Observer-based output-feedback control for linear systems

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1 State observer design

• To begin with, let us consider the following linear state-space model:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

• **Assumption:** The system matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{p\times n}$ are known and can be used to design a state observer.

- In most cases, a so-called Luenberger observer is utilized for estimating the state:

$$\begin{cases} \dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu(t) - L\big(y(t) - \tilde{y}(t)\big) \\ \tilde{y}(t) = C\tilde{x}(t). \end{cases} \tag{2}$$

where $\tilde{x}(t)$ denotes the estimated state.

- The matrix $L \in \mathbb{R}^{n \times p}$ is called the observer gain to be designed later.

- Now, let us define the estimation error as

$$e(t) = x(t) - \tilde{x}(t).$$

- Then, our aim is to design the observer gain L that achieve

$$\lim_{t \to \infty} e(t) \to 0. \tag{3}$$

- To accomplish this aim, we need to derive the error dynamic system model from (1) and (2):

$$\begin{aligned} \dot{e}(t) &= Ae(t) + L\big(y(t) - \tilde{y}(t)\big) \\ &= \Big(A + LC\Big)e(t). \end{aligned} \tag{4}$$

The following theorem provides LMI-based observer design conditions obtained in the sense of Lyapunov stability.

Theorem 1. Convergence condition (3) holds if there exist matrices $P = P^T \in \mathbb{R}^{n\times n}$ and $\bar{L} \in \mathbb{R}^{n\times p}$, such that

$$P > 0$$

$$\mathbf{He}\left\{PA + \bar{L}C\right\} < 0.$$

Furthermore, the observer gain can be reconstructed in this manner:

$$L = P^{-1}\bar{L}.$$