnet DC (PMDC) motor drive. MATLAB/SIMULINK was used in the analysis for the GUI environment.

The performance of PI controller for speed or position regulation degrades under external disturbances and machine parameter variations. Furthermore, the PI controller gains have to be carefully selected in order to obtain a desired response. This can be solved by advanced control techniques such as LQR and MPC.

Model Predictive Control has advantage over conventional PID control schemes is its constraints handling capacity for control inputs as well as system states. This paper presents Model Predictive Control, LQR and PID controller which applied to control the speed of a DC motor. The rest of the paper is presented as follows: at first the plant model is described. The next section describes the PID technique, the design of LQR and Model Predictive Control. Then simulation results are presented. Finally, the last section contains paper conclusion.

II. DC MOTOR MODEL

The speed of a DC motor is proportional to the voltage applied to it. While, its torque is proportional to the motor current. Speed control can be achieved by variable battery tapings, variable supply voltage, resistors or electronic controls.

A simple motor model is shown in Fig.1. The armature circuit consist of a resistance (Ra) connected in series with an inductance (La), and a voltage source (eb) representing the back emf (back electromotive force) induced in the armature when during rotation. [4].

The motor torque Tm is related to the armature current, ia , by a torque constant Ki;

$$T_m = K_i i_a \qquad (1)$$

The back emf, eb, is relative to angular velocity by;

$$e_b = k_b \omega_m = k_b \frac{d\theta}{dt} \qquad (2)$$

978-1-4678-6503-1/15/$31.00 © 2015 IEEE

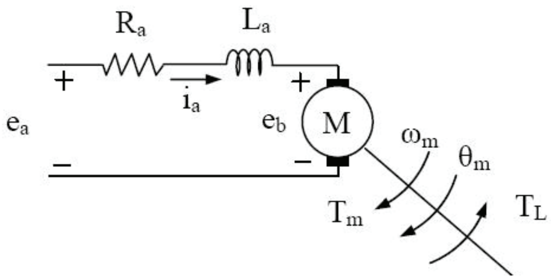


Fig. 1 DC Motor Model [4]

From Fig. 1 we can write the following equations based on the Newton's law combined with the Kirchoff's law:

$$e_a = i_a R_a + L_a \frac{di_a}{dt} + e_a \tag{3}$$

$$L_a \frac{di_a}{dt} + R_a i_a = e_a - K_b \frac{d\theta}{dt} \tag{4}$$

$$J_m \frac{d^2\theta}{dt^2} + B_m \frac{d\theta}{dt} = K_i i_a \tag{5}$$

There are several different ways to describe a system of linear differential equations. The plant model will be introduced in the form of state-space representation and given by the equations:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \tag{6}$$

According to equations from (2) to (5), the state space model will be:

$$\begin{bmatrix} \dot{i_a} \\ \dot{\omega_m} \\ \dot{\theta_m} \end{bmatrix} = \begin{bmatrix} -R_a/L_a & -K_b/L_a & 0 \\ K_i/J_m & -B_m/J_m & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix} + \begin{bmatrix} 1/L_a \\ 0 \\ 0 \end{bmatrix} e_a \tag{7}$$

$$\omega_m = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix} \tag{8}$$

The DC motor data taken for this work are shown in table 1:

Table 1 for DC motor data [5]

Symbol		Value and unit
E	=	12volt
$\mathbf{J_m}$	=	$0.01kgm^2$
$\mathbf{B_m}$	=	$0.00003kgm^2/s$
$\mathbf{K_i}$	=	0.023Nm/A
$\mathbf{K_b}$	=	0.023V/rad/s
$\mathbf{R_a}$	=	1Ω
$\mathbf{L_a}$	=	0.5H

III. LQR & STATE FEEDBACK OBSERVER CONTROLLER

LQR control that designed is classified as optimal control systems. This is an important function of control engineering.

The performance of a control system can be represented by integral performance measures. Therefore, the design of the system must be based on minimizing a performance index, such as the integral of the squared error (ISE).

The specific form of the performance index can be given as in eq.(9), where xT indicates the transpose of the x matrix, then, in terms of the state vector, is

$$J = \int_0^{t_f} (x^T x) dt \tag{9}$$

Where x equals the state vector, and tf equals the final time.

Then the design steps are as follows:

1. Determine the matrix P that satisfies eq.(10), where A is known.

$$A^T P + PA = -I \tag{10}$$

2. Minimize J by determining the minimum of eq.(11) by adjusting one or more unspecified system parameters.

$$J = \int_0^{\infty} x^T x \, dt = x^T(0) P x(0) \tag{11}$$

Upon examining the performance index (eq.11), recognizing that the reason the magnitude of the control signal is not accounted for in the original calculation is that U (equals the control vector) is not included within the expression for the performance index.

To account for the expenditure of the energy of the control signal, it will be utilize the performance index

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt \tag{12}$$

Where Q is a positive definite or positive semi definite Hermitian matrix and R is a positive definite Hermitian matrix. Q & R are weighing factors. The performance index J can be minimized when

$$K = R^{-1} B^T P \tag{13}$$

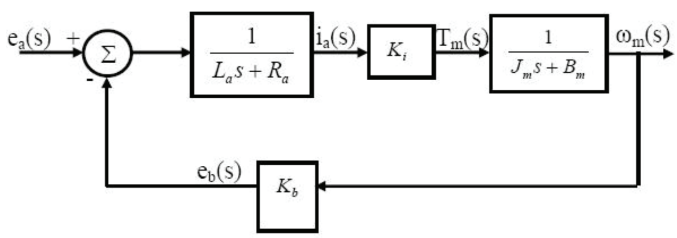


Fig. 2- DC-Motor System Block Diagram for speed control

From Fig.3 the state variable feedback will be represented by

$$u = -Kx \tag{14}$$

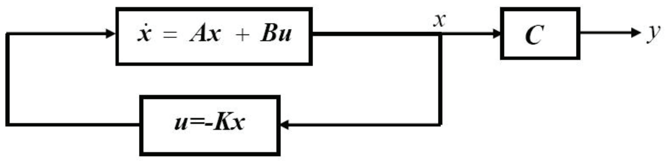


Fig. 3- Linear Quadratic Structure

The n×n matrix p is determined from the solution of equation

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{15}$$

The two matrices Q and R are selected by design engineer by trial and error. Generally speaking, selecting Q large means that, to keep J small. On the other hand selecting R large means that the control input u must be smaller to keep J small.

Equation (15) can be easily programmed for a computer, or solved using MATLAB. Popularly equation (15) is known as Algebraic Riccati Equation.

IV. MODEL PREDICTIVE CONTROL

Model Predictive Control is an Optimal Control Strategy based on the dynamic system model. Since its advent it has become popular in industries for constrained control of the systems. Anti wind-up using PID can at the most provide constraints on the control input.

The basic idea standing behind Model Predictive Control is depicted on figure 4. Having defined a reference trajectory for the output of the plant $r(t)$ we want to track it in an optimal way. Namely, we want to balance between the index referring to the tracking error and other performance indexes such as the aggressiveness of the control action. In the current sampling instant an output prediction trajectory $z(t|k)$ is being calculated. It represents the fashion in which the reference trajectory $r(t)$ should be reached by the output signal $y(t)$.

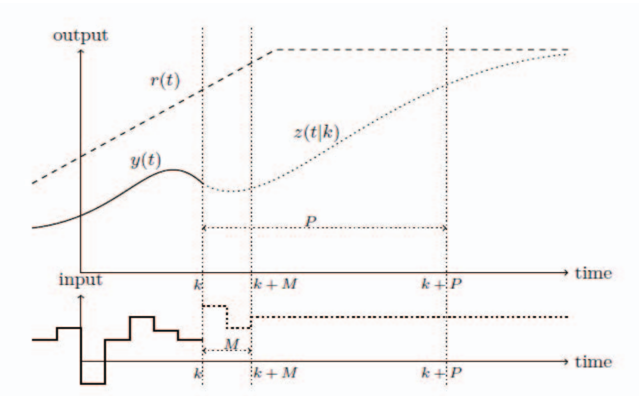


Fig. 4 Basic Idea Behind Mode Predictive Control [6]

It is defined over a certain number of future samples known as the prediction horizon P. The plant's inputs, which will be responsible for driving the system along the prediction trajectory $z(t|k)$, are assumed to be changing over a certain number of samples, called the control horizon M, and stay constant afterwards.

With the knowledge of the plant's dynamics, in a form of a model, a formula describing system's outputs evolution, over a given prediction horizon P is derived. Next a cost function, describing the optimal, from the designers point of view, balance between certain characteristics of the plant's behavior, is defined.

It is often not an easy task since the individual control objectives which are reflected there are often opposite in nature. The value of the decision vector that minimizes the cost function is,

$$J = \sum_{j=1}^{P} [\hat{y}(k+j) - r(k+j)]^2 + \rho \sum_{j=1}^{M} [\Delta u(k+j-1)]^2 \qquad \ldots (16)$$

the one that will allow the system to follow the prediction trajectory $z(t|k)$ in the way that is most satisfactory. It typically consists of the values of the current and future inputs to the systems $U(k)$ but can also contain other variables too.

Once a set of future inputs to the system has been computed, only the first one is applied to the plant. In the next iteration of the algorithm (at time k + 1) the cycle will repeat.

Both prediction P and control M horizons will be shifted forward in time (but will preserve their length) by one sample, new set of future inputs will be obtained and again only the first one will be used. This approach is called the receding horizon strategy.

V. SIMULATION RESULTS

The simulation procedure may be summarized as follows:

I. First input the DC motor data

II. Write the differential equations for the model then get the state space representation as in equation (7).

III. Get the open loop transfer function and the closed loop step response.

IV. Finally performing the performance of PID controller, LQR controller, Observer based controller [7], model predictive control and then compare the results.

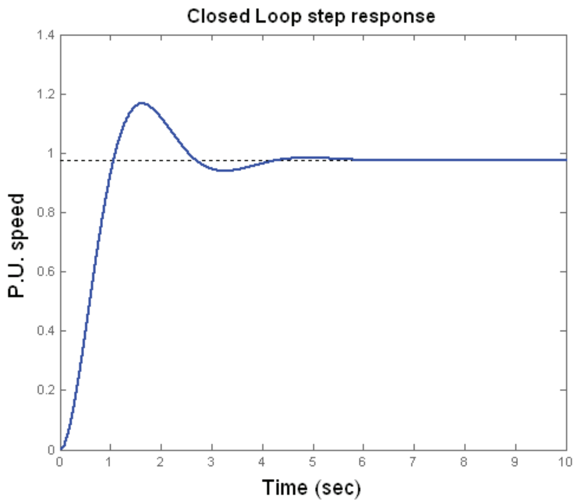
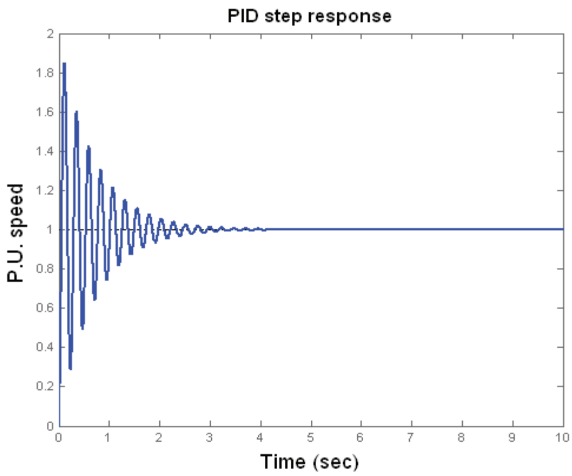


Fig. 5-Closed loop step response



A.

Fig. 6-PID step response

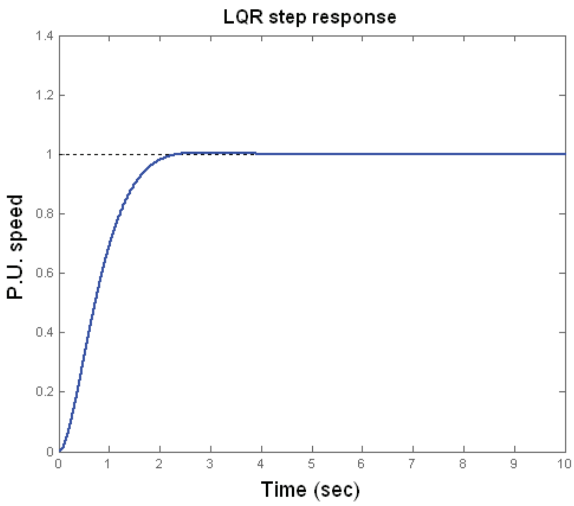


Fig. 7-LQR step response

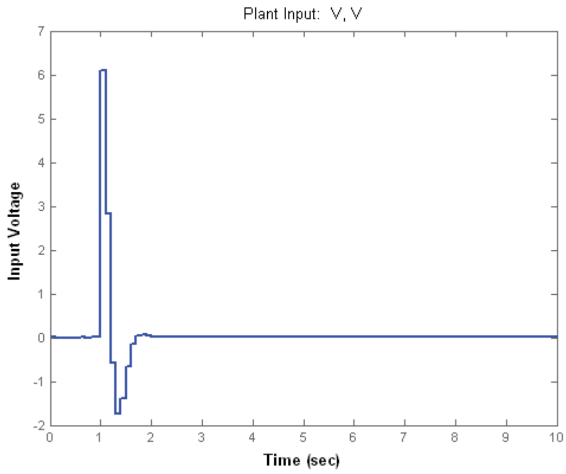


Fig. 8- Constraint Input to the System

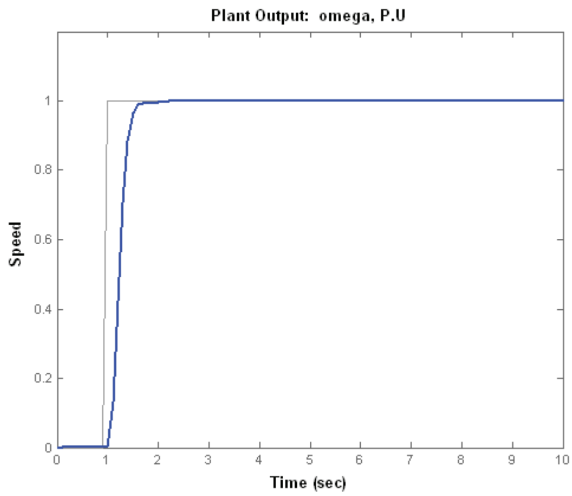


Fig. 9- step response with MPC controller

Simulation results of the MPC is presented in Figure 8 and 9. Figure 8 shows motor input voltage is given between -10v to +10V to a separately excited DC motor. In figure 9, when we gave a step input to the MPC controller, it exactly tracks the input reference.

VI. CONCLUSION

Speed control of a DC motor is an important issue, so this paper presents a design method to determine the optimal speed control using different controllers. From the obtained results, which is shown in Table 2, we find LQR controller has smaller overshoot and model predictive controller has zero overshoot.MPC has shorter settling time and Peak amplitude compared to the other controllers.

TABLE-2 COMPARISON OF SIMULATION RESULTS

Different Controllers	Settling Time In sec	Peak Amplitude	Over Shoot In %
Closed Loop With unity Feedback	3.83	1.17	19.5
PID Controller	2.76	1.85	84.8
LQR Controller	1.99	1.00	0.525
ModelPredictive Control	1.56	1.00	0.126

REFERENCES

[1] H. Neenu Thomas and Dr. P.Poongodi, "Position Control of DC Motor Using Genetic Algorithm Based PID Controller", Proceedings of the World Congress on Engineering 2009 Vol. II, WCE2009, July 1-3, 2009, London, U.K

[2] Hadi Delavari, Ghasem Alizadeh and Mohammad Bagher Bannane Sharifian, "Optimal Integral State Feedback Controller for a DC Motor", 2006, paper identification number 440.

[3] Sharaf, A.M., Elbakush, E., Altas, I.H., "Novel Control Strategies for Photovoltaic Powered PMDC Motor Drives', IEEE, 2007

[4] Katsuhiko Ogata: Modern Control Engineering; Prentice Hall International, Inc. Fourth Edition 2002.

[5] Tutorial 12 about DC motor, instrumentation and process control by Hung Nguyen, 2006.

[6] Predictive Control with constraints, J.M. Maciejowski, Prentice Hall,2001

[7] Ashish Tiwari: Modern Control Design With Matlab and Simulink; John WILEY & Sons, Ltd., Singapore.