# HOME WORK #6

Pham Thi Ngoc Thao
20215435

1. Do trajectory planning with a cubic polynomial for the two joints of the manipulator

$\theta_1(0) = 30°$ $\quad$ $\theta_1(t_f) = 150°$ $\quad$ $t_f = 1$ sec

$\theta_2(0) = 150°$ $\quad$ $\theta_2(t_f) = 30°$

$\dot\theta_1(0) = 0$ $\quad$ $\dot\theta_1(t_f) = 0$

$\dot\theta_2(0) = 0$ $\quad$ $\dot\theta_2(t_f) = 0$

By the initial condition:

$$\theta(0) = \begin{bmatrix} 30° \\ 150° \end{bmatrix} = a_0 \quad ; \quad \dot\theta(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = a_1$$

By the final condition:

$$\theta(1) = \begin{bmatrix} 150° \\ 30° \end{bmatrix} \quad ; \quad \dot\theta(1) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

+ The 4 equations describing this general cubic are:

$$\theta_0 = a_0 \qquad \dot\theta_0 = a_1$$

$$\theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 \quad ; \quad \dot\theta_f = a_1 + 2a_2 t_f + 3a_3 t_f^2$$

Solve these equations, we obtain:

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) - \frac{2}{t_f}\dot\theta_0 - \frac{1}{t_f}\dot\theta_f$$

$$= \frac{3}{1^2}\left(\begin{bmatrix} 150 \\ 30 \end{bmatrix} - \begin{bmatrix} 30 \\ 150 \end{bmatrix}\right) - \frac{2}{1}\begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{1}{1}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 3.\begin{bmatrix} 120 \\ -120 \end{bmatrix} = \begin{bmatrix} 360 \\ -360 \end{bmatrix}$$

$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0) + \frac{1}{t_f^2}(\dot\theta_f + \dot\theta_0) = -\frac{2}{1}\left(\begin{bmatrix} 150 \\ 30 \end{bmatrix} - \begin{bmatrix} 30 \\ 150 \end{bmatrix}\right) + \frac{1}{1}\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -240 \\ 240 \end{bmatrix}$$

Thus, we got the result:

$$\theta(t) = \begin{bmatrix} 30^\circ \\ 150^\circ \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} t + \begin{bmatrix} 360^\circ \\ -360^\circ \end{bmatrix} t^2 + \begin{bmatrix} -240 \\ 240 \end{bmatrix} t^3$$

$$\theta(t) = \begin{bmatrix} 30^\circ \\ 150^\circ \end{bmatrix} + \begin{bmatrix} 360^\circ \\ -360^\circ \end{bmatrix} t^2 + \begin{bmatrix} -240 \\ 240 \end{bmatrix} t^3$$

$$\dot{\theta}(t) = \begin{bmatrix} 720^\circ \\ -720^\circ \end{bmatrix} t + \begin{bmatrix} -720^\circ \\ 720^\circ \end{bmatrix} t^2 = 720\left( \begin{bmatrix} 1 \\ -1 \end{bmatrix} t + \begin{bmatrix} -1 \\ 1 \end{bmatrix} t^2 \right)$$

$$\ddot{\theta}(t) = 720^\circ \left( \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix} t \right)$$

* Torque trajectory:

$$\tau = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta)$$

$$M(\theta) = \begin{bmatrix} l_2^2 m_2 + 2 l_1 l_2 m_2 c_2 + l_1^2 (m_1 + m_2) & l_2^2 m_2 + l_1 l_2 m_2 c_2 \\ l_2^2 m_2 + l_1 l_2 m_2 c_2 & l_2^2 m_2 \end{bmatrix}$$

$$V(\theta, \dot{\theta}) = \begin{bmatrix} -m_2 l_1 l_2 s_2 \dot{\theta}_2^2 - 2 m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 \\ m_2 l_1 l_2 s_2 \dot{\theta}_1^2 \end{bmatrix}$$

$$G(\theta) = \begin{bmatrix} m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{bmatrix}$$

**2. Perform PD control simulation.**

```
clc
clear all
close all
syms time
m1=10;
m2=5;
l=0.5;
tt01=30*pi/180;
tt02=150*pi/180;
ttf1=150*pi/180;
ttf2=30*pi/180;


a01=pi/6;
a02=5*pi/6;
a21=pi;
a22=-pi;
a31=-4*pi/3;
a32=4*pi/3;
g=9.81;
i=1;
deltat=0.01;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%                P_D Controller                        %%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
tt1_r=30*pi/180;
tt2_r=150*pi/180;
dtt1_r=0;
dtt2_r=0;
e1=0;
e2=0;

Kp1=9000;
Kp2=7800;
Kv1=0.1;
Kv2=2.5;
hold off;
for t=0:deltat:1
    tt1=a01+a21*t*t+a31*t^3;
    tt2=a02+a22*t*t+a32*t^3;
    dtt1=2*a21*t+3*a31*t^2;
    dtt2=2*a22*t+3*a32*t^2;
    ddtt1=2*a21+6*a31*t;
    ddtt2=2*a22+6*a32*t;

    tque1=-Kv1*dtt1_r+Kp1*e1;
    tque2=-Kv2*dtt2_r+Kp2*e2;


    T=[tque1;tque2];
    M=[l^2*m2+2*l^2*m2*cos(tt2_r)+l^2*(m1+m2)
l^2*m2+l^2*m2*cos(tt2_r);l^2*m2+l^2*m2*cos(tt2_r) l^2*m2];
```

```
    V=[-m2*l^2*sin(tt2_r)*dtt2_r^2-
2*m2*l^2*sin(tt2_r)*dtt1_r*dtt2_r;m2*l^2*sin(tt2_r)*dtt1^2];

G=[m2*l*g*cos(tt1_r+tt2_r)+(m1+m2)*l*g*cos(tt1_r);m2*l*g*cos(tt1_r+tt2_r)];
    Minv=M^-1;

    ddtt=Minv*(T-V-G)
    ddtt1_r=ddtt(1)
    ddtt2_r=ddtt(2)
    dtt1_r=dtt1_r+deltat*ddtt1_r;
    dtt2_r=dtt2_r+deltat*ddtt2_r;
    tt1_r=tt1_r+dtt1_r*deltat+0.5*deltat^2*ddtt1_r;
    tt2_r=tt2_r+dtt2_r*deltat+0.5*deltat^2*ddtt2_r;
    e1=tt1-tt1_r;
    e2=tt2-tt2_r;

    figure(1)
    plot(t,tt1,'blue*');
    hold on
    plot(t,tt1_r,'red*');
    hold on
    plot(t,e1,'black*')
    legend({'Desired', 'Real', 'Error'}, 'FontSize', 12);
    title('PD Joint#1');

    figure(2)
    plot(t,tt2,'blue*');
    hold on
    plot(t,tt2_r,'red*');
    hold on
    plot(t,e2,'black*');
    legend({'Desired', 'Real', 'Error'}, 'FontSize', 12);
    title('PD Joint#2');

    figure(3)
    plot(t,e1,'red*')
    hold on
    plot(t,e2,'blue*');
    legend({'Error#1', 'Error#2'}, 'FontSize', 12);
    title('PD Error');

    i=i+1;
end
```

The results obtained:

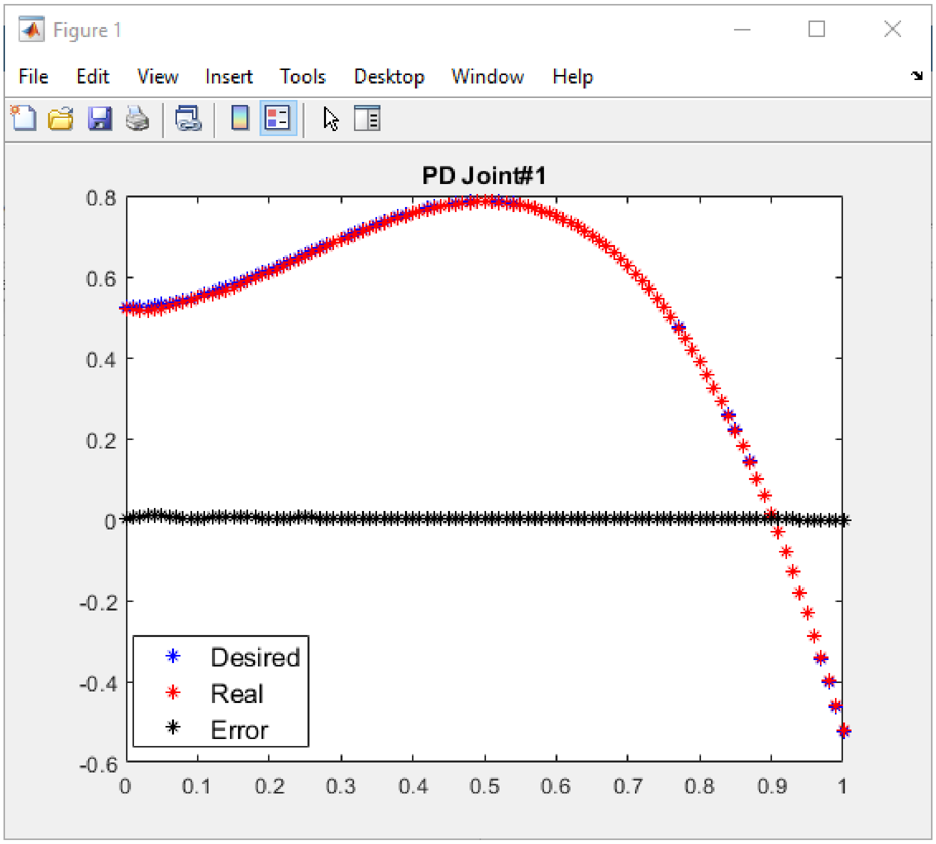


Figure 1
File Edit View Insert Tools Desktop Window Help
PD Joint#1
Desired
Real
Error
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

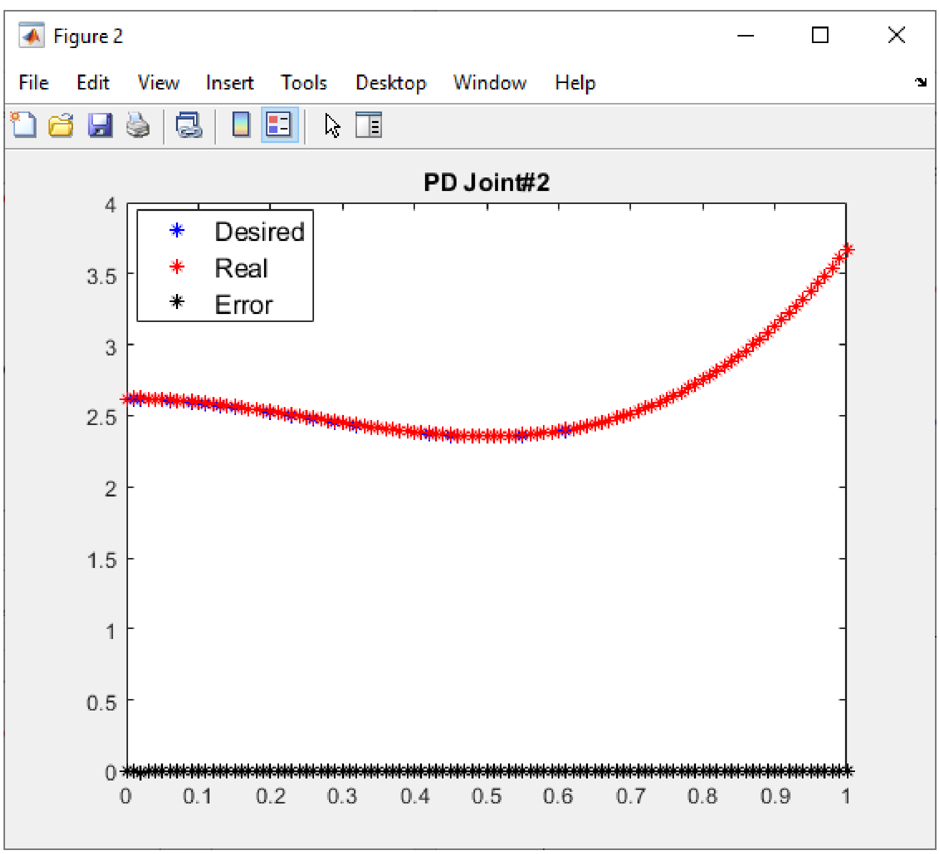
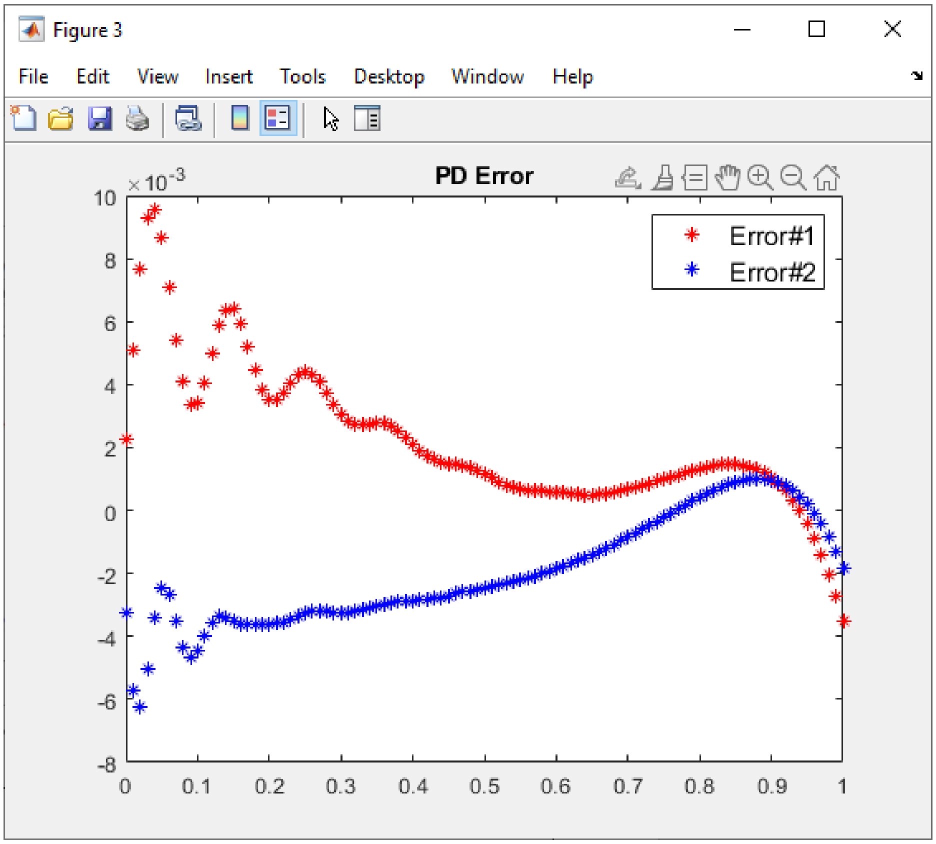


Figure 2
File Edit View Insert Tools Desktop Window Help
PD Joint#2
Desired
Real
Error
4
3.5
3
2.5
2
1.5
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1




**3. Perform PD + gravity control simulation.**

Code in Matlab

```
tt1_r=30*pi/180;
tt2_r=150*pi/180;
dtt1_r=0;
dtt2_r=0;
e1=0;
e2=0;
Kp1=9000;
Kp2=7800;
Kv1=0.1;
Kv2=2.5;
for t=0:deltat:1
    tt1=a01+a21*t*t+a31*t^3;
    tt2=a02+a22*t*t+a32*t^3;
    dtt1=2*a21*t+3*a31*t^2;
    dtt2=2*a22*t+3*a32*t^2;
    ddtt1=2*a21+6*a31*t;
    ddtt2=2*a22+6*a32*t;
    g1=m2*l*g*cos(tt1+tt2)+(m1+m2)*l*g*cos(tt1);
    g2=m2*l*g*cos(tt1+tt2);



    tque1=g1-Kv1*dtt1_r+Kp1*e1;
    tque2=g2-Kv2*dtt2_r+Kp2*e2;
```

```
    T=[tque1;tque2];
    M=[l^2*m2+2*l^2*m2*cos(tt2_r)+l^2*(m1+m2)
l^2*m2+l^2*m2*cos(tt2_r);l^2*m2+l^2*m2*cos(tt2_r) l^2*m2];
    V=[-m2*l^2*sin(tt2_r)*dtt2_r^2-
2*m2*l^2*sin(tt2_r)*dtt1_r*dtt2_r;m2*l^2*sin(tt2_r)*dtt1^2];

G=[m2*l*g*cos(tt1_r+tt2_r)+(m1+m2)*l*g*cos(tt1_r);m2*l*g*cos(tt1_r+tt2_r)];
    Minv=M^-1;
    ddtt=Minv*(T-V-G)
    ddtt1_r=ddtt(1)
    ddtt2_r=ddtt(2)
    dtt1_r=dtt1_r+deltat*ddtt1_r;
    dtt2_r=dtt2_r+deltat*ddtt2_r;
    tt1_r=tt1_r+dtt1_r*deltat+0.5*deltat^2*ddtt1_r;
    tt2_r=tt2_r+dtt2_r*deltat+0.5*deltat^2*ddtt2_r;
    e1=tt1-tt1_r;
    e2=tt2-tt2_r;

    figure(4)
    plot(t,tt1,'blue*');
    hold on
    plot(t,tt1_r,'red*');
    hold on
    plot(t,e1,'black*')
    legend({'Desired', 'Real', 'Error'}, 'FontSize', 12);
    title('PD + Gravity (Joint#1)');

    figure(5)
    plot(t,tt2,'blue*');
    hold on
    plot(t,tt2_r,'red*');
    hold on
    plot(t,e2,'black*')
    legend({'Desired', 'Real', 'Error'}, 'FontSize', 12);
    title('PD + Gravity (Joint#2)');

    figure(6)
    plot(t,e1,'red*')
    hold on
    plot(t,e2,'*')
    hold on
    legend({'Desired', 'Real', 'Error'}, 'FontSize', 12);
    title('PD + Gravity (Error)');

    i=i+1;
end
```

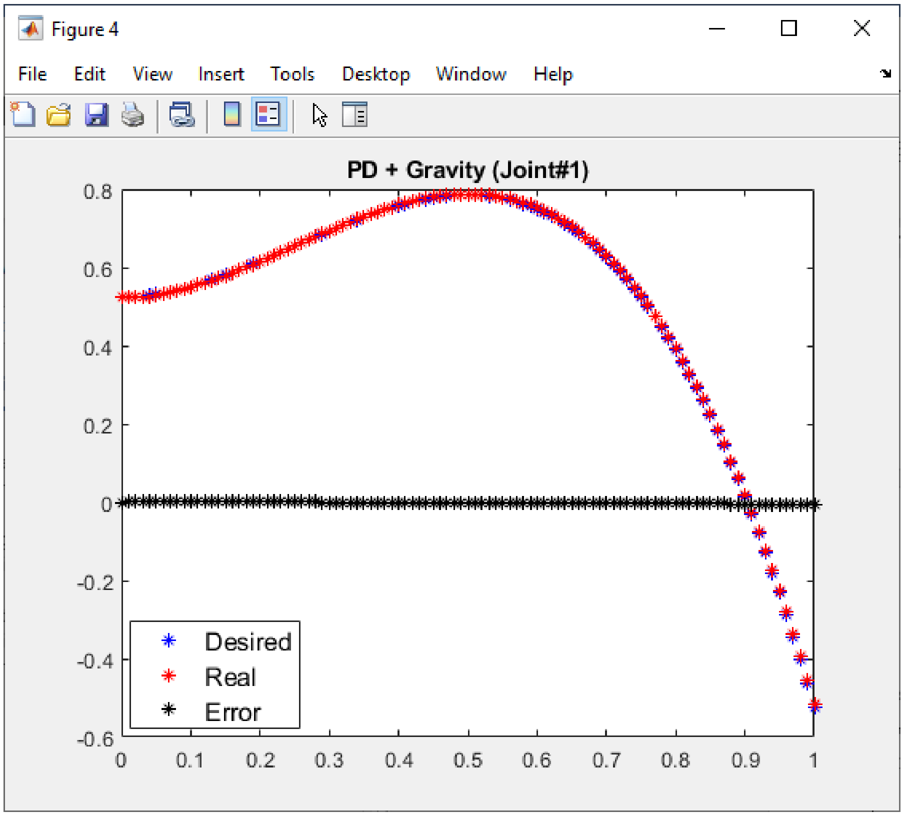


Figure 4
File Edit View Insert Tools Desktop Window Help
PD + Gravity (Joint#1)
Desired
Real
Error
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

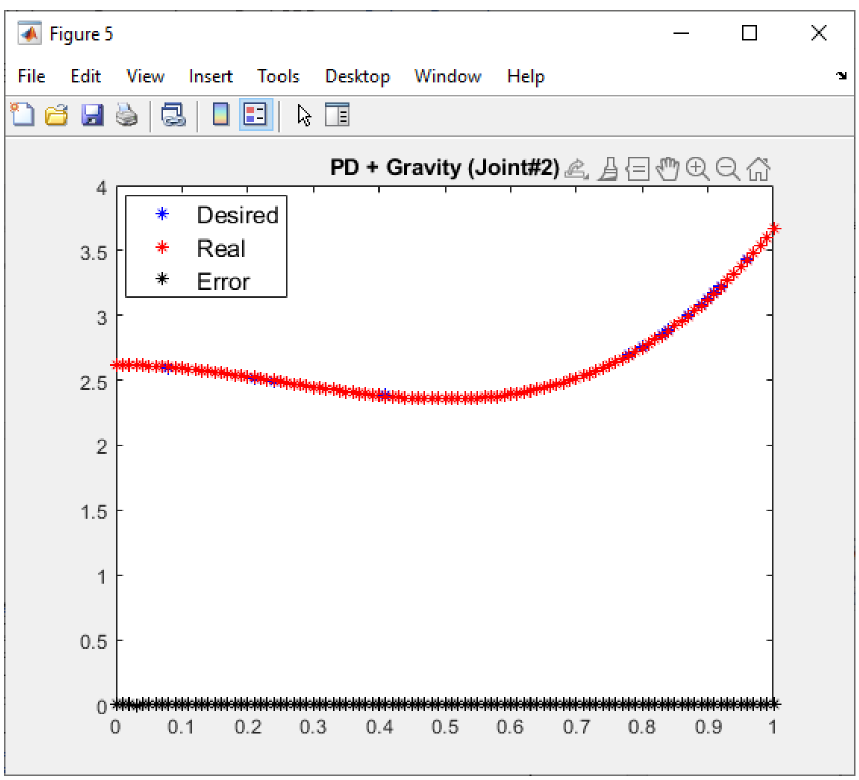
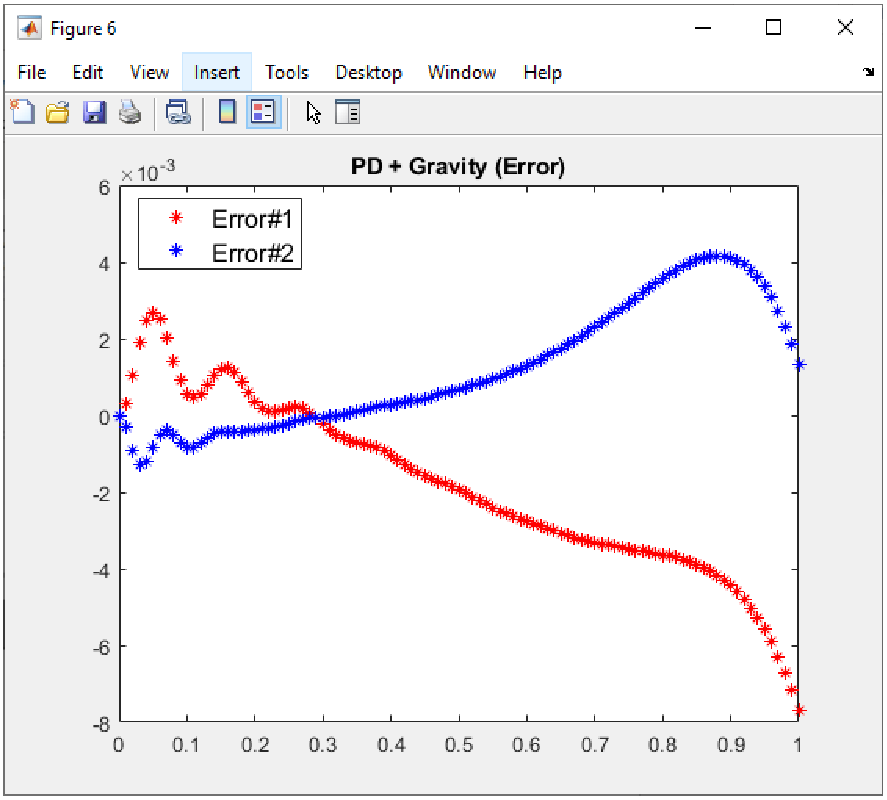


Figure 5
File Edit View Insert Tools Desktop Window Help
PD + Gravity (Joint#2)
Desired
Real
Error
4
3.5
3
2.5
2
1.5
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1




**4. Perform computed torque control simulation.**

Code in Matlab

```
tt1_r=30*pi/180;
tt2_r=150*pi/180;
dtt1_r=0;
dtt2_r=0;
e1=0;
e2=0;
de1=0;
de2=0;
Kp1=7800;
Kp2=5800;
Kv1=0.1;
Kv2=0.5;

for t=0:deltat:1
    tt1=a01+a21*t*t+a31*t^3;
    tt2=a02+a22*t*t+a32*t^3;
    dtt1=2*a21*t+3*a31*t^2;
    dtt2=2*a22*t+3*a32*t^2;
    ddtt1=2*a21+6*a31*t;
    ddtt2=2*a22+6*a32*t;

    tque_c1=ddtt1+Kp1*e1+Kv1*de1
    tque_c2=ddtt2+Kp2*e2+Kv2*de2
```

```
    alpha=[l^2*m2+2*l^2*m2*cos(tt2)+l^2*(m1+m2)
l^2*m2+l^2*m2*cos(tt2);l^2*m2+l^2*m2*cos(tt2) l^2*m2];
    M=alpha;



    V=[-m2*l^2*sin(tt2_r)*dtt2_r^2-
2*m2*l^2*sin(tt2_r)*dtt1_r*dtt2_r;m2*l^2*sin(tt2_r)*dtt1^2];

G=[m2*l*g*cos(tt1_r+tt2_r)+(m1+m2)*l*g*cos(tt1_r);m2*l*g*cos(tt1_r+tt2_r)];
    beta=V+G;
    T=alpha*[tque_c1;tque_c2]+beta
    ddtt=M\(T-V-G);

    ddtt1_r=ddtt(1);
    ddtt2_r=ddtt(2);
    dtt1_r=dtt1_r+deltat*ddtt1_r;
    dtt2_r=dtt2_r+deltat*ddtt2_r;
    tt1_r=tt1_r+dtt1_r*deltat+0.5*deltat^2*ddtt1_r;
    tt2_r=tt2_r+dtt2_r*deltat+0.5*deltat^2*ddtt2_r;
    e1=tt1-tt1_r;
    e2=tt2-tt2_r;
    de1=dtt1-dtt1_r;
    de2=dtt2-dtt2_r;

    figure(7)
    plot(t,tt1,'blue*');
    hold on
    plot(t,tt1_r,'red*');
    hold on
    plot(t,e1,'black*')
    legend({'Desired', 'Real', 'Error'}, 'FontSize', 12);
    title('Torque Control (Joint#1)');

    figure(8)
    plot(t,tt2,'blue*');
    hold on
    plot(t,tt2_r,'red*');
    hold on
    plot(t,e2,'black*')
    legend({'Desired', 'Real', 'Error'}, 'FontSize', 12);
    title('Torque Control (Joint#2)');

    figure(9)
    plot(t,e1,'red*')
    hold on
    plot(t,e2,'blue*')
    legend({'Error#1', 'Error#2'}, 'FontSize', 12);
    title('Torque Control (Error)');

    i=i+1;
end
```

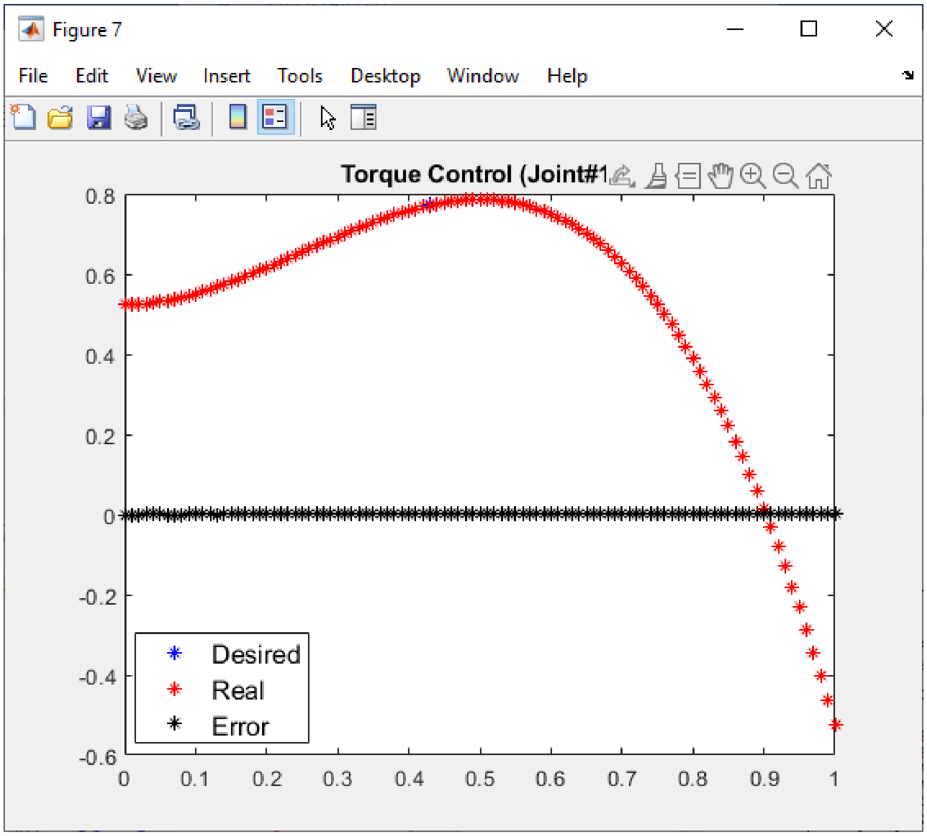


Figure 7
File Edit View Insert Tools Desktop Window Help
Torque Control (Joint#1
Desired
Real
Error
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

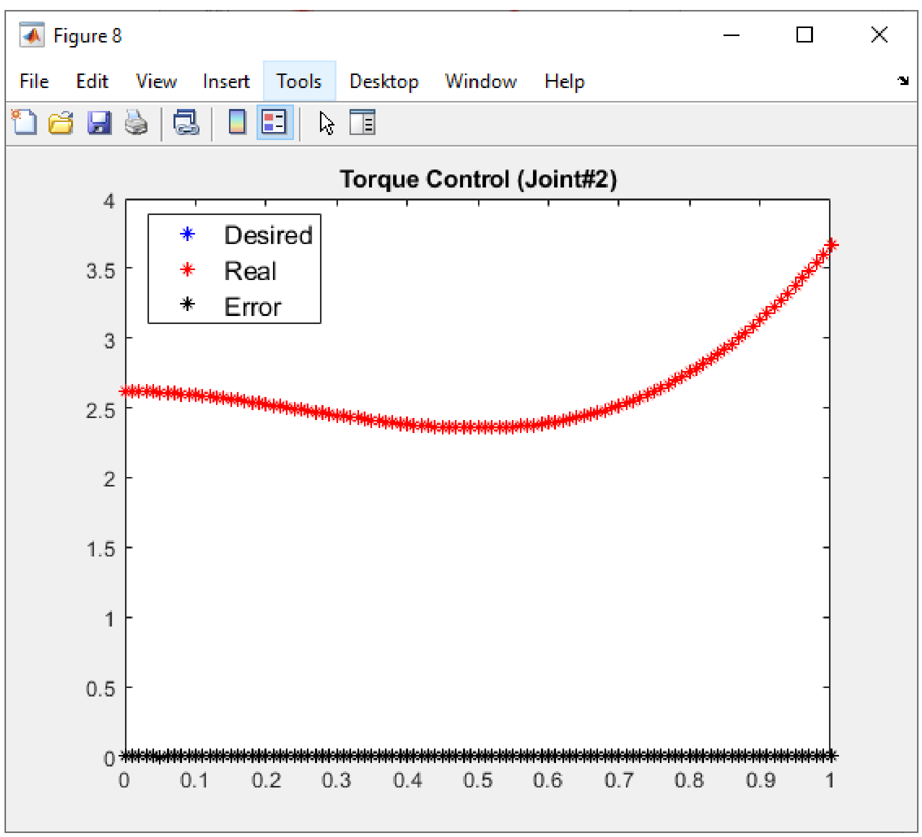
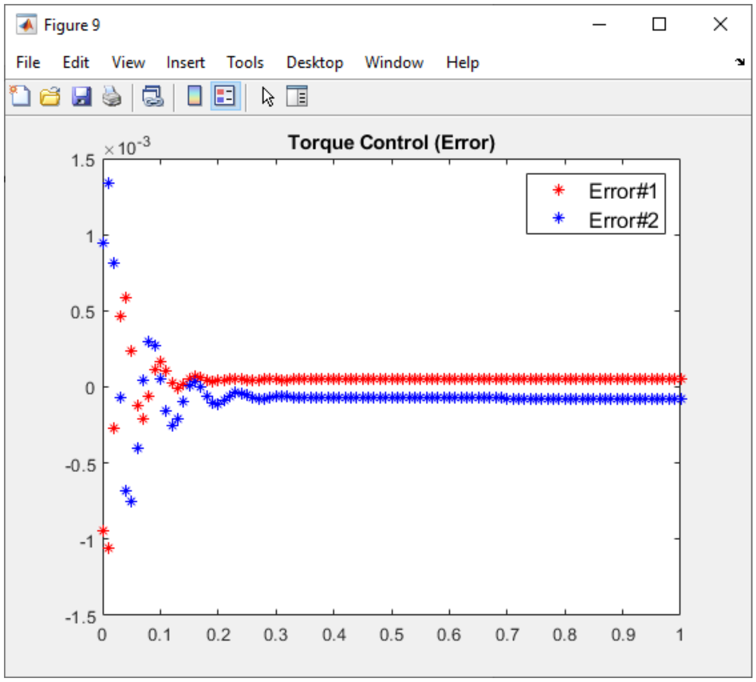


Figure 8
File Edit View Insert Tools Desktop Window Help
Torque Control (Joint#2)
Desired
Real
Error
4
3.5
3
2.5
2
1.5
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1




**5. Compare errors of 3 cases above.**