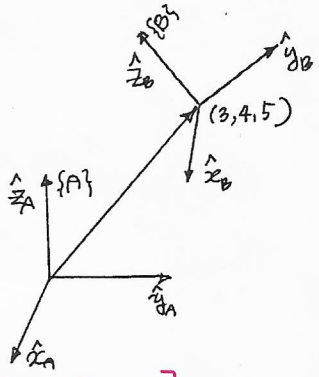
2013 Manipulator Mechanics Final Exam.

(Closed Book, No Question)

1. For {A} -> {B}, Find transformation matrix(${}^A_B T$).

where ${}^A(\hat{x}_B) = [0.43, -0.50, 0.75]^T$, ${}^A(\hat{y}_B) = [0.25, 0.87, 0.43]^T$

(10pts)



$${}^A_B R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} {}^A\hat{X}_B & {}^A\hat{Y}_B & {}^A\hat{Z}_B \end{bmatrix} = \begin{bmatrix} 0.43 & 0.25 & r_{13} \\ -0.5 & 0.87 & r_{23} \\ 0.75 & 0.43 & r_{33} \end{bmatrix}$$

$$r_{11}^2 + r_{12}^2 + r_{13}^2 = 1 \qquad r_{11}r_{21} + r_{12}r_{22} + r_{13}r_{23} = 0$$

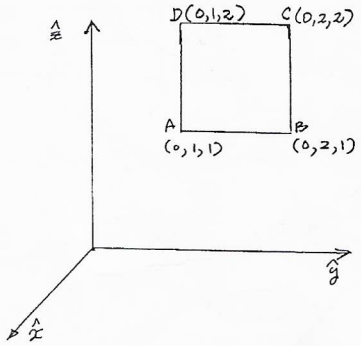
$$r_{21}^2 + r_{22}^2 + r_{23}^2 = 1 \qquad r_{11}r_{31} + r_{12}r_{32} + r_{13}r_{33} = 0$$

$$r_{31}^2 + r_{32}^2 + r_{33}^2 = 1 \qquad r_{21}r_{31} + r_{22}r_{32} + r_{23}r_{33} = 0$$

$${}^A_B T = \left[\begin{array}{c|c} {}^A_B R & P_{BORG} \\ \hline 0\,0\,0 & 1 \end{array}\right]$$

2.

1) When plate ABCD is sequentially rotated by 30° about $\hat{x}$ and rotated by 30° about $\hat{y}$ shown in the right Fig.
Answer the corresponding coordinates of C (10pts) $R_y(30)R_x(30).C$



2) When plate ABCD is sequentially rotated by 30° about $\hat{x}$ and rotated by 30° about $\hat{y}$ in Euler angle convention
Answer the corresponding coordinates of D (10pts) $R_x(30)R_y(30).D$

3) When plate ABCD is rotated by 30° about $\hat{k} = [\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}]^T$, $R_K(30).A$

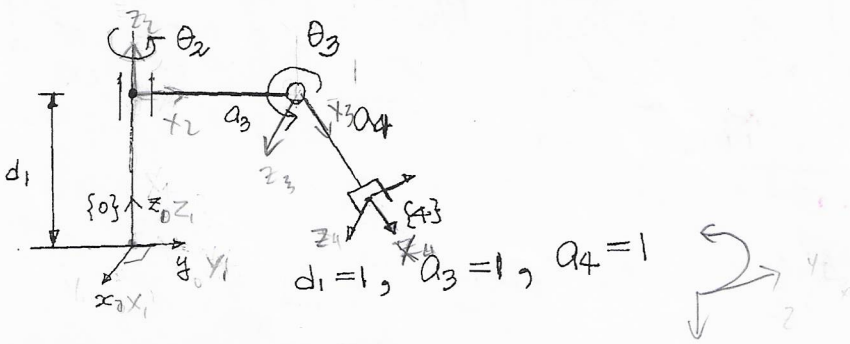
Answer the corresponding coordinates of A (10pts)

Equivalent angle Angle and Axis:

$$R_K(\theta) = \begin{bmatrix} k_x k_x v\theta + c\theta & k_x k_y v\theta - k_z s\theta & k_x k_z v\theta + k_y s\theta \\ k_x k_y v\theta + k_z s\theta & k_y k_y v\theta + c\theta & k_y k_z v\theta - k_x s\theta \\ k_x k_z v\theta - k_y s\theta & k_y k_z v\theta + k_x s\theta & k_z k_z v\theta + c\theta \end{bmatrix}$$

Where $c\theta = \cos\theta$, $s\theta = \sin\theta$, $v\theta = 1 - \cos\theta$, and ${}^A K = [k_x\ k_y\ k_z]^T$

3.



1) For the following PRR Manipulator, Assign the required coordinate frames, and fill out the corresponding DH parameter table. The current configuration is shown as when $d_1 = 1$,$\Theta_2 = 30°$, $\Theta_3 = 90°$ (20pts)

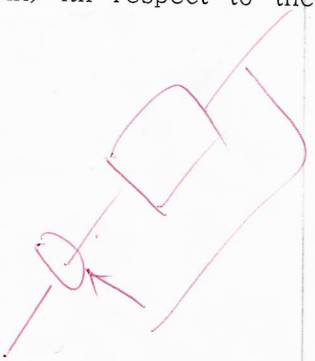
$z_{i-1} \to z_i$, $\hat{x}_{i-1}$ $\quad z_{i-1} \to z_i$ / $\hat{x}_{i-1}$ $\quad x_{i-1} \to x_i$, $\hat{z}_i$ $\quad x_{i-1} \to x_i$, $\hat{z}_i$

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	0
2	0	0	d_1	(90+θ_2) θ_2
3	90	a_3	0	θ_3
4	0	a_4	0	0

2) When $d_1 = 1$,$\Theta_2 = 30°$, $\Theta_3 = 90°$, Find the position of robot tip point with respect to the {0}-coordinate frame (10pts)

3) When $d_1 = 1$,$\Theta_2 = 30°$, $\Theta_3 = 90°$, Find the Jacobian (6 by 3 matrix) ith respect to the {0}-coordinate frame.(10pts)

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



4. Robot Dynamic Equation is given as $\tau = M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta)$
For the desired joint trajectory($\theta_d(t)$),

1) draw the Block diagram for Independent Joint PID control (10pts)
2) draw the Block diagram of Computed torque control (10pts)
3) discuss about merits and demerits of above controllers (10pts)