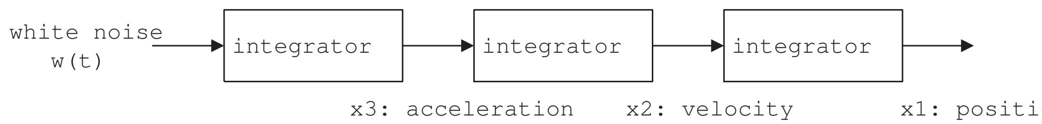
Lecture 6: Discrete Kalman filter - position estimation

Homework

A generic process model for vehicle movement is three integrators in cascade as shown in the following figure.



Let $x(t)$ be defined by

$$x(t) \triangleq \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix},$$

and the continuous process model is given by

$$\dot{x}(t) = Ax(t) + Bw(t)$$

where $w(t)$ is a zero-mean white Gaussian process with variance q_1. Suppose we measure movement of the vehicle every $T = 0.01$ sec using an accelerometer whose measurement noise can be modelled as a zero-mean white Gaussian process with covariance R. Then we can write the discrete system model as follows:

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} 1 & T_1 & \frac{T_1^2}{2} \\ 0 & 1 & T_1 \\ 0 & 0 & 1 \end{bmatrix} x_k + w_k \\ z_k &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x_k + v_k \end{aligned}$$

where w_k and v_k are zero-mean white Gaussian noises with covariance Q and R, respectively.

$$Q = 0.05 \begin{bmatrix} \frac{T_1^5}{20} & \frac{T_1^4}{8} & \frac{T_1^3}{6} \\ \frac{T_1^4}{8} & \frac{T_1^3}{3} & \frac{T_1^2}{2} \\ \frac{T_1^3}{6} & \frac{T_1^2}{2} & T_1 \end{bmatrix}, \quad R = 0.01$$

(1) Do simulation to generate x_k and z_k with initial position 0.
(2) With a Kalman filter, find $\hat{x}_k$ from z_k. Assume that you know the exact initial position 0.
(3) Draw plot of e_k, $P_k(1,1)$, $P_k(2,2)$, $P_k(3,3)$.