Homework #1

1. A vector ${}^A P$ is rotated about $\hat{X}_A$ by ϕ degrees and is subsequently rotated about $\hat{Z}_A$ by θ degrees.

   (a) Give the rotation matrix which accomplishes these rotations in the given order.

   (b) What is the result if $\phi = 30°$ and $\theta = 45°$?

2. A frame {B} is located as follows: initially coincident with a frame {A} we rotate {B} about $\hat{Z}_B$ by θ and then we rotate the resulting frame about $\hat{X}_B$ by ϕ degrees.

   (a) Give the rotation matrix, ${}^A_B R$ which will change the description of vectors from ${}^B P$ to ${}^A P$.

   (b) What is the result if $\phi = 30°$ and $\theta = 45°$?

   (c) What is ${}^A \hat{Y}_B$?

3. A vector is given by

$$ {}^B P = \begin{bmatrix} 1.0 \\ 5.0 \\ 10.0 \end{bmatrix}. $$

   Given

$$ {}^B_A T = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 3.0 \\ 0.0 & 0.6 & 0.8 & -1.0 \\ 0.0 & -0.8 & 0.6 & 1.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}, $$

   compute ${}^A P$.

4. Given the following 3x3 matrix,

$$ R = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}. $$

   (a) Show that it is a rotation matrix.

   (b) Determine a unit vector that defines this axis of rotation and the angle (in degrees) of rotation.

   (c) What are the Euler parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ of R?

   (d) What is the quaternian expression for R?

5. Starting with the rotation matrix with Euler parmeters, Derive the rotation matrix in terms of equivalent angle-axis seen in class.