Mathematical Description of Linear Systems with various constraints

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2 State-space Model

The linear time-invariant (LTI) state and output equations can be described as the following state-space form:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{1}$$

where

- $x(t)$ is the state vector belonging to the set $\mathbb{R}^n$,
- $u(t)$ is the control input vector belonging to the set $\mathbb{R}^m$, and
- $y(t)$ is the output vector belonging to the set $\mathbb{R}^p$.

Accordingly, the system matrices satisfy

$$A \in \mathbb{R}^{n\times n},\ B \in \mathbb{R}^{n\times m},\ C \in \mathbb{R}^{p\times n},\ \text{and}\ D \in \mathbb{R}^{p\times m}.$$

Examples

Example 2.1. Let us consider a simple RLC electrical circuit given in Figure 1.

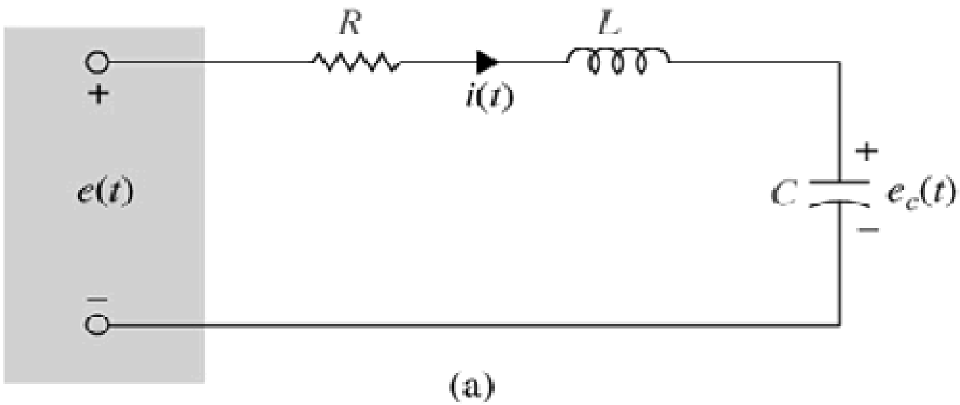
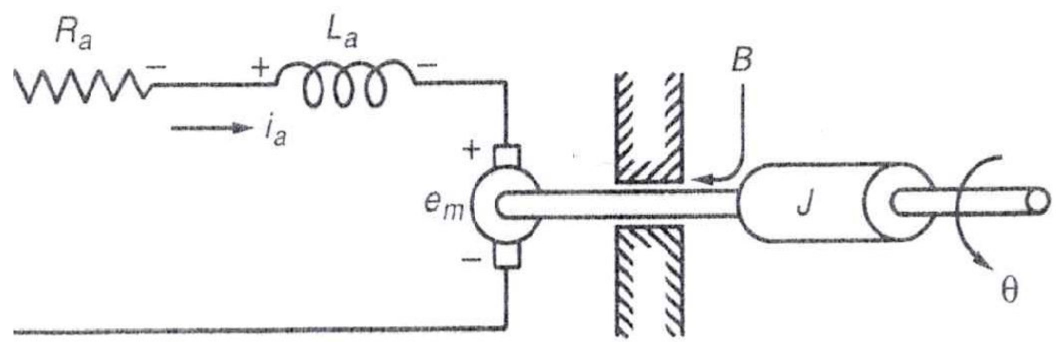


Figure 1: RLC electrical circuit.

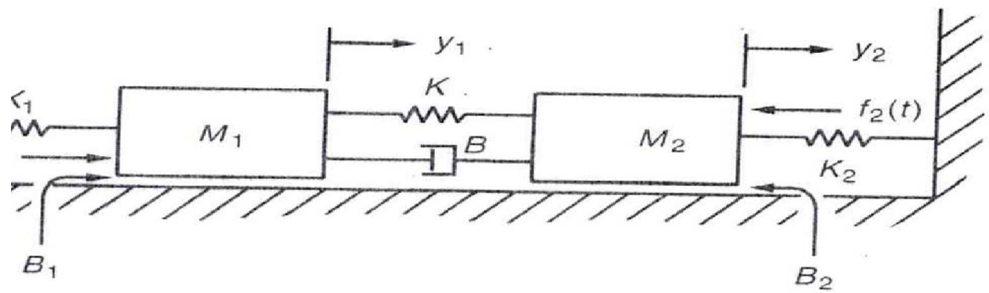
Example 2.2. Let us consider a simplified model of an armature voltage-controlled dc servomotor consisting of a stationary field and a rotating armature and load.



Assignment 2.1. Newton's second law yields the following coupled second-order ordinary differential equations:

$$
\begin{aligned}
M_1\ddot{y}_1 + (B+B_1)\dot{y}_1 + (K+K_1)y_1 - B\dot{y}_2 - Ky_2 &= f_1(t) \\
M_2\ddot{y}_2 + (B+B_2)\dot{y}_2 + (K+K_2)y_2 - B\dot{y}_1 - Ky_1 &= -f_2(t)
\end{aligned}
\tag{6}
$$

with initial data $y_1(0)$, $y_2(0)$, $\dot{y}_1(0)$, and $\dot{y}_2(0)$.



Please express (6) as a state-space model.