Analysis and Control of RM with Redundancy

T. Yoshikawa → 'IJRT Robotics Research'

2. Measure of Manipulability

Need a quantitative measure of manipulating ability of robot arms in positioning and orienting the end-effectors

$r = f(\theta)$ $\quad$ r: m-dimensional task space

θ: n-dimensional joint space

$\dot{r} = J\dot{\theta}$ $\quad$ J: $m \times n$ matrix

$\max_{\theta} \operatorname{rank} J(\theta) = m$

$\operatorname{rank}(J) = m$ 일때 $\quad$ Manipulator has $(n-m)$ degree of redundancy

$\operatorname{rank}(J) < m$ 일때 $\quad$ Manipulator is in singular state

Proposing the following quantitative measure of manipulatability

$w = \sqrt{\det(JJ^T)}$: measure of manipulatability

From singular value decomposition (SVD)

$$J = U \Sigma V^T$$

$$\Sigma = \left[\begin{array}{cccc|c} \sigma_1 & & & & \\ & \sigma_2 & & & \\ & & \ddots & & 0 \\ & & & \sigma_m & \end{array}\right] \in R^{m \times n}$$

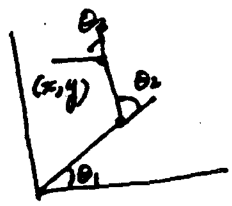
$\sigma_1 \geq \sigma_2 \geq \sigma_3 \cdots \geq \sigma_m \geq 0$ $\quad$ σ_i : singular value

$w = \sigma_1 \sigma_2 \cdots \sigma_m$

when $\|\dot{\theta}\| = \theta_1^2 + \theta_2^2 + \cdots + \theta_m^2 \leq 1$, $\dot{r}$ 는 $\sigma_1 u_1, \sigma_2 u_2 \cdots \sigma_m u_m$ 을

principal axes로 갖는 ellipsoid를 형성하는데

이 ellipsoid를 manipulatability ellipsoid라 한다.



$$\dot{r} = J\dot{\theta}$$

$$J = \begin{bmatrix} l_1 C_1 + l_2 C_{12} + l_3 C_{123} & l_2 C_{12} + l_3 C_{123} & l_3 C_{123} \\ -l_1 S_1 - l_2 S_{12} - l_3 S_{123} & -l_2 S_{12} - l_3 S_{123} & l_3 S_{123} \end{bmatrix}$$

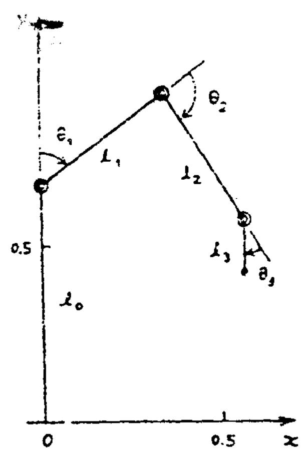


Figure 1
Three degrees of freedom arm, $l_0 = 0.67$, $l_1 = l_2 = 0.432$, $l_3 = 0.15$ (m).

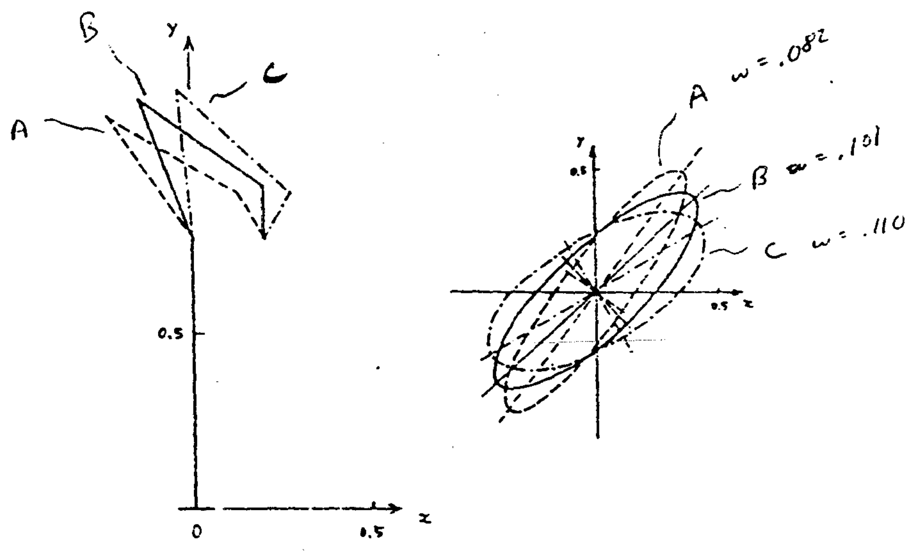


Figure 2
Arm posture and manipulatability ellipsoid: —, $\theta = \theta_a = [-34.1, 155.9, 28.2]^T$, $w = 0.082$; ———, $\theta = \theta_b = [-20.1, 146.4, 53.7]^T$, $w = 0.101$; -·-·-, $\theta = \theta_c = [-4.7, 138.8, 75.9]^T$, $w = 0.110$.

4. Utilization of Redundancy of Optimizing Given Performance Criterion

$$p = q(\theta). \tag{4.1}$$

] want to maximize p

maximize performance criterion

The second subtask is described as keeping the value of this criterion as large as possible. A control algorithm for achieving this task will be given in this section.

The main idea used here is quite the same as that used in [12] for developing a steering law for a control moment gyro system with 6 degrees of freedom in controlling a spacecraft orientation (3 degrees of freedom). Three redundant degrees of freedom of the control moment gyro system were utilized for the momentum distribution to avoid the singularities.

The general solution of (2.2) is given by $\dot{x} = J\dot{\theta}$ (2.2)

$$\boxed{\dot{\theta} = J^{+}\dot{r} + (I - J^{+}J)k,} \tag{4.2}$$

joint space

where J^{+} is the pseudoinverse of the Jacobian matrix J [13] and $k \in R^{n}$ is an arbitrary constant vector. The second term of the right-hand side of (4.2) represents the redundancy left after performing the first subtask.

The time derivative of p is given by

$$\dot{p} = \xi^{\mathrm{T}}\dot{\theta}, \quad \rightarrow \quad \dot{p} = \frac{\partial q(\theta)}{\partial \xi_i}\dot{\theta} = \frac{\partial q(\theta)}{\partial \xi_1}\dot{\theta}_1 + \cdots + \frac{\partial q(\theta)}{\partial \xi_n}\dot{\theta}_n \tag{4.3}$$

where

$$\xi = [\xi_1, \xi_2, \ldots, \xi_n]^{\mathrm{T}}, \tag{4.4}$$

$$\xi_i = \frac{\partial q(\theta)}{\partial \theta_i}, \quad i = 1, 2, \ldots, n. \tag{4.5}$$

substitutes i for i

Under the assumption that the first subtask is perfectly performed, we obtain from (4.3) and (4.2)

$$\dot{p} = \xi^{\mathrm{T}}J^{+}\dot{r}^{*} + \xi^{\mathrm{T}}(I - J^{+}J)k. \tag{4.6}$$

In order to achieve the second subtask, we propose to select k as

$$k = \xi k_1, \tag{4.7}$$

where k_1 is a positive constant. Hence the basic equation for the control algorithm is given by

$$\dot{\theta} = J^{+}\dot{r}^{*} + (I - J^{+}J)\xi k_1. \tag{4.8}$$

The reason for the selection of (4.7) is as follows. From (4.6) and (4.7) we obtain

$$\cdots + \xi^{\mathrm{T}}(I - J^{+}J)\xi k_1 \tag{4.9}$$

and the second term in the right-hand side of (4.9) becomes nonnegative due to the fact that $(I - J^{+}J)$ is idempotent, contributing to an increase of the value p.

The selection (4.7) can also be characterized as follows. Let

idempotent: (I

$$\dot{\theta}_0 = \dot{\theta}|_{\dot{r}=0} = (I - J^{+}J)k,$$

$$\dot{p}_0 = \dot{p}|_{\dot{r}=0} = \xi^{\mathrm{T}}(I - J^{+}J)k;$$

then the value of k which maximizes $\dot{p}_0$ under the condition

$$\|\dot{\theta}_0\| \leqslant k_1[\xi^{\mathrm{T}}(I - J^{+}J)\xi]^{1/2}$$

is given by (4.7). In practical applications, in order to prevent $\dot{\theta}$ from becoming excessive, a limitation for k_1 should be given. One canditate for this will be $\|\dot{\theta}_0\| \leqslant k_3\dot{\theta}_H$, i.e.,

$$k_1 \leqslant [\xi^{\mathrm{T}}(I - J^{+}J)\xi]^{-1/2}k_3\dot{\theta}_H,$$

where $\dot{\theta}_H$ is the hardware limit for the joint angle rate and k_3 is a constant $(0 \leqslant k_3 \leqslant 1)$.

Note that the algorithm of the form (4.8) was originally given in [12] for the attitude control spacecrafts. Essentially the same algorithm appeared also in [6] in the context of redundant mechanical systems including manipulators.

5. Singularity Avoidance

- try not only to avoid the real singularities but also to keep the ability of manipulation as much as possible

- $G = JJ^T = [g_{ij}]$

  $P = \sqrt{\det G}$ as performance criterion

$$\xi_l = \frac{1}{2\sqrt{\det G}} \sum_{i,j=1}^{m} \Delta_{ij} \cdot {}_l g'_{ij}$$
$$= \frac{1}{2}\sqrt{\det G} \sum_{i,j=1}^{m} [G^{-1}]_{ij} ({}_l J'_i J_j^{\mathrm{T}} + {}_l J'_j J_i^{\mathrm{T}}),$$

where

Δ_{ij} = cofactor of a_{ij} for G,
${}_l g'_{ij} = \partial(g_{ij}(\theta))/\partial\theta_l$,
$[G^{-1}]_{ij}$ = the (i,j) element of the inverse of G,
J_i = the ith row of J,
${}_l J'_i$ = the derivative of J_i with respect to θ_l.

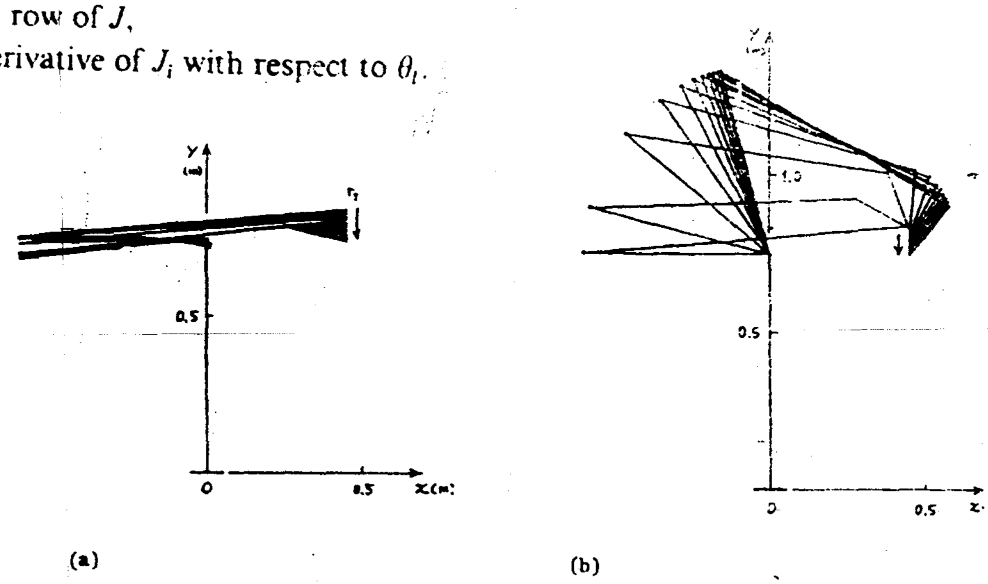


Figure 3.
Simulation result for singularity avoidance: (a) control law $\dot{\theta} = J^+ \dot{r}^*$; (b) control law $\dot{\theta} = J^+ \dot{r}^* + [I - J^+ J]\xi k_1$ with $k = 5.0$.

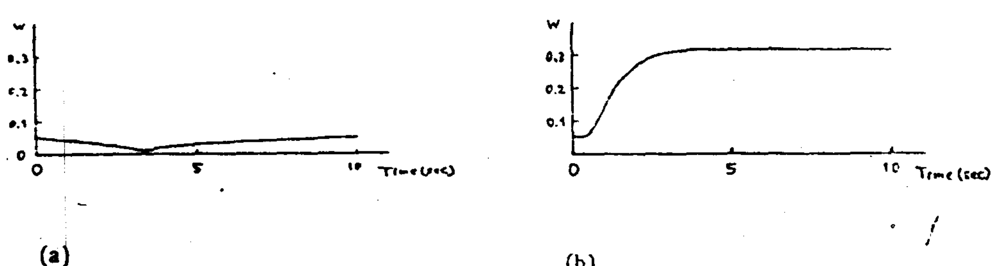


Figure 4
Measure of manipulatability: (a) control law $\dot{\theta} = J^+ \dot{r}^*$; (b) control law $\dot{\theta} = J^+ \dot{r}^* + [I - J^+ J]\xi k_1$.

6. Obstacle Avoidance

PM, teach in advance just one constant arm posture θ_r, which is desirable for avoiding collision with the obstacle

Let the performance criterion for the second subtask be

$$p = g(\theta) = \tfrac{1}{2}(\theta - \theta_r)^{\mathrm{T}} H_2(\theta - \theta_r), \tag{6.1}$$

where $H_2 = \mathrm{diag}(h_{2i}) \in R^{n \times n}$, and $h_{2i} > 0$ are constants. The condition (6.1) means that the arm should try to come close to the taught arm posture θ_r as much as possible by utilizing the redundancy left after the realization of the first subtask.

From (6.1) we have

$$\xi = \partial g(\theta)/\partial \theta = -H_2(\theta - \theta_r). \quad +? \tag{6.2}$$

From (4.8) and (6.2) we obtain

$$\dot{\theta} = J^{+} \dot{r}^{*} - (I - J^{+} J) H_2(\theta - \theta_r) k_1. \tag{6.3}$$

$+?$

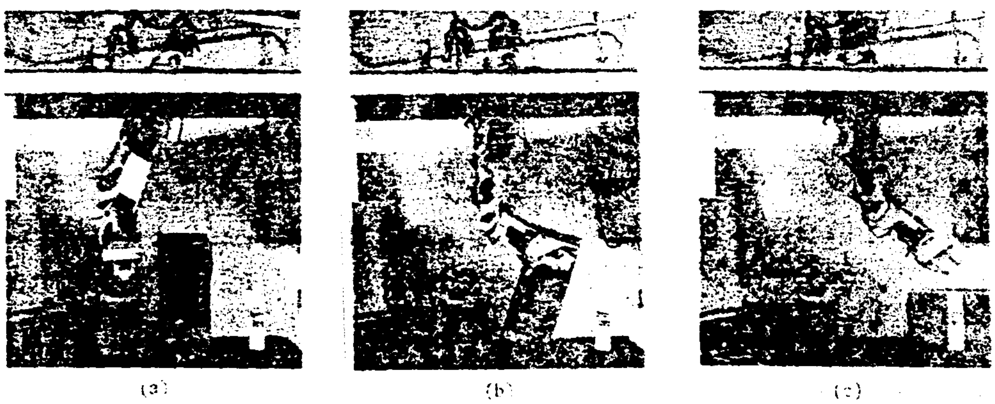


Figure 6
Trajectory control without provision for obstacle avoidance

Figure 7
Reference arm posture θ_r.

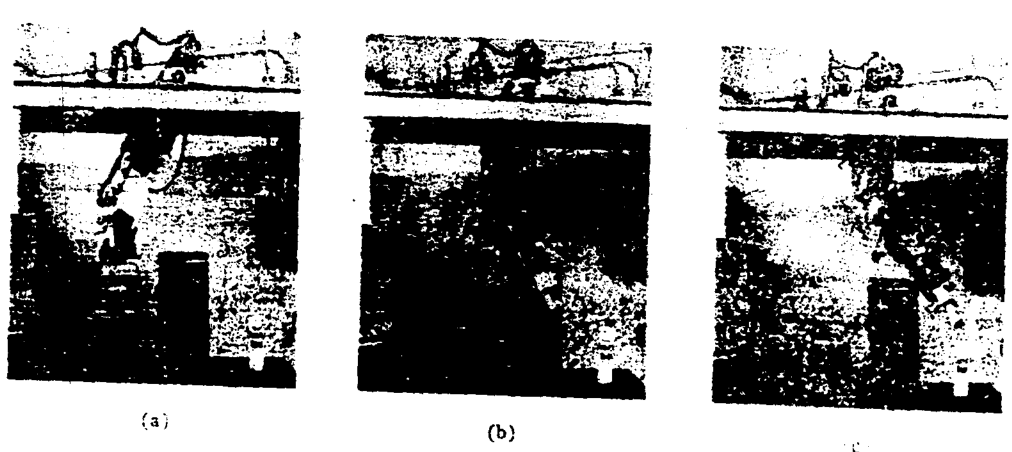


Figure 8
Trajectory control with provision for obstacle avoidance.