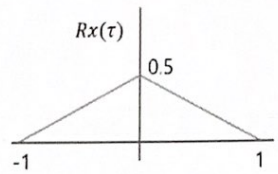
2024 Estimation Theory Midexam

Warning: There is a penalty for the wrong answer (minus score). Be careful!

(1) Let X be a uniform random variable on $[0, 10]$. Find the mean and variance of X.

(2) The autocorrelation of a random variable X is given by as follows:



Find $\mathrm{E}\{(x(t) + x(t+2))^2\}$.

$E[v_1^2] = 1 = E[v_2^2]$

(3) Suppose there are two measurement z_1 and z_2:

$$\begin{aligned} z_1 &= x + v_1 \\ z_2 &= x + v_2 \end{aligned}$$

where $v_1 \sim N(0,1)$ and $v_2 \sim N(0,1)$, $\mathrm{E}\{v_1 v_2\} = 0.5$. Suppose we use an estimator $\hat{x} = \alpha z_1 + (1-\alpha) z_2$. Compute $\mathrm{E}\{(x - \hat{x})^2\}$.

(4) Find the condition that the following matrix is postive definite.

$$\begin{bmatrix} 2 & a \\ a & 2 \end{bmatrix}$$

(5) Let P_k^- in the Kalman filter is given by

$$P_k^- = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Let the measurement equation is given by

$$z_k = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{H} x_k + v_k$$

where $v_k \sim N(0,1)$. Compute P_k.

(6) Let P_k^- in the Kalman filter is given by

$$P_k^- = \begin{bmatrix} 0.1 & 0 \\ 0 & \infty \end{bmatrix}.$$

Let the measurement equation is given by

$$z_k = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}} x_k + v_k$$

where $v_k \sim N(0,1)$. Compute P_k using the information form.

(7) Compute the following integral:

$$\int_0^t \exp\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} r\right) \begin{bmatrix} 2 \\ 3 \end{bmatrix} dr.$$