Mechanics and Control of Robot Manipulators

Chapter 1 Solutions

This manual contains solutions to almost all exercises in Chapters 1 - 9 and solutions to the programming problems for Chapters 2 - 9.

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CHAPTER 1

INTRODUCTION

EXERCISES

1.1) HERE'S JUST AN EXAMPLE OF A REASONABLE RESPONSE: (REF. [6] IN CHAP. 1)

1955 DENAVIT & HARTENBERG DEVELOPED METHODOLOGY FOR DESCRIBING LINKAGES.

1961 GEORGE DEVOL PATENTS DESIGN OF FIRST ROBOT

1961 FIRST UNIMATE ROBOT INSTALLED.

1968 SHAKEY ROBOT DEVELOPED AT S.R.I.

1975 ROBOT INSTITUTE OF AMERICA FORMED.

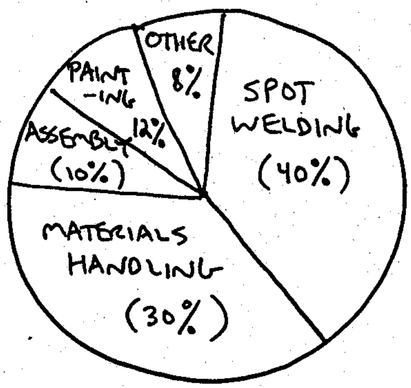
1975 UNIMATION BECOMES FIRST ROBOT CO. TO BE PROFITABLE.

1978 FIRST PUMA ROBOT SHIPPED TO GM.

1985 TOTAL U.S. MARKET EXCEEDS 500 MILLION DOLLARS (ANNUAL REVENUE).

DEVELOPMENTS MIGHT BE SPLIT INTO A TECHNICAL LIST AND A BUSINESS LIST.

1.2)

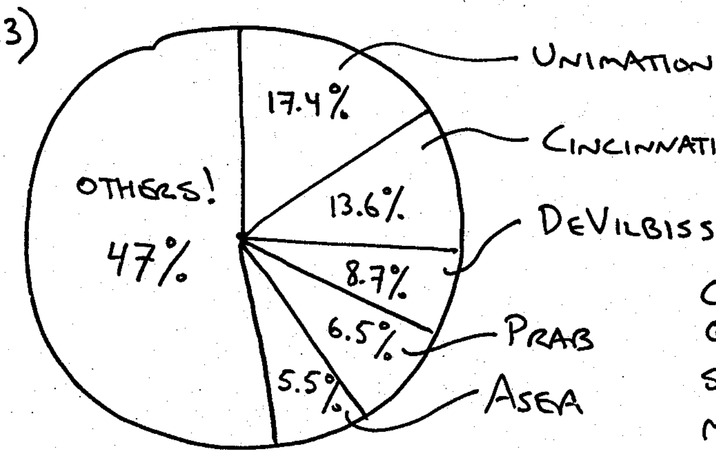


(BASED ON 1981 NUMBERS)

SOURCE:

L. CONIGLIARO, "ROBOTICS PRESENTATION, INSTITUTIONAL INVESTORS CONF.", MAY 28, 1981, BACHE NEWSLETTER 81-249.

1.3)



1983

CHANGING QUICKLY, SO SEE LATEST MATERIALS.

1.4) <u>KINEMATICS</u> IS THE STUDY OF POSITION AND DERIVATIVES OF POSITION WITHOUT REGARD TO FORCES WHICH CAUSE THE MOTION. <u>WORKSPACE</u> IS THE LOCUS OF POSITIONS AND ORIENTATIONS ACHIEVABLE BY THE END-EFFECTOR OF A MANIPULATOR. <u>TRAJECTORY</u> IS A TIME BASED FUNCTION WHICH SPECIFIES THE POSITION (AND HIGHER DERIVATIVES) OF THE ROBOT MECHANISM FOR ANY VALUE OF TIME.

1.5) FRAME IS A COORDINATE SYSTEM, USUALLY SPECIFIED IN POSITION AND ORIENTATION RELATIVE TO SOME IMBEDDING FRAME. DEGREES OF FREEDOM IS THE NUMBER OF INDEPENDENT VARIABLES WHICH MUST BE SPECIFIED IN ORDER TO COMPLETELY LOCATE ALL MEMBERS OF A (RIGID-BODY) MECHANISM. POSITION CONTROL IMPLIES THE USE OF A CONTROL SYSTEM, USUALLY IN A CLOSED-LOOP MANNER, TO CONTROL THE POSITION OF ONE OR MORE MOVING BODIES.

1.6) FORCE CONTROL IS THE USE OF (USUALLY CLOSED-LOOP) ALGORITHMS TO CONTROL THE FORCES OF CONTACT GENERATED WHEN A ROBOT TOUCHES ITS WORK ENVIRONMENT. A ROBOT PROGRAMMING LANGUAGE IS A PROGRAMMING LANGUAGE INTENDED FOR USE IN SPECIFYING MANIPULATOR ACTIONS.

1.7) SEE REFERENCES. IN 1985 AVERAGE LABOR COSTS OF $15 TO $20 ARE REASONABLE DEPENDING HOW FRINGE BENEFITS ARE CALCULATED.

1.8) IT'S INCREASED A LOT! SEE REFERENCES. COMBINE 1.7 & 1.8 ANSWERS TO SEE WHY THE ECONOMICS OF ROBOT USE HAS CHANGED DRAMATICALLY.

CHAPTER 2

SPATIAL DESCRIPTIONS AND TRANSFORMATIONS.

EXERCISES

2.1) $$R = \text{ROT}(\hat{X},\phi)\,\text{ROT}(\hat{Z},\theta)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix} \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta & -s\theta & 0 \\ c\phi s\theta & c\phi c\theta & -s\phi \\ s\phi s\theta & s\phi c\theta & c\phi \end{bmatrix}$$

2.2) $$R = \text{ROT}(\hat{X},45^\circ)\,\text{ROT}(\hat{Y},30^\circ)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & .707 & -.707 \\ 0 & .707 & .707 \end{bmatrix} \begin{bmatrix} .866 & 0 & .5 \\ 0 & 1 & 0 \\ -.5 & 0 & .866 \end{bmatrix}$$

$$= \begin{bmatrix} .866 & 0 & .5 \\ .353 & .707 & -.612 \\ -.353 & .707 & .612 \end{bmatrix}$$

2.3) SINCE ROTATIONS ARE PERFORMED ABOUT AXES OF THE FRAME BEING ROTATED, THESE ARE EULER-ANGLE STYLE ROTATIONS:

$$R = \mathrm{ROT}(\hat{z}, \theta)\ \mathrm{ROT}(\hat{x}, \phi)$$

WE MIGHT ALSO USE THE FOLLOWING REASONING:

$$
\begin{aligned}
{}^{A}_{B}R(\theta,\phi) &= {}^{B}_{A}R^{-1}(\theta,\phi) \\
&= \left[\mathrm{ROT}(\hat{x}, -\phi)\ \mathrm{ROT}(\hat{z}, -\theta)\right]^{-1} \\
&= \mathrm{ROT}^{-1}(\hat{z}, -\theta)\ \mathrm{ROT}^{-1}(\hat{x}, -\phi) \\
&= \mathrm{ROT}(\hat{z}, \theta)\ \mathrm{ROT}(\hat{x}, \phi)
\end{aligned}
$$

YET ANOTHER WAY OF VIEWING THE SAME OPERATION:

1^{ST} ROTATE BY $\mathrm{ROT}(\hat{z}, \theta)$

2^{ND} ROTATE BY $\mathrm{ROT}(\hat{z}, \theta)\ \mathrm{ROT}(\hat{x}, \phi)\ \mathrm{ROT}^{-1}(\hat{z}, \theta)$

(THIS IS A SIMILARITY TRANSFORM)

COMPOSING THESE TWO ROTATIONS:

$$= \mathrm{ROT}(\hat{z}, \theta)\ \mathrm{ROT}(\hat{x}, \phi)\ \mathrm{ROT}^{-1}(\hat{z}, \theta) \cdot \mathrm{ROT}(\hat{z}, \theta)$$

$$= \mathrm{ROT}(\hat{z}, \theta)\ \mathrm{ROT}(\hat{x}, \phi)$$

$$= \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

$$= \begin{bmatrix} c\theta & -s\theta c\phi & s\theta s\phi \\ s\theta & c\theta c\phi & -c\theta s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

2.4) THIS IS THE SAME AS 2.3 ONLY WITH NUMBERS.

$$R = ROT(\hat{z}, 30°)\, ROT(\hat{x}, 45)$$

$$= \begin{bmatrix} .866 & -.353 & .353 \\ .50 & .612 & -.612 \\ 0 & .707 & .707 \end{bmatrix}$$

2.5) IF V_i IS AN EIGENVECTOR OF R, THEN

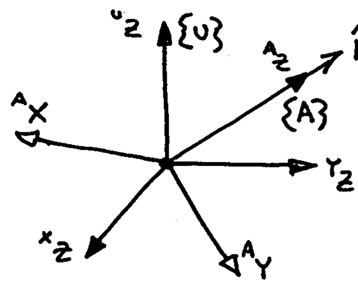
$$R V_i = \lambda V_i$$

IF THE EIGENVALUE ASSOCIATED WITH V_i IS 1, THEN

$$R V_i = V_i$$

HENCE THE VECTOR IS NOT CHANGED BY THE ROTATION R. SO V_i IS THE AXIS OF ROTATION.

2.6) IMAGINE A FRAME $\{A\}$ WHOOSE $\hat{Z}$ AXIS IS ALIGNED WITH THE DIRECTION $\hat{K}$:



THEN, THE ROTATION WITH ROTATES VECTORS ABOUT $\hat{K}$ BY θ DEGREES COULD BE WRITTEN:

$$R = {}^{U}_{A}R \;\; \mathrm{ROT}({}^{A}\hat{Z}, \theta) \;\; {}^{A}_{U}R \qquad [1]$$

WE WRITE THE DESCRIPTION OF $\{A\}$ IN $\{U\}$ AS:

$$ {}^{U}_{A}R = \begin{bmatrix} A & D & K_x \\ B & E & K_Y \\ C & F & K_Z \end{bmatrix} $$

IF WE MULTIPLY OUT EQ. [1] ABOVE, AND THEN SIMPLIFY USING $A^2+B^2+C^2=1$, $D^2+E^2+F^2=1$, $[A\ B\ C]\cdot[D\ E\ F]=0$, $[A\ B\ C]\otimes[D\ E\ F]=[K_x\ K_Y\ K_Z]$ WE ARRIVE AT EQ. (2.77) IN THE BOOK. ALSO, SEE [R. PAUL]* PAGE 25.

* REFERENCE [4] IN CHAPTER 2.

2.7) LET $R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$

① COMPUTE $R_{11} + R_{22} + R_{33} = N$

② IF $N = 3$, THEN $\theta = \text{ACOS}\left(\frac{N-1}{2}\right) = 0°$.
SINCE ROTATION IS ZERO, $\hat{K}$ IS ARBITRARY.

③ IF $N = -1$, THEN $\theta = \text{ACOS}(-1) = 180°$. IN THIS CASE EQ. (2.77) BECOMES:

$$\text{ROT}(\hat{K}, 180°) = \begin{bmatrix} 2K_X^2 - 1 & 2K_X K_Y & 2K_X K_Z \\ 2K_X K_Y & 2K_Y^2 - 1 & 2K_Y K_Z \\ 2K_X K_Z & 2K_Y K_Z & 2K_Z^2 - 1 \end{bmatrix}$$

SO:

$$2K_X^2 - 1 = R_{11} \implies K_X = \pm\sqrt{(R_{11}+1)/2}$$

$$2K_X K_Y = R_{21} \implies K_Y = R_{12}/2K_X$$

$$2K_X K_Z = R_{31} \implies K_Z = R_{31}/2K_X$$

HOWEVER, IF $K_X \cong 0$, THEN THIS IS ILL-DEFINED, SO USE A DIFFERENT COLUMN FOR SOLUTION (NOT THE FIRST COLUMN AS ABOVE).

④ IF $-1 < N < 3$ (SO THAT $0 < \theta < 180°$) THE USE EQ. (2.80) IN BOOK.

2.8) PROCEDURE RMTOAA IS GIVEN ESSENTIALLY IN THE SOLUTION TO 2.7. HOWEVER, WRITING CLEAN CODE TO CHECK THE VARIOUS CASES IS A GOOD EXERCISE IN ITSELF. PROCEDURE AATORM IS GIVEN BY EQ. (2.77) AND IS EASY.

2.9) THE SUBROUTINES ENCODE EQ. (2.57) AND EQUATIONS (2.59), (2.60), AND (2.61).

2.10) THE SUBROUTINES FOLLOW FROM EQ. (2.69) AND EQUATIONS (2.71), (2.72), AND (2.73).

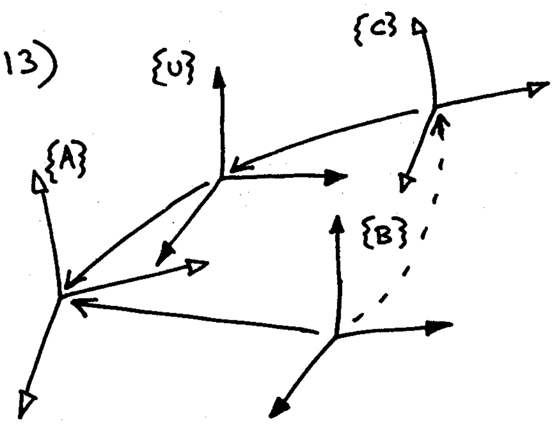
2.11) WHEN THEY REPRESENT ROTATIONS ABOUT THE SAME AXIS.

2.12) VELOCITY IS A "FREE VECTOR" AND ONLY WILL BE AFFECTED BY ROTATION, AND NOT BY TRANSLATION:

$$ {}^{A}V = {}^{A}_{B}R\,{}^{B}V = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} $$

$$ {}^{A}V = \begin{bmatrix} -1.34 & 22.32 & 30.0 \end{bmatrix}^{T} $$

2.13)



BY JUST FOLLOWING ARROWS, AND REVERSING (BY INVERSION) WHERE NEEDED, WE HAVE:

$$ {}^{B}_{C}T = {}^{B}_{A}T \; {}^{U}_{A}T^{-1} \; {}^{C}_{U}T^{-1} $$

INVERTING A TRANSFORM IS DONE USING EQ. (2.40) IN BOOK. REST IS BORING

2.14) THIS ROTATION CAN BE WRITTEN AS:

$$ {}^{A}_{B}T = TRANS({}^{A}\hat{P}, |{}^{A}P|) \; ROT(\hat{K},\theta) \; TRANS(-{}^{A}\hat{P}, |{}^{A}P|) $$

WHERE $ROT(\hat{K},\theta)$ IS WRITTEN AS IN EQ. (2.77),

AND $TRANS({}^{A}\hat{P}, |{}^{A}P|) = \begin{bmatrix} 1 & 0 & 0 & P_X \\ 0 & 1 & 0 & P_Y \\ 0 & 0 & 1 & P_Z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

AND $TRANS(-{}^{A}\hat{P}, |{}^{A}P|) = \begin{bmatrix} 1 & 0 & 0 & -P_X \\ 0 & 1 & 0 & -P_Y \\ 0 & 0 & 1 & -P_Z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

MULTIPLYING OUT WE GET:

$$
{}^{A}_{B}T = \left[\begin{array}{ccc|c} R_{11} & R_{12} & R_{13} & Q_X \\ R_{21} & R_{22} & R_{23} & Q_Y \\ R_{31} & R_{32} & R_{33} & Q_Z \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
$$

WHERE THE R_{ij} ARE GIVEN BE EQ. (2.77).

AND:

$$
Q_X = P_X - P_X(K_X^2 V\theta + C\theta) - P_Y(K_X K_Y V\theta - K_Z S\theta) - P_Z(K_X K_Z V\theta + K_Y S\theta)
$$

$$
Q_Y = P_Y - P_X(K_X K_Y V\theta + K_Z S\theta) - P_Y(K_Y^2 V\theta + C\theta) - P_Z(K_Y K_Z V\theta + K_X S\theta)
$$

$$
Q_Z = P_Z - P_X(K_X K_Z V\theta - K_Y S\theta) - P_Y(K_Y K_Z V\theta + K_X S\theta) - P_Z(K_Z^2 V\theta + C\theta)
$$

2.15) RECALL THAT $e^X = 1 + X + \frac{X^2}{2!} + \frac{X^3}{3!} + \cdots$

SO $e^{K\theta} = I + K\theta + \frac{1}{2!}K^2\theta^2 + \frac{1}{3!}K^3\theta^3 + \cdots$

$$
K^2 = \begin{bmatrix} -K_Y^2 - K_Z^2 & K_X K_Y & K_X K_Z \\ K_X K_Y & -K_X^2 - K_Z^2 & K_Y K_Z \\ K_X K_Z & K_Y K_Z & -K_X^2 - K_Y^2 \end{bmatrix}
$$

WRITING OUT THE (1,2) ELEMENT OF $e^{K\theta}$ (AS AN EXAMPLE) WE HAVE:

$$\left(e^{K\theta}\right)_{1,2} = 0 + (-K_z)\theta + \frac{1}{2!}(K_xK_Y)\theta^2 + \frac{1}{3!}K_z\theta^3 + \cdots$$

RECALL THAT:

$$S\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

$$C\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$$

$$V\theta = 1 - C\theta = \frac{\theta^2}{2!} - \frac{\theta^4}{4!} + \frac{\theta^6}{6!} - \cdots$$

WE CAN WRITE:

$$\left(e^{K\theta}\right)_{1,2} = (-K_z)\theta + \frac{1}{3!}K_z\theta^3 - \frac{1}{5!}K_z\theta^5 + \cdots$$

$$+ \frac{1}{2!}(K_xK_Y)\theta^2 - \frac{1}{4}(K_xK_Y)\theta^4 + \cdots$$

OR:

$$\left(e^{K\theta}\right)_{1,2} = -K_zS\theta + K_xK_YV\theta$$

WHICH IS THE VALUE GIVEN IN (2.77). OTHER ELEMENTS MAY BE CHECKED SIMILARLY.

2.16) IN METHOD 1, THE MULTIPLICATION OF TWO 3X3 MATRICES REQUIRES 27 MULTIPLICATIONS AND 18 ADDITIONS. THE COMPUTATION OF ${}^{A}_{B}R$ TAKES TWO MATRIX MULTIPLIES, OR

57 MULTIPLICATIONS, 36 ADDITIONS

THE COMPUTATION OF ${}^{A}_{B}R\,{}^{B}P$ IS 9 MULT. AND 6 ADDITIONS. HENCE, IN ONE SECOND THIS METHOD WILL REQUIRE:

$$30 \times \text{COMPUTATION OF } {}^{A}_{D}R = 30 \times 54 \text{ MULT.}$$
$$30 \times 36 \text{ ADD.}$$

$$100 \times \text{COMPUTATION OF } {}^{A}P = 100 \times 9 \text{ MULT.}$$
$$100 \times 6 \text{ ADD.}$$

TOTAL = $\underline{2520 \text{ MULT.}, 1680 \text{ ADD.}}$

IN METHOD TWO, COMPUTATION OF ${}^{C}_{D}R\,{}^{D}P$ REQUIRES 9 MULT. AND 6 ADD.; LIKEWISE THE COMPUTATION OF ${}^{B}_{C}R\,{}^{C}P$ AND ${}^{A}_{B}R\,{}^{B}P$, FOR A TOTAL OF 27 MULT. AND 18 ADD. THESE MUST OCCUR 100 TIMES/SEC., SO IN ONE SECOND WE HAVE:

$$27 \times 100 \text{ MULT.} = \underline{2700 \text{ MULT.}}$$
$$18 \times 100 \text{ ADD.} = \underline{1800 \text{ ADD.}}$$

THEREFORE, METHOD 1 IS SUPERIOR, BUT NOT BY MUCH.

2.17) $${}^{A}P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} R\cos\theta \\ R\sin\theta \\ z \end{bmatrix}$$

2.18) $${}^{A}P = \begin{bmatrix} P_X \\ P_Y \\ P_Z \end{bmatrix} = \begin{bmatrix} R\cos\alpha\cos\beta \\ R\sin\alpha\cos\beta \\ R\sin\beta \end{bmatrix}$$

2.19) IN THE Z-Y-Z EULER ANGLE SET, THE FIRST ROTATION IS:

$$R_1 = \mathrm{ROT}(\hat{Z}, \alpha)$$

THE SECOND ROTATION EXPRESSED IN FIXED COORDINATES IS:

$$R_2 = \mathrm{ROT}(\hat{Z},\alpha)\ \mathrm{ROT}(\hat{Y},\beta)\ \mathrm{ROT}^{-1}(\hat{Z},\alpha)$$

THE THIRD IS:

$$R_3 = (R_2 R_1)\ \mathrm{ROT}(\hat{Z},\gamma)\ (R_2 R_1)^{-1}$$

THE RESULT IS:

$$R = R_3 R_2 R_1 = \mathrm{ROT}(\hat{Z},\alpha)\,\mathrm{ROT}(\hat{Y},\beta)\,\mathrm{ROT}(\hat{Z},\gamma)$$

WHICH CAN BE MULTIPLIED OUT TO GIVE THE RESULT OF (2.69).

2.20) THIS IS EASILY DERIVED IF YOU WORK BACKWARDS. I.E, SUBSTITUTE INTO RODRIQUEZ'S FORMULA WHEREVER $\hat{K} \otimes \hat{Q}$ OR $\hat{K} \cdot \hat{Q}$ OCCUR, COLLECT TERMS, AND YOU'LL GET (2.77).

CHAPTER 3

MANIPULATOR KINEMATICS

EXERCISES

3.1)

α_{i-1}	a_{i-1}	d_i
0	0	0
0	L_1	0
0	L_2	0

$$ {}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & L_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^2_3T = \begin{bmatrix} c_3 & -s_3 & 0 & L_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

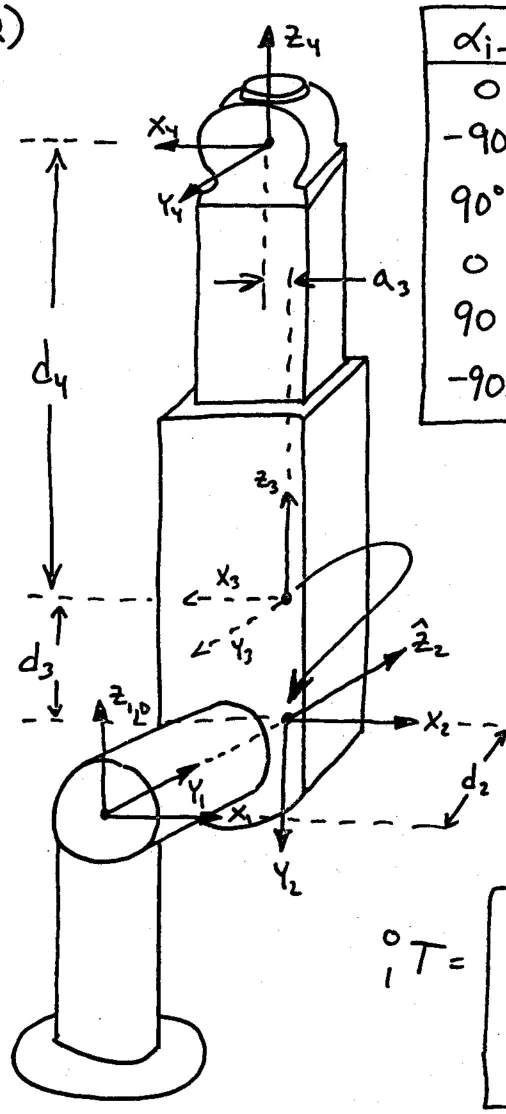
$$ {}^0_3T = {}^0_1T\,{}^1_2T\,{}^2_3T = \begin{bmatrix} C_{123} & -S_{123} & 0 & L_1c_1 + L_2C_{12} \\ S_{123} & C_{123} & 0 & L_1S_1 + L_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

WHERE:

$$ C_{123} = \cos(\theta_1 + \theta_2 + \theta_3) $$

$$ S_{123} = \sin(\theta_1 + \theta_2 + \theta_3) \quad , \text{ ETC.} $$

3.2)



α_{i-1}	a_{i-1}	d_i	θ_i
0	0	0	θ_1
$-90°$	0	d_2	θ_2
$90°$	0	d_3	$180°$
0	a_3	d_4	θ_4
90	0	0	θ_5
-90	0	0	θ_6

WHEN $d_3 = 0$ THE ORIGINS OF FRAMES 2 AND 3 COINCIDE. FRAME 3 IS FIXED TO LINK 3.

$$ {}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^2_3T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{4}T = \begin{bmatrix} c_4 & -s_4 & 0 & a_3 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{4}_{5}T = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{5}_{6}T = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_6 & -c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{0}_{6}T = {}^{0}_{3}T\,{}^{3}_{6}T $$

$$ {}^{0}_{3}T = \begin{bmatrix} -c_1c_2 & s_1 & c_1s_2 & -d_2s_1 + d_3c_1s_2 \\ -s_1c_2 & -c_1 & s_1s_2 & d_2c_1 + d_3s_1s_2 \\ s_2 & 0 & c_2 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{6}T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -(c_4c_5s_6 + s_4c_6) & -c_4s_5 & a_3 \\ (s_4c_5c_6 + c_4s_6) & -s_4c_5s_6 + c_4c_6 & -s_4s_5 & 0 \\ s_5c_6 & -s_5s_6 & c_5 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$
{}^{0}_{6}T = \begin{bmatrix} R_{11} & R_{12} & R_{13} & P_X \\ R_{21} & R_{22} & R_{23} & P_Y \\ R_{31} & R_{32} & R_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

WHERE:

$$R_{11} = -C_1C_2C_4C_5C_6 + C_1C_2S_4S_6 + S_1S_4C_5C_6 + S_1C_4S_6 + C_1S_2S_5S_6$$

$$R_{12} = C_1C_2C_4C_5S_6 + C_1C_2S_4C_6 - S_1S_4C_5S_6 + S_1C_4C_6 - S_1S_2S_5S_6$$

$$R_{13} = C_1C_2C_4S_5 - S_1S_4S_5 + C_1S_2C_5$$

$$R_{21} = -S_1C_2C_4C_5C_6 + S_1C_2S_4S_6 - C_1S_4C_5C_6 - C_1C_4S_6 + S_1S_2S_5C_6$$

$$R_{22} = S_1C_2C_4C_5S_6 + S_1C_2S_4C_6 + C_1S_4C_5S_6 - C_1C_4C_6 - S_1S_2S_5S_6$$

$$R_{23} = S_1C_2C_4S_5 + C_1S_4S_5 + S_1S_2C_5$$

$$R_{31} = S_2C_4C_5C_6 - S_2S_4S_6 + C_2S_5C_6$$

$$R_{32} = -S_2C_4C_5S_6 - S_2S_4C_6 - C_2S_5S_6$$

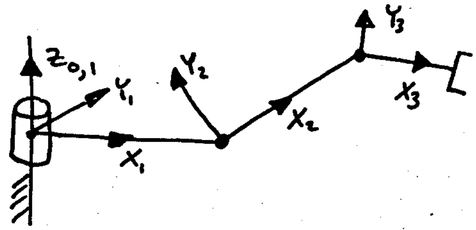
$$R_{33} = -S_2C_4C_5 + C_2C_5$$

$$P_X = -d_2S_1 + (d_3 + d_4)C_1S_2 - a_3C_1C_2$$

$$P_Y = d_2C_1 + (d_3 + d_4)S_1S_2 - a_3S_1C_2$$

$$P_z = (d_3 + d_4)C_2 + a_3S_2$$

3.3)



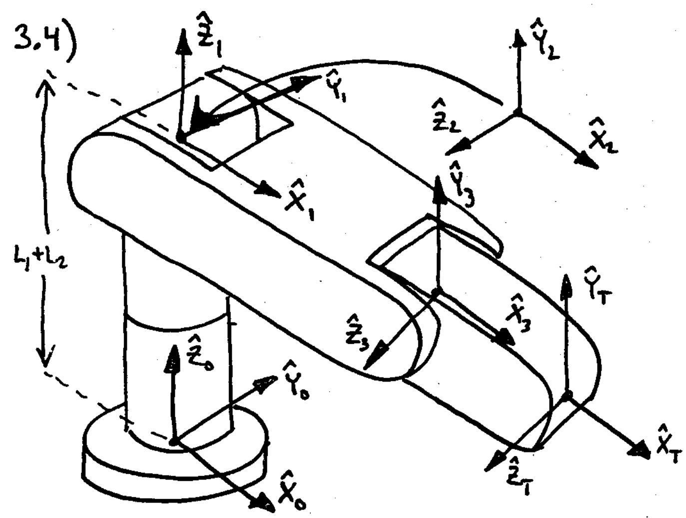
α_{i-1}	a_{i-1}	d_i
0	0	0
90°	L_1	0
0	L_2	0

$$ {}^0_1T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1_2T = \begin{bmatrix} C_2 & -S_2 & 0 & L_1 \\ 0 & 0 & -1 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2_3T = \begin{bmatrix} C_3 & -S_3 & 0 & L_2 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^B_WT = {}^0_3T = {}^0_1T\,{}^1_2T\,{}^2_3T $$

$$ {}^B_WT = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & L_1C_1 + L_2C_1C_2 \\ S_1C_{23} & -S_1S_{23} & -C_1 & L_1S_1 + L_2S_1C_2 \\ S_{23} & C_{23} & 0 & L_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

3.4)



$\{0\} = \{S\}$

$\{4\} = \{T\}$

α_{i-1}	a_{i-1}	d_i	θ_i
0	0	L_1+L_2	θ_1
$90°$	0	0	θ_2
0	L_3	0	θ_3
0	L_4	0	0

$$ {}^{0}_{1}T = \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & L_1+L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} c2 & -s2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s2 & c2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{2}_{3}T = \begin{bmatrix} c3 & -s3 & 0 & L3 \\ s3 & c3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

3.5) THE SUBROUTINE COMPUTES EQ. (3.13). BY CAREFULLY GROUPING COMMON TERMS, THE MULTIPLICATION COUNT CAN BE REDUCED BELOW THE OBVIOUS BRUTE-FORCE CALCULATION.

3.6) THE SUBROUTINE COMPUTES THE PRODUCT OF MATRICES GIVEN IN EQ(3.7). (ABOUT 30 MULTS)

3.7) THE SUBROUTINE COMPUTES ${}^{B}_{W}T$ GIVEN IN THE SOLUTION TO EXERCISE 3.3. (ABOUT 43 MULTS)

3.8) WHEN $\{G\} = \{T\}$ WE HAVE:

$$ {}^{B}_{W}T \; {}^{W}_{T}T = {}^{B}_{S}T \; {}^{S}_{G}T $$

SO,

$$ {}^{W}_{T}T = {}^{B}_{W}T^{-1} \; {}^{B}_{S}T \; {}^{S}_{G}T $$

3.9) ${}^{0}P_{TIP} = {}^{0}_{2}T \; {}^{2}P_{TIP}$; ${}^{2}P_{TIP} = \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}$

$$ {}^{0}P_{TIP} = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & L_1c_1 \\ s_1c_2 & -s_1s_2 & -c_1 & L_1s_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} $$

$$ = \begin{bmatrix} L_1c_1 + L_2c_1c_2 \\ L_1s_1 + L_2s_1c_2 \\ L_2s_2 \end{bmatrix} $$

3.10) THE COMPUTATION CAN BE STRUCTURED AS FOLLOWS. KK5, KK6, K8, ETC. ARE PRE-COMPUTED CONSTANTS.

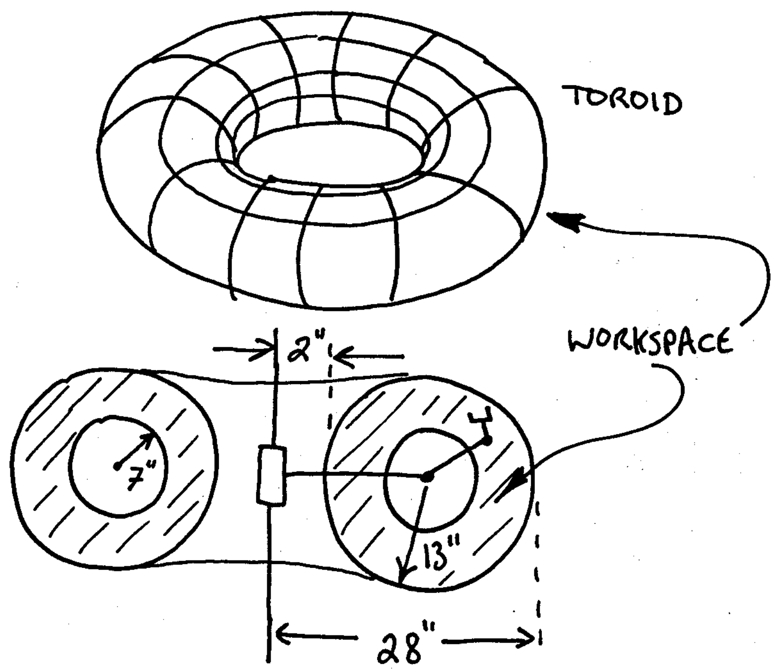
```
BEGIN
  cp2:=sk2*a[2]+lambda2;
  cp2:=(cp2*cp2+kk5)/kk6;
  cp3:=sk3*a[3]+lambda3;
  cp3:=(cp3*cp3+kk7)/kk8;
  sp2:=sqrt(1.0-cp2*cp2);
  sp3:=sqrt(1.0-cp3*cp3);
  t[3,4]:=kk1*cp2+kk2*sp2+kk3*cp3-kk4*sp3+hb;
  q:=-kk2*cp2+kk1*sp2+kk4*cp3+kk3*sp3;
  theta1:=sk1*a[1]+omega1;
  c1:=cos(theta1);
  s1:=sin(theta1);
  t[1,4]:=c1*q;
  t[2,4]:=s1*q;
  theta234:=k8-sk4*a[4];
  theta5:=k9-sk5*a[5];
  c234:=cos(theta234);
  s234:=sin(theta234);
  c5:=cos(theta5);
  s5:=sin(theta5);
  x1:=c1*c5;
  x2:=s1*s5;
  x3:=c1*s5;
  x4:=s1*c5;
  t[1,1]:=x1*c234-x2;
  t[2,1]:=x4*c234+x3;
  t[3,1]:=-c5*s234;
  t[1,2]:=-x3*c234-x4;
  t[2,2]:=-x2*c234+x1;
  t[3,2]:=s5*s234;
  t[1,3]:=c1*s234;
  t[2,3]:=s1*s234;
  t[3,3]:=c234;
END;  { atox }
```

CHAPTER 4

INVERSE MANIPULATOR KINEMATICS

EXERCISES

4.1)



4.2) THIS PROBLEM CAN HAVE DIFFERENT SOLUTIONS DEPENDING HOW IT IS INTERPRETED. I INTENDED THAT A GOAL IS SPECIFIED WHICH INCLUDES A DESIRED ORIENTATION OF THE LAST LINK. IN THIS CASE, THE SOLUTION IS FAIRLY EASY.

$^S_T T$ IS GIVEN, SO COMPUTE:

$$ {}^B_W T = {}^B_S T \; {}^S_T T \; {}^W_T T^{-1} $$

NOW ${}^{B}_{W}T = {}^{0}_{3}T$ WHICH WE WRITE OUT AS:

$$ {}^{0}_{3}T = \begin{bmatrix} R_{11} & R_{12} & R_{13} & P_x \\ R_{21} & R_{22} & R_{23} & P_Y \\ R_{31} & R_{32} & R_{33} & P_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

FROM THE SOLUTION OF EXERCISE 3 FROM CHAPTER 3 WE HAVE:

$$ {}^{0}_{3}T = \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & S_1 & C_1(C_2 L_2 + L_1) \\ S_1 C_{23} & -S_1 S_{23} & -C_1 & S_1(C_2 L_2 + L_1) \\ S_{23} & C_{23} & 0 & S_2 L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

EQUATE ELEMENTS $(1,3)$: $S_1 = R_{13}$

EQUATE ELEMENTS $(2,3)$: $-C_1 = R_{23}$

$$ \therefore \quad \boxed{\theta_1 = \text{ATAN2}(R_{13}, -R_{23})} $$

IF BOTH $R_{13} = 0$ AND $R_{23} = 0$ THE GOAL IS UNATTAINABLE.

EQUATE ELEMENTS $(1,4)$: $P_X = C_1(C_2 L_2 + L_1)$

EQUATE ELEMENTS $(2,4)$: $P_Y = S_1(C_2 L_2 + L_1)$

IF $C_1 \neq 0$ THEN $C_2 = \frac{1}{L_2}\left(\frac{P_X}{C_1} - L_1\right)$

ELSE $C_2 = \frac{1}{L_2}\left(\frac{P_Y}{S_1} - L_1\right)$

EQUATE ELEMENTS (3,4): $P_Z = S_2 L_2$

SO,

$$\boxed{\theta_2 = \mathrm{ATAN2}\left(P_Z/L_2 \,,\, C_2 \right)}$$

EQUATE ELEMENTS (3,1): $S_{23} = R_{31}$

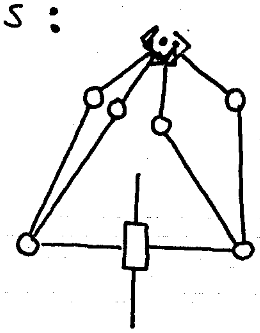
EQUATE ELEMENTS (3,2): $C_{23} = R_{32}$

SO,

$$\boxed{\theta_3 = \mathrm{ATAN2}\left(R_{31}, R_{32}\right) - \theta_2}$$

IF BOTH R_{31} AND R_{32} ARE ZERO, THE GOAL IS UNATTAINABLE.

A SECOND INTERPRETATION OF THE PROBLEM IS THAT ONLY A DESIRED POSITION IS GIVEN (NO ORIENTATION). IN THIS THERE MAY BE UP TO FOUR SOLUTIONS:



ASSUME ${}^{3}P_{TOOL} = L_3 \hat{X}_3$, THEN

$$
{}^{0}P_{TOOL} = \begin{bmatrix} P_X \\ P_Y \\ P_Z \end{bmatrix} = \begin{bmatrix} L_1 C_1 + L_2 C_1 C_2 + L_3 C_1 C_{23} \\ L_1 S_1 + L_2 S_1 C_2 + L_3 S_1 C_{23} \\ L_2 S_2 + L_3 S_{23} \end{bmatrix}
$$

FIRST,

$$
S_1 = \frac{P_Y}{L_1 + L_2 C_2 + L_3 C_{23}} \qquad C_1 = \frac{P_X}{L_1 + L_2 C_2 + L_3 C_{23}}
$$

SO,

$$
\boxed{\theta_1 = \mathrm{ATAN2}(P_Y, P_X) \quad \text{OR} \quad \mathrm{ATAN2}(-P_Y, -P_X)}
$$

SINCE THE SIGN OF THE "$L_1 + L_2 C_2 + L_3 C_{23}$" TERM MAY BE + OR −.

NEXT, DEFINE:

$$
\alpha = \begin{cases} \dfrac{P_X}{C_1} - L_1 & \text{IF } C_1 \neq 0 \\[2ex] \dfrac{P_Y}{S_1} - L_1 & \text{IF } S_1 \neq 0 \end{cases}
$$

AND WE HAVE:

$$
L_2 C_2 + L_3 C_{23} = \alpha
$$

$$
L_2 S_2 + L_3 S_{23} = P_Z
$$

SQUARE AND ADD THESE TWO EQUATIONS TO GET:

$$
L_2^2 + L_3^2 + 2 L_2 L_3 C_3 = \alpha^2 + P_Z^2
$$

$$
C_3 = \frac{1}{2 L_2 L_3} \left(\alpha^2 + P_Z^2 - L_2^2 - L_3^2 \right)
$$

$$
S_3 = \pm \sqrt{1 - C_3^2} \; ; \quad \boxed{\theta_3 = \mathrm{ATAN2}(S_3, C_3)}
$$

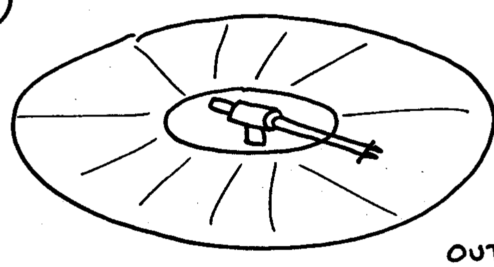
FINALLY,

$$L_3 C_{23} = \alpha - L_2 C_2$$
$$L_3 S_{23} = P_z - L_2 S_2$$

SO,

$$\boxed{\theta_2 = \mathrm{ATAN2}\left(P_z - L_2 S_2, \alpha - L_2 C_2\right) - \theta_3}$$

4.3)



A FLAT DISK OF ZERO THICKNESS WITH INNER RADIUS EQUAL TO MINIMUM EXTENSION, AND OUTER RADIUS EQUAL TO MAXIMUM EXTENSION.

4.4) FOR AN ARM LIKE THIS IT IS REASONABLE TO SPECIFY A GOAL BY GIVING THE DESIRED X & Y COORDINATES OF THE TIP AND A VALUE FOR θ_3, THE WRIST ROLL. AFTER TRANSFORMING BACK TO P_X & P_Y OF THE WRIST (I.E. REMOVE OFFSET DUE TO L_3) THE SOLUTION IS SIMPLE AND CORRESPONDS TO THE CONVERSION BETWEEN CARTESIAN AND POLAR COORDINATES.

4.5) TURN THE RESULTS OF SECTION 4.61 INTO A COMPUTER ALGORITHM, PLUS CHECK EACH SOLUTION TO SEE IF JOINTS ARE IN RANGE.

4.6) TO DERIVE THE "NEAREST" SOLUTION, WE WOULD LIKE TO MINIMIZE THE ROTATION OF EACH JOINT. DENOTE THE STARTING ANGLE OF EACH JOINT AS θ_{jo} (FOR j-TH JOINT), AND THE FINAL POSITION OF EACH JOINT AS θ_{jF}.
FOR EACH PROPOSED SOLUTION, COMPUTE:

$$S = \sum_{j=1}^{N} |\theta_{jF} - \theta_{jo}|$$

AND CHOOSE THE "NEAREST" AS THE ONE WHICH MINIMIZES S. SOMETIMES A WEIGHTING FACTOR IS USED (TO PENALIZE MOTION OF "LARGE" JOINTS, FOR EXAMPLE) AND SO THE SCORE FOR EACH PROPOSED SOLUTION IS:

$$S = \sum_{j=1}^{N} W_j \, |\theta_{jF} - \theta_{jo}|$$

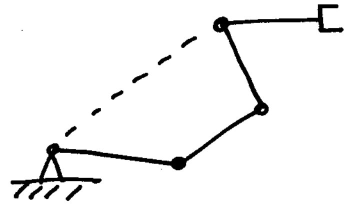
4.7) REPEATABILITY IS AFFECTED BY:

1) STEADY STATE ERROR IS SERVO SYSTEM
2) FLEXIBILITY OF LINKS
3) BACKLASH IN GEARS
4) LOOSENESS IN BEARINGS
5) NOISE IN SENSOR READINGS
6) THERMAL EFFECTS

ACCURACY IS AFFECTED BY ALL OF THE ABOVE, PLUS:

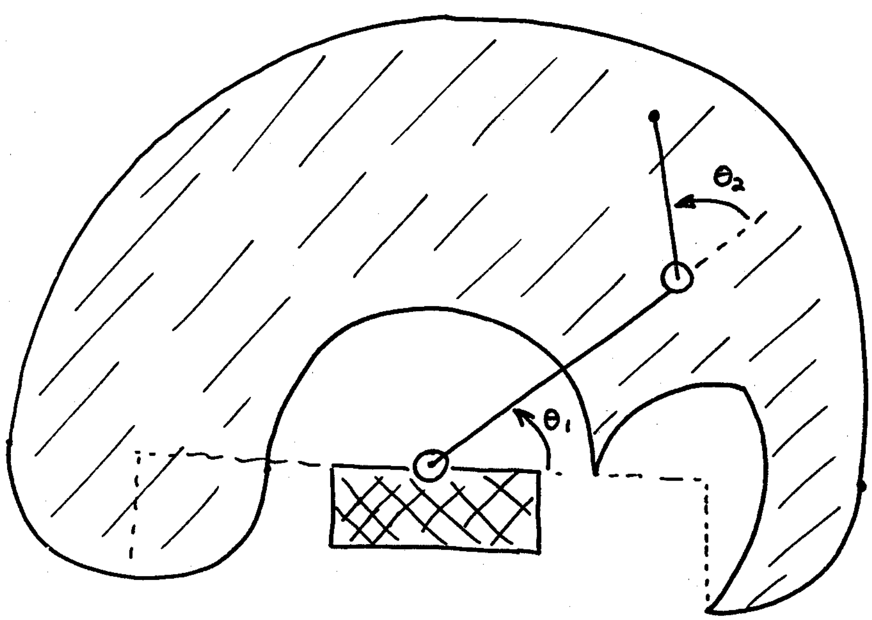
1) IMPRECISE KNOWLEDGE OF D.H. PARAMETERS.

4.8) THERE ARE AN INFINITE NUMBER OF SOLUTIONS. IMAGINE FIXING THE LAST LINK IN POSITION AND ORIENTATION:



THEN, THE FIRST 3 LINKS FORM A 4-BAR LINKAGE WHICH CAN TAKE ON AN INFINITY OF POSITIONS SINCE IT HAS A DEGREE OF FREEDOM.

4.9) THIS IS SLIGHTLY TRICKIER THAN IT LOOKS AT FIRST. APPROXIMATELY:



4.10) THIS SUBSPACE WITH BE GIVEN IN TERMS OF AN EXPRESSION FOR ${}^0_3T$ WHICH IS A FUNCTION OF THREE INDEPENDANT VARIABLES X, Y, AND θ. AS THESE VARIABLES RUN THROUGHOUT THEIR RANGES, A SUBSPACE IS SWEPT OUT.

AS IN EXAMPLE 4.2 THE ORIGIN OF THE END-EFFECTOR FRAME (HERE, {3}) MUST HAVE ZERO Z-COMPONENT. ALSO, $\hat{Z}_3$ MUST HAVE NO Z-COMPONENT AND IT'S DIRECTION IS GIVEN BY THE COORDINATES OF ${}^0P_{3ORG}$. SO WE HAVE:

$$ {}^0_3T = \begin{bmatrix} A & D & \alpha & X \\ B & E & \beta & Y \\ C & F & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

WHERE:

$$ \alpha = \frac{X}{\sqrt{X^2+Y^2}} \quad \text{AND} \quad \beta = \frac{Y}{\sqrt{X^2+Y^2}} $$

TO FIND EXPRESSIONS FOR A, B, C, D, E, F AS A FUNCTION OF INDEPENDANT VARIABLE, θ, CONSIDER ROTATING THE VECTOR $[0\ 0\ 1]^T$ ABOUT $\hat{Z}_3$ (GIVEN BY $[\alpha\ \beta\ 0]^T$) BY AMOUNT θ. THIS SERVES AS OUR EXPRESSION FOR $\hat{X}_3$ AND CAN BE FOUND FROM EQ. (2.77) USING $K = [\alpha\ \beta\ 0]^T$, AND JUST TAKING THE 3RD COLUMN.

THIS YIELDS :

$$\hat{X}_3 = \begin{bmatrix} -\beta c\theta & \alpha c\theta & s\theta \end{bmatrix}^T$$

THEN WE COMPUTE $\hat{Y}_3 = \hat{Z}_3 \otimes \hat{X}_3$ TO GET

$$ {}^0_3T = \begin{bmatrix} -\beta c\theta & \beta s\theta & \alpha & X \\ \alpha c\theta & -\alpha s\theta & \beta & Y \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

WHERE: $\alpha = \dfrac{X}{\sqrt{X^2+Y^2}} \qquad \beta = \dfrac{Y}{\sqrt{X^2+Y^2}}$

CHAPTER 5

JACOBIANS: VELOCITIES AND STATIC FORCES

EXERCISES

5.1) THE JACOBIAN IN FRAME $\{0\}$ IS:

$$ {}^{0}J(\underline{\theta}) = \begin{bmatrix} -L_1 S_1 - L_2 S_{12} & -L_2 S_{12} \\ L_1 C_1 + L_2 C_{12} & L_2 C_{12} \end{bmatrix} $$

$$ \begin{aligned} \mathrm{DET}\left({}^{0}J(\underline{\theta})\right) &= -(L_2 C_{12})(L_1 S_1 + L_2 S_{12}) + (L_2 S_{12})(L_1 C_1 + L_2 C_{12}) \\ &= -L_1 L_2 S_1 C_{12} - L_2^2 S_{12} C_{12} + L_1 L_2 C_1 S_{12} + L_2^2 S_{12} C_{12} \\ &= L_1 L_2 C_1 S_{12} - L_1 L_2 S_1 C_{12} = L_1 L_2 (C_1 S_{12} - S_1 C_{12}) \\ &= L_1 L_2 S_2 \end{aligned} $$

$\therefore$ THE SAME RESULT AS WHEN YOU START WITH ${}^{3}J(\underline{\theta})$, NAMELY, THE SINGULAR CONFIGURATIONS ARE $\theta_2 = 0°$ OR $180°$.

5.2) FROM EXERCISE 3.3 WE HAVE:

$$ {}^{0}_{3}T = \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & S_1 & L_1 C_1 + L_2 C_1 C_2 \\ S_1 C_{23} & -S_1 S_{23} & -C_1 & L_1 S_1 + L_2 S_1 C_2 \\ S_{23} & C_{23} & 0 & L_2 S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

AND:

$$ {}^{3}_{4}T = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \qquad {}^{0}_{4}T = {}^{0}_{3}T \; {}^{3}_{4}T $$

WE COULD THEN FIND ${}^{0}J(\underline{\theta})$ QUITE EASILY BY DIFFERENTIATING ${}^{0}P_{4ORG}$. FINALLY, ${}^{4}J(\underline{\theta})$ CAN BE CALCULATED AS ${}^{4}_{0}R \; {}^{0}J(\underline{\theta})$. THIS MIGHT BE TEDIOUS, SO LETS TRY "STANDARD" VELOCITY PROPAGATION AS DONE IN THE TEXT:

$$ {}^{1}\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} \qquad {}^{1}V_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

$$ {}^{2}\omega_2 = {}^{2}_{1}R \; {}^{1}\omega_1 + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix} = \begin{bmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix} $$

$$ {}^{2}\omega_2 = \begin{bmatrix} s_2\dot\theta_1 \\ c_2\dot\theta_1 \\ \dot\theta_2 \end{bmatrix} \qquad {}^{2}V_2 = {}^{2}_{1}R \left({}^{1}V_1 + {}^{1}\omega_1 \times {}^{1}P_2 \right) $$

$$ {}^{2}V_2 = \begin{bmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ L_1\dot\theta_1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \\ -L_1\dot\theta_1 \end{bmatrix} $$

$$ {}^{3}\omega_3 = {}^{3}_{2}R \; {}^{2}\omega_2 + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_3 \end{bmatrix} = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_2\dot\theta_1 \\ c_2\dot\theta_1 \\ \dot\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_3 \end{bmatrix} $$

$$ {}^{3}\omega_3 = \begin{bmatrix} s_{23}\dot\theta_1 \\ c_{23}\dot\theta_1 \\ \dot\theta_2 + \dot\theta_3 \end{bmatrix} \qquad {}^{3}V_3 = {}^{3}_{2}R \left({}^{2}V_2 + {}^{2}\omega_2 \times {}^{2}P_3 \right) $$

$$ {}^{3}V_3 = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ -L_1\dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ L_2\dot\theta_2 \\ -L_2 c_2\dot\theta_1 \end{bmatrix} \right) $$

$$^{3}V_3 = \begin{bmatrix} S_3 L_2 \dot{\theta}_2 \\ C_3 L_2 \dot{\theta}_2 \\ -L_1\dot{\theta}_1 - L_2 C_2 \dot{\theta}_1 \end{bmatrix} \qquad {}^{4}\omega_4 = {}^{3}\omega_3$$

$$^{4}V_4 = {}^{4}_{3}R\left({}^{3}V_3 + {}^{3}\omega_3 \times {}^{3}P_4\right) = {}^{3}V_3 + {}^{3}\omega_3 \times {}^{3}P_4$$

$$= \begin{bmatrix} S_3 L_2 \dot{\theta}_2 \\ C_3 L_2 \dot{\theta}_2 \\ -L_1\dot{\theta}_1 - L_2 C_2 \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ L_3(\dot{\theta}_2 + \dot{\theta}_3) \\ -L_3 C_{23}\dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} S_3 L_2 \dot{\theta}_2 \\ C_3 L_2 \dot{\theta}_2 + L_3(\dot{\theta}_2 + \dot{\theta}_3) \\ -L_1\dot{\theta}_1 - L_2 C_2 \dot{\theta}_1 - L_3 C_{23}\dot{\theta}_1 \end{bmatrix}$$

$$\therefore \quad {}^{4}J(\underline{\theta}) = \begin{bmatrix} 0 & S_3 L_2 & 0 \\ 0 & C_3 L_2 + L_3 & L_3 \\ (-L_1 - L_2 C_2 - L_3 C_{23}) & 0 & 0 \end{bmatrix}$$

5.3) FIRST, VELOCITY ANALYSIS: (THIS PART IS SAME AS EXER. 5.2)

$$^{1}\omega_1 = {}^{1}_{0}R\,{}^{0}\omega_0 + \dot{\theta}_1\,{}^{1}\hat{Z}_1 = \dot{\theta}_1 \hat{Z}$$

$$^{1}V_1 = {}^{1}_{0}R\left({}^{0}V_0 + {}^{0}\omega_0 \times {}^{0}P_1\right) = 0$$

$$^{2}\omega_2 = {}^{2}_{1}R\,{}^{1}\omega_1 + \dot{\theta}_2\,{}^{2}\hat{Z}_2$$

$$^{2}\omega_2 = \begin{bmatrix} C_2 & 0 & S_2 \\ -S_2 & 0 & C_2 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} S_2\dot{\theta}_1 \\ C_2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$^{2}V_2 = {}^{2}_{1}R\left({}^{1}V_1 + {}^{1}\omega_1 \times {}^{1}P_2\right)$$

$$^{2}V_2 = \begin{bmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \times \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$^{2}V_2 = \begin{bmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ L_1\dot{\theta}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -L_1\dot{\theta}_1 \end{bmatrix}$$

$$^{3}\omega_3 = {}^{3}_{2}R\,{}^{2}\omega_2 + \dot{\theta}_3\,{}^{3}\hat{z}_3$$

$$^{3}\omega_3 = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_2\dot{\theta}_1 \\ c_2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} s_2c_3\dot{\theta}_1 + c_2s_3\dot{\theta}_1 \\ -s_2s_3\dot{\theta}_1 + c_2c_3\dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$$

$$^{3}\omega_3 = \begin{bmatrix} s_{23}\dot{\theta}_1 \\ c_{23}\dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$$

$$^{3}V_3 = {}^{3}_{2}R\left({}^{2}V_2 + {}^{2}\omega_2 \times {}^{2}P_3\right)$$

$$^{3}V_3 = {}^{3}_{2}R\left(\begin{bmatrix} 0 \\ 0 \\ -L_1\dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} s_2\dot{\theta}_1 \\ c_2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \times \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$^{3}V_3 = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ L_2\dot{\theta}_2 \\ -L_1\dot{\theta}_1 - L_2c_2\dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} L_2s_3\dot{\theta}_2 \\ L_2c_3\dot{\theta}_2 \\ -L_1\dot{\theta}_1 - L_2c_2\dot{\theta}_1 \end{bmatrix}$$

$$ {}^4\omega_4 = {}^4_3R\,{}^3\omega_3 + 0 \quad ; \quad {}^4_3R = I \quad ; \quad {}^4\omega_4 = {}^3\omega_3 $$

$$ {}^4V_4 = {}^4_3R\left({}^3V_3 + {}^3\omega_3 \times {}^3P_4\right) $$

$$ {}^4V_4 = {}^4_3R\left(\begin{bmatrix} L_2 S_3 \dot\theta_2 \\ L_2 C_3 \dot\theta_2 \\ -L_1\dot\theta_1 - L_2 C_2 \dot\theta_1 \end{bmatrix} + \begin{bmatrix} S_{23}\dot\theta_1 \\ C_{23}\dot\theta_1 \\ \dot\theta_2 + \dot\theta_3 \end{bmatrix} \times \begin{bmatrix} L_3 \\ 0 \\ 0 \end{bmatrix}\right) $$

$$ {}^4V_4 = \left(\begin{bmatrix} L_2 S_3 \dot\theta_2 \\ L_2 C_3 \dot\theta_2 \\ -L_1\dot\theta_1 - L_2 C_2 \dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ L_3(\dot\theta_2 + \dot\theta_3) \\ -L_3 C_{23}\dot\theta_1 \end{bmatrix}\right) $$

$$ {}^4V_4 = \begin{bmatrix} L_2 S_3 \dot\theta_2 \\ L_2 C_3 \dot\theta_2 + L_3(\dot\theta_2 + \dot\theta_3) \\ -L_1\dot\theta_1 - L_2 C_2 \dot\theta_1 - L_3 C_{23}\dot\theta_1 \end{bmatrix} $$

$$ {}^4V_4 = {}^4J(\underline{\theta})\,\underline{\dot\theta} $$

$$ \therefore\ {}^4J(\underline{\theta}) = \begin{bmatrix} 0 & L_2 S_3 & 0 \\ 0 & L_2 C_3 + L_3 & L_3 \\ -L_1 - L_2 C_2 - L_3 C_{23} & 0 & 0 \end{bmatrix} $$

NEXT, USING FORCE ANALYSIS:

$$ {}^4F_4 = \begin{bmatrix} f_x \\ f_Y \\ f_Z \end{bmatrix} \qquad {}^4N_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

$$ {}^{3}F_{3} = {}^{3}_{4}R\, {}^{4}F_{4} = \begin{bmatrix} f_X \\ f_Y \\ f_Z \end{bmatrix} $$

$$ {}^{3}N_{3} = {}^{3}_{4}R\, {}^{4}N_{4} + {}^{3}P_{4} \times {}^{3}F_{3} = \begin{bmatrix} L_3 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} f_X \\ f_Y \\ f_Z \end{bmatrix} = \begin{bmatrix} 0 \\ -L_3 f_Z \\ L_3 f_Y \end{bmatrix} $$

$$ {}^{2}F_{2} = {}^{2}_{3}R\, {}^{3}F_{3} = \begin{bmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_X \\ f_Y \\ f_Z \end{bmatrix} = \begin{bmatrix} c_3 f_X - s_3 f_Y \\ s_3 f_X + c_3 f_Y \\ f_Z \end{bmatrix} $$

$$ {}^{2}N_{2} = {}^{2}_{3}R\, {}^{3}N_{3} + {}^{2}P_{3} \times {}^{2}F_{2} $$

$$ {}^{2}N_{2} = \begin{bmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -L_3 f_Z \\ L_3 f_Y \end{bmatrix} + \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} c_3 f_X - s_3 f_Y \\ s_3 f_X + c_3 f_Y \\ f_Z \end{bmatrix} $$

$$ {}^{2}N_{2} = \begin{bmatrix} L_3 s_3 f_Z \\ -L_2 f_Z - L_3 c_3 f_Z \\ L_2 (s_3 f_X + c_3 f_Y) + L_3 f_Y \end{bmatrix} $$

$$ {}^{1}F_{1} = {}^{1}_{2}R\, {}^{2}F_{2} = \begin{bmatrix} c_2 & -s_2 & 0 \\ 0 & 0 & -1 \\ s_2 & c_2 & 0 \end{bmatrix} \begin{bmatrix} c_3 f_X - s_3 f_Y \\ s_3 f_X + c_3 f_Y \\ f_Z \end{bmatrix} $$

$$ {}^{1}F_{1} = \begin{bmatrix} c_2 (c_3 f_X - s_3 f_Y) - s_2 (s_3 f_X + c_3 f_Y) \\ -f_Z \\ s_2 (c_3 f_X - s_3 f_Y) + c_2 (s_3 f_X + c_3 f_Y) \end{bmatrix} $$

$${}^{1}N_1 = {}^{1}_{2}R\,{}^{2}N_2 + {}^{1}P_2 \times {}^{1}F_1$$

$${}^{1}N_1 = \begin{bmatrix} c_2 & -s_2 & 0 \\ 0 & 0 & -1 \\ s_2 & c_2 & 0 \end{bmatrix} \begin{bmatrix} l_3 s_3 f_z \\ -l_2 f_z - l_3 c_3 f_z \\ l_2(s_3 f_x + c_3 f_Y) + l_3 f_Y \end{bmatrix} + \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \times {}^{1}F_1$$

$${}^{1}N_1 = \begin{bmatrix} c_2 l_3 s_3 f_z + s_2 l_2 f_z + l_2 s_2 c_3 f_z \\ -l_2(s_3 f_x + c_3 f_Y) - l_3 f_Y \\ l_3 s_2 s_3 f_z - l_2 c_2 f_z - l_3 c_2 c_3 f_z \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ -l_1 s_2 (c_3 f_x - s_3 f_Y) - l_2 c_2 (s_3 f_x + c_3 f_Y) \\ -l_1 f_z \end{bmatrix}$$

TO COMPUTE TORQUES, TAKE THE z-COMPONENT OF THE ${}^{i}N_i$:

$$\tau_1 = \left[-l_1 - l_2 c_2 + l_3(s_2 s_3 - c_2 c_3)\right] f_z$$

$$\tau_2 = l_2 s_3 f_x + (l_2 c_3 + l_3) f_Y$$

$$\tau_3 = l_3 f_Y$$

$$\underline{\tau} = {}^{4}J^{T}(\underline{\theta}) \begin{bmatrix} f_x \\ f_Y \\ f_z \end{bmatrix}$$

WHICH LEADS TO SAME EXPRESSION AS BEFORE FOR ${}^{4}J(\underline{\theta})$.

FINALLY, BY DIFFERENTIATION OF KINEMATIC EQUATIONS:

$$
{}^{0}P_{4ORG} = \begin{bmatrix} L_1 C_1 + L_2 C_1 C_2 + L_3 C_1 C_{23} \\ L_1 S_1 + L_2 S_1 C_2 + L_3 S_1 C_{23} \\ L_2 S_2 + L_3 S_{23} \end{bmatrix} \triangleq P
$$

$$
{}^{0}J(\underline{\theta}) = \begin{bmatrix} \frac{\partial P_x}{\partial \theta_1} & \frac{\partial P_x}{\partial \theta_2} & \frac{\partial P_x}{\partial \theta_3} \\ \frac{\partial P_Y}{\partial \theta_1} & \frac{\partial P_Y}{\partial \theta_2} & \frac{\partial P_Y}{\partial \theta_3} \\ \frac{\partial P_z}{\partial \theta_1} & \frac{\partial P_z}{\partial \theta_2} & \frac{\partial P_z}{\partial \theta_3} \end{bmatrix}
$$

$$
{}^{0}J(\underline{\theta}) = \begin{bmatrix} -L_1 S_1 - L_2 S_1 C_2 - L_3 S_1 C_{23} & -L_2 C_1 S_2 - L_3 C_1 S_{23} & -L_3 C_1 S_{23} \\ L_1 C_1 + L_2 C_1 C_2 + L_3 C_1 C_{23} & L_2 S_1 S_2 - L_3 S_1 S_{23} & -L_3 S_1 S_{23} \\ 0 & L_2 C_2 + L_3 C_{23} & L_3 C_{23} \end{bmatrix}
$$

$$
{}^{0}V_4 = {}^{0}J(\underline{\theta})\,\dot{\underline{\theta}} \quad ; \quad {}^{4}V_4 = \underbrace{{}^{4}_{0}R \; {}^{0}J(\underline{\theta})}_{{}^{4}J(\underline{\theta})}\,\dot{\underline{\theta}}
$$

$$
{}^{4}_{0}R = \begin{bmatrix} C_1 C_{23} & S_1 C_{23} & S_{23} \\ -C_1 S_{23} & -S_1 S_{23} & C_{23} \\ S_1 & -C_1 & 0 \end{bmatrix}
$$

MULTIPLYING OUT ${}^{4}_{0}R \; {}^{0}J(\underline{\theta})$ IS TEDIOUS, BUT SURE ENOUGH, IT LEADS AGAIN TO THE SAME EXPRESSION FOR ${}^{4}J(\underline{\theta})$.

5.4) THE MAPPING WHICH POTENTIALLY CAN BE SINGULAR IS: $\underline{V} = J(\theta)\,\underline{\dot{\theta}}$ FOR THE "POSITION DOMAIN", AND $\underline{\tau} = J^T(\underline{\theta})\,\underline{F}$ FOR THE "FORCE DOMAIN". NOW SINCE TRANSPOSITION HAS NOTHING TO DO WITH THE RANK OF A (SQUARE) MATRIX, ITS CLEAR THAT THE SINGULARITIES OF $J(\underline{\theta})$ ARE THE SAME AS THOSE OF $J^T(\underline{\theta})$.

5.5) SEE "DIFFERENTIAL KINEMATIC CONTROL EQUATIONS FOR SIMPLE MANIPULATORS" BY R. PAUL, B. SHIMANO, AND G. MAYER IN IEEE TRANS. ON SYSTEMS, MAN, AND CYBERNETICS, VOL. SMC-11, NO. 6, JUNE, 1981. THE ANSWER IS GIVEN IN EQUATION (18) IN THAT PAPER, BUT THERE ARE SOME TYPOS, SO I HAVE INCLUDED A PASCAL LISTING BELOW WHICH IS CORRECTED. ACTUALLY, IN THE PUMA 560 THERE IS SOME MECHANICAL COUPLING BETWEEN JOINTS 4, 5, AND 6 WHICH IS NOT TAKEN INTO ACCOUNT IN THESE FORMULATIONS.

```
Procedure Jacobian(VAR THETA:VECT6; VAR J:MAT6);
VAR
  c2,c3,c4,c5,c6 : real;
  s2,s3,s4,s5,s6 : real;
  s23,c23 : real;
  k1,k2,k3,k4,k5,k6,k7,k8,k9,k10,k11: real;
  J: mat6;
BEGIN

  fsc(theta[2],s2,c2);    {TABLE LOOKUP OF SINE-COSINE}
  fsc(theta[3],s3,c3);    {PAIRS. FIRST ARGUMENT IS INPUT,}
  fsc(theta[4],s4,c4);    {SECOND TWO ARE OUTPUT.}
  fsc(theta[5],s5,c5);
  fsc(theta[6],s6,c6);
  s23:=s2*c3+c2*s3;
  c23:=c2*c3-s2*s3;

  k1:=d4*s23+a3*c23+a2*c2;                  {NOTE: a3 HERE}
  k2:=c4*c5*c6-s4*s6;                       {HAS OPPOSITE SIGN}
  k3:=c4*c5*s6+s4*s6;                       {OF a3 IN TEXT!}
  k4:=a3+a2*c3;            { corrected }
  k5:=d4+a2*s3;            { corrected }
  k6:=s4*c5*c6+c4*s6;
  k7:=-s4*c5*s6+c4*c6;
  k8:=s5*c6;
  k9:=s5*s6;
  k10:=s4*s5;
  k11:=c4*s5;

  j[1,1]:=k1*k6-d3*(c23*k2-s23*k8);
  j[1,2]:=k4*k8+k5*k2;
  j[1,3]:=a3*k8+d4*k2;                      { corrected }
  j[1,4]:=0.0;
  j[1,5]:=0.0;
  j[1,6]:=0.0;

  j[2,1]:=k1*k7-d3*(-c23*k3+s23*k9);
  j[2,2]:=-k4*k9-k5*k3;
  j[2,3]:=-a3*k9-d4*k3;                     { corrected }
  j[2,4]:=0.0;
  j[2,5]:=0.0;
  j[2,6]:=0.0;

  j[3,1]:=k1*k10-d3*(c23*k11+s23*c5);
  j[3,2]:=k5*k11-k4*c5;
  j[3,3]:=-a3*c5+d4*k11;                    { corrected }
  j[3,4]:=0.0;
  j[3,5]:=0.0;
  j[3,6]:=0.0;

  j[4,1]:=-(s23*k2+c23*k8);
  j[4,2]:=k6;
  j[4,3]:=k6;
  j[4,4]:=-k8;
  j[4,5]:=s6;
  j[4,6]:=0.0;

  j[5,1]:=s23*k3+c23*k9;
  j[5,2]:=k7;
```

```
  j[5,3]:=k7;
  j[5,4]:=k9;
  j[5,5]:=c6;
  j[5,6]:=0.0;

  j[6,1]:=-s23*k11+c23*c5;                  { corrected }
  j[6,2]:=k10;
  j[6,3]:=k10;
  j[6,4]:=c5;
  j[6,5]:=0.0;
  j[6,6]:=1.0;

END;   { Jacobian }
```

5.6) I THINK THIS IS TRUE! SEE B. SHIMANO, "THE KINEMATIC DESIGN AND FORCE CONTROL OF COMPUTER CONTROLLED MANIPULATORS", STANFORD A.I. LAB, MEMO #313, 1978.

5.7) SEE FIGURE 9.10 – IT MUST BE A PURELY CARTESIAN MANIPULATOR:

$$\underline{V} = J\dot{\underline{\theta}} \quad ; \quad \underline{V} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{\underline{\theta}}$$

MUST HAVE 3 ORTHOGONAL PRISMATIC JOINTS, SO $\dot{\underline{\theta}} = [\dot{d}_1 \ \dot{d}_2 \ \dot{d}_3]^T$.

5.8) THE JACOBIAN OF THIS 2-LINK IS:

$$^3J(\underline{\theta}) = \begin{bmatrix} L_1 S_2 & 0 \\ L_1 C_2 + L_2 & L_2 \end{bmatrix}$$

AN ISOTROPIC POINT EXISTS IF $^3J = \begin{bmatrix} L_2 & 0 \\ 0 & L_2 \end{bmatrix}$

SO, $\quad L_1 S_2 = L_2$

$$L_1 C_2 + L_2 = 0$$

OR, $S_2 = \frac{L_2}{L_1}$ $\quad C_2 = \frac{-L_2}{L_1}$

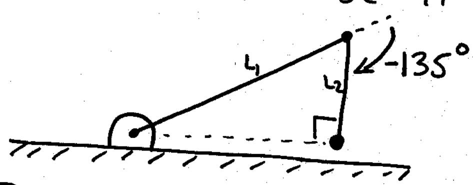
NOW $S_2^2 + C_2^2 = 1$, SO $\left(\frac{L_2}{L_1}\right)^2 + \left(\frac{-L_2}{L_1}\right)^2 = 1$

OR $L_1^2 = 2L_2^2 \longrightarrow L_1 = \sqrt{2}\, L_2$

UNDER THIS CONDITION $S_2 = \frac{1}{\sqrt{2}} = \pm .707$

AND $C_2 = -.707$

$\therefore$ AN ISOTROPIC POINT EXISTS IF $L_1 = \sqrt{2}\, L_2$ AND IN THAT CASE IT EXISTS WHEN $\theta_2 = \pm 135°$



IN THIS CONFIGURATION, THE MANIPULATOR LOOKS MOMENTARILY LIKE A CARTESIAN MANIPULATOR.

5.9) UNSOLVED. A SMALL PART OF THE ANSWER CAN BE FOUND IN REFERENCE [12] OF CHAP. 10.

5.10) $\underline{\tau} = {}^3J^T\ {}^3\underline{F} \quad \therefore\ {}^3\underline{F} = {}^3J^{-T}\,\underline{\tau}$

$${}^3J = \begin{bmatrix} L_1S_2 & 0 \\ L_1C_2 + L_2 & L_2 \end{bmatrix}$$

$${}^3J^T = \begin{bmatrix} L_1S_2 & L_1C_2 + L_2 \\ 0 & L_2 \end{bmatrix}$$

so,

$$^{3}J^{-T} = \frac{1}{L_1 L_2 S_2} \begin{bmatrix} L_2 & -L_1 C_2 - L_2 \\ 0 & L_1 S_2 \end{bmatrix}$$

5.11) FROM (5.67):

$$^{B}\nu = \begin{bmatrix} {}^{B}_{A}R & -{}^{B}_{A}R \; {}^{A}P\times \\ 0 & {}^{B}_{A}R \end{bmatrix} {}^{A}\nu$$

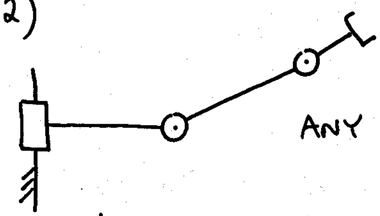
$$ {}^{B}_{A}R \; {}^{A}P\times = \begin{bmatrix} 0.86 & 0.5 & 0 \\ -0.5 & 0.86 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -5 & 0 \\ 5 & 0 & -10 \\ 0 & 10 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2.5 & -4.3 & -5.0 \\ 4.3 & 2.5 & -8.6 \\ 0 & 10 & 0 \end{bmatrix}$$

$$^{B}\nu = \begin{bmatrix} 0.86 & 0.5 & 0 & -2.5 & 4.3 & 5.0 \\ -0.5 & 0.86 & 0 & -4.3 & -2.5 & 8.6 \\ 0 & 0 & 1 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0.86 & 0.5 & 0 \\ 0 & 0 & 0 & -0.5 & 0.86 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -3 \\ 1.41 \\ 1.41 \\ 0 \end{bmatrix}$$

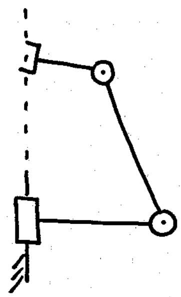
$$^{B}\nu = \begin{bmatrix} 3.52 & -7.80 & -17.1 & 1.91 & 0.51 & 0 \end{bmatrix}^{T}$$

5.12)



"WORKSPACE BOUNDARY"

ANY ANGLE SET : $\{\Theta_1, \Theta_2, 0\}$



"WORKSPACE INTERIOR"

ANY ANGLE SET SUCH THAT :

$$L_1 + L_2 C_2 + L_3 C_{23} = 0$$

(Θ_1 IS ARBITRARY)

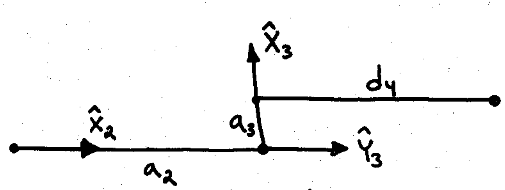
5.13) $\underline{\tau} = {}^0J^T(\underline{\Theta})\ {}^0\underline{F}$

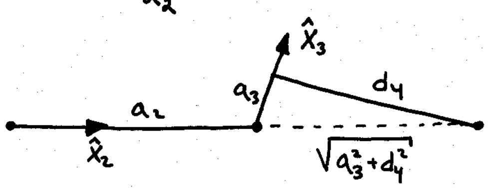
$$\underline{\tau} = \begin{bmatrix} -L_1 S_1 - L_2 S_{12} & L_1 C_1 + L_2 C_{12} \\ -L_2 S_{12} & L_2 C_{12} \end{bmatrix} \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\tau_1 = -10 S_1 L_1 - 10 L_2 S_{12}$$

$$\tau_2 = -10 L_2 S_{12}$$

5.14)





(SINGULAR)

SO, AT SINGULAR POSITION:

$$\theta_3 = -\text{ATAN2}(d_4, a_3)$$

IF $a_3 \to 0$, THEN $\theta_3 \to -\text{ATAN2}(d_4, 0)$

WHICH IS $-90°$.

5.15) THE KINEMATICS CAN BE DONE EASILY TO OBTAIN:

$$^0P_{4ORG} = \begin{bmatrix} (d_2 + L_2 + L_3)\, s_1 \\ -(d_2 + L_2 + L_3)\, c_1 \\ 0 \end{bmatrix}$$

$$^0\underline{v} = {}^0J\, \dot{\underline{\theta}}$$

$$^0J = \begin{bmatrix} \frac{\partial\, ^0P_{4ORGX}}{\partial\theta_1} & \frac{\partial\, ^0P_{4ORGX}}{\partial\theta_2} & \frac{\partial\, ^0P_{4ORGX}}{\partial\theta_3} \\ \frac{\partial\, ^0P_{4ORGY}}{\partial\theta_1} & \frac{\partial\, ^0P_{4ORGY}}{\partial\theta_2} & \frac{\partial\, ^0P_{4ORGY}}{\partial\theta_3} \\ \frac{\partial\, ^0P_{4ORGZ}}{\partial\theta_1} & \frac{\partial\, ^0P_{4ORGZ}}{\partial\theta_2} & \frac{\partial\, ^0P_{4ORGZ}}{\partial\theta_3} \end{bmatrix}$$

SO,

$$^0J = \begin{bmatrix} (d_2 + L_2 + L_3)\, c_1 & s_1 & 0 \\ (d_2 + L_2 + L_3)\, s_1 & -c_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$