

Sol. of Assignment 1: Letting

$$x_1 = y_1, \quad x_2 = \dot{y}_1, \quad x_3 = y_2, \quad x_4 = \dot{y}_2, \quad u_1 = f_1, \quad u_2 = f_2$$

the state-space model of (2) is given as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1+K}{M_1} & -\frac{B_1+B}{M_1} & \frac{K}{M_1} & \frac{B}{M_1} \\ 0 & 0 & 0 & 1 \\ \frac{K}{M_2} & \frac{B}{M_2} & -\frac{K+K_2}{M_2} & -\frac{B+B_2}{M_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{M_1} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{M_2} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

with initial state $x(0) = [x_1(0) \quad x_2(0) \quad x_3(0) \quad x_4(0)]^T$. Furthermore, if it is assumed that y_1 and y_2 are measurable, then the output vector is given as

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t).$$