HOME WORK #6

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1. Do trajectory planning with a cubic polynomial for the two joints of the manipulator.

$\theta_1(0) = 30°$ $\quad$ $\theta_1(t_f) = 150°$ $\quad$ $t_f = 1$ sec

$\theta_2(0) = 150°$ $\quad$ $\theta_2(t_f) = 30°$

$\dot{\theta}_1(0) = 0$ $\quad$ $\dot{\theta}_1(t_f) = 0$

$\dot{\theta}_2(0) = 0$ $\quad$ $\dot{\theta}_2(t_f) = 0$

By the initial condition:

$$\theta(0) = \begin{bmatrix} 30° \\ 150° \end{bmatrix} = a_0 \quad ; \quad \dot{\theta}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = a_1$$

By the final condition:

$$\theta(1) = \begin{bmatrix} 150° \\ 30° \end{bmatrix} \quad ; \quad \dot{\theta}(1) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

+ The 4 equations describing this general cubic are:

$$\theta_0 = a_0 \quad ; \quad \dot{\theta}_0 = a_1$$

$$\theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 \quad ; \quad \dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 t_f^2$$

Solve these equations, we obtain:

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f$$

$$= \frac{3}{1^2}\left(\begin{bmatrix} 150 \\ 30 \end{bmatrix} - \begin{bmatrix} 30 \\ 150 \end{bmatrix}\right) - \frac{2}{1}\begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{1}{1}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 3 \cdot \begin{bmatrix} 120 \\ -120 \end{bmatrix} = \begin{bmatrix} 360 \\ -360 \end{bmatrix}$$

$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0) + \frac{1}{t_f^2}(\dot{\theta}_f + \dot{\theta}_0) = -\frac{2}{1}\left(\begin{bmatrix} 150 \\ 30 \end{bmatrix} - \begin{bmatrix} 30 \\ 150 \end{bmatrix}\right) + \frac{1}{1}\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -240 \\ 240 \end{bmatrix}$$

Thus, we got the result:

$$\theta(t) = \begin{bmatrix} 30^\circ \\ 150^\circ \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} t + \begin{bmatrix} 360^\circ \\ -360^\circ \end{bmatrix} t^2 + \begin{bmatrix} -240 \\ 240 \end{bmatrix} t^3$$

$$\theta(t) = \begin{bmatrix} 30^\circ \\ 150^\circ \end{bmatrix} + \begin{bmatrix} 360^\circ \\ -360^\circ \end{bmatrix} t^2 + \begin{bmatrix} -240 \\ 240 \end{bmatrix} t^3$$

$$\dot\theta(t) = \begin{bmatrix} 720^\circ \\ -720^\circ \end{bmatrix} t + \begin{bmatrix} -720^\circ \\ 720^\circ \end{bmatrix} t^2 = 720\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} t + \begin{bmatrix} -1 \\ 1 \end{bmatrix} t^2\right)$$

$$\ddot\theta(t) = 720^\circ\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix} t\right)$$

* Torque trajectory:

$$\tau = M(\theta)\ddot\theta + V(\theta,\dot\theta) + G(\theta)$$

$$M(\theta) = \begin{bmatrix} l_2^2 m_2 + 2l_1 l_2 m_2 c_2 + l_1^2(m_1+m_2) & l_2^2 m_2 + l_1 l_2 m_2 c_2 \\ l_2^2 m_2 + l_1 l_2 m_2 c_2 & l_2^2 m_2 \end{bmatrix}$$

$$V(\theta,\dot\theta) = \begin{bmatrix} -m_2 l_1 l_2 s_2 \dot\theta_2^2 - 2m_2 l_1 l_2 s_2 \dot\theta_1 \dot\theta_2 \\ m_2 l_1 l_2 s_2 \dot\theta_1^2 \end{bmatrix}$$

$$G(\theta) = \begin{bmatrix} m_2 l_2 g c_{12} + (m_1+m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{bmatrix}$$