$\boxed{\text{Robust LQR}}$ state-feedback control: $\dot{x}(t) = (A+\Delta A)x(t) + (B+\Delta B)u(t)$

where $\Delta A = E\gamma H_1$, $\Delta B = E\gamma H_2$, $u(t) = Kx(t)$

$\Rightarrow \dot{x}(t) = (\bar{A} + E\gamma\bar{H})x(t)$, where $\bar{A} = A + BK$, $\bar{H} = H_1 + H_2K$

We have $\dot{V}(t) + x^TQx(t) + u^T(t)Ru(t) < 0$, where $V(t) = x^T(t)Px(t)$

$\Rightarrow \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + x^T(t)Qx(t) + x^T(t)K^TRKx(t) < 0$

$\Rightarrow x^T(t)\left[(\bar{A}+E\gamma\bar{H})^TP + P(\bar{A}+E\gamma\bar{H}) + Q + K^TRK\right]x(t) < 0$

$$\Rightarrow \begin{cases} P > 0 \\ \mathrm{He}\{P(\bar{A}+E\gamma\bar{H})\} + Q + K^TRK < 0 \end{cases}$$

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Example 4.1

$$\Rightarrow \mathrm{He}\{P\bar{A}\} + \underbrace{\mathrm{He}\{PE\gamma\bar{H}\}}_{} + Q + K^TRK < 0$$

$$\leq \varepsilon PEE^TP + \varepsilon^{-1}\bar{H}^T\gamma^T\gamma\bar{H} = \varepsilon PEE^TP + \varepsilon^{-1}\bar{H}^T\bar{H}$$

$$\Rightarrow \mathrm{He}\{P\bar{A}\} + \varepsilon PEE^TP + \underbrace{\varepsilon^{-1}\bar{H}^T\bar{H}}_{-\bar{H}^T(-\varepsilon^{-1} I)\bar{H}} + Q + K^TRK < 0$$

$$\Rightarrow \underbrace{\left[\begin{array}{c|c} \mathrm{He}\{P\bar{A}\} + \varepsilon PEE^TP + Q + K^TRK & \bar{H}^T \\ \hline \bar{H} & -\varepsilon I \end{array}\right]}_{} < 0$$

$$\Rightarrow \underbrace{\begin{bmatrix} P^{-1} & 0 \\ 0 & I \end{bmatrix}}_{M^T} \underbrace{\begin{bmatrix} & \\ & \end{bmatrix}}_{\Omega} \underbrace{\begin{bmatrix} P^{-1} & 0 \\ 0 & I \end{bmatrix}}_{M} < 0$$

$$\Rightarrow \left[\begin{array}{c|c} \mathrm{He}\{\bar{A}P^{-1}\} + \varepsilon EE^T + P^{-1}QP^{-1} + P^{-1}K^TRKP^{-1} & P^{-1}\bar{H}^T \\ \hline \bar{H}P^{-1} & -\varepsilon I \end{array}\right] < 0$$

Define $\bar{K} = KP^{-1}$, $\bar{P} = P^{-1}$

$\bar{A}P^{-1} = (A+BK)P^{-1} = AP^{-1} + BKP^{-1} = A\bar{P} + B\bar{K}$

$\bar{H}P^{-1} = (H_1+H_2K)P^{-1} = H_1\bar{P} + H_2\bar{K}$

$$\Rightarrow \left[\begin{array}{c:c} \mathrm{He}\{A\bar{P} + B\bar{K}\} + \varepsilon E E^T + \bar{P} Q \bar{P} + \bar{K}^T R \bar{K} & \bar{P} H^T \\ \hdashline H_1 \bar{P} + H_2 \bar{K} & -\varepsilon I \end{array}\right] < 0$$

$$\bar{P} Q Q^{-1} Q \bar{P}$$

$$\Rightarrow \left[\begin{array}{c:c} \mathrm{He}\{A\bar{P} + B\bar{K}\} + \varepsilon E E^T + \bar{K}^T R \bar{K} & \bar{P} H^T \\ \hdashline H_1 \bar{P} + H_2 \bar{K} & -\varepsilon I \end{array}\right] - \left[\begin{array}{c} \bar{P} Q \\ \hdashline 0 \end{array}\right] \left[-Q^{-1}\right] \left[\begin{array}{c:c} Q\bar{P} & 0 \end{array}\right] < 0$$

$$\Rightarrow \left[\begin{array}{c:c:c} \mathrm{He}\{A\bar{P} + B\bar{K}\} + \varepsilon E E^T + \bar{K}^T R \bar{K} & \bar{P} H^T & \bar{P} Q \\ \hdashline H_1 \bar{P} + H_2 \bar{K} & -\varepsilon I & 0 \\ \hdashline Q \bar{P} & 0 & -Q \end{array}\right] < 0$$

$$\bar{K}^T R R^{-1} R \bar{K}$$

$$\Rightarrow \left[\begin{array}{c:c:c} \mathrm{He}\{A\bar{P} + B\bar{K}\} + \varepsilon E E^T & \bar{P} H^T & \bar{P} Q \\ \hdashline H_1 \bar{P} + H_2 \bar{K} & -\varepsilon I & 0 \\ \hdashline Q \bar{P} & 0 & -Q \end{array}\right] - \left[\begin{array}{c} \bar{K}^T R \\ \hdashline 0 \\ \hdashline 0 \end{array}\right] \left[-R^{-1}\right] \left[\begin{array}{c:c:c} R\bar{K} & 0 & 0 \end{array}\right] < 0$$

$$\Rightarrow \begin{cases} \left[\begin{array}{c:c:c:c} \mathrm{He}\{A\bar{P} + B\bar{K}\} + \varepsilon E E^T & \bar{P} H^T & \bar{P} Q & \bar{K}^T R \\ \hdashline H_1 \bar{P} + H_2 \bar{K} & -\varepsilon I & 0 & 0 \\ \hdashline Q \bar{P} & 0 & -Q & 0 \\ \hdashline R \bar{K} & 0 & 0 & -R \end{array}\right] < 0 \\ \bar{P} > 0 \end{cases}$$

$$\left|\begin{array}{l} K = \bar{K} \bar{P}^{-1} \\ P = \bar{P}^{-1} \end{array}\right.$$