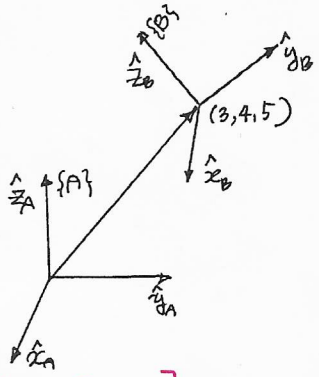
2013 Manipulator Mechanics Final Exam.

(Closed Book, No Question)

1. For {A} -> {B}, Find transformation matrix(${}^A_B T$).

where ${}^A(\hat{x}_B) = [0.43, -0.50, 0.75]^T$, ${}^A(\hat{y}_B) = [0.25, 0.87, 0.43]^T$

(10pts)



$${}^A_B R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} {}^A\hat{X}_B & {}^A\hat{Y}_B & {}^A\hat{Z}_B \end{bmatrix} = \begin{bmatrix} 0.43 & 0.25 & r_{13} \\ -0.5 & 0.87 & r_{23} \\ 0.75 & 0.43 & r_{33} \end{bmatrix}$$

$$r_{11}^2 + r_{12}^2 + r_{13}^2 = 1 \qquad r_{11}r_{21} + r_{12}r_{22} + r_{13}r_{23} = 0$$

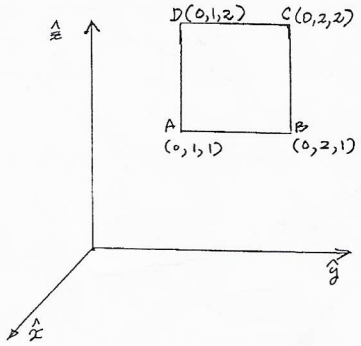
$$r_{21}^2 + r_{22}^2 + r_{23}^2 = 1 \qquad r_{11}r_{31} + r_{12}r_{32} + r_{13}r_{33} = 0$$

$$r_{31}^2 + r_{32}^2 + r_{33}^2 = 1 \qquad r_{21}r_{31} + r_{22}r_{32} + r_{23}r_{33} = 0$$

$${}^A_B T = \left[\begin{array}{c|c} {}^A_B R & P_{BORG} \\ \hline 0\,0\,0 & 1 \end{array}\right]$$

2.

1) When plate ABCD is sequentially rotated by 30° about $\hat{x}$ and rotated by 30° about $\hat{y}$ shown in the right Fig.
Answer the corresponding coordinates of C (10pts) $R_y(30)R_x(30).C$



2) When plate ABCD is sequentially rotated by 30° about $\hat{x}$ and rotated by 30° about $\hat{y}$ in Euler angle convention
Answer the corresponding coordinates of D (10pts) $R_x(30)R_y(30).D$

3) When plate ABCD is rotated by 30° about $\hat{k} = [\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}]^T$, $R_K(30).A$

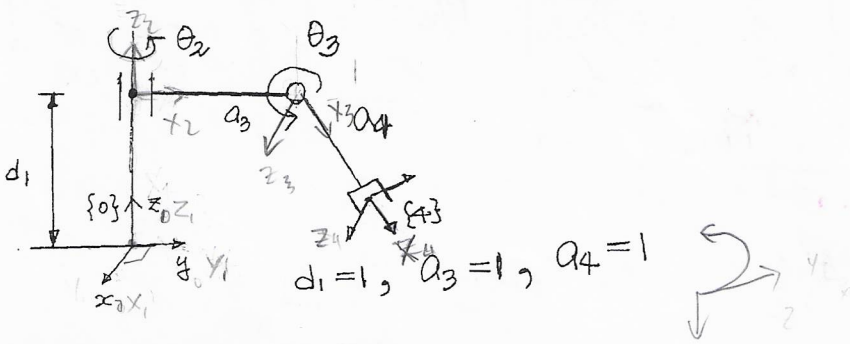
Answer the corresponding coordinates of A (10pts)

Equivalent angle Angle and Axis:

$$R_K(\theta) = \begin{bmatrix} k_x k_x v\theta + c\theta & k_x k_y v\theta - k_z s\theta & k_x k_z v\theta + k_y s\theta \\ k_x k_y v\theta + k_z s\theta & k_y k_y v\theta + c\theta & k_y k_z v\theta - k_x s\theta \\ k_x k_z v\theta - k_y s\theta & k_y k_z v\theta + k_x s\theta & k_z k_z v\theta + c\theta \end{bmatrix}$$

Where $c\theta = \cos\theta$, $s\theta = \sin\theta$, $v\theta = 1 - \cos\theta$, and ${}^A K = [k_x\ k_y\ k_z]^T$

3.



1) For the following PRR Manipulator, Assign the required coordinate frames, and fill out the corresponding DH parameter table. The current configuration is shown as when $d_1 = 1$,$\Theta_2 = 30°$, $\Theta_3 = 90°$ (20pts)

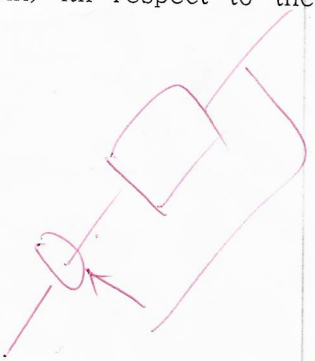
$z_{i-1} \to z_i$, $\hat{x}_{i-1}$ $\quad z_{i-1} \to z_i$ / $\hat{x}_{i-1}$ $\quad x_{i-1} \to x_i$, $\hat{z}_i$ $\quad x_{i-1} \to x_i$, $\hat{z}_i$

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	0
2	0	0	d_1	(90+θ_2) θ_2
3	90	a_3	0	θ_3
4	0	a_4	0	0

2) When $d_1 = 1$,$\Theta_2 = 30°$, $\Theta_3 = 90°$, Find the position of robot tip point with respect to the {0}-coordinate frame (10pts)

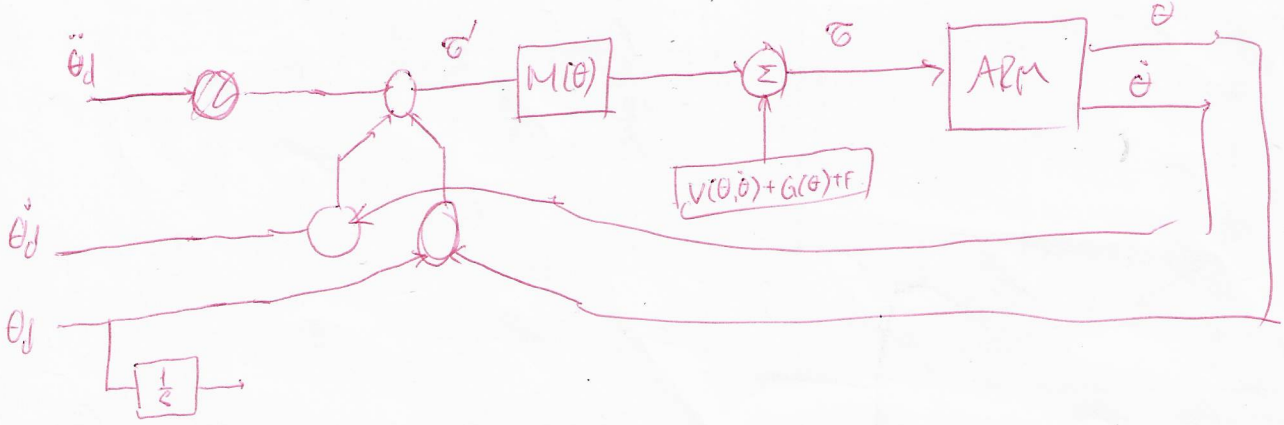
3) When $d_1 = 1$,$\Theta_2 = 30°$, $\Theta_3 = 90°$, Find the Jacobian (6 by 3 matrix) ith respect to the {0}-coordinate frame.(10pts)

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



4. Robot Dynamic Equation is given as $\tau = M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta)$
For the desired joint trajectory($\theta_d(t)$),

1) draw the Block diagram for Independent Joint PID control (10pts)
2) draw the Block diagram of Computed torque control (10pts)
3) discuss about merits and demerits of above controllers (10pts)



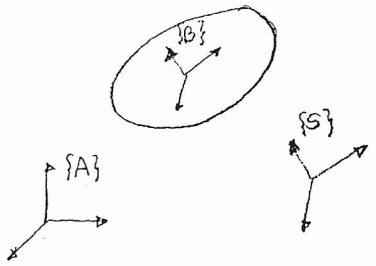
$$-\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$$

2015 MMC Final Exam(Closed Book, No Question)

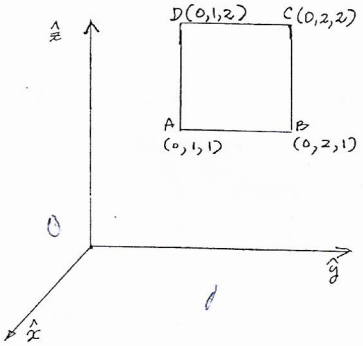
1. For the right figure,

$${}^A_B T = \begin{bmatrix} 0 & 0 & -1 & 1 \\ -0.5 & 0.866 & 0 & 2 \\ 0.866 & -0.5 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^S_B T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find ${}^A_S T$ (10 pts)



2. 1) When plate ABCD is rotated about y-axes by 60° and then, rotated about x-axes by 30°(Fixed angle rotation), descrbe the coordinates of the points A, C (10 pts)



2) When plate ABCD is rotated by 45° about $\overrightarrow{K} = [1,2,2]^T$, Answer the corresponding coordinates of B, D (10 pts)

$$R_K(\theta) = \begin{bmatrix} k_x k_x v\theta + c\theta & k_x k_y v\theta - k_z s\theta & k_x k_z v\theta + k_y s\theta \\ k_x k_y v\theta + k_z s\theta & k_y k_y v\theta + c\theta & k_y k_z v\theta - k_x s\theta \\ k_x k_z v\theta - k_y s\theta & k_y k_z v\theta + k_x s\theta & k_z k_z v\theta + c\theta \end{bmatrix}$$

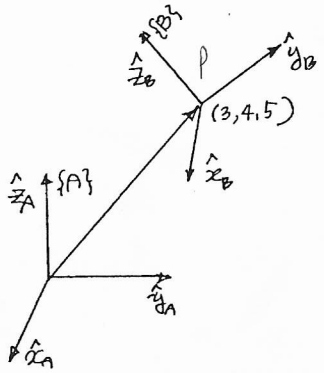
Where $c\theta = \cos\theta$, $s\theta = \sin\theta$, $v\theta = 1 - \cos\theta$, and ${}^A\hat{K} = [k_x\ k_y\ k_z]^T$.

45° 45 45 1 2 2

3. For {A} -> {B}, Find transformation matrix(${}^A_B T$).

where ${}^A(\hat{x_B}) = [\frac{1}{\sqrt{2}}\ 0\ -\frac{1}{\sqrt{2}}]^T$, ${}^A(\hat{z_B}) = [\frac{1}{2}\ \frac{1}{\sqrt{2}}\ \frac{1}{2}]^T$.

(10pts)



4. 1) For the following Puma 560 Manipulator, Assign the required coordinate frames, and fill out the corresponding DH parameter table up to wrist center point, (15pts)

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	0	θ_2
3	0	a_2	d_3	θ_3
4	-90°	a_3	d_4	θ_4

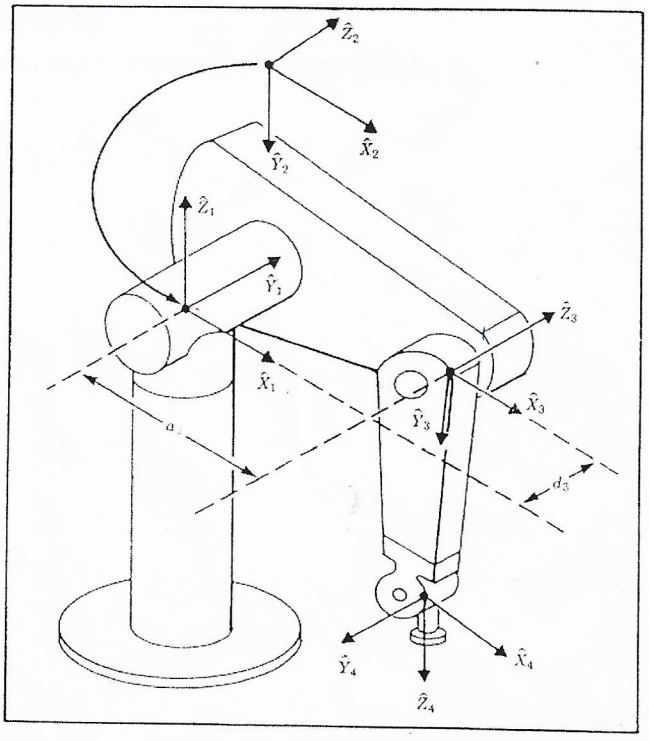


FIGURE 3.18 Some kinematic parameters and frame assignments for the PUMA 560 manipulator.

2) When $\Theta_1 = 0°$, $\Theta_2 = -30°$, $\Theta_3 = 30°$, $a_2 = 1000$, $d_3 = 200$ Find the position of 3rd coordinate origin point with respect to the {0}-coordinate frame (15 pts)

3) When $\Theta_1 = 0°$, $\Theta_2 = 30°$, $\Theta_3 = 30°$, $a_2 = 1000$, $d_3 = 200$, $a_3 = 1000$, $d_4 = 200$ Find the second column(6 by 1 vector) of the Jacobian(6 by 3 matrix) about the wrist center point with respect to the {0}-coordinate frame.(10pts)

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

4) When $\Theta_1 = 0°$, $\Theta_2 = 30°$, $\Theta_3 = 30°$, $\dot{\Theta}_1 = \dot{\Theta}_2 = \dot{\Theta}_3 = 2\,rad/\sec$. $a_2 = 1000$, $d_3 = 200$, $a_3 = 1000$, $d_4 = 200$ Find the linear velocities of the origin of the 3rd coordinate frame with respect to the {3}-coordinate frame.(15pts). Find the linear velocities of the origin of the 3rd coordinate frame {0}-coordinate frame(5 pts).

$$ {}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}, \qquad {}^{i+1}v_{i+1} = {}^{i+1}_{i}R({}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1}) $$

5. $\theta(0) = 30° \;\; \theta(t_f) = 90°$ $\qquad t_f = 1\sec$ (10pts)
$\dot{\theta}(0) = 0 \;\; \dot{\theta}(t_f) = 0$

1) Do trajectory planning with a cubic polynomial for this joint control

6. Robot Dynamic Equation is given as $\tau = M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta)$
For the desired joint trajectory($\theta_d(t)$), the desired cartesian trajectory($x_d(t)$)

1) draw the Block diagram for Independent Joint PID control (5pts)
2) draw the Block diagram of Computed torque control (5pts)
3) For Cartesian based control, Find equivalent model M_x, V_x, G_x (5pts)
4) draw the Block diagram of Cartesian based Jacobian Transpose control(5pts)