Midterm Examination

1. (10pts) The random variable X has the following probability mass function: $P\{X = 1\} = \frac{1}{2}$, $p\{X = 2\} = \frac{1}{3}$, $\{X = 24\} = \frac{1}{6}$. Find E[X].

2. (10pts) If the density of X is defined below, find $P\{X > 20\}$.

$$f(x) = \begin{cases} 10/x^2, & \text{for } x > 10. \\ 0, & \text{otherwise.} \end{cases}$$

3. (25pts) Let X be a Poisson random variables with mean λ.

   a. (5pt) Find the probability that $X = x$

   b. (5pt) Find the moment generation function of random variable X.

   c. (5pt) Find the expectation of X, $E[X]$.

   d. (5pt) Find the second moment of X, $E[X^2]$.

   e. (5pt) Find the variance of X.

4. (30pts) Let X and W be the working and subsequent repair times of a certain machine. Let $Y = X + W$ and suppose that the joint probability density of X and Y is

$$f_{X,Y}(x, y) = \lambda^2 e^{-\lambda y} \ 0 < x < y < \infty$$

   a. (5pt) Find the density of X.

   b. (5pt) Find the density of Y.

   c. (10pt) Find the joint density of X and W.

   d. (10pt) Find the density of W.

5. (15pts) The joint density of X and Y is given by the below, find the conditional probability density function $f_{X|Y=y}(x|y)$ for given $Y = y$.

$$f_{X,Y}(x, y) = \frac{e^{-y}}{y} \ 0 < x < y, 0 < y < \infty$$

6. (15pts) If $f(x) = 0$ for $x < 0$, then, for any $\alpha > 0$, prove $P\{X \geq \alpha\} \leq \frac{E[X]}{\alpha}$.