Stability analysis of linear systems

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2.4 $\mathcal{H}_\infty$ stability

- This stability requires to simultaneously analyze **(i)** the stability and **(ii)** the $\mathcal{H}_\infty$ performance for the following linear systems with external disturbances:

$$\begin{cases} \dot{x}(t) = Ax(t) + Nw(t) \\ z(t) = Gx(t) \end{cases} \tag{18}$$

- As mentioned, $w(t) \in \mathbb{R}^d$ denotes the external disturbances, and $z(t) \in \mathbb{R}^q$ denotes the performance output.

- To consider the $\mathcal{H}_\infty$ performance, it is essential to assume that $w(t) \in \mathcal{L}_2$, that is,

$$\int_0^\infty w^T(\tau)w(\tau)d\tau < \infty.$$

Definition 3. If the following two conditions hold:

(i) for $x(0) \neq 0$ and $w(t) = 0$,

$$\dot{V}(t) < 0 \quad :\sim \text{stability criterion}$$

(ii) for $x(0) = 0$ and $w(t) \neq 0$,

$$\int_0^\infty z^T(\tau)z(\tau)d\tau \leq \gamma^2 \int_0^\infty w^T(\tau)w(\tau)d\tau \quad :\sim \mathcal{H}_\infty \text{ performance}$$

then it can be said that (18) is stable and has an $\mathcal{H}_\infty$ disturbance attenuation level γ.

The following lemma provides the $\mathcal{H}_\infty$ stability criterion.

Lemma 5. If it holds that

$$\dot{V}(t) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) < 0 \tag{19}$$

then system (18) is stable and has an $\mathcal{H}_\infty$ disturbance attenuation level γ.

The following theorem provides a set of linear matrix inequalities (LMIs) for (19).

Theorem 2. System (18) is stable and has an $\mathcal{H}_\infty$ disturbance attenuation level γ, if there exist a symmetric matrix $P \in \mathbb{R}^{n\times n}$ and a positive scalar γ satisfying

$$P > 0 \tag{22}$$

$$\begin{bmatrix} \mathbf{He}\{PA\} + G^T G & PN \\ N^T P & -\gamma^2 I \end{bmatrix} < 0. \tag{23}$$

Assignment:

Obtain a set of linear matrix inequalities (LMIs) that ensures the robust $\mathcal{H}_\infty$ stability of

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + Nw(t) \\ z(t) = Gx(t). \end{cases}$$