State-feedback control systems

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4 Robust state-feedback control

- The considered system model is as follows:

$$\dot{x}(t) = \big(A + \Delta A\big)x(t) + \big(B + \Delta B\big)u(t). \tag{21}$$

- The parameter uncertainties $\Delta A \in \mathbb{R}^{n\times n}$ and $\Delta B \in \mathbb{R}^{n\times m}$ are described as, respectively,

$$\Delta A = E\Upsilon H_1,\ \Delta B = E\Upsilon H_2 \tag{22}$$

where $E \in \mathbb{R}^{n\times p}$, $H_1 \in \mathbb{R}^{q\times n}$, and $H_2 \in \mathbb{R}^{q\times m}$ are known real constant matrices; and $\Upsilon \in \mathbb{R}^{p\times q}$ is an unknown matrix but satisfies

$$\Upsilon^T\Upsilon \leq I. \tag{23}$$

- Let us employ the following state-feedback control law:

$$u(t) = Kx(t) \tag{24}$$

where $K \in \mathbb{R}^{m\times n}$ denotes the control gain to be designed later.

- As a result, the closed-loop control system is described as follows:

$$\begin{aligned}\dot{x}(t) &= \big(A + E\Upsilon H_1\big)x(t) + \big(B + E\Upsilon H_2\big)Kx(t) \\ &= \big(A + BK + E\Upsilon(H_1 + H_2K)\big)x(t) \\ &= \big(\bar{A} + E\Upsilon\bar{H}\big)x(t) \end{aligned}\tag{25}$$

by defining

$$\bar{A} = A + BK,\ \ \bar{H} = H_1 + H_2K.$$

Theorem 3. System (21) is robustly stable at the origin if there exist matrices $\bar{P} = \bar{P}^T \in \mathbb{R}^{n\times n}$, $\bar{K} \in \mathbb{R}^{m\times n}$, and a scalar $\epsilon > 0$ satisfying

$$\bar{P} > 0 \tag{26}$$

$$\left[\begin{array}{c|c} \mathbf{He}\{A\bar{P} + B\bar{K}\} + \epsilon E E^T & \star \\ \hline H_1\bar{P} + H_2\bar{K} & -\epsilon I \end{array}\right] < 0. \tag{27}$$

Furthermore, the control gain can be reconstructed as follows:

$$K = \bar{K}\bar{P}^{-1}.$$

Example 4.1. Obtain the linear matrix inequalities (LMI) needed to design a robust LQR state-feedback controller for system (21).

5 $\mathcal{H}_\infty$ state-feedback control

• This state-feedback control is designed with the aim of minimizing the effect of disturbances on the closed-loop system while using feedback of the state variables.

• To begin with, let us consider the following linear system with external disturbances belonging to the $\mathcal{L}_2$ space:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Nw(t) \\ z(t) = Cx(t) + Du(t). \end{cases} \tag{34}$$

- Let us employ the following state-feedback control law:

$$u(t) = Kx(t)$$

where $K \in \mathbb{R}^{m \times n}$ is the control gain to be designed later.

- As a result, the closed-loop control system is described as follows:

$$\begin{cases} \dot{x}(t) = \bar{A}x(t) + Nw(t) \\ z(t) = \bar{C}x(t) \end{cases} \tag{35}$$

where

$$\bar{A} = A + BK, \ \bar{C} = C + DK.$$

Theorem 4. System (34) is stable and has an $\mathcal{H}_\infty$ disturbance attenuation level γ, if there exist symmetric matrices $\bar{P} \in \mathbb{R}^{n\times n}$ and $\bar{K} \in \mathbb{R}^{m\times n}$, and a scalar $\gamma > 0$ satisfying

$$\bar{P} > 0 \tag{36}$$

$$\left[\begin{array}{cc|c} \mathbf{He}\{A\bar{P} + B\bar{K}\} & \star & \star \\ N^T & -\gamma^2 I & 0 \\ \hline C\bar{P} + D\bar{K} & 0 & -I \end{array}\right] < 0. \tag{37}$$

Furthermore, the control gain can be reconstructed as follows:

$$K = \bar{K}\bar{P}^{-1}.$$