From Example 2.1, it is obtained that

$$A = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \qquad n = 2, \; m = 1$$

$$\frac{1}{L} \longrightarrow \frac{1}{L_0} + \delta \quad \text{subject to } \delta \in [-0.1, 0.1]$$

untrue but bounded

$$\Downarrow$$

$$\begin{bmatrix} -R\left(\frac{1}{L_0}+\delta\right) & -\left(\frac{1}{L_0}+\delta\right) \\ \frac{1}{C} & 0 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{R}{L_0} & -\frac{1}{L_0} \\ \frac{1}{C} & 0 \end{bmatrix}}_{A:=} + \underbrace{\begin{bmatrix} -R\delta & -\delta \\ 0 & 0 \end{bmatrix}}_{\Delta A:=}$$

$$B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{L_0}+\delta \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{L_0} \\ 0 \end{bmatrix}}_{B:=} + \underbrace{\begin{bmatrix} \delta \\ 0 \end{bmatrix}}_{\Delta B:=}$$

Example 4.1. In Example 2.1, let us consider the case where

$$\frac{1}{L} = \frac{1}{L_0} + \delta, \quad \delta \in [-0.1,\ 0.1].$$

Please define ΔA and ΔB; and express them in the following form:

$$\Delta A = E\Upsilon H_1, \quad \Delta B = E\Upsilon H_2$$

subject to

$$\Upsilon^T\Upsilon \le I,$$

$\Upsilon \in \mathbb{R}^1$ scalar

$$\Delta A = \begin{bmatrix} -R\delta & -\delta \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \times \underbrace{\delta}_{\Upsilon} \times \begin{bmatrix} R & 1 \end{bmatrix}$$

-0.1, 10

$\delta \in [-0.1, 0.1]$

$\times 10$

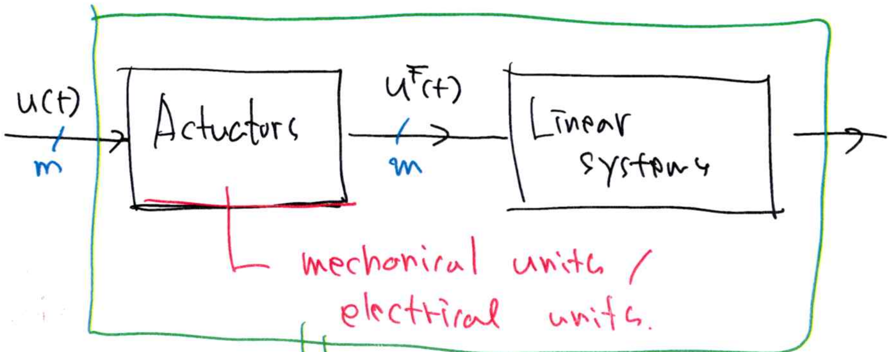
$$= \begin{bmatrix} -0.1 \\ 0 \end{bmatrix} 10\cdot\delta \begin{bmatrix} R & 1 \end{bmatrix}$$

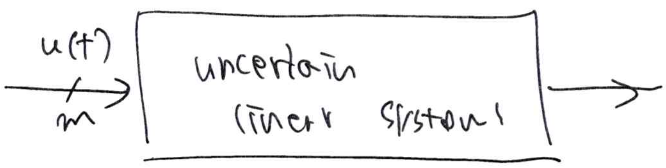
$$\Upsilon^T\Upsilon \le I \;\Downarrow\; \Upsilon^2 \le 1 \iff \Upsilon \in [-1, 1]$$

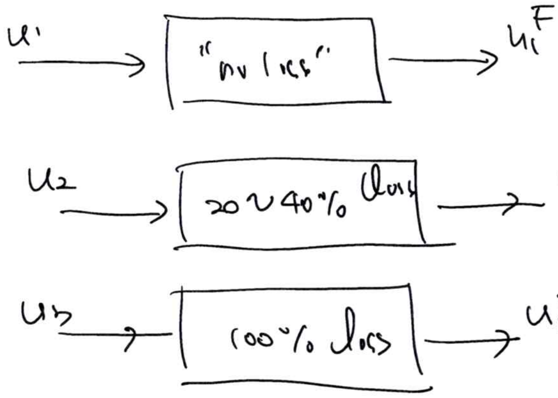
$$= \underbrace{\begin{bmatrix} -1 \\ 0 \end{bmatrix}}\; 10\cdot\delta\,[\,0.1R \quad 0.1\,]$$

$$\Delta A = \underbrace{\begin{bmatrix} -0.1 \\ 0 \end{bmatrix}}_{E}\; \underbrace{10\cdot\delta}_{r}\; \underbrace{[\,R \quad 1\,]}_{H_1}$$

$$\Delta B = \begin{bmatrix} \delta \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} -0.1 \\ 0 \end{bmatrix}}_{E}\; \underbrace{10\cdot\delta}_{r}\; \underbrace{[\,-1\,]}_{H_2}$$







$u_1^F = \alpha_1 u_1 \qquad \alpha_1 = 1$

$u_2^F = \alpha_2 u_2$

$\alpha_2 \in [\,0.6 \quad 0.8\,]$

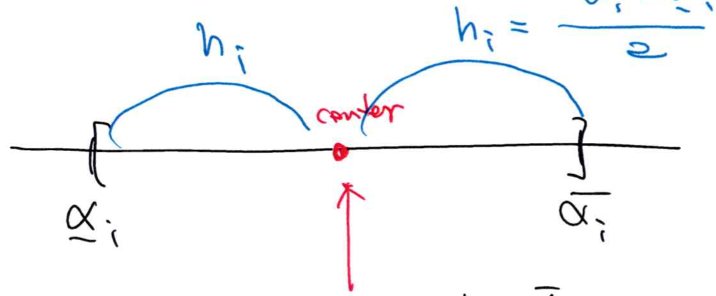
$u_3^F = \alpha_3 u_3$

$\alpha_3 = 0$

$N < N < \bar{N}$

$u_3 = 0$

$$\underline{\alpha}_i \leq \alpha_i \leq \overline{\alpha}_i$$



$$e_i = \frac{\underline{\alpha}_i + \overline{\alpha}_i}{2}$$

$$\alpha_i = e_i + \delta_i \quad \text{subject to} \quad \delta_i \in [-h_i, h_i]$$

$$= e_i + h_i \overline{\delta}_i$$

$$\overline{\delta}_i := \frac{\delta_i}{h_i} \in [-1, 1]$$

$$\delta_i = h_i \overline{\delta}_i$$

$$\Downarrow$$

$$\alpha = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_m)$$

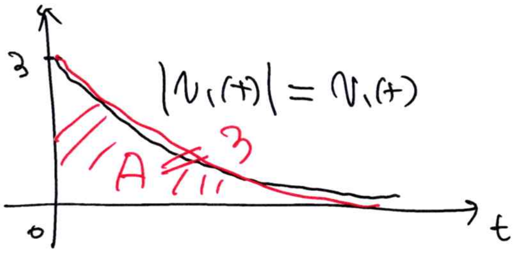
$$= \begin{bmatrix} \alpha_1 & & & 0 \\ & \alpha_2 & & \\ & & \ddots & \\ 0 & & & \alpha_m \end{bmatrix} = \begin{bmatrix} e_1 + h_1 \overline{\delta}_1 & & & \\ & e_2 + h_2 \overline{\delta}_2 & & \\ & & \ddots & \\ & & & e_m + h_m \overline{\delta}_m \end{bmatrix}$$

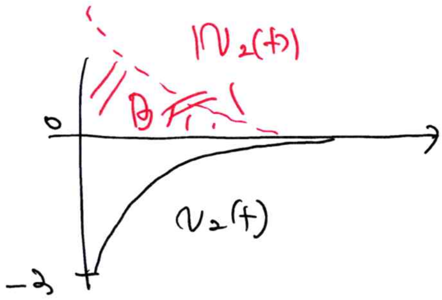
$$= \underbrace{\begin{bmatrix} e_1 & & & 0 \\ & e_2 & & \\ & & \ddots & \\ 0 & & & e_m \end{bmatrix}}_{\text{nominal term}} + \underbrace{\begin{bmatrix} \overline{\delta}_1 & & & 0 \\ & \overline{\delta}_2 & & \\ & & \ddots & \\ 0 & & & \overline{\delta}_m \end{bmatrix}}_{\Gamma} \begin{bmatrix} h_1 & & & 0 \\ & h_2 & & \\ & & \ddots & \\ & & & h_m \end{bmatrix}$$

$$\Gamma^T \Gamma \leq I$$

Example 5.1. Find the $\mathcal{L}_1$-norm of $v(t)$ and show that $v(t)$ belongs to the $\mathcal{L}_1$-space:

$$v(t) = \begin{bmatrix} 3e^{-t} \\ -3e^{-3t} \end{bmatrix} \in \mathbb{R}^2. \quad , \ t \geq 0.$$





$$v_1(t) = 3e^{-t}$$

$$v_2(t) = -3e^{-3t}$$

$$\|v\|_1 = \sum_{i=1}^{2} \int_0^\infty |v_i(t)| \, dt$$

$$= \int_0^\infty |v_1(t)| \, dt + \int_0^\infty |v_2(t)| \, dt$$

$$= \int_0^\infty 3e^{-t} \, dt + \int_0^\infty 3e^{-3t} \, dt$$

$$= \left[-3e^{-t}\right]_0^\infty + \left[-e^{-3t}\right]_0^\infty$$

$$= 0 - (-3) + 0 - (-1)$$

$$= 4.$$

$$\|v\|_1 = 4 \implies \boxed{v(t) \in \mathcal{L}_1}$$

$$||v(t)||_2 = \left(\sum_{i=1}^{w}\int_0^\infty v_i^2(t)dt\right)^{\frac{1}{2}} = \left(\int_0^\infty \boxed{v^T(t)v(t)}dt\right)^{\frac{1}{2}}.$$

$$= \int_0^\infty \sum_{i=1}^{\omega} v_i^2(t)\,dt$$

$$v(t) = \begin{bmatrix} v_1 \\ \vdots \\ v_\omega \end{bmatrix}$$

$$v^T v = [v_1 \; .. \; v_\omega] \begin{bmatrix} v_1 \\ \\ v_\omega \end{bmatrix}$$

Example 5.2. Find the $\mathcal{L}_2$-norm of $v(t)$ and show that $v(t)$ belongs to the $\mathcal{L}_2$-space:

$$v(t) = \begin{bmatrix} 3e^{-t} \\ -3e^{-3t} \end{bmatrix} \in \mathbb{R}^2.$$

$$||v(t)||_2^2 = \sum_{i=1}^{2}\int_0^\infty v_i^2(t)\,dt$$

$$= \int_0^\infty v_1^2(t)\,dt + \int_0^\infty v_2^2(t)\,dt$$

$$= \int_0^\infty 9e^{-2t}\,dt + \int_0^\infty 9e^{-6t}\,dt$$

$$= \left[-\frac{9}{2}e^{-2t}\right]_0^\infty + \left[-\frac{9}{6}e^{-6t}\right]_0^\infty$$

$$= 0 - \left(-\frac{9}{2}\right) + 0 - \left(-\frac{9}{6}\right)$$

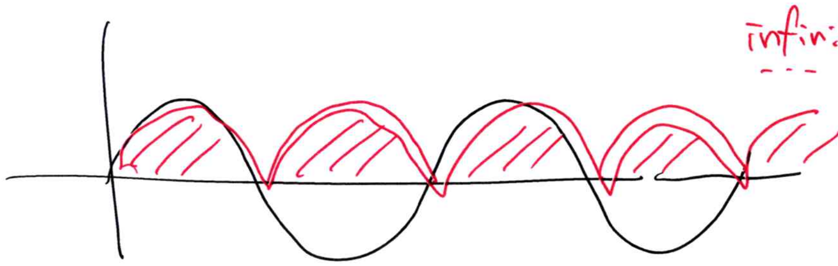
$$= \frac{9}{2} + \frac{\cancel{9}^{3}}{\cancel{6}_{2}} = \frac{12}{2} = 6$$

$$||v||_2^2 = 6 \implies ||v||_2 = \sqrt{6}$$

$v \in \mathcal{L}_e$

How about $v(t) = \sin \pi t$??

$\| v_1 \| = ??$



$v_1 \notin \mathcal{L}_1$

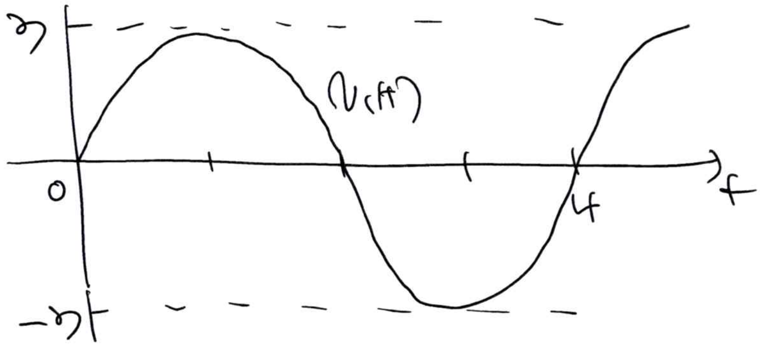
Example 5.3. Find the $\mathcal{L}_\infty$-norm of $v(t)$ and show that $v(t)$ belongs to the $\mathcal{L}_\infty$-space:

$$v(t) = \begin{bmatrix} 3\sin\frac{\pi}{2}t \\ -5\cos\pi t \end{bmatrix} \in \mathbb{R}^2.$$

$$||v(t)||_\infty := \max_{i=1,2,\ldots,w} \left(\sup_{t\geq 0} |v_i(t)| \right).$$

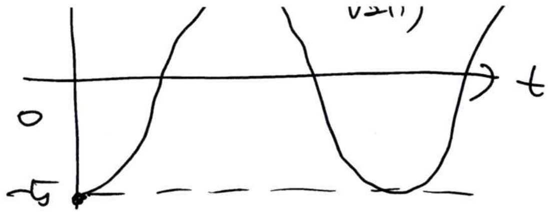
$v_1(t) = 3 \sin \frac{\pi}{2} t$

$v_2(t) = -5 \cos \pi t$



$\sup_{t\geq 0} |v_1(t)| = 3$

$\sup_{t\geq 0} |v_2(t)| = 5$



$$\sup_{t \geq 0} |v_2(t)| = 5$$

$$\|v(t)\|_\infty = 5 < \infty$$

$$v(t) \in \mathcal{L}_\infty \text{ "}$$

How about $v(t) = t$.