Nguyen Nguyen Anh
Id: 20205107

Homework #5

1. Set up cubic trajectory:

$\theta_1(0) = 30^0 = \pi/6$ $\theta_1(t_f) = 150^0 = 5\pi/6$
$\theta_2(0) = 150^0 = 5\pi/6$ $\theta_2(t_f) = 30^0 = \pi/6$

$$\begin{cases} \theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \\ \dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2 \\ \ddot{\theta}(t) = 2a_2 + 6a_3 t \end{cases}$$

By the innitial condition:

$$\theta(0) = a_0 = \begin{bmatrix} \frac{\pi}{6} \\ \frac{5\pi}{6} \end{bmatrix}$$

$$\dot{\theta}(0) = a_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

By the final condition:

$$\theta(1) = \begin{bmatrix} \frac{5\pi}{6} \\ \frac{\pi}{6} \end{bmatrix}$$

$$\dot{\theta}(1) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Then:

$$a_0 = \theta(0) = \begin{bmatrix} \frac{\pi}{6} \\ \frac{5\pi}{6} \end{bmatrix}$$

$$a_1 = \dot{\theta}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a_2 = \frac{3}{{t_f}^2}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f = \frac{3}{1^2} * \begin{bmatrix} \frac{2\pi}{3} \\ -\frac{2\pi}{3} \end{bmatrix} = \begin{bmatrix} 2\pi \\ -2\pi \end{bmatrix}$$

$$a_3 = -\frac{2}{{t_f}^3}(\theta_f - \theta_0) + \frac{1}{{t_f}^2}(\dot{\theta}_f + \dot{\theta}_0) = -\frac{2}{1^2}\begin{bmatrix} \frac{2\pi}{3} \\ -\frac{2\pi}{3} \end{bmatrix} = \begin{bmatrix} -\frac{4\pi}{3} \\ \frac{4\pi}{3} \end{bmatrix}$$

We got the result:

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$$\begin{cases} \theta(t) = \begin{bmatrix} \frac{\pi}{6} + 2\pi t^2 - \frac{4\pi}{3}t^3 \\ \frac{5\pi}{6} - 2\pi t^2 + \frac{4\pi}{3}t^3 \end{bmatrix} \\ \dot{\theta}(t) = \begin{bmatrix} 4\pi t - 4\pi t^2 \\ -4\pi t + 4\pi t^2 \end{bmatrix} \\ \ddot{\theta}(t) = \begin{bmatrix} 4\pi - 8\pi t \\ -4\pi + 8\pi t \end{bmatrix} \end{cases}$$

Find Torque Trajectory for the obtained trajectory:

$$\tau = M(\Theta)\ddot{\Theta} + V(\Theta,\dot{\Theta}) + G(\Theta)$$

$$\tau_1 = m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2 c_2(2\ddot{\theta}_1 + \ddot{\theta}_2) + (m_1+m_2) l_1^2 \ddot{\theta}_1 - m_2 l_1 l_2 s_2 \dot{\theta}_2^2$$
$$-2 m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 + m_2 l_2 g c_{12} + (m_1+m_2) l_1 g c_1$$
$$\tau_2 = m_2 l_1 l_2 c_2 \ddot{\theta}_1 + m_2 l_1 l_2 s_2 \dot{\theta}_1^2 + m_2 l_2 g c_{12} + m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2)$$

**Use Matlab to calculate, we got this result as follow:

2. Perform PD control:

```
tt1_r=30*pi/180;
tt2_r=150*pi/180;
dtt1_r=0;
dtt2_r=0;
e1=0;
e2=0;

Kp1=6800;
Kp2=5800;
Kv1=0.01;
Kv2=1.5;
for t=0:deltat:1
tt1=a01+a21*t*t+a31*t^3;
tt2=a02+a22*t*t+a32*t^3;
dtt1=2*a21*t+3*a31*t^2;
dtt2=2*a22*t+3*a32*t^2;
ddtt1=2*a21+6*a31*t;
ddtt2=2*a22+6*a32*t;



tque1=-Kv1*dtt1_r+Kp1*e1;
tque2=-Kv2*dtt2_r+Kp2*e2;
```

```
T=[tque1;tque2];
M=[l^2*m2+2*l^2*m2*cos(tt2_r)+l^2*(m1+m2)
l^2*m2+l^2*m2*cos(tt2_r);l^2*m2+l^2*m2*cos(tt2_r)
l^2*m2];
V=[-m2*l^2*sin(tt2_r)*dtt2_r^2-
2*m2*l^2*sin(tt2_r)*dtt1_r*dtt2_r;m2*l^2*sin(tt2_r)*dtt1^
2];
G=[m2*l*g*cos(tt1_r+tt2_r)+(m1+m2)*l*g*cos(tt1_r);m2*l*g*
cos(tt1_r+tt2_r)];
Minv=M^-1;
ddtt=Minv*(T-V-G)
ddtt1_r=ddtt(1)
ddtt2_r=ddtt(2)
dtt1_r=dtt1_r+deltat*ddtt1_r;
dtt2_r=dtt2_r+deltat*ddtt2_r;
tt1_r=tt1_r+dtt1_r*deltat+0.5*deltat^2*ddtt1_r;
tt2_r=tt2_r+dtt2_r*deltat+0.5*deltat^2*ddtt2_r;
e1=tt1-tt1_r;
e2=tt2-tt2_r;
```

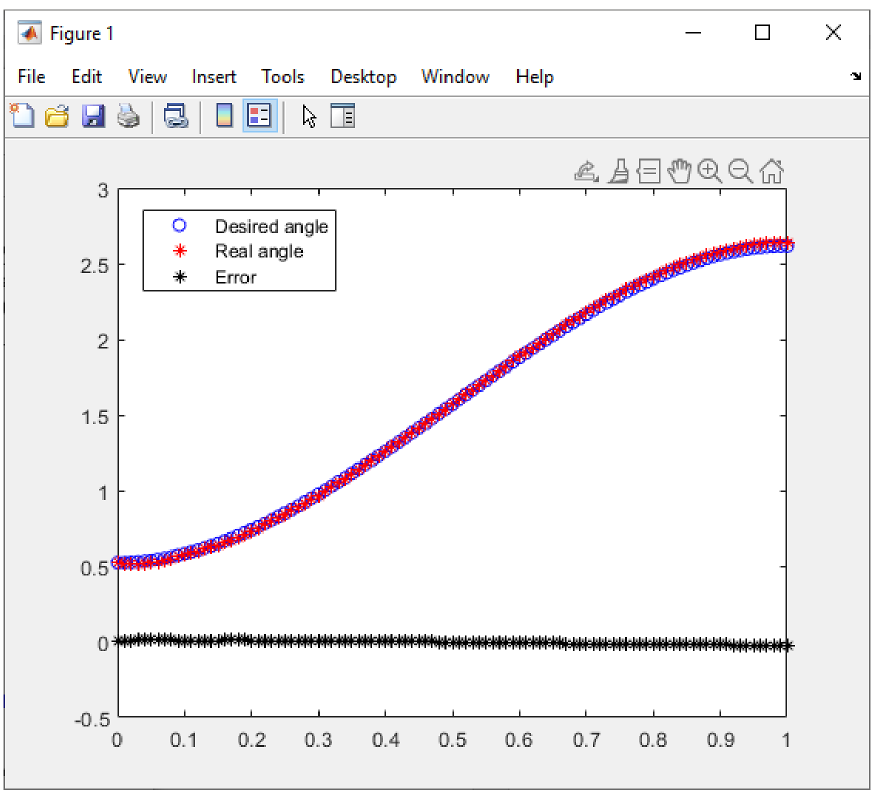

Figure 1. Angle trajectory of joint 1

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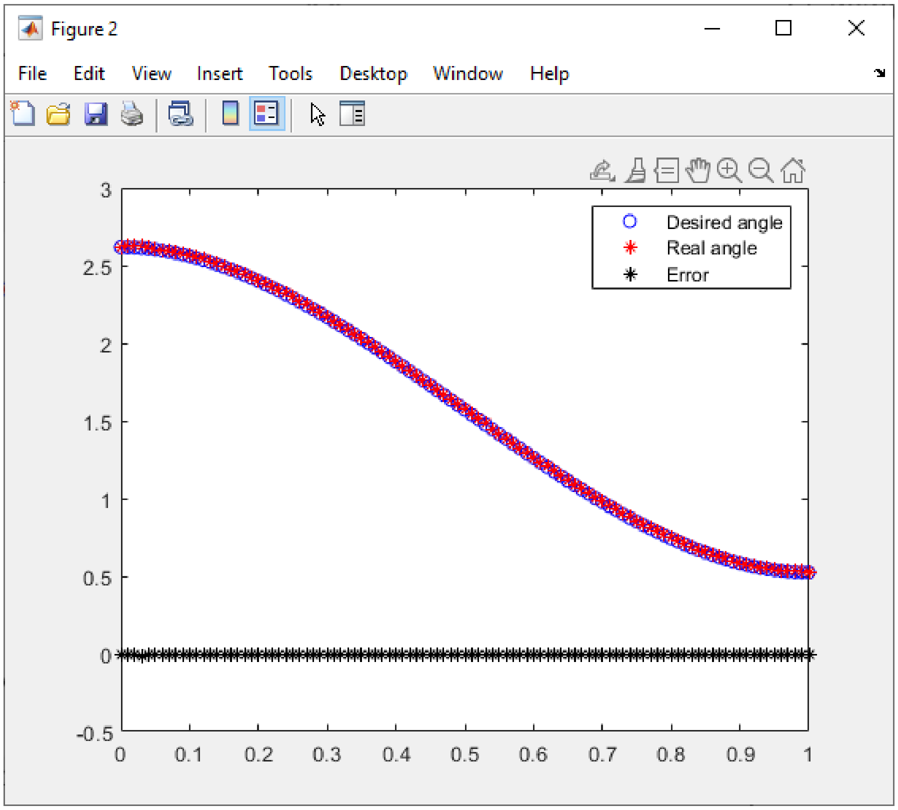


Figure 2. Angle trajectory of joint 2

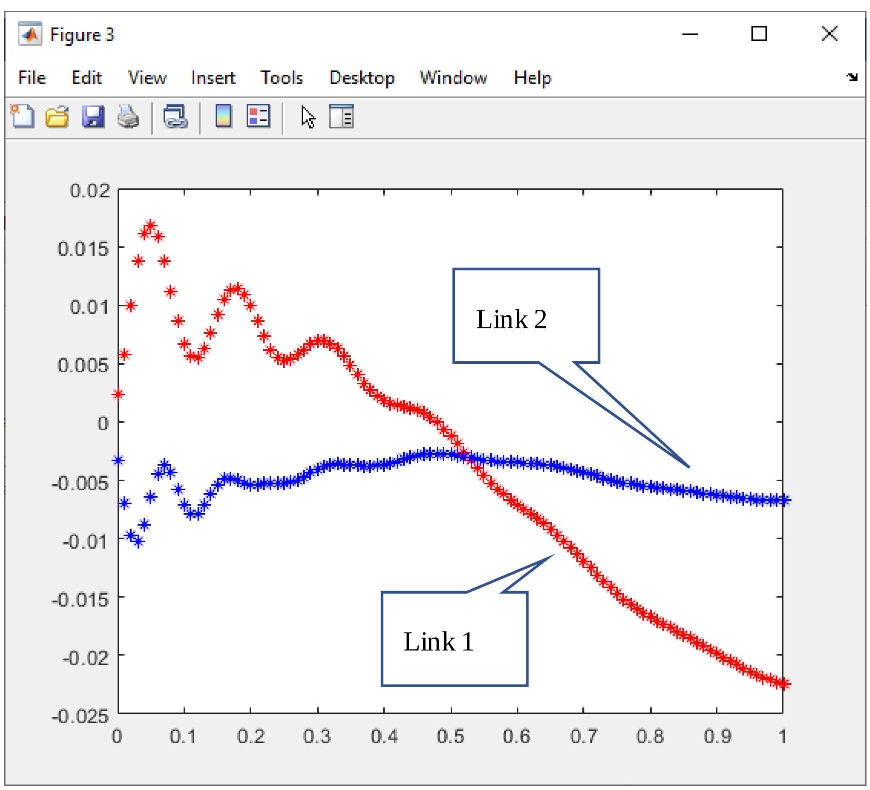


Figure 3. Error of the joint angle

3. Perform of PD + gravity control:

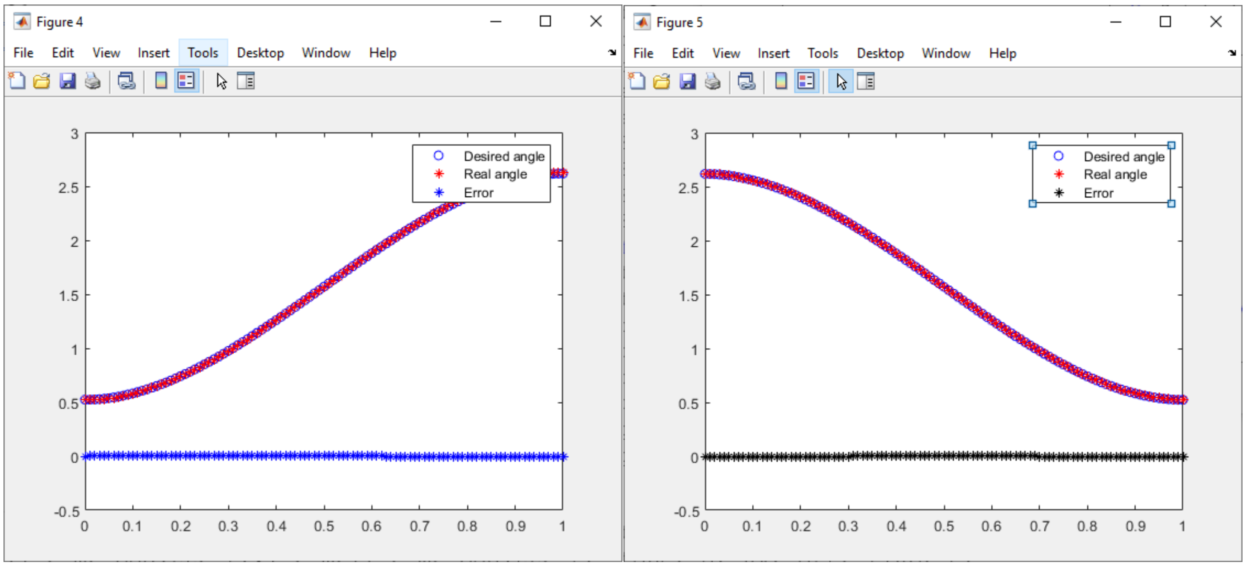
```
tt1_r=30*pi/180;
tt2_r=150*pi/180;
dtt1_r=0;
dtt2_r=0;
e1=0;
e2=0;
Kp1=6800;
Kp2=5800;
Kv1=0.01;
Kv2=1.5;
for t=0:deltat:1
tt1=a01+a21*t*t+a31*t^3;
tt2=a02+a22*t*t+a32*t^3;
dtt1=2*a21*t+3*a31*t^2;
dtt2=2*a22*t+3*a32*t^2;
ddtt1=2*a21+6*a31*t;
ddtt2=2*a22+6*a32*t;
g1=m2*l*g*cos(tt1+tt2)+(m1+m2)*l*g*cos(tt1);
g2=m2*l*g*cos(tt1+tt2);


tque1=g1-Kv1*dtt1_r+Kp1*e1;
tque2=g2-Kv2*dtt2_r+Kp2*e2;




T=[tque1;tque2];
M=[l^2*m2+2*l^2*m2*cos(tt2_r)+l^2*(m1+m2)
l^2*m2+l^2*m2*cos(tt2_r);l^2*m2+l^2*m2*cos(tt2_r)
l^2*m2];
V=[-m2*l^2*sin(tt2_r)*dtt2_r^2-
2*m2*l^2*sin(tt2_r)*dtt1_r*dtt2_r;m2*l^2*sin(tt2_r)*dtt1^
2];
G=[m2*l*g*cos(tt1_r+tt2_r)+(m1+m2)*l*g*cos(tt1_r);m2*l*g*
cos(tt1_r+tt2_r)];
Minv=M^-1;
ddtt=Minv*(T-V-G)
ddtt1_r=ddtt(1)
ddtt2_r=ddtt(2)
dtt1_r=dtt1_r+deltat*ddtt1_r;
dtt2_r=dtt2_r+deltat*ddtt2_r;
tt1_r=tt1_r+dtt1_r*deltat+0.5*deltat^2*ddtt1_r;
```

```
tt2_r=tt2_r+dtt2_r*deltat+0.5*deltat^2*ddtt2_r;
e1=tt1-tt1_r;
e2=tt2-tt2_r;
```



(a) (b)

Figure 4. Angle trajectory of PD control: a) angle of joint 1, b) angle of joint 2

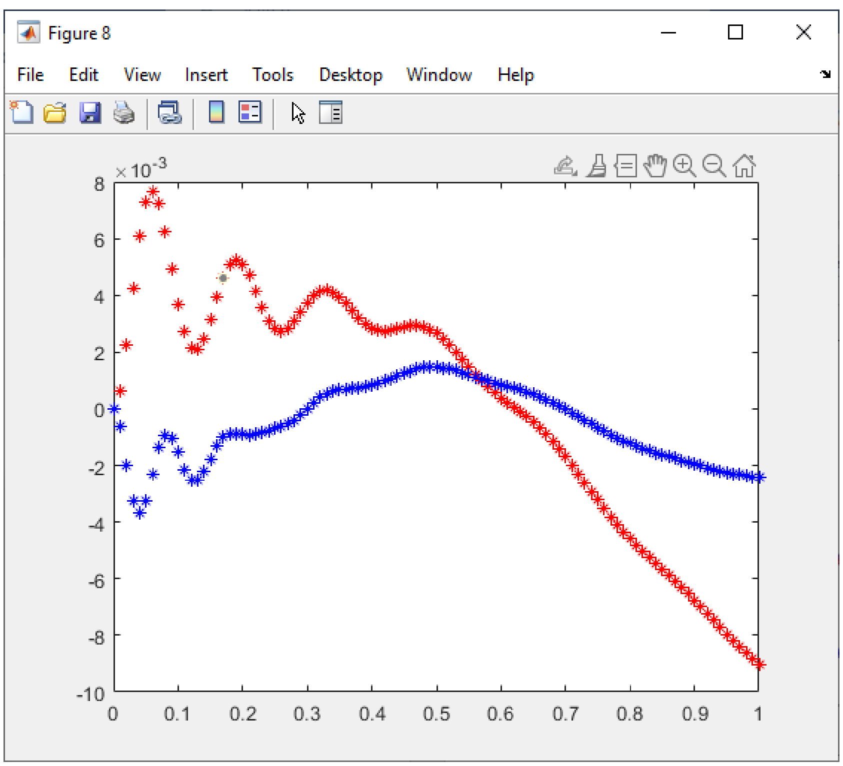


Figure 5. Error of the joint angles

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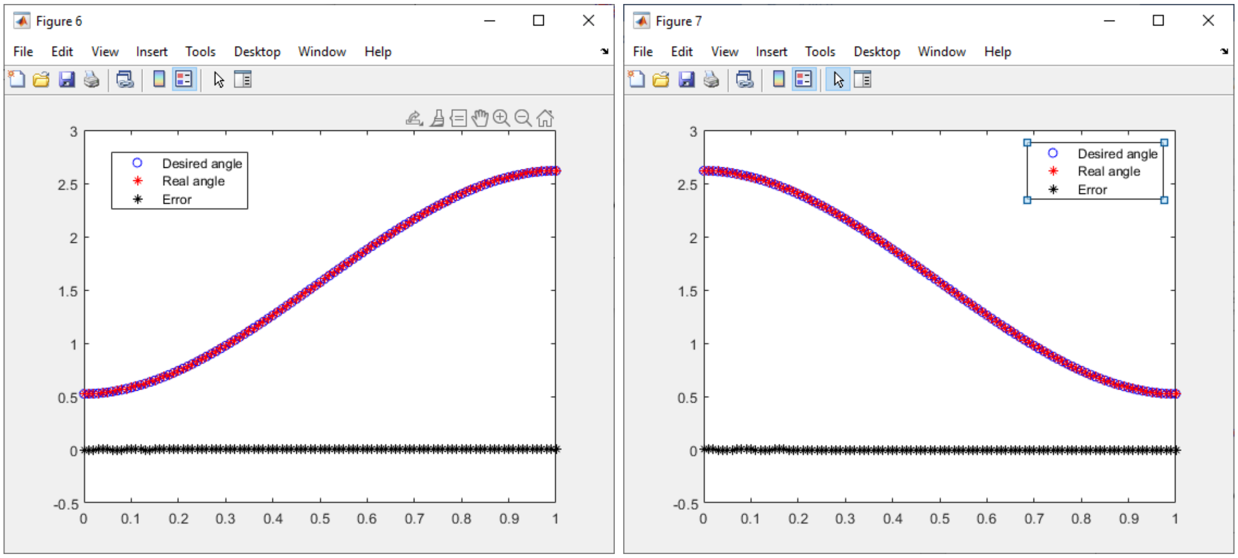
4. Perform compute torque control

```
tt1_r=30*pi/180;
tt2_r=150*pi/180;
dtt1_r=0;
dtt2_r=0;
e1=0;
e2=0;
de1=0;
de2=0;
Kp1=7800;
Kp2=5800;
Kv1=0.1;
Kv2=0.5;

for t=0:deltat:1
tt1=a01+a21*t*t+a31*t^3;
tt2=a02+a22*t*t+a32*t^3;
dtt1=2*a21*t+3*a31*t^2;
dtt2=2*a22*t+3*a32*t^2;
ddtt1=2*a21+6*a31*t;
ddtt2=2*a22+6*a32*t;

tque_c1=ddtt1+Kp1*e1+Kv1*de1
tque_c2=ddtt2+Kp2*e2+Kv2*de2

anpha=[l^2*m2+2*l^2*m2*cos(tt2)+l^2*(m1+m2)
l^2*m2+l^2*m2*cos(tt2);l^2*m2+l^2*m2*cos(tt2) l^2*m2];
M=anpha;


V=[-m2*l^2*sin(tt2_r)*dtt2_r^2-
2*m2*l^2*sin(tt2_r)*dtt1_r*dtt2_r;m2*l^2*sin(tt2_r)*dtt1^2];
G=[m2*l*g*cos(tt1_r+tt2_r)+(m1+m2)*l*g*cos(tt1_r);m2*l*g*cos(tt1
_r+tt2_r)];
beta=V+G;
T=anpha*[tque_c1;tque_c2]+beta
ddtt=M\(T-V-G);

ddtt1_r=ddtt(1);
ddtt2_r=ddtt(2);
```

```
dtt1_r=dtt1_r+deltat*ddtt1_r;
dtt2_r=dtt2_r+deltat*ddtt2_r;
tt1_r=tt1_r+dtt1_r*deltat+0.5*deltat^2*ddtt1_r;
tt2_r=tt2_r+dtt2_r*deltat+0.5*deltat^2*ddtt2_r;
e1=tt1-tt1_r;
e2=tt2-tt2_r;
de1=dtt1-dtt1_r;
de2=dtt2-dtt2_r;
```



(a) (b)

Figure 6. Angle trajectory of PD compute torque control: a) angle of joint 1, b) angle of joint 2

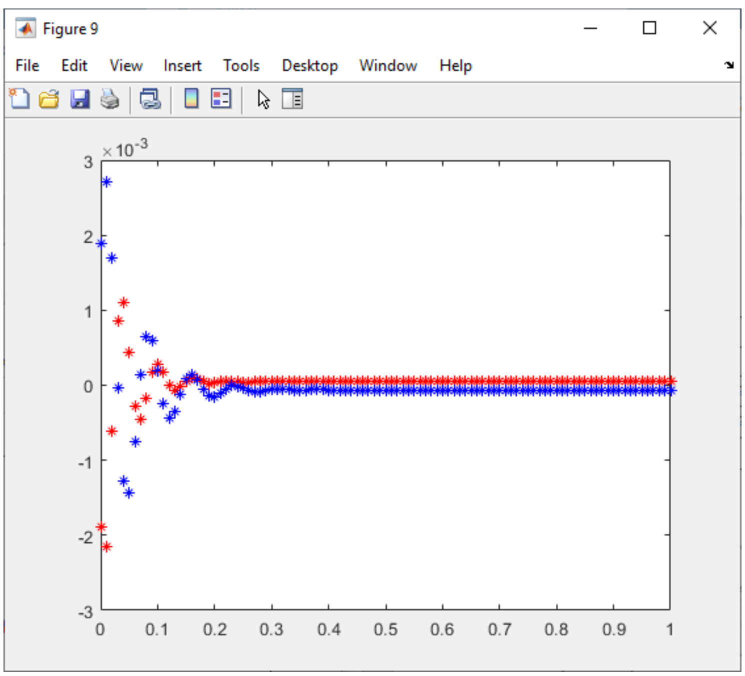
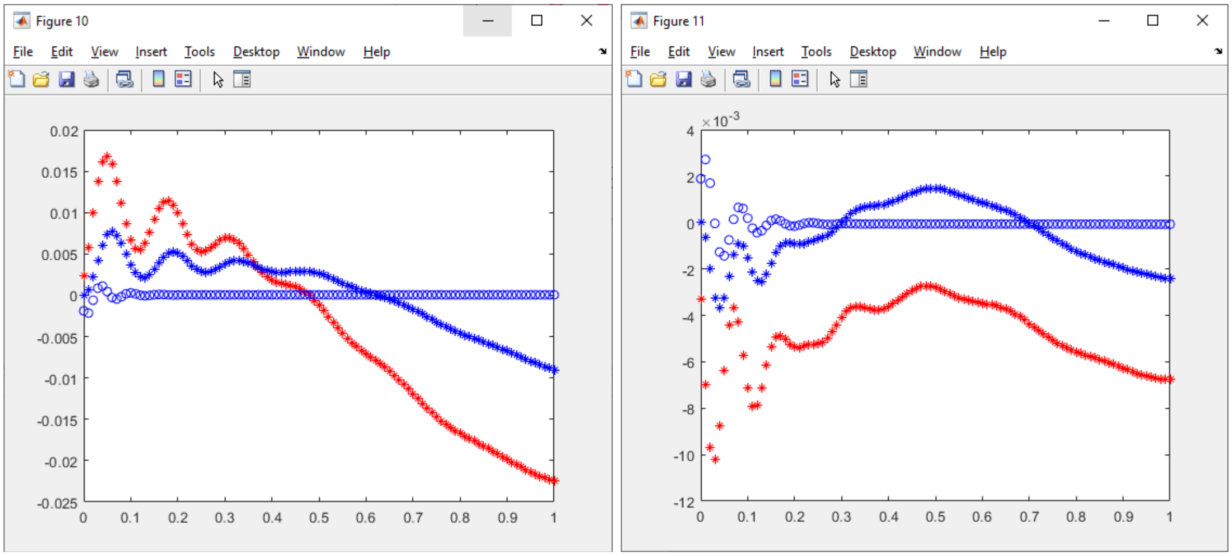


Figure 7. Error of the joint angles

5. Comparing from the result of the controllers:
   We compare the result by draw the angles's error of each jont on the same figure:



(a) (b)

Figure 8. Error of the joint angles :a) joint 1, b) joint 2

In both cases. The blue O line is the error of torque compute control method
The blue * line is the error of PD+gravity control method
The red* line is the error of PD method control method
We can say that the PD control method is not as good as PD+ gravity method
The compute torque control method is better than PD+ gravity method
(Definitely the result show that in the case of no controller, it is the worst case)