Ch. 3

- **Manipulator Kinematics**

- DOF (degree of freedom)

Point (3dof)

single body (bdof)

Constrained single body (1dof)

ball joint (3dof)

- Mobility criteria (# of dof)

$$M = 6(n-1) - \sum_{j=1}^{m}(6 - f_j)$$

(3차원 공간)

Where:

n: number of rigid bodies (including ground)

m: number of joints

f_j : number of dof at jth joint

$$M = 3(n-1) - \sum_{j=1}^{m}(3 - f_j)$$

- **Joint type**

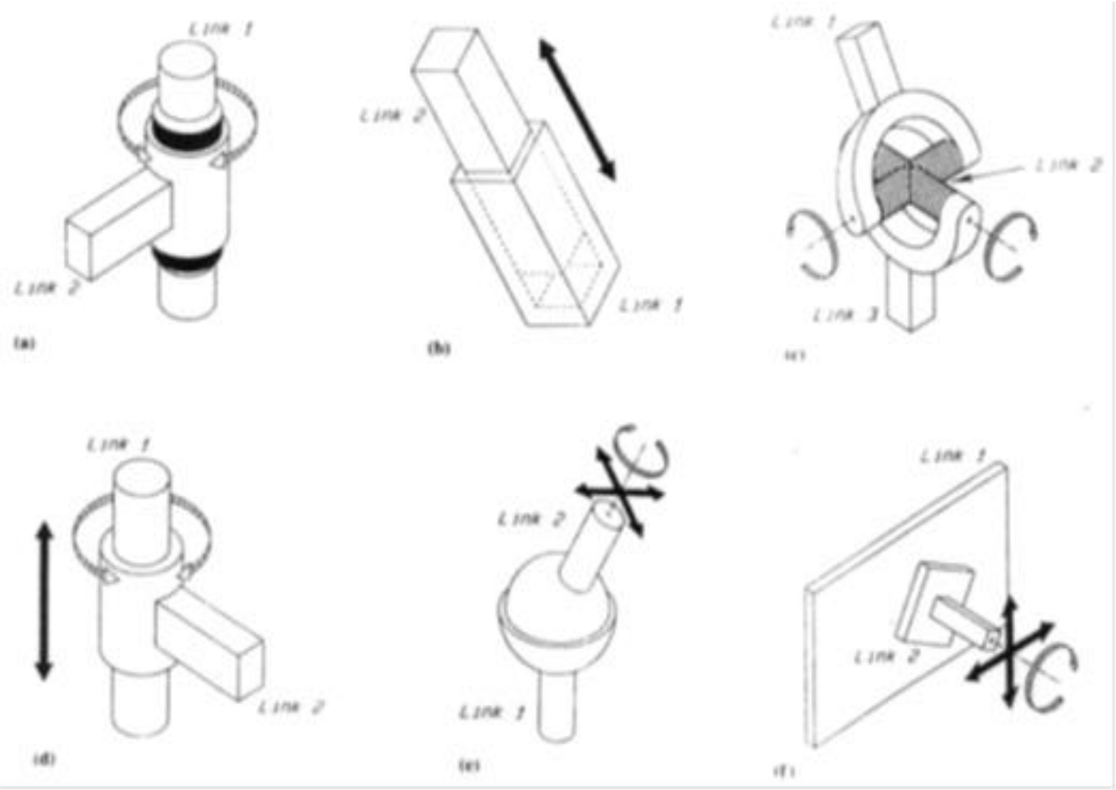
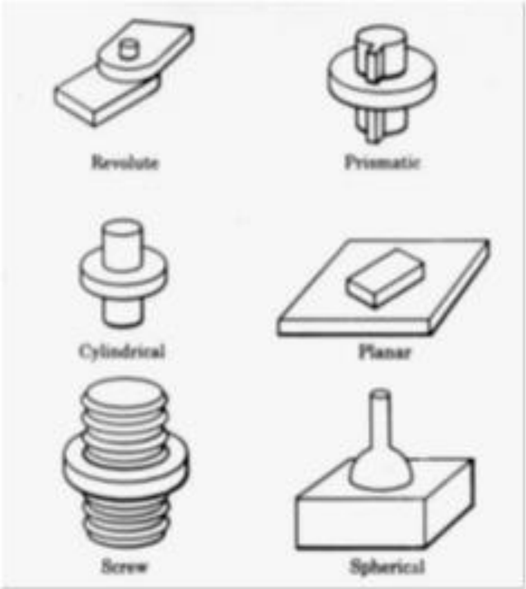
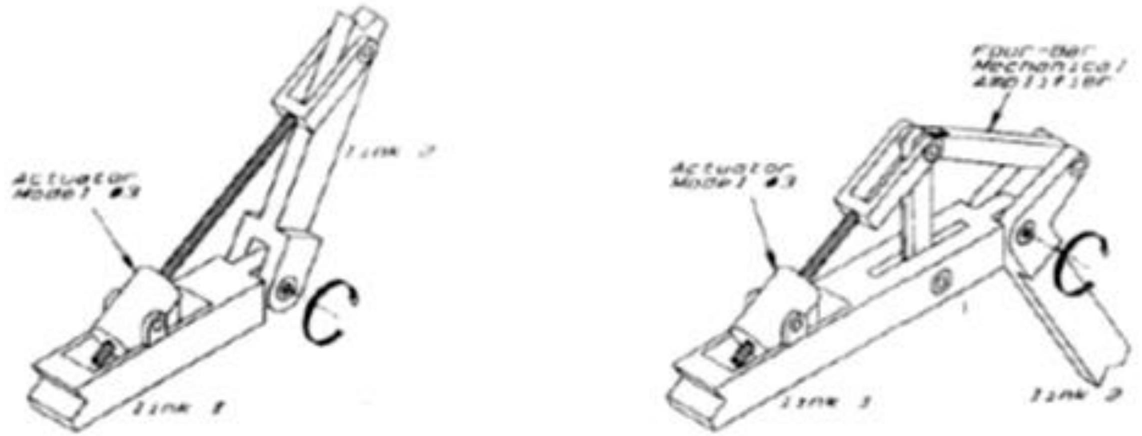


FIGURE. 12. Undriven structural joints



3.1 The six possible lower pair joints



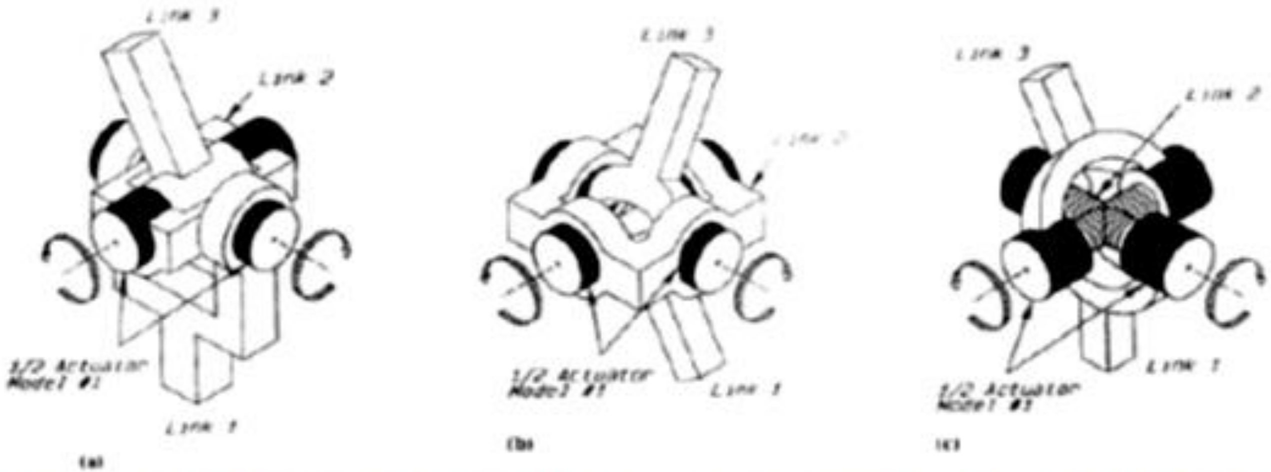
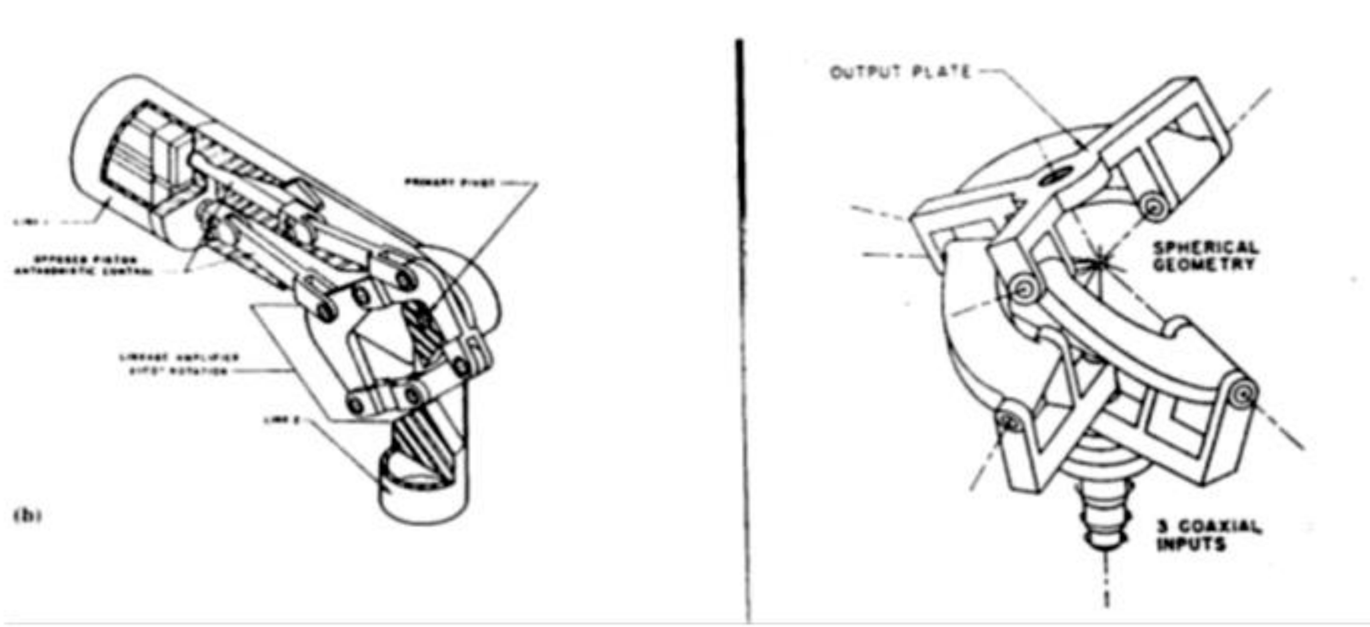
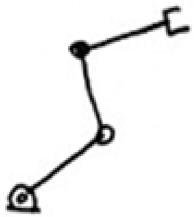


FIG. 16. Two-DOF knuckles with perpendicular axes (a-c, left to right): glimbal-based knuckle; exterior frame knuckle (based on universal joint); interior cross knuckle (based on universal joint)



- Example for mobility of system

$$M = 3(n-1) - \sum_{j=1}^{m}(3 - f_j)$$



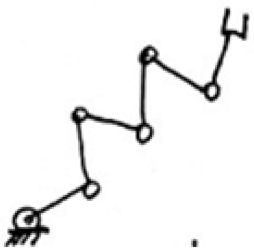
N=4 including ground

M=3

$f_{1,2,3} = 1$

M=3(4 – 1) – 3 x 2 = 3dof

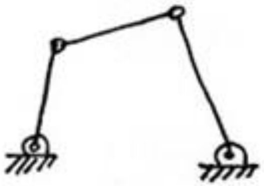
On the plane



$$M = 6(n-1) - \sum_{j=1}^{m}(6 - f_j)$$

$$= 6(7-1) - 6 \times 5 = 6\text{dof}$$

In space



n = 4

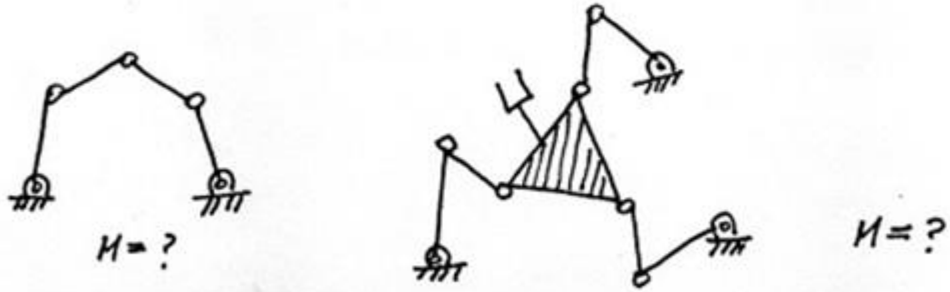
m = 4

$f_{1-4} = 1$

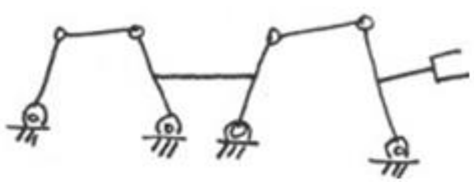
M = 3(4 – 1) – 4 x 2 = 1dof

On the plane

과제)



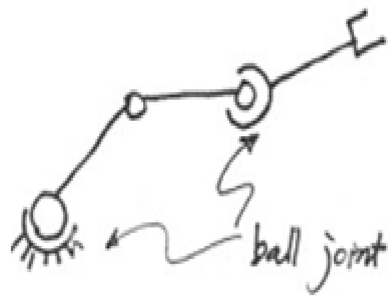
과제)



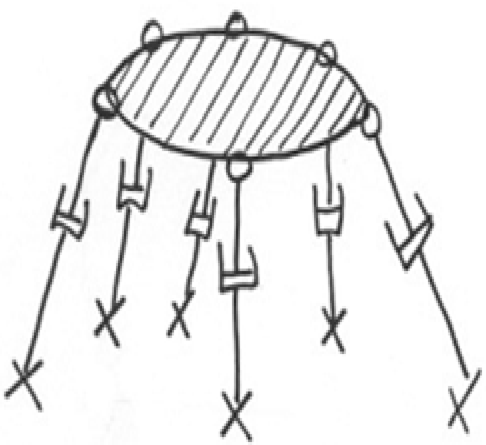
n=?

m=?

M=?



M=?



o : dof ball joint

X : 2dof kmickle joint

: 1dof prismatic joint

M = ?

Link Description

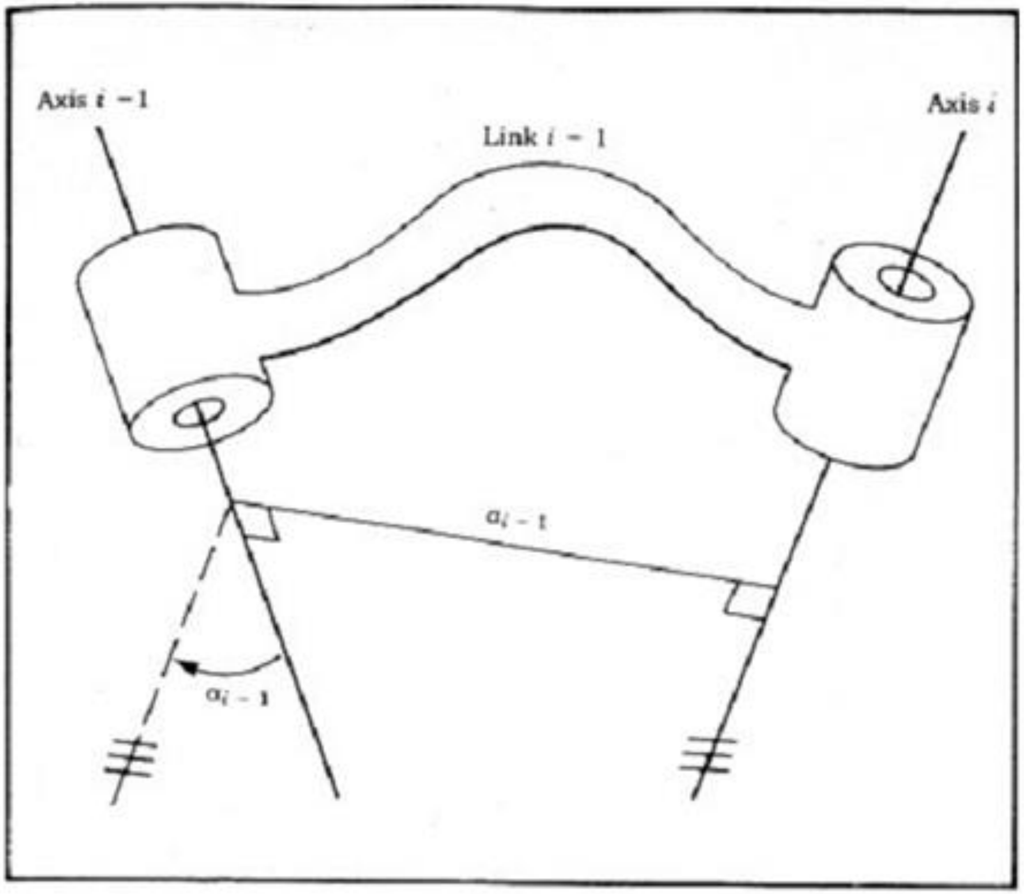


FIGURE 3.2 The kinematic function of a link is to maintain a fixed relationship between the two joint axes it supports. This relationship can be described with two parameters, the link length, a, and the link twist, α.

Link length (a)
: mutually perpendicular line always exists and is unique
Link twist (a)
: relative location of two axes

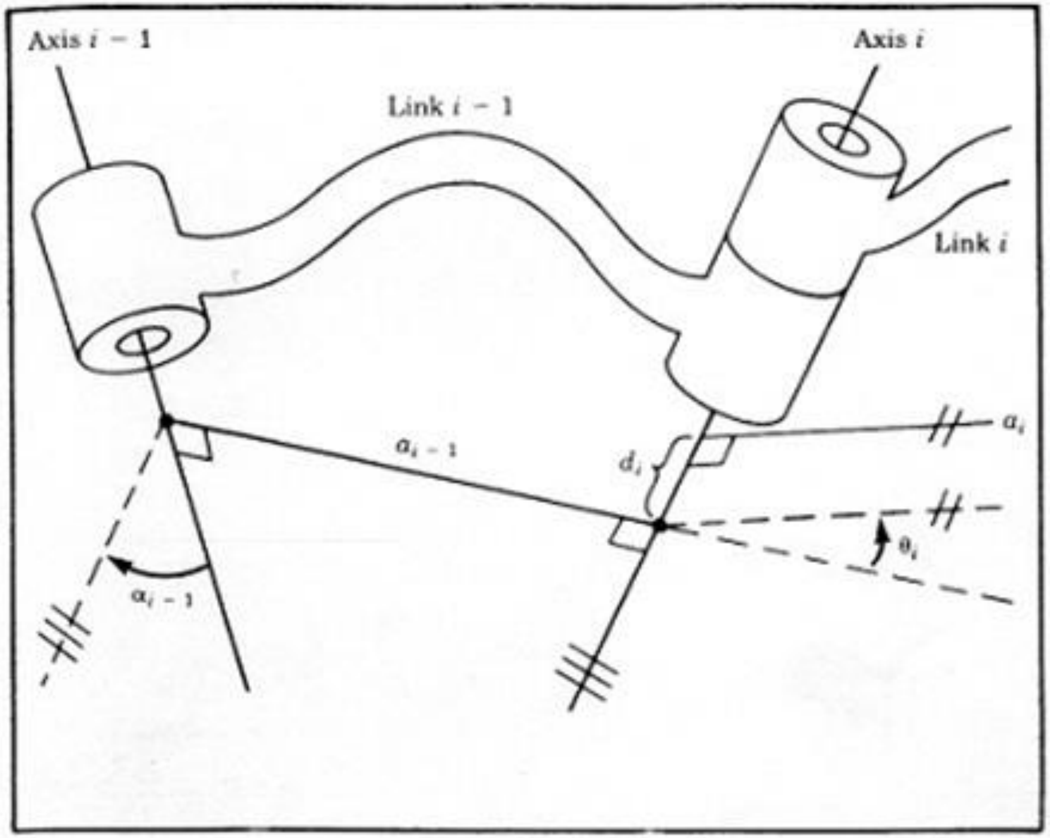


FIGURE 3.4 The link offset, d, and the joint angle, θ, are two parameters which may be used to describe the nature of the connection between neighboring links.

Link offset (d)
Joint variable (θ)
=> 4 parameter convention

: Danavit-Hartenburg parameter

- Affixing frames to links

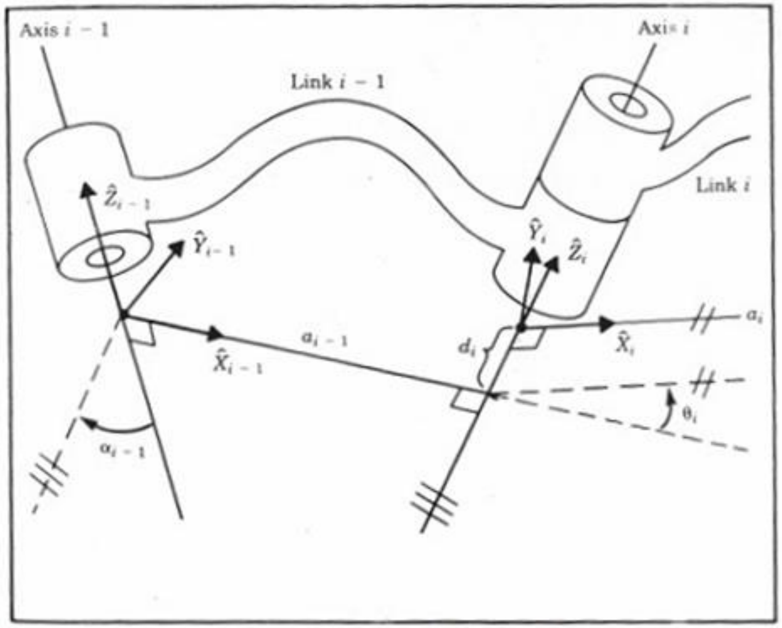


Fig 3.5 Link frames are attached so that frame {i} is attached rigidly to link i.

- Definition of the link parameters : Craig's way

a_i = the distance from $\hat{Z}_i$ to $\hat{Z}_{i+1}$measured along $\hat{X}_i$;

α_i= the angle between $\hat{Z}_i$ and $\hat{Z}_{i+1}$measured about $\hat{X}_i$

d_i= the distance from $\hat{X}_{i-1}$to $\hat{X}_i$measured along $\hat{Z}_i$;and

θ_i= the angle between $\hat{X}_{i-1}$and $\hat{X}_i$ measured about $\hat{Z}_i$.

- Link frames attachment procedure

1. Identify the joint axes and imagine (or draw) infinite lines along them. For steps 2 through 5 below, consider two of these neighboring lines (at axes i and $i+1$).
2. Identify the common perpendicular between them, or point of intersection. At the point of intersection, or at the point where the common perpendicular meets the ith axis, assign the link frame origin.
3. Assign the $\hat{Z}_i$ axis pointing along the ith joint axis.
4. Assign the axis $\hat{X}_i$pointing along the common perpendicular, or of the axes intersect, assign $\hat{X}_i$to be normal to the plane containing the two axes.
5. Assign the $\hat{Y}_i$ axis to complete a right-hand coordinate system.
6. Assign {0} to match {1} when the first joint variable is zero. For {N} choose an origin location and $\hat{X}_N$ direction freely, but generally so as to cause as many linkage parameters as possible to become zero.

Example 1)

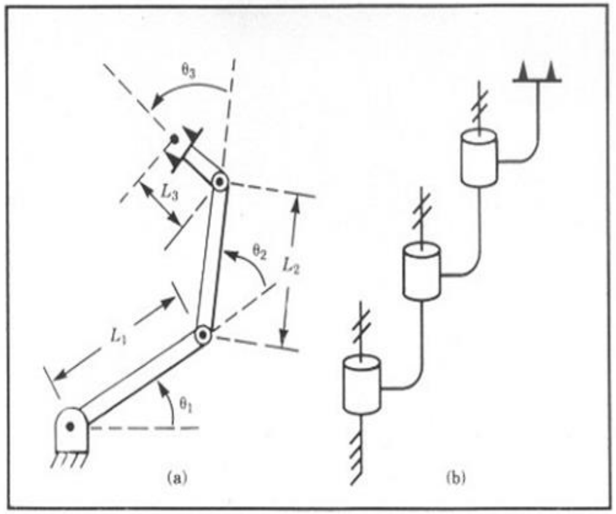


Fig. 3.6 A three-link planar arm. On the right we show the same manipulator by means of a simple schematic notation. Hash marks on the axes indicate that they are mutually parallel.

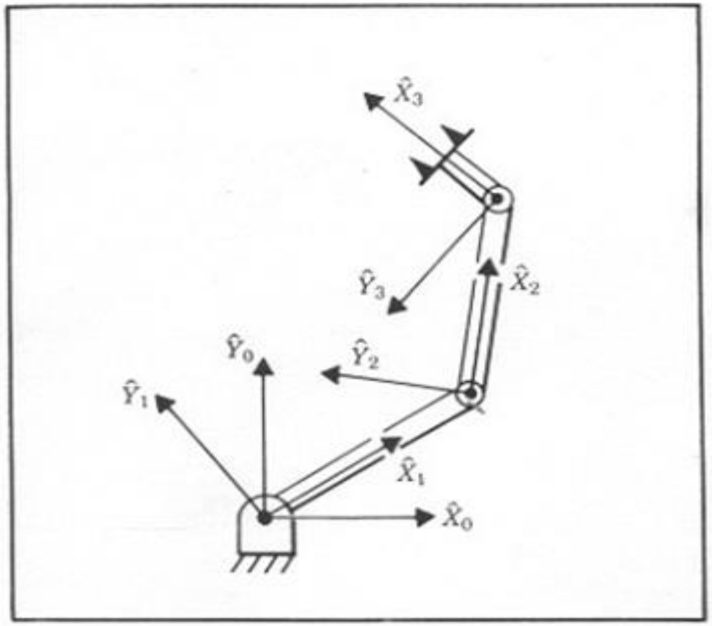


Fig 3.7 link frame assignments.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

Fig 3.8 Link parameters of the three-link planar manipulators.

Ex 2)

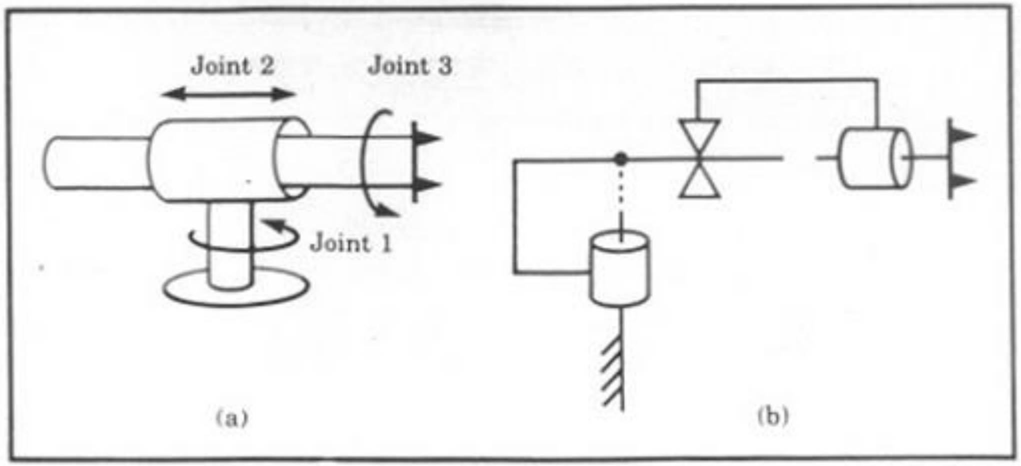


Fig 3.9 manipulator having three degrees of freedom and one prismatic joint.

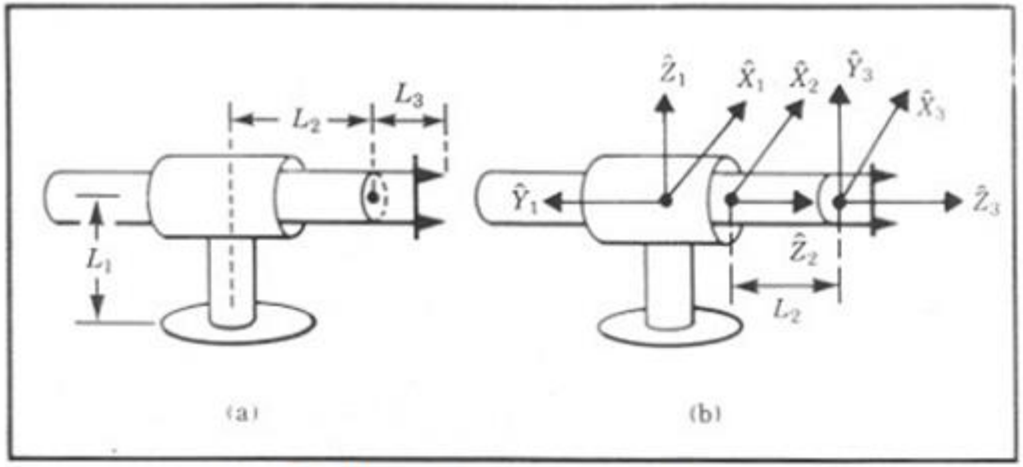


Fig 3.10 Link frame assignments.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

Fig 3.11 Link parameters for the RPR manipulator of Example 3.4.

Ex 3) Demonstration of non-uniqueness of frame assignments

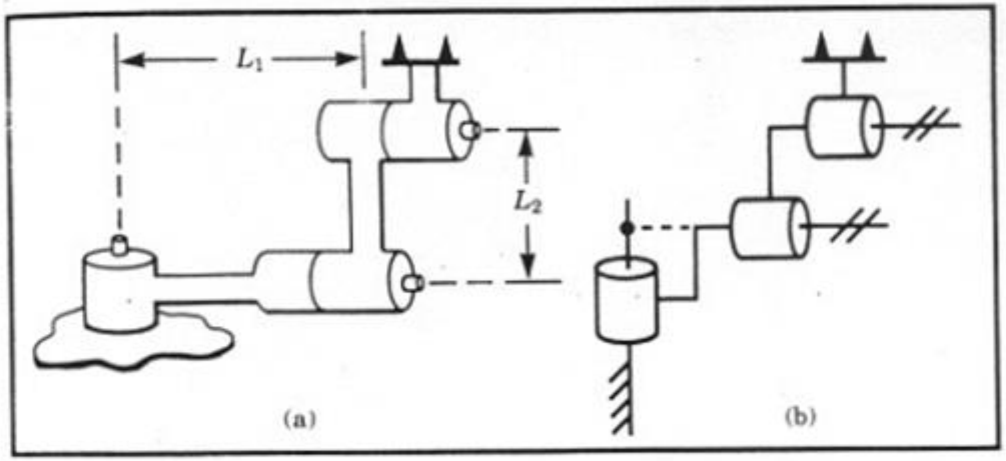


Fig 3.12 Three-link, non-planar manipulator.

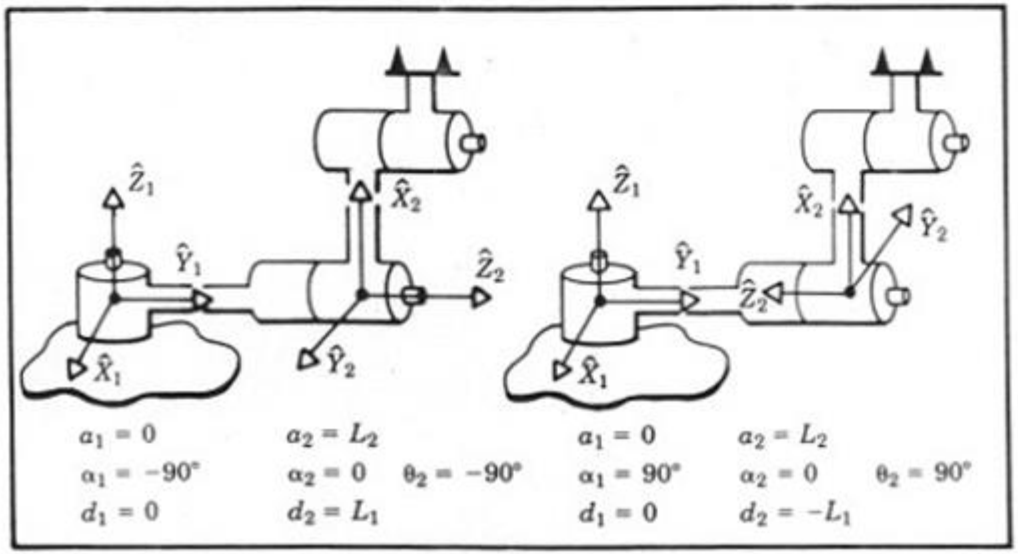


Fig 3.13 Two possible frame assignments.

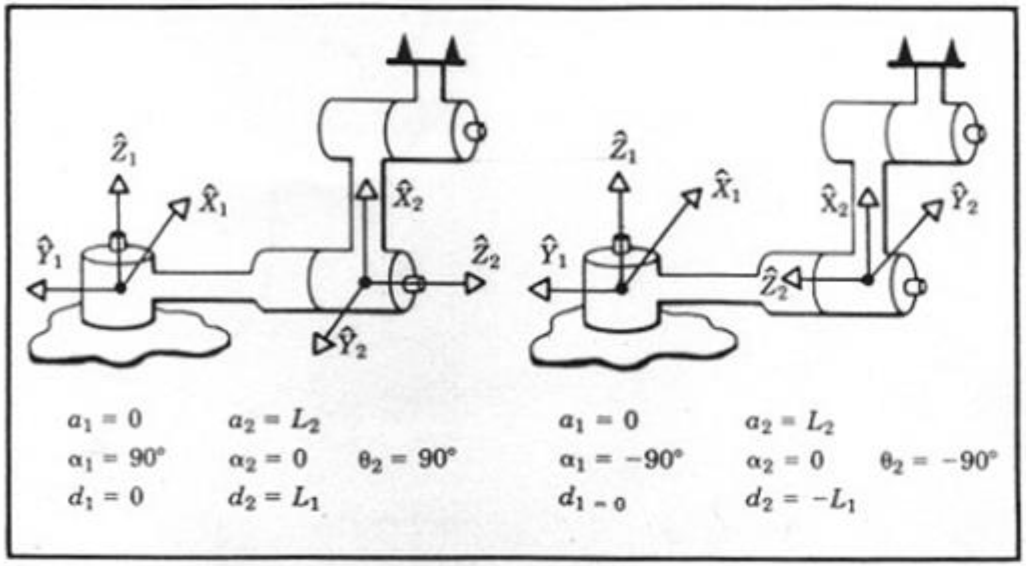


Fig 3.14 Two more possible frame assignments.

- Derivation of link transformation

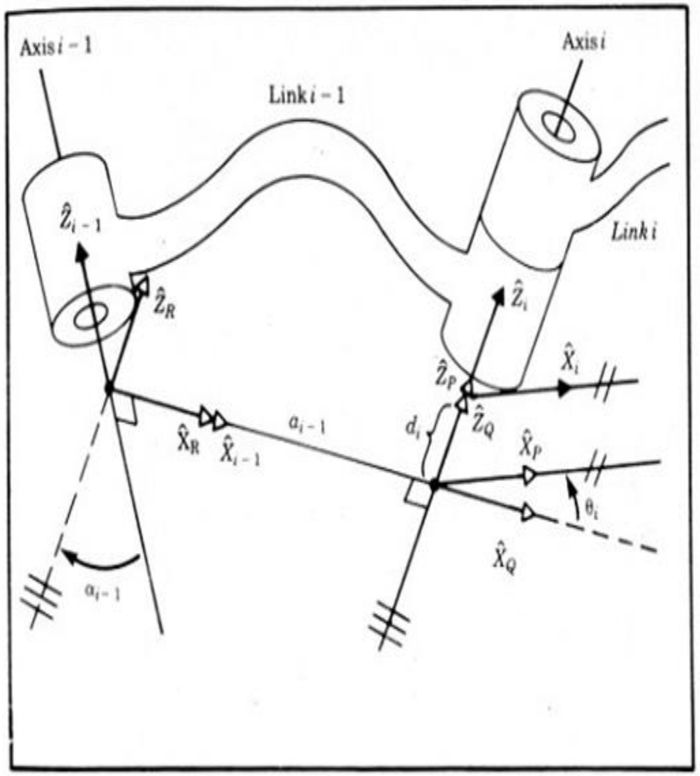


Fig 3.15 Location of intermediate frame {P}, {Q}, and {R}.

$$ {}^{i-1}P = {}^{i-1}_{R}T\,{}^{R}_{Q}T\,{}^{Q}_{P}T\,{}^{P}_{i}T\,{}^{i}P, \qquad (3.1)$$

Or

$$ {}^{i-1}P = {}^{i-1}_{i}T\,{}^{i}P, \qquad (3.2)$$

where

$$ {}^{i-1}_{i}T = {}^{i-1}_{R}T\,{}^{R}_{Q}T\,{}^{Q}_{P}T\,{}^{P}_{i}T. \qquad (3.3)$$

Considering each of these transformations, we see that (3.3) may be written:

$$ {}^{i-1}_{i}T = R_X(\alpha_{i-1})D_X(a_{i-1})R_Z(\theta_i)D_Z(d_i), \qquad (3.4)$$

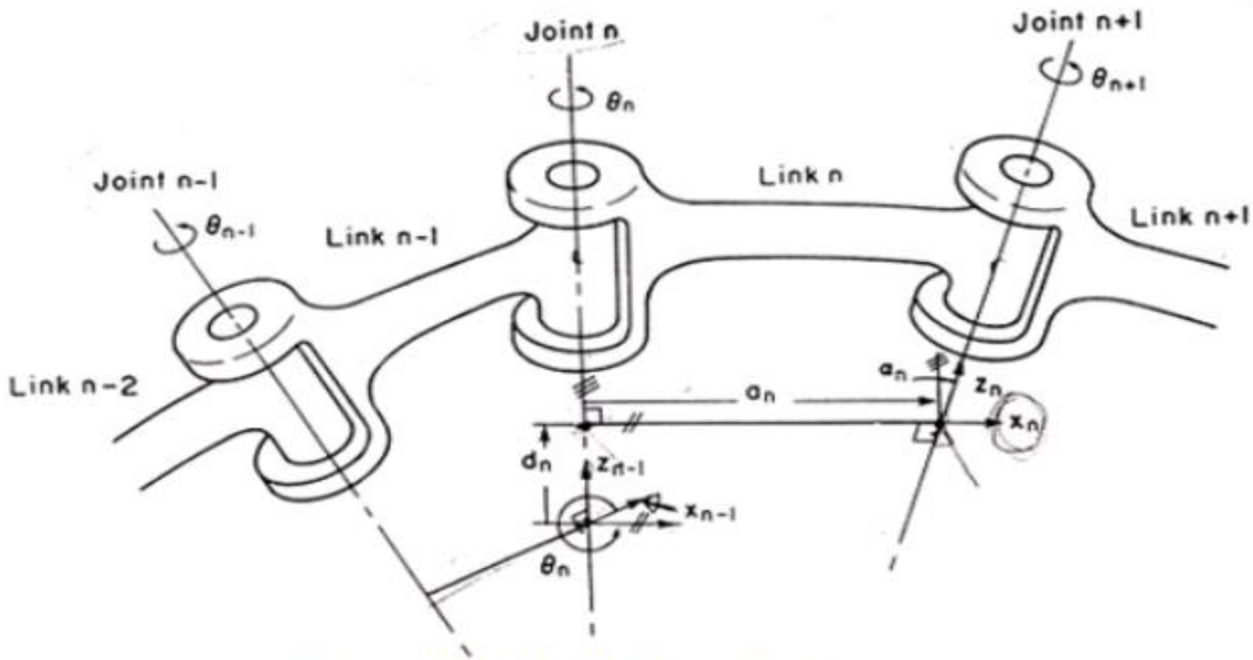
or

$$ {}^{i-1}_{i}T = Screw_X(a_{i-1},\alpha_{i-1})Screw_Z(d_i,\theta_i), \qquad (3.5)$$

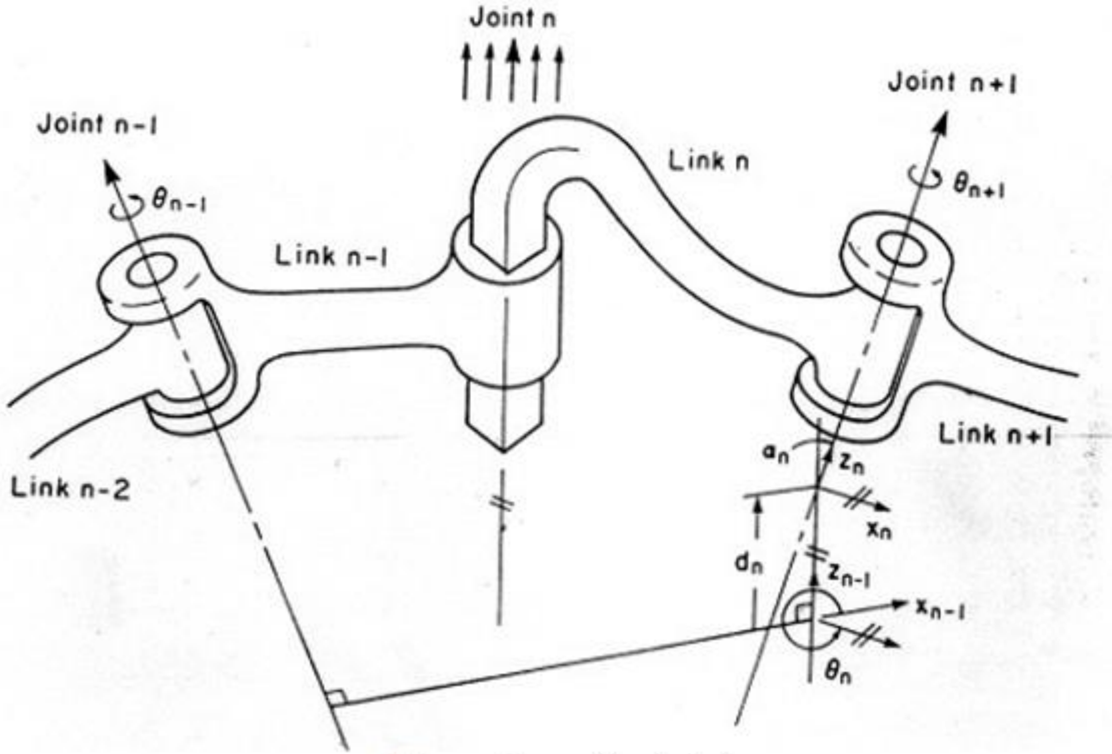
Where the notation $Screw_Q(r,\emptyset)$ stands for a translation along an axis $\hat{Q}$ by a distance r, and a rotation about the same axis by an angle $\emptyset$. Multiplying out (3.4) we obtain the general form of ${}^{i-1}_{i}T$:

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (3.6)$$

- Paul’s way for link frame attachment



For revolute joint: $\theta_n \rightarrow d_i \rightarrow a_n \rightarrow \alpha_n$



For prismatic joint

Definition of the link parameters: Paul's way

Rotate about z_{n-1} an angle θ_n;
Translate along z_{n-1}, a distance d_n;
Translate along rotated $x_{n-1} = x_n$ a length a_n;
Rotate about x_n, the twist angle α_n.

Link transformation

$$A_n^{n-1} = Rot(z,\theta)Trans(0,0,d)Tran(a,0,0)Rot(x,\alpha)$$

$$A_n^{n-1} = \begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\alpha & -sin\alpha & 0 \\ 0 & sin\alpha & cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_n^{n-1} = \begin{bmatrix} cos\theta & -sin\theta cos\alpha & sin\theta sin\alpha & acos\alpha \\ sin\theta & cos\theta cos\alpha & -cos\theta sin\alpha & asin\alpha \\ 0 & sin\alpha & cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 1

Denavit-hanrtenburg parameter

3.11)

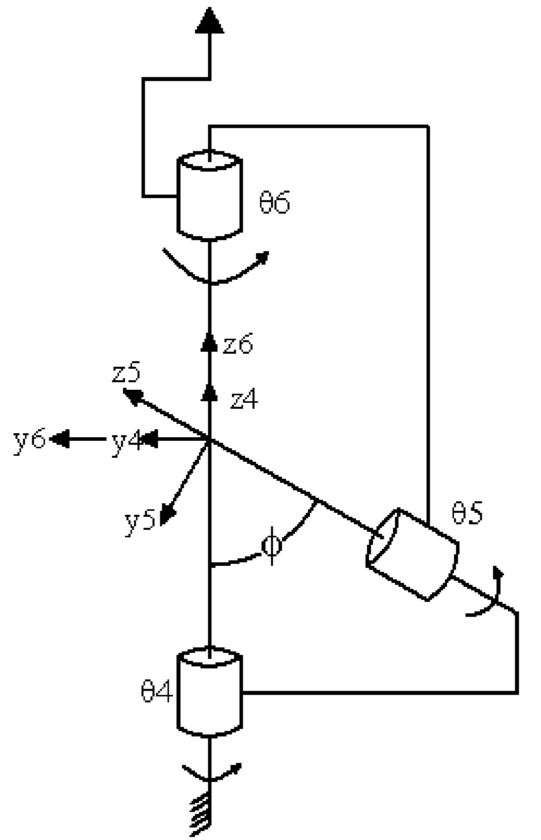


Fig. 3R non-orthogonal axis robot

α	a	d	θ
.	.	.	θ_4
$-\phi$	0	0	θ_5
ϕ	0	0	θ_6

3.12) No, need ? parameters

3.15)

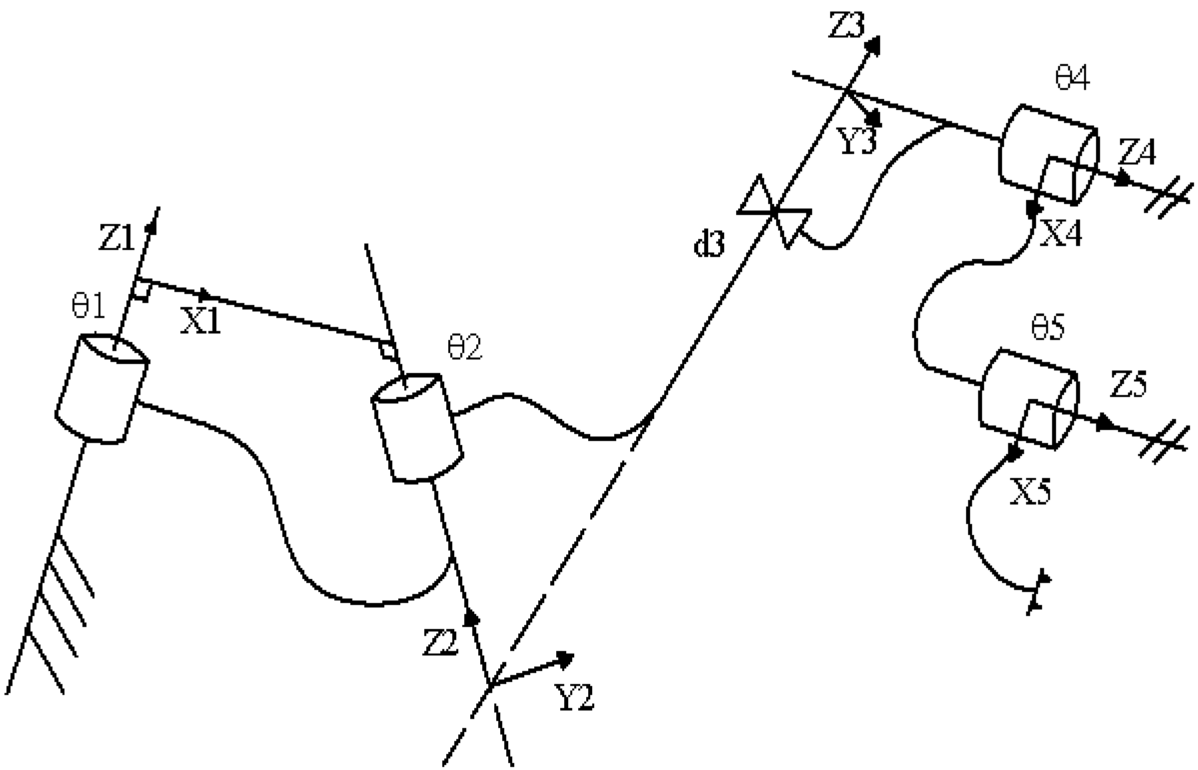


Fig 3.34 Schematic of a 2RP2R manipulator

- **Concatenating link transformation:**
  {0} frame → {n} frame

$$ {}^{0}_{n}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T \; \ldots\ldots \; {}^{n-1}_{n}T $$

- **Space:**

Actuator space ⇄ Joint space ⇄ Cartesian space / Task Oriented space / Operational Space

Actualtor parameter vector → Actuator space

Joint variable vector → Joint space

$$\begin{bmatrix} \theta 1 \\ \\ \theta n \end{bmatrix}$$

Cartesian Vector → Cartesian space

$$\begin{bmatrix} X \\ Y \\ Z \\ \theta x \\ \theta y \\ \theta z \end{bmatrix}$$

N > 6 : redundant manipulator

- **Kinematics for Puma 560 manipulator**

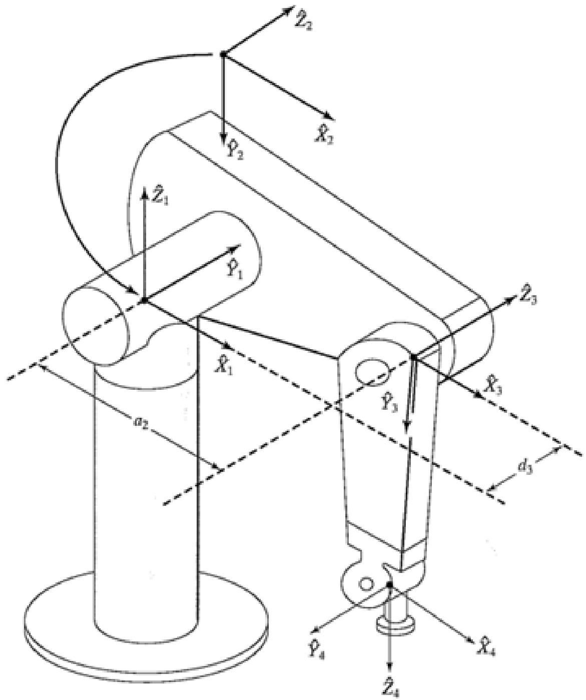


FIGURE 3.18. Some kinematic parameters and frame assignments for the Puma 560 manipulator

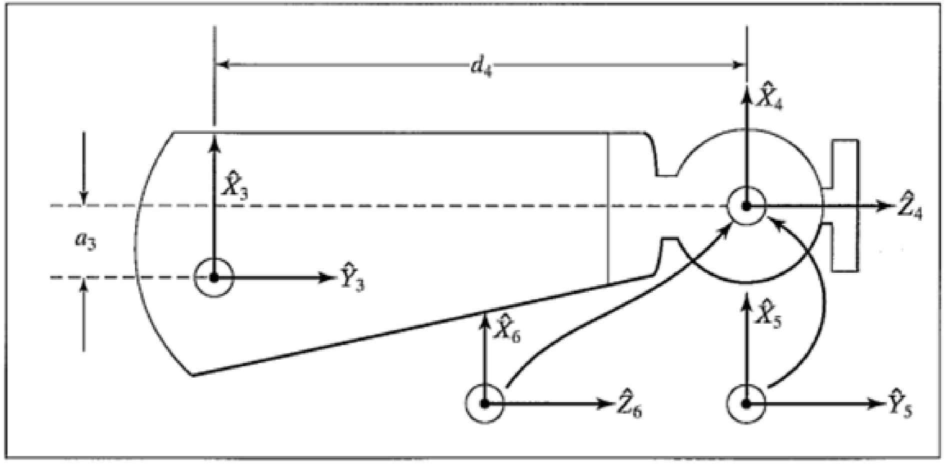


FIGURE 3.19. Kinematic parameters and frame assignments for the forearm of the PUMA 560 manipulator

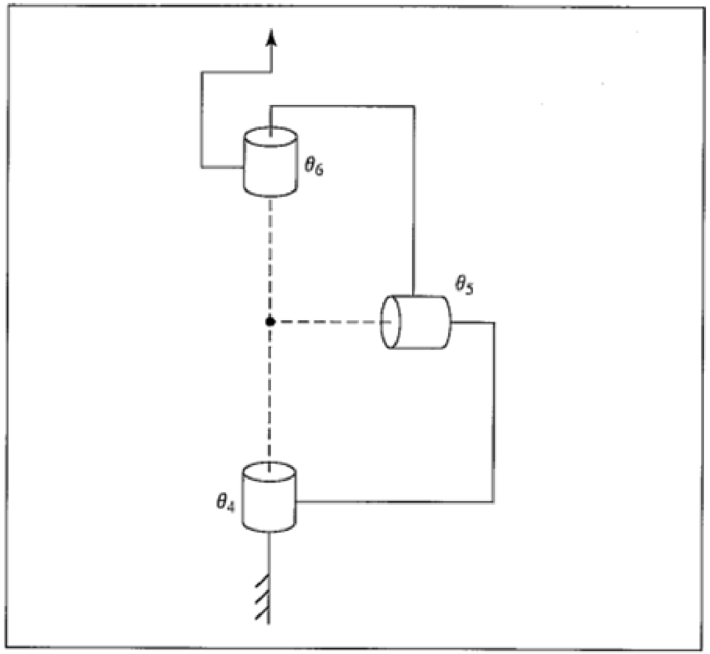


FIGURE 3.20 Schematic of a 3R wrist in which all three axes intersect at a point and are mutually orthogonal. This design is used in the PUMA 560 manipulator and many other industrial robots.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90^0	0	0	θ_2
3	0	a_2	d_3	θ_3
4	-90^0	a_3	d_4	θ_4
5	90^0	0	0	θ_5
6	-90^0	0	0	θ_6

$${}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4_5T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^5_6T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_6 & -c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}^4_6T = {}^4_5T\,{}^5_6T = \begin{bmatrix} c_5c_6 & -c_5s_6 & -s_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ s_5c_6 & -s_5s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Where c_5 is the shorthand for $\cos(\theta_5)$, s_5 for $\sin(\theta_5)$ and so on. Then we have

$$ {}^3_6T = {}^3_4T\,{}^4_6T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & -c_4s_5 & a_3 \\ s_5c_6 & -s_5s_6 & c_5 & d_4 \\ -s_4c_5c_6 - c_4s_6 & s_4s_5c_6 - c_4c_6 & s_4s_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_3T = {}^1_2T\,{}^2_3T = \begin{bmatrix} c_{23} & -s_{23} & 0 & a_2c_2 \\ 0 & 0 & 1 & d_3 \\ -s_{23} & -c_{23} & 0 & -a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Where we have used the sum of angles formulas(from appendix A)

$c_{23} = c_2c_3 - s_2s_3$

$s_{23} = c_2s_3 + s_2c_3$

Then we have

$$
{}^{1}_{6}T = {}^{1}_{3}T\,{}^{3}_{6}T = \begin{bmatrix} {}^{1}r_{11} & {}^{1}r_{12} & {}^{1}r_{13} & {}^{1}p_x \\ {}^{1}r_{21} & {}^{1}r_{22} & {}^{1}r_{23} & {}^{1}p_y \\ {}^{1}r_{31} & {}^{1}r_{32} & {}^{1}r_{33} & {}^{1}p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{aligned}
{}^{1}r_{11} &= c_{23}[c_4c_5c_6 - s_4s_6] - s_{23}s_5s_6, \\
{}^{1}r_{21} &= -\,s_4c_5c_6 - c_4s_6, \\
{}^{1}r_{31} &= -s_{23}[c_4c_5c_6 - s_4s_6] - s_{23}s_5c_6, \\
{}^{1}r_{12} &= -c_{23}[c_4c_5s_6 + s_4c_6] + s_{23}s_5s_6, \\
{}^{1}r_{22} &= -\,s_4c_5s_6 - c_4c_6, \\
{}^{1}r_{32} &= -\,s_{23}[c_4c_5s_6 + s_4c_6] + c_{23}s_5s_6, \\
{}^{1}r_{13} &= -\,c_{23}c_4s_5 - s_{23}c_5\,, \\
{}^{1}r_{23} &= s_4s_5\,, \\
{}^{1}r_{33} &= \ s_{23}c_4s_5 - c_{23}c_5, \\
{}^{1}p_x &= \ \alpha_2c_2 + \alpha_3c_{23} - d_4s_{23}\,, \\
{}^{1}p_y &= \ d_3\,, \\
{}^{1}p_z &= -\,\alpha_3s_{23} - \alpha_2s_2 - d_4c_{23}\,,
\end{aligned}
$$

Finally, we obtain the production of all six link transforms

$$ {}^{0}_{6}T = {}^{0}_{1}T\,{}^{1}_{6}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

Where

$$ r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6] + s_1(s_4c_5c_6 + c_4s_6), $$

$$ r_{21} = s_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6] - c_1(s_4c_5c_6 + c_4s_6), $$

$$ r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6, $$

$$ r_{12} = c_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6), $$

$$ r_{22} = s_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6), $$

$$ r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6, $$

$$ r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5, $$

$$ r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5, $$

$$ r_{33} = s_{23}c_4s_5 - c_{23}c_5, $$

$$ p_x = c_1[\alpha_2c_2 + \alpha_3c_{23} - d_4s_{23}] - d_3s_1, $$

$$ p_y = s_1[\alpha_2c_2 + \alpha_3c_{23} - d_4s_{23}] + d_3c_1, $$

$$ p_z = -\alpha_3s_{23} - \alpha_2s_2 - d_4c_{23}, $$

❖ Frames with standard names

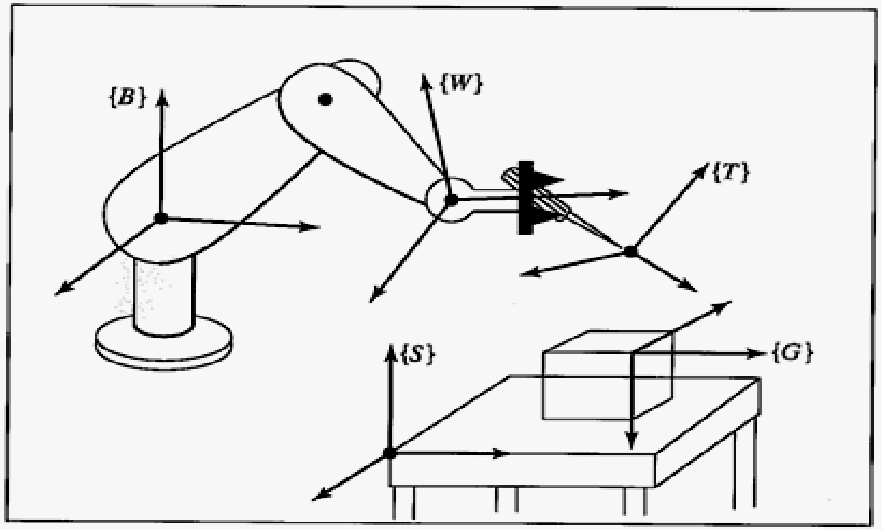


FIGURE 3.27: The standard frames.

Base Frame {B}	:	{0} link frame
Wrist Frame {W}	:	{N} link frame $\quad {}^{B}_{W}T = {}^{0}_{N}T$
Tool Frame {T}	:	the end of any tool
Station Frame {S}	:	${}^{B}_{S}T$ always known
Goal Frame {G}	:	tool frame should coincide with goal frame

$$ {}^{B}_{W}T\,{}^{W}_{T}T\,{}^{T}_{G}T = {}^{B}_{S}T\,{}^{S}_{G}T $$

$$ {}^{T}_{G}T = {}^{W}_{T}T^{-1}\,{}^{B}_{W}T^{-1}\;\;{}^{B}_{S}T\,{}^{S}_{G}T $$