(1) $E[X] = \int_0^{10} \frac{1}{10} x \, dx = \frac{1}{10} \left[\frac{1}{2}x^2\right]_0^{10} = 5$

$E[(X - E[X])^2] = \int_0^{10} \frac{1}{10}(x-5)^2 \, dx = \frac{1}{30}(x-5)^3 \Big|_0^{10}$

$= \frac{1}{30}(125 + 125) = \frac{250}{30} = \frac{25}{3}$

(2) $E[(X(t) + X(t+2))^2]$

$= E[X^2(t)] + E[X^2(t+2)] + 2E[X(t)X(t+2)]$

$= 1$

(3) $E[(-\alpha V_1 - (1-\alpha) V_2)^2]$

$= \alpha^2 E[V_1^2] + (1-\alpha)^2 E[V_2^2] + 2\alpha(1-\alpha) E[V_1 V_2]$

$= \alpha^2 + (1-\alpha)^2 + \alpha(1-\alpha)$

$= \alpha^2 + \alpha^2 - 2\alpha + 1 - \alpha^2 + \alpha = \alpha^2 - \alpha + 1$

(4) $\left| \lambda I - \begin{bmatrix} 2 & a \\ a & 2 \end{bmatrix} \right| = \left| \begin{bmatrix} \lambda - 2 & -a \\ -a & \lambda - 2 \end{bmatrix} \right|$

$= (\lambda - 2)^2 - a^2 = 0 \qquad \lambda^2 - 4\lambda + 4 - a^2 = 0$

eigenvalue $> 0 \Rightarrow 4 - a^2 > 0 \qquad a^2 < 4$

$|a| < 2$

(5) $K_k = P_k^- H' (H P_k^- H' + R)^{-1}$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 1 \right)^{-1} = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}$$

$$P_k = (I - K_k H) P_k^- = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}$$

(6) $(P_k^-)^{-1} = \begin{bmatrix} 10 & 0 \\ 0 & 0 \end{bmatrix}$

$$(P_k)^{-1} = \begin{bmatrix} 10 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 1 \\ 1 & 1 \end{bmatrix}$$

$$P_k = \frac{1}{11-1} \begin{bmatrix} 1 & -1 \\ -1 & 11 \end{bmatrix} = \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & 1.1 \end{bmatrix}$$

(7) $\int_0^t \begin{bmatrix} 1 & r \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} dr = \int_0^t \begin{bmatrix} 2+3r \\ 3 \end{bmatrix} dr$

$$= \begin{bmatrix} 2r + \frac{3}{2} r^2 \\ 3r \end{bmatrix}_0^t = \begin{bmatrix} 2t + \frac{3}{2} t^2 \\ 3t \end{bmatrix}$$