Manipulator Dynamics

- **Robot dynamic equation**:

$$\tau = M(\theta)\ddot{\theta} + V\left(\theta, \dot{\theta}\right) + G(\theta) + \Delta(t) \qquad (1)$$

$$\Rightarrow \quad \ddot{\theta} = M(\theta)^{-1}[\tau - V\left(\theta, \dot{\theta}\right) - G(\theta) - \Delta(t)] \qquad (2)$$

Where:

$\theta \in R^n$ is the state vector

τ is the torque produced by actuators

$M(q) \in R^{n \times n}$ is the mass matrix

$V_m(q, \dot{q}) \in R^n$ is the vector of centrifugal and Coriolis terms

$G(q) \in R^n$ is the vector of gravity terms

$\Delta(t) \in R^n$ is the unmodelled error of system

Model manipulator

- From Eq. (2), we can rewrite in state space form as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + g(x)\boldsymbol{\tau} - d(x) \\ y &= x_1 \end{aligned} \qquad (3)$$

Where

$x_1 = \theta \in R^n$

$x_2 = \dot{\theta} \in R^n$

$f(x) = M(\theta)^{-1}[-V(\theta, \dot{\theta}) - G(\theta)]$

$g(x) = M^{-1}(\theta)$

$d(x) = M^{-1}(\theta)[\Delta(t)]$ is the uncertainty of the system.

The uncertainty is bounded: $|M^{-1}(\theta)[\Delta(t)]| \leq$ D

Design of Sliding mode control

- The design procedure of the sliding mode control includes two main steps:

  - The first step involves the construction of the desired sliding surface, which is chosen such that when it converges to zero, the desired control is achieved.

  - The next step is to select a control law that forces the system state to reach the sliding surface in a finite time.

Design of Sliding mode control

- The first step is to choose a proper switching surface:

$$s = \dot{e} + \lambda e \qquad (4)$$

Where

$e = x_d - x_1$

x_d is the desired trajectory

λ is a strictly positive constant.

- The second step, to ensure the trajectories of the system approach the sliding surface, the derivative of the sliding surface $\dot{s} = 0$ should be satisfied such that

$$\begin{aligned}\dot{s} &= \ddot{e} + \lambda\dot{e} \\ &= \ddot{x}_d - \dot{x}_2 + \lambda\dot{e} \\ &= \ddot{x}_d + \lambda\dot{e} - f(x) - g(x)\boldsymbol{\tau} + d(x)\end{aligned}$$

Design of Sliding mode control

- According to the sliding mode design procedure, we choose:

$$\tau = \tau_{eq} + \tau_{SMC} \qquad (5)$$

  - The equivalent control signal τ_{eq} is obtained by equation $\dot{s} = 0$ without considering the presence of the system uncertainties. :

$$\tau_{eq} = g(x)^{-1}[\ddot{x}_d + \lambda\dot{e} - f(x)] \qquad (6)$$

  - τ_{SMC} is the term that compensates for the effect of the uncertainties:

$$\tau_{SMC} = g(x)^{-1}\rho sign(s) \qquad (7)$$

where ρ is a constant chosen based on the upper bound of the modeling uncertainties in the system.

- So that:

$$\tau = g(x)^{-1}[\ddot{x}_d + \lambda\dot{e} - f(x) + \rho sign(s)] \qquad (8)$$

$$\tau = M(\theta)[\ddot{x}_d + \lambda\dot{e} + M(\theta)^{-1}\left[V\left(\theta,\dot{\theta}\right) + G(\theta)\right] + \rho sign(s)] \qquad (9)$$

Lyapunov function

- Define a Lyapunov function candidate as $V = \frac{1}{2}s^2$, its time derivative given by

$$\dot{V} = s\dot{s} = s[\ddot{x}_d + \lambda\dot{e} - f(x) - g(x)\tau + d(x)]$$

$$= s(\ddot{x}_d + \lambda\dot{e} - f(x) - g(x)g(x)^{-1}[\ddot{x}_d + \lambda\dot{e} - f(x) + \rho sign(s)] + d(x))$$

$$= s[-\rho sign(s) + d(x)]$$

$$\leq s[-\rho sign(s) + D]$$

- If $\rho \geq D$ is satisfied, then $\dot{V} \leq 0$ is sufficiently ensured. This means that the system is stable.

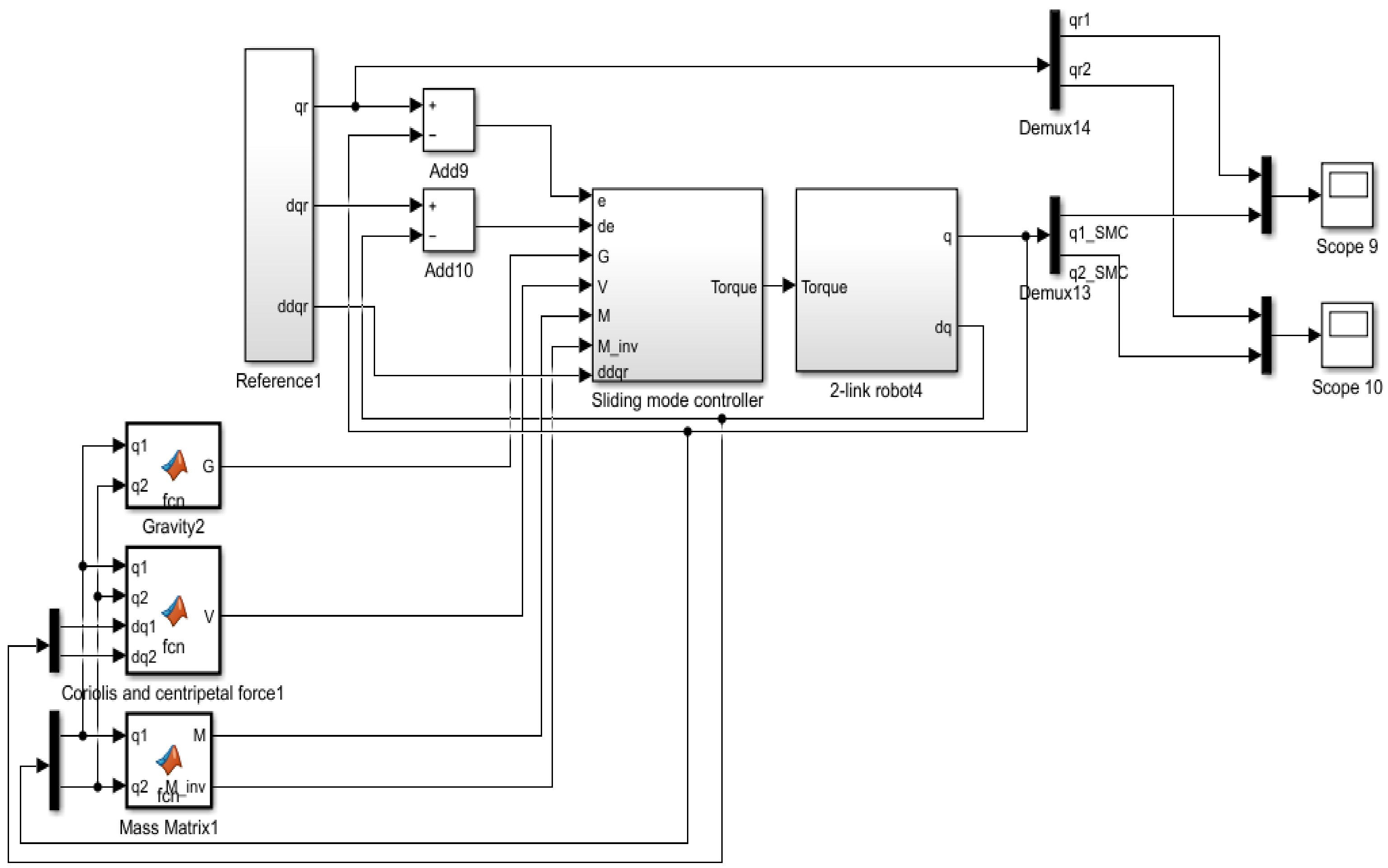
Lyapunov function

- The major drawback in the practical realization of SMC is chattering. To avoid chattering, various methods have been proposed to “soften” the chattering.

- For example, the continuous approximation method in which the $\operatorname{sign}(s)$ function is replaced by a continuous approximation $\frac{s}{|s|+\epsilon}$ where ϵ is a small positive number.

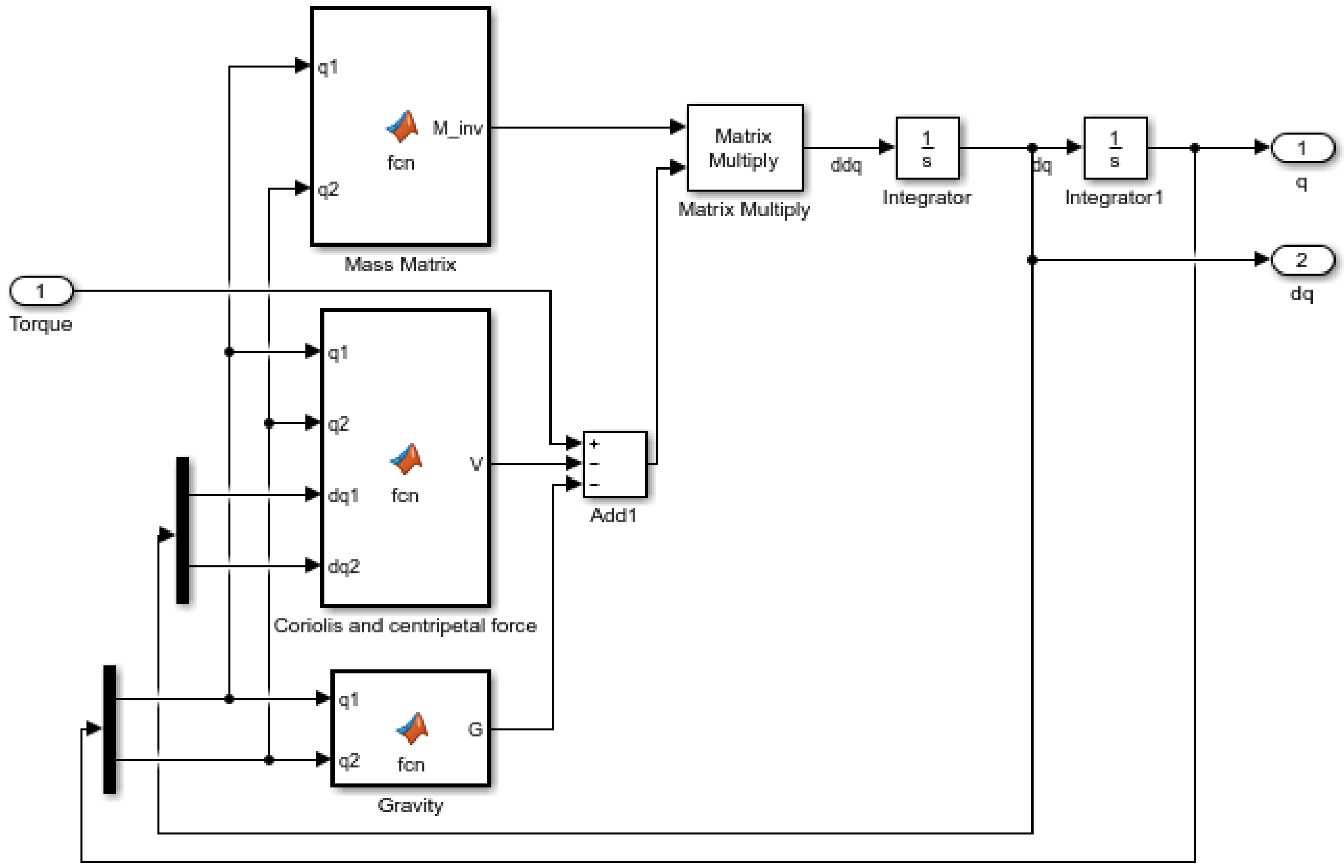
Simulation Results

Choose the constant λ=20 and sliding gain ρ=50.

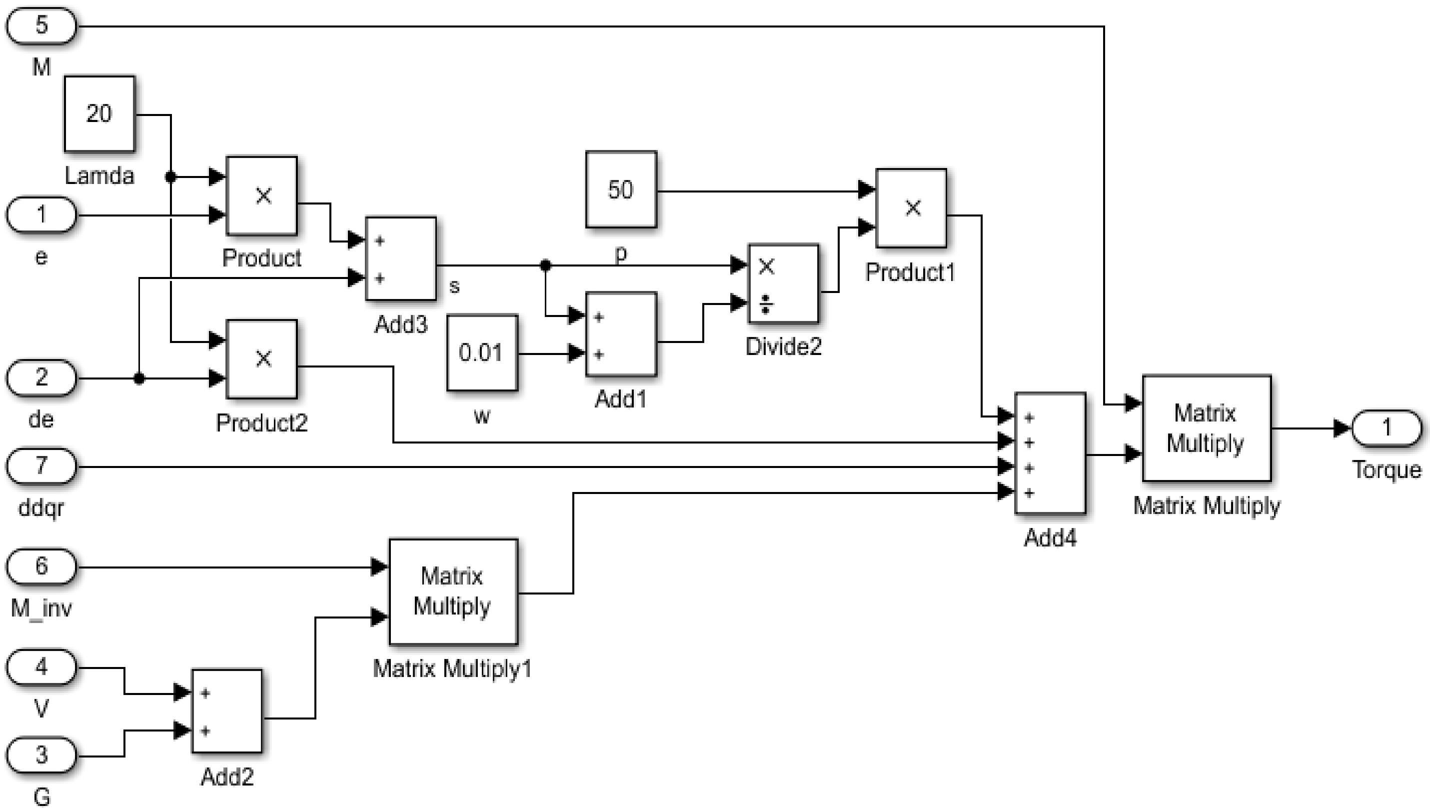


Simulation Results

2-link robot:

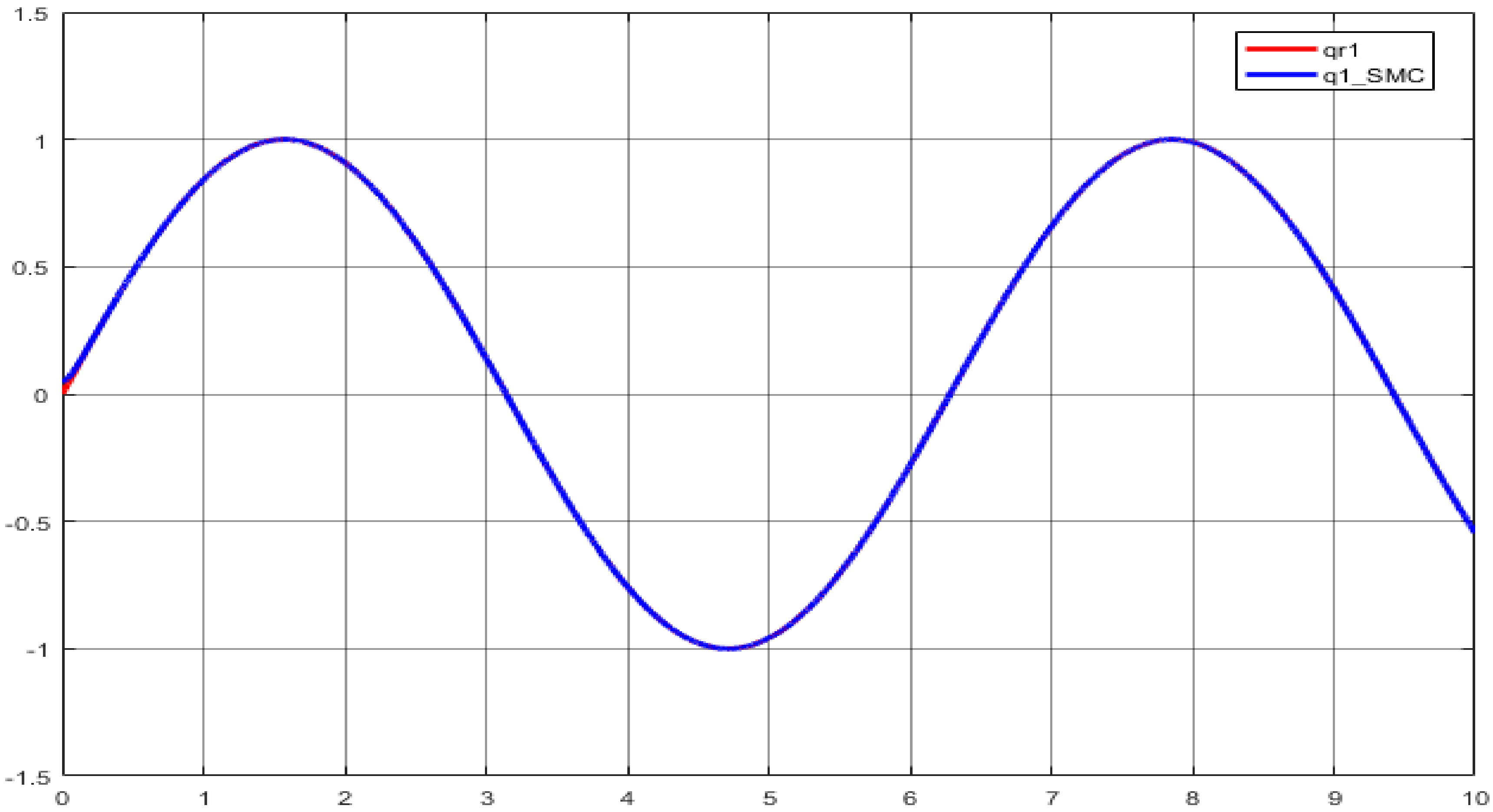


Sliding mode controller



Simulation Results

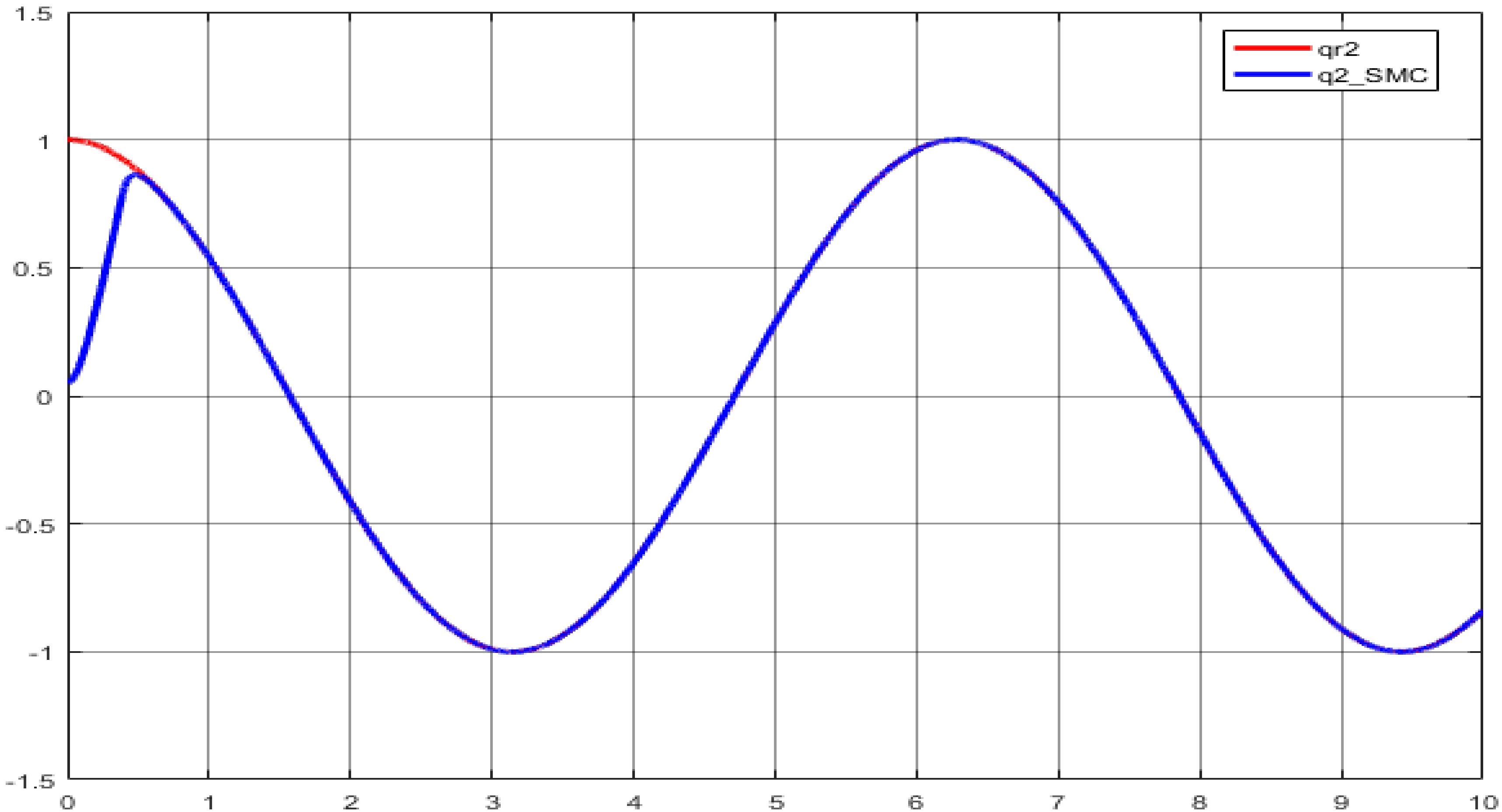
Result:



Joint 1

Simulation Results

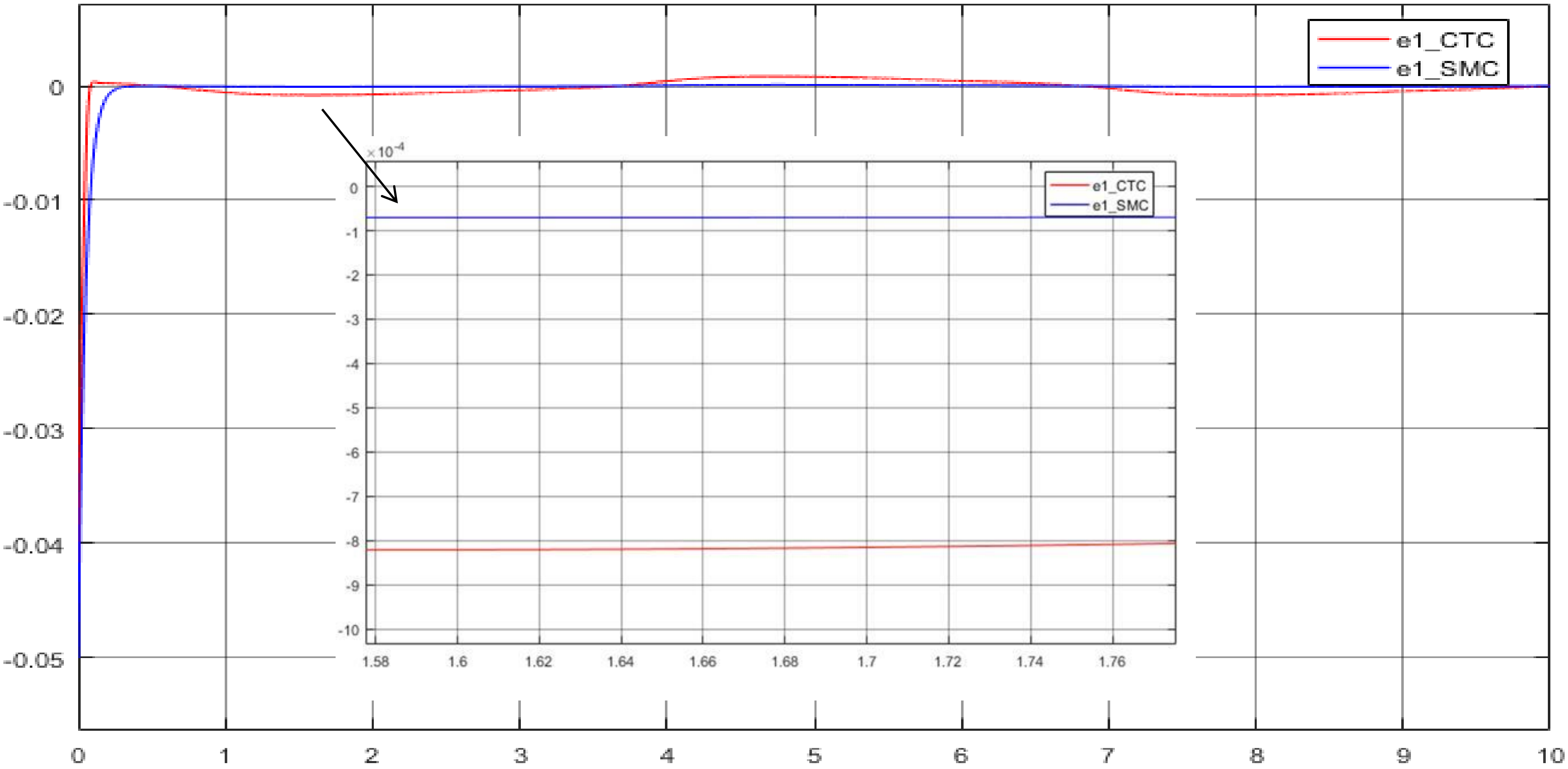
Result:



Joint 2

Simulation Results

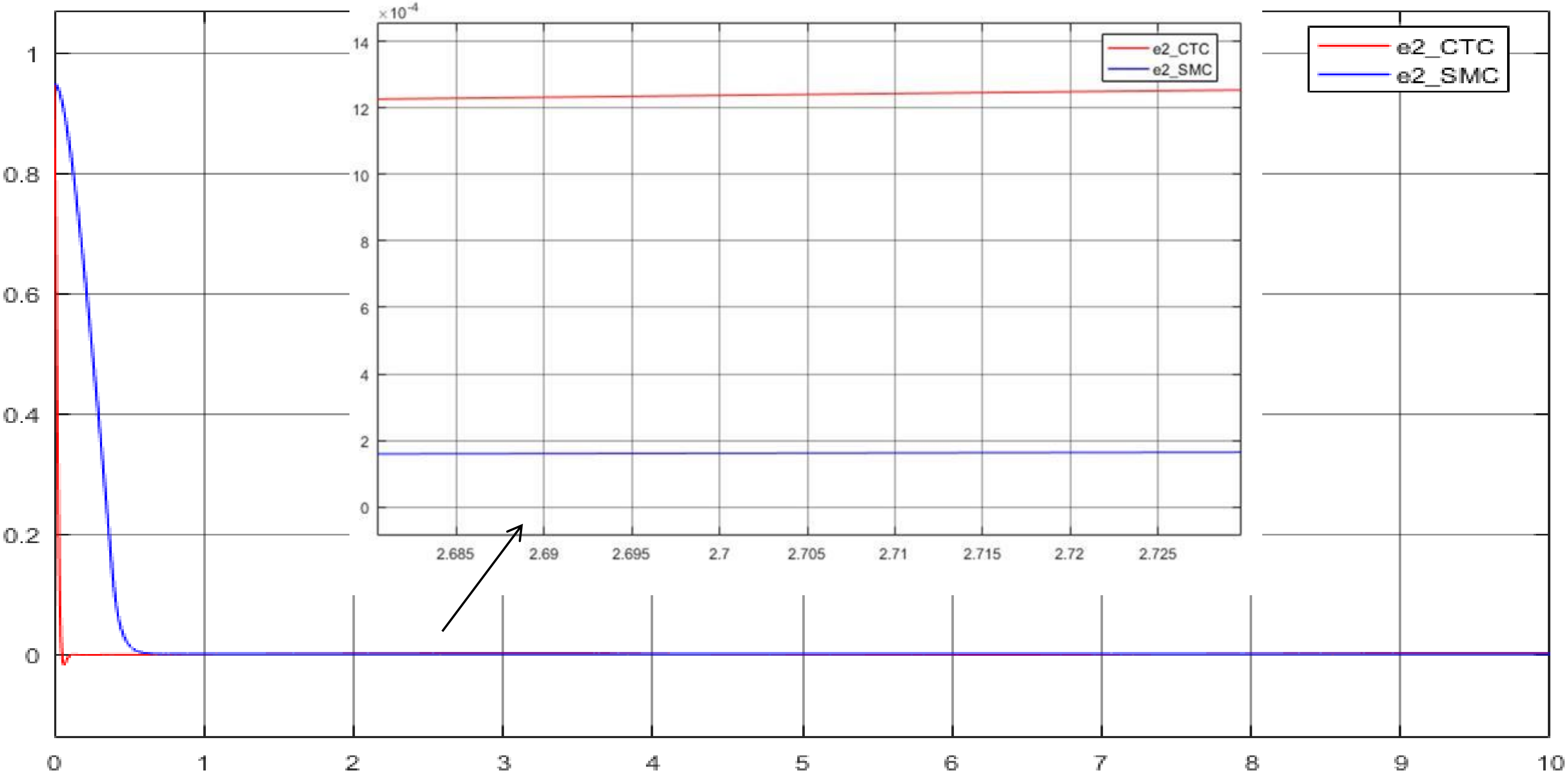
Compare with Computed torque control



Joint 1

Simulation Results

Compare with Computed torque control



Joint 2

Sliding mode controller

- The second order system: $\ddot{\theta} = f\left(\theta, \dot{\theta}\right) + bu$
- Let : $x_1 = \theta$, $x_2 = \dot{\theta}$, the system is described

$$\dot{x_2} = f(x_1, x_2) + bu$$

- The trajectory command is denoted as : x_d , then the error: $e = x_d - x_1$
- The sliding surface: $s = ce + \dot{e}$, and we easily get:

$$\begin{aligned}\dot{s} &= c\dot{e} + \ddot{e} \\ &= c\dot{e} + \ddot{x}_d - \ddot{x}_1 \\ &= c\dot{e} + \ddot{x}_d - (f(x_1, x_2) + bu)\end{aligned}$$

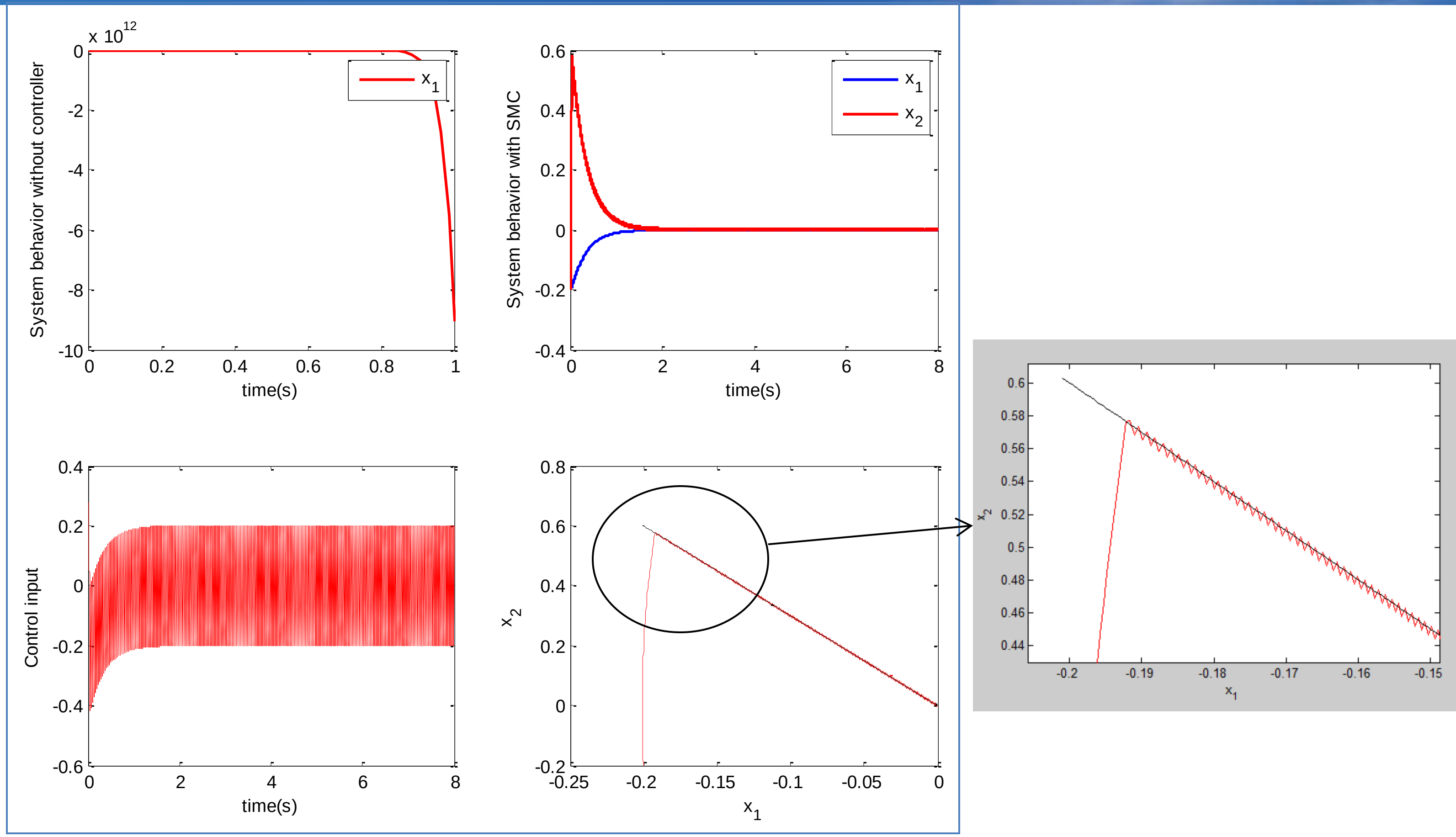
- Following Utkin's theory, we have the equivalent control part and the switching part as follows:

$$u_{eq} = \frac{1}{b}[c\dot{e} + \ddot{x}_d - f(x_1, x_2)]$$

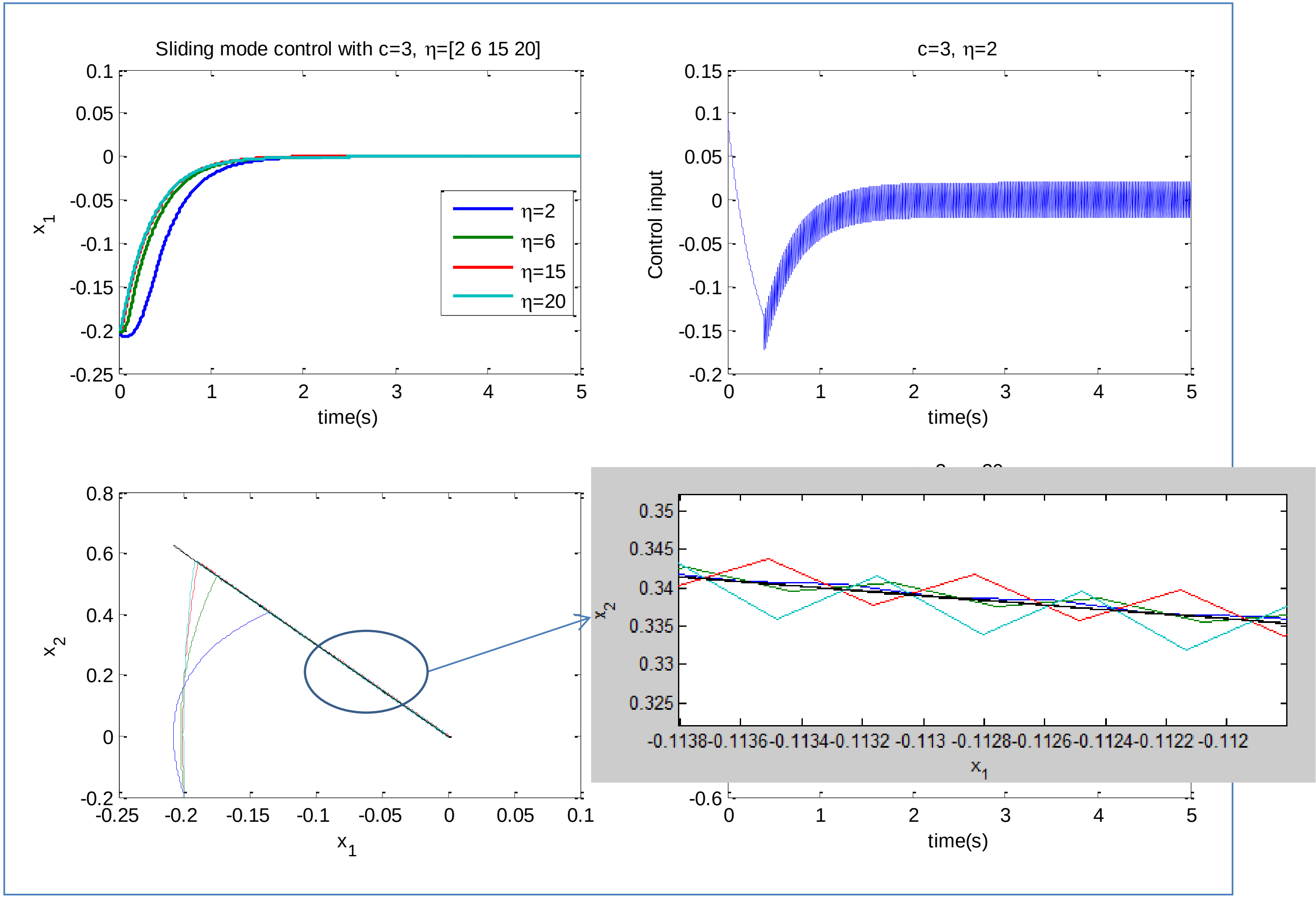
$$u_{sw} = \frac{1}{b}\eta . sgn(s)$$

- The total control signal: $u_{SMC} = u_{eq} + u_{sw}$ that guarantees : $s\dot{s} = -\eta|s| < 0$

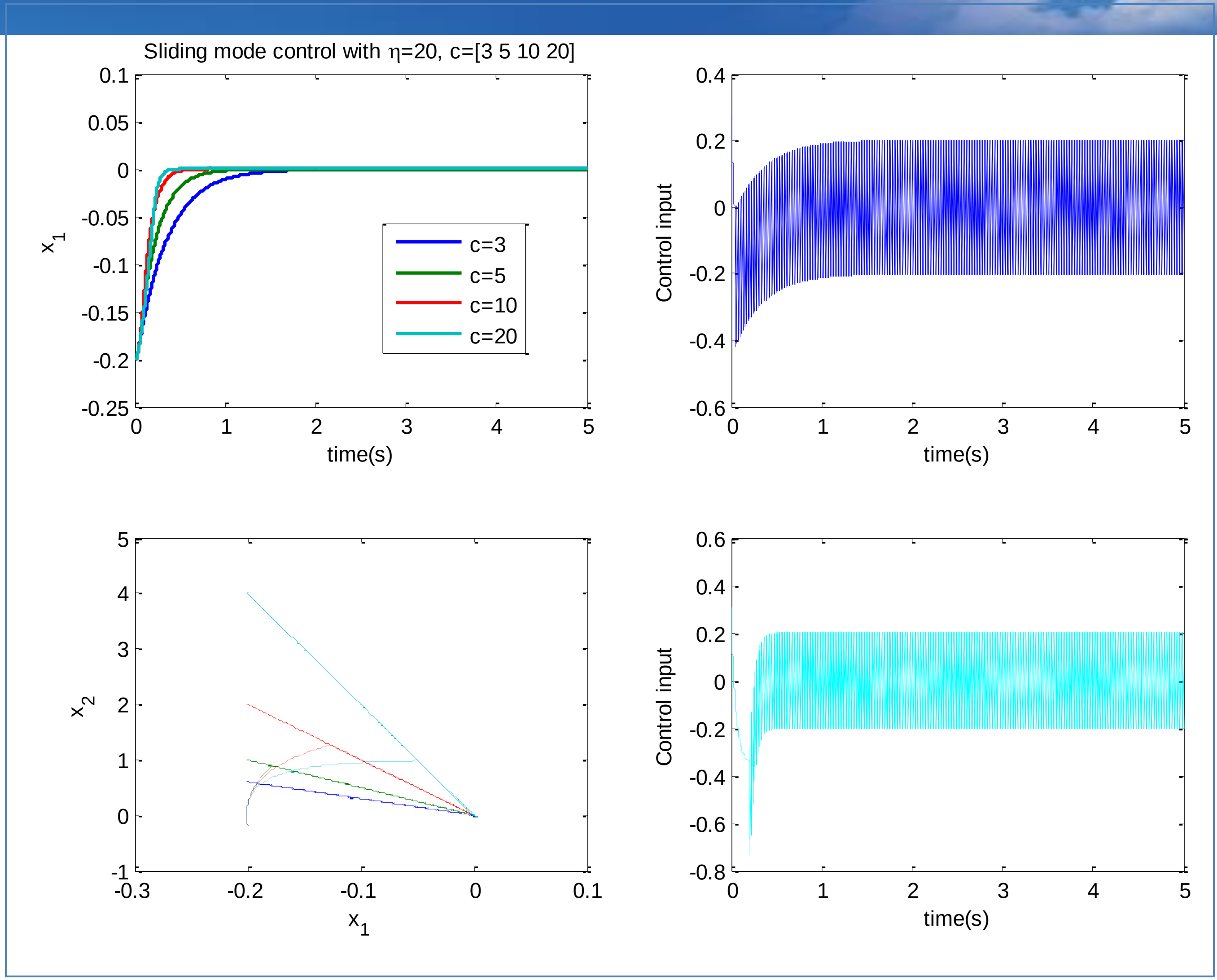
The system: $\ddot{\theta} = 35\dot{\theta} + 100u$ with initial condition [x1,x2]=[-0.2,-0.2]



1. The original system is unstable at equilibrium point x1=0
2. The sliding mode controller makes system stable at its equilibrium point
3. Because the control signal includes the high switching part, so it causes chattering phenomenon

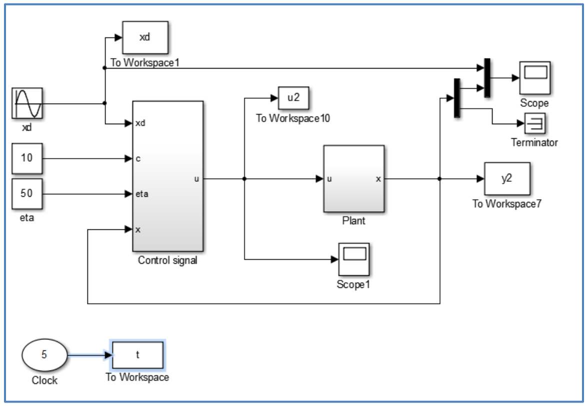


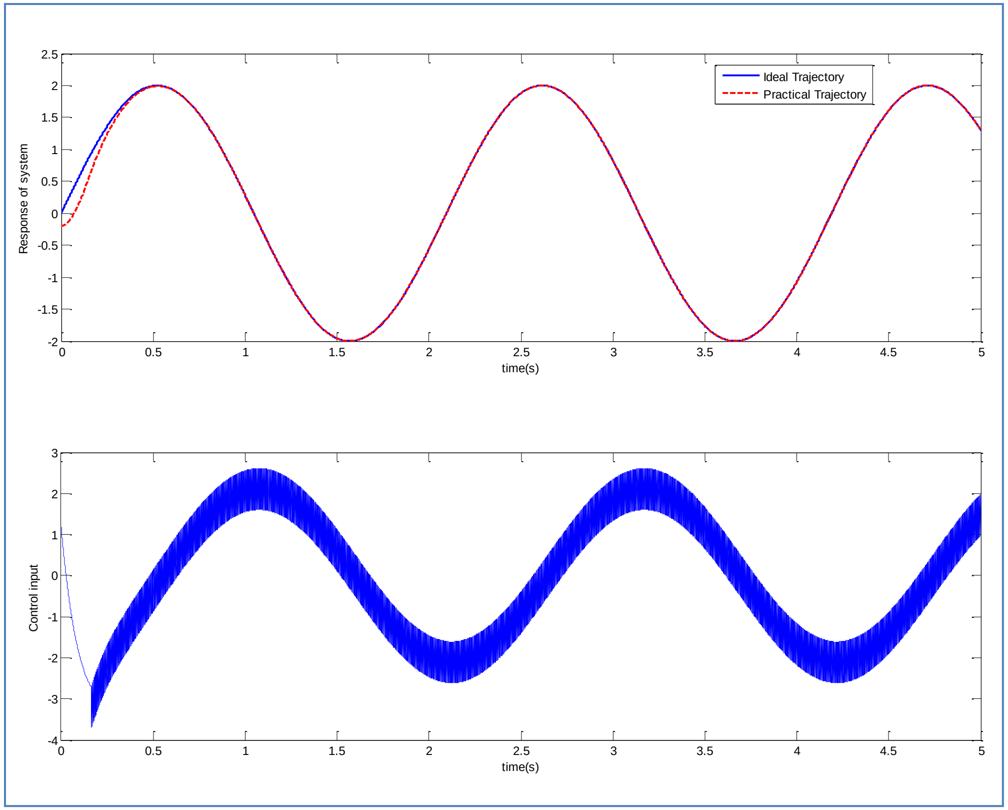
1. The effect of eta parameter to system behavior. An fundamental trade-off between the speed of convergence and control signal, chattering level



1. The effect of ‘c’(λ) parameter to system behavior. An fundamental trade-off between the speed of convergence and control signal, chattering level

A simulation results for trajectory tracking control problem





High gain observer based Adaptive Sliding Mode Control for robot manipulators

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Professor: **Hee-Jun Kang**

Contents

- Preliminaries
- High-gain observer
- Proposed control scheme
- Stability analysis
- Simulation and results.

Preliminaries

- The dynamics of an n-joint robotic manipulator can be described by the following equation:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \tau_d = \tau \qquad (1)$$

- The manipulator dynamics has the following properties:
  - The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric matrix.
  - The dynamics of robotic manipulator can be linearly parameterized as follows:

$$\boldsymbol{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau - \tau_d = Y(q,\dot{q},\ddot{q})\theta} \qquad (2)$$

Where $Y(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n\times p}$ is the regression matrix, and $\theta \in \mathbb{R}^p$ is the constant vector of system parameters

- Assumption 1: The disturbance torques τ_d are bounded

$$|\tau_{di}| \le D_i \qquad i = 1,2,\dots,n \qquad (3)$$

Preliminaries

- The dynamic equation of robot manipulator can be written in the state space as follow:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}x_2\\ f(x_1,x_2)+\Delta f\end{bmatrix} + \begin{bmatrix}0\\ g(x_1)\end{bmatrix}\tau$$

$$y = x_1 \quad (4)$$

where $x_1 = q \epsilon \mathbb{R}^n$ is the vector of joint position, and

$$f(x_1,x_2) = M(x_1)^{-1}\big(-C(x_1,x_2) - G(x_1) - F(x_2)\big)$$

$$g(x_1) = M(x_1)^{-1}.$$

- Problem: Trajectory tracking control for robot manipulator :
  - Unknown system parameters
  - External disturbance torques
  - Without velocity measurements

Output feedback control ->
Velocity observer design

High-gain Observer

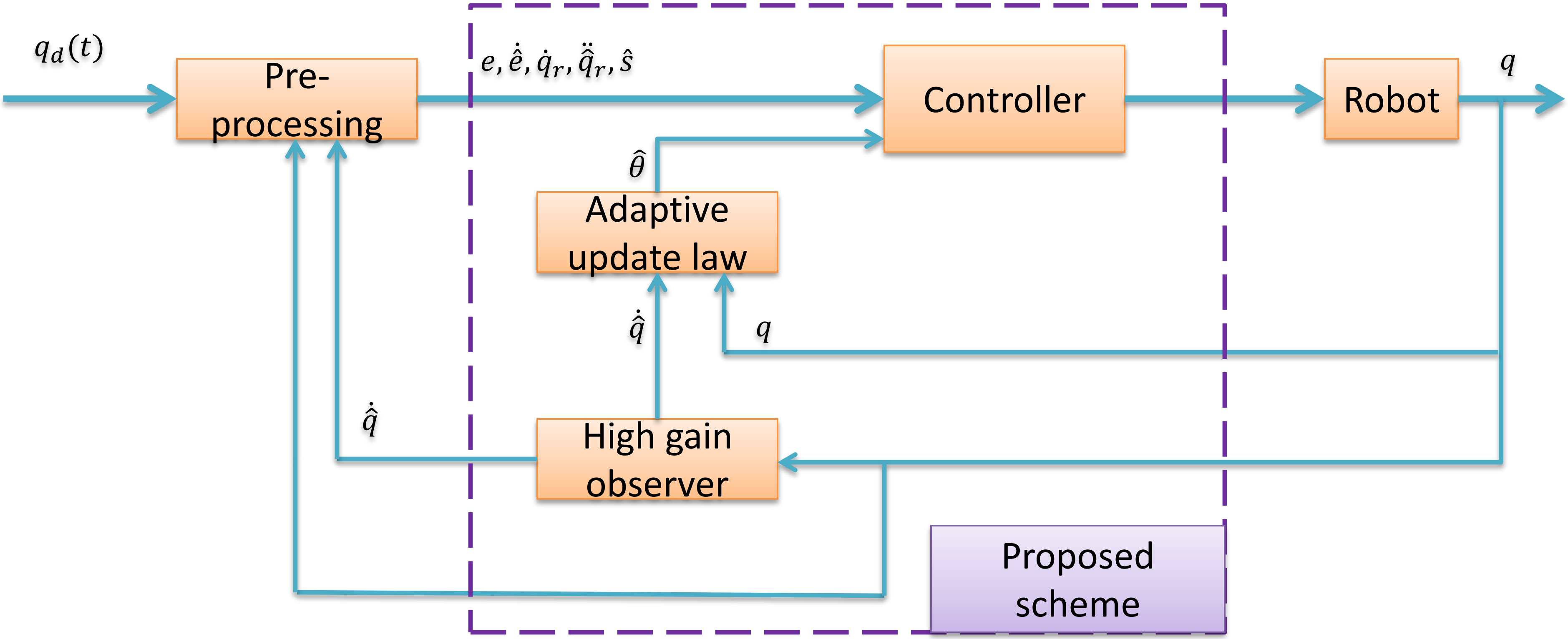
- High-gain observer robustly estimates velocity with fast convergence in the absence of measurement noise.
- Consider the robot manipulator (1), $x_1 = q$ is the measurement output. The high-gain observer is designed as:

$$\dot{\hat{x}}_1 = \hat{x}_2 - \frac{1}{\epsilon} L_p (\hat{x}_1 - x_1)$$

$$\dot{\hat{x}}_2 = -\frac{1}{\epsilon^2} L_v (\hat{x}_1 - x_1) \tag{5}$$

where $\hat{x}_1, \hat{x}_2$ denote the estimated values of x_1, x_2 respectively, ϵ is a small positive parameter, $L_p = diag(l_{pi}), Lv = diag(l_{vi})$ are positive definite matrices chosen such that $H = \begin{bmatrix} -L_p & I \\ -L_v & 0_{n\times n} \end{bmatrix}$ is a Hurwitz matrix.

Proposed control Scheme



Proposed control scheme

- For the system (1), The desired trajectories denote: $q_d(t)$, so the tracking error $e = q - q_d$. We also denote:

$$\dot{q}_r = \dot{q}_d - \Lambda e, \text{ where } \Lambda = diag([\lambda_1, \lambda_2, \dots, \lambda_n]), \lambda_i > 0$$

- Define the sliding surface as: $s = \dot{e} + \Lambda e = \dot{q} - \dot{q}_r$

- The observing sliding mode variables: $\hat{s} = \dot{\hat{e}} + \Lambda e$

- For eq (2), we denote $\hat{\theta}$ is the estimation of θ, and from (2) we have

$$\widetilde{M}(q)\ddot{q}_d + \tilde{C}(q,\dot{q})\dot{q}_d + \tilde{G}(q) + \tilde{F}(\dot{q}) = Y(q,\dot{q},\dot{q}_d,\ddot{q}_d)\tilde{\theta}$$

where

$$\widetilde{\mathrm{M}}(q) = \widehat{M}(q) - M(q)$$
$$\tilde{C}(q,\dot{q}) = \hat{C}(q,\dot{q}) - C(q,\dot{q})$$
$$\tilde{G}(q) = \hat{G}(q) - G(q)$$
$$\tilde{F}(\dot{q}\ ) = \hat{F}(\dot{q}) - F(\dot{q})$$
$$\tilde{\theta} = \hat{\theta} - \theta$$

Proposed control scheme

- We also have

$$\tilde{M}(q)\ddot{q}_r + \tilde{C}(q,\dot{q})\dot{q}_r + \tilde{G}(q) + \tilde{F}(\dot{q}) = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\theta}$$

- The proposed controller is given by:

$$\boldsymbol{\tau} = \widehat{\boldsymbol{M}}(\boldsymbol{q})\ddot{\hat{\boldsymbol{q}}}_r + \widehat{\boldsymbol{C}}\left(\boldsymbol{q},\dot{\hat{\boldsymbol{q}}}\right)\dot{\boldsymbol{q}}_r + \widehat{\boldsymbol{G}}(\boldsymbol{q}) + \widehat{\boldsymbol{F}}(\dot{\hat{\boldsymbol{q}}}) - \boldsymbol{K}\hat{\boldsymbol{s}} - \boldsymbol{\eta}\boldsymbol{sign}(\hat{\boldsymbol{s}}) \qquad (6)$$

cf) Inertia related adaptive control $\tau = \hat{M}(\theta)\ddot{\theta}_r + \hat{V}_m(\theta,\dot{\theta})\dot{\theta}_r + \hat{G}(\theta) + K_D r$

with : $K = diag([k_1, k_2, \dots, k_n]), k_i > 0,$

the update law : $\dot{\hat{\theta}} = -\Gamma Y(q,\dot{\hat{q}},\dot{q}_r,\ddot{\hat{q}}_r)^T \hat{s}, \Gamma^{-1} = diag\left(\left[\gamma_1,\gamma_2,\dots,\gamma_p\right]\right), \gamma_j > 0$

$\eta_i = D_i + \xi_i, \xi_i > 0$

Stability analysis

- We consider the Lyapunov function candidate as:

$V = \frac{1}{2} s^T M s + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$ cf) $V = ½ r^T M(q) r + ½ \tilde{\varphi}^T \Gamma^{-1} \tilde{\varphi}$

- Therefore, we have:

$$\begin{aligned}
\dot{V}(t) &= s^T M \dot{s} + \frac{1}{2} s^T \dot{M} s + \tilde{\theta} \Gamma^{-1} \dot{\tilde{\theta}} \\
&= s^T (M(q)\ddot{q} - M(q)\ddot{q}_r) + \frac{1}{2} s^T \dot{M} s + \tilde{\theta} \Gamma^{-1} \dot{\tilde{\theta}} \\
&= s^T (\tau - C(q,\dot{q})\dot{q} - G(q) - F(\dot{q}) - \tau_d - M(q)\ddot{q}_r) + \frac{1}{2} s^T \dot{M} s + \tilde{\theta} \Gamma^{-1} \dot{\tilde{\theta}} \\
&= s^T (\tau - C(q,\dot{q})(s + \dot{q}_r) - G(q) - F(\dot{q}) - \tau_d - M(q)\ddot{q}_r) + \frac{1}{2} s^T \dot{M} s + \tilde{\theta} \Gamma^{-1} \dot{\tilde{\theta}} \\
&= s^T (\tau - C(q,\dot{q})\dot{q}_r - G(q) - F(\dot{q}) - \tau_d - M(q)\ddot{q}_r) + \frac{1}{2} s^T (\dot{M} - 2C(q,\dot{q})) s + \tilde{\theta} \Gamma^{-1} \dot{\tilde{\theta}}
\end{aligned} \tag{7}$$

- Substituting (6) into (7) and base on the convergence of the high-gain observer, we have:

$$\dot{V}(t) \le -s^T K s - \sum \xi_i |s_i| \le 0$$

Thus, The system stability is guaranteed in the sense of Lyapunov theorem.

Simulation results

- Consider a direct drive vertical robot manipulator with 2DOF (Fernando Reyes and Rafael Kelly (Robotica-1997)that has the parameters and the entries of robot dynamics as follows :

$$M(\boldsymbol{q}) = \begin{bmatrix} \theta_1 + 2\theta_2\cos(q_2) & \theta_3 + \theta_2\cos(q_2) \\ \theta_3 + \theta_2\cos(q_2) & \theta_3 \end{bmatrix},$$

$$C(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{bmatrix} -2\theta_2\sin(q_2)\dot{q}_2 & -\theta_2\sin(q_2)\dot{q}_2 \\ \theta_2\sin(q_2)\dot{q}_1 & 0 \end{bmatrix},$$

$$\boldsymbol{g}(\boldsymbol{q}) = \begin{bmatrix} \theta_4\sin(q_1) + \theta_5\sin(q_1+q_2) \\ \theta_5\sin(q_1+q_2) \end{bmatrix},$$

$$\theta_1 = m_1 l_{c1}^2 + m_2 l_1^2 + m_2 l_{c2}^2 + I_1 + I_2$$

$$\theta_2 = l_1 m_2 l_{c2}$$

$$\theta_3 = m_2 l_{c2}^2 + I_2$$

$$\theta_4 = g(l_{c1} m_1 + m_2 l_1)$$

$$\theta_5 = g m_2 l_{c2}$$

$$\tau_d = \begin{bmatrix} 2sin(2t) \\ 3sin(\pi t) \end{bmatrix}$$

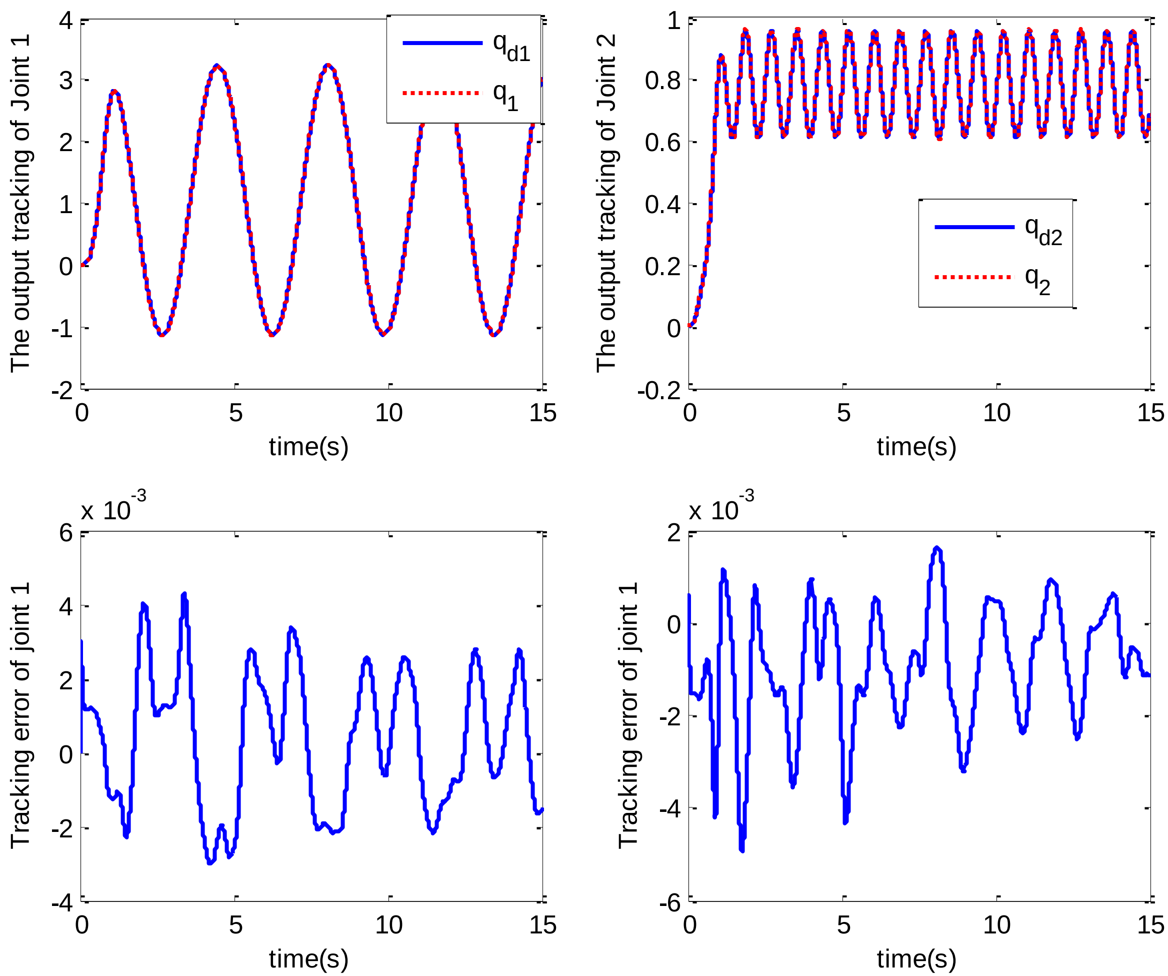
Table I. Parameter values.

Parameter	Notation	Value	Unit
Length link 1	l_1	0.45	m
Mass link 1	m_1	23.902	Kg
Mass link 2	m_2	3.880	Kg
Link (1) center of mass	l_{c1}	0.091	m
Link (2) center of mass	l_{c2}	0.048	m
Inertia link 1	I_1	1.266	Kg m^2
Inertia link 2	I_2	0.093	Kg m^2

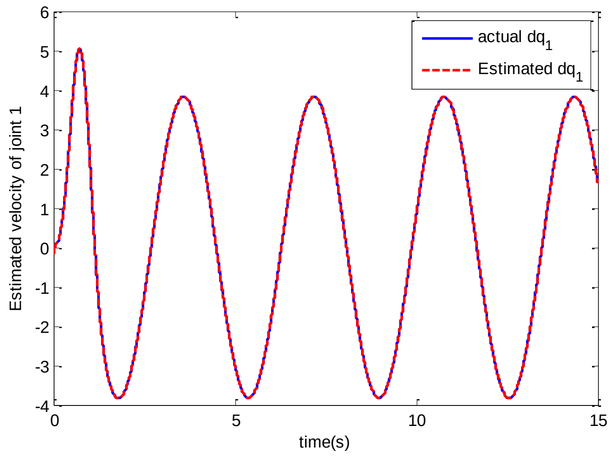
- The desired trajectory used in our simulation (Dawson DM, Carroll JJ, IEEE Transaction on control system 1994)

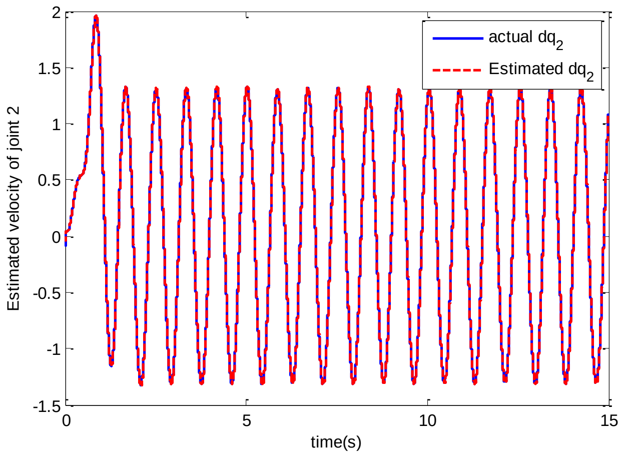
$$q_d(t) = \begin{bmatrix} 1.0472\left(1 - e^{-1.8t^3}\right) + 2.1816\left(1 - e^{-1.8t^3}\right)\sin(1.75t) \\ 0.7854\left(1 - e^{-2.0t^3}\right) + 2.1816\left(1 - e^{-2.0t^3}\right)\sin(7.5t) \end{bmatrix}(rad)$$

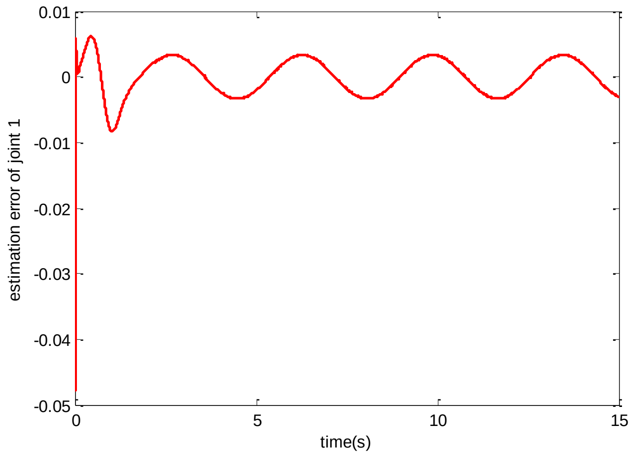
Simulation results

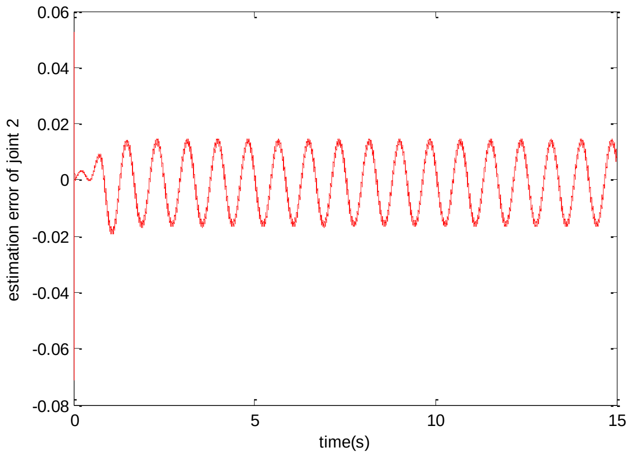


Simulation results

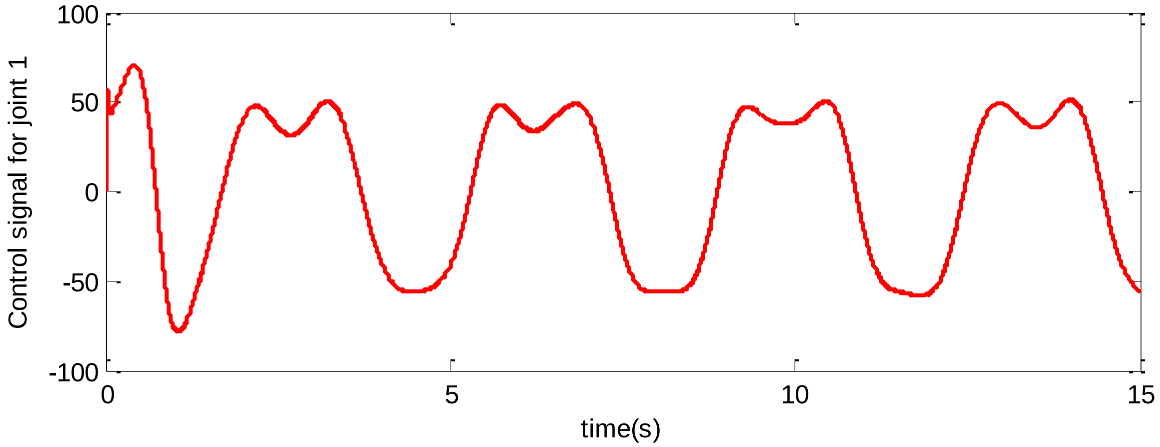


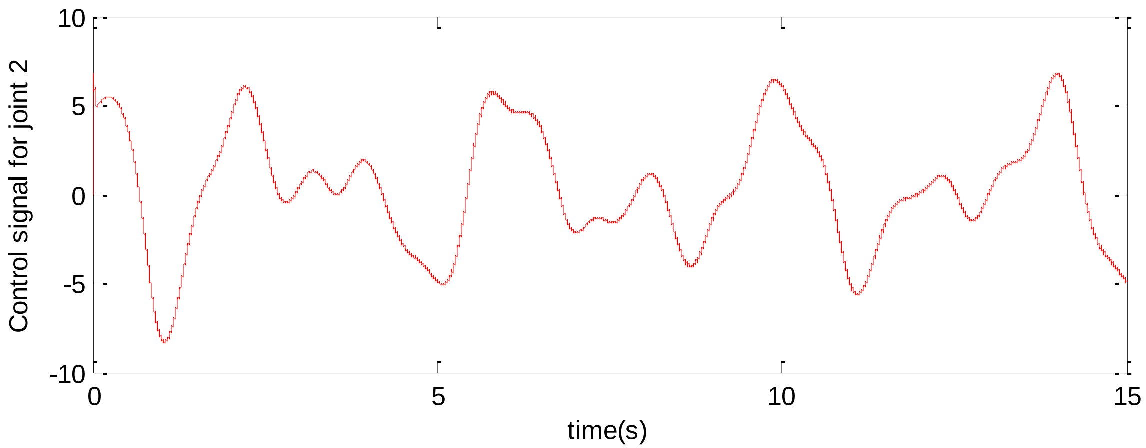




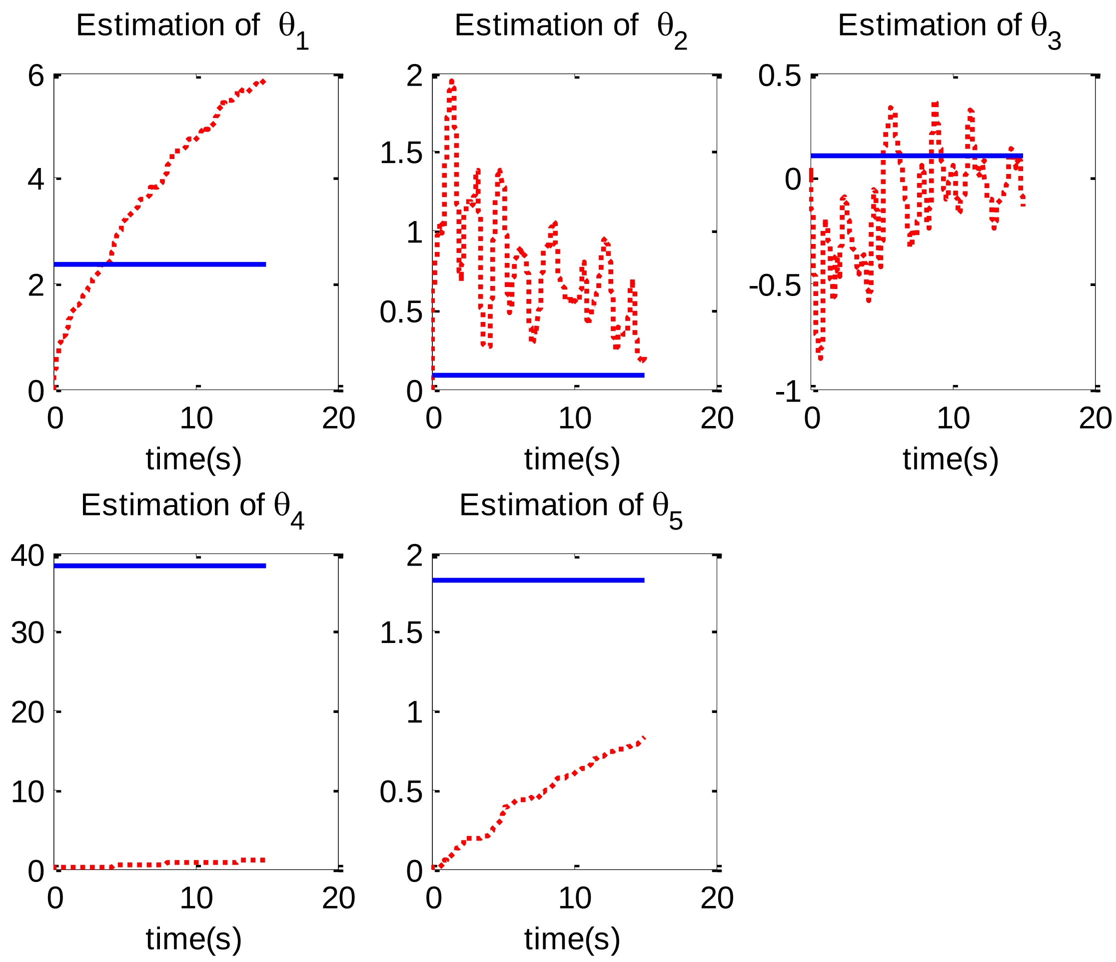


Simulation results





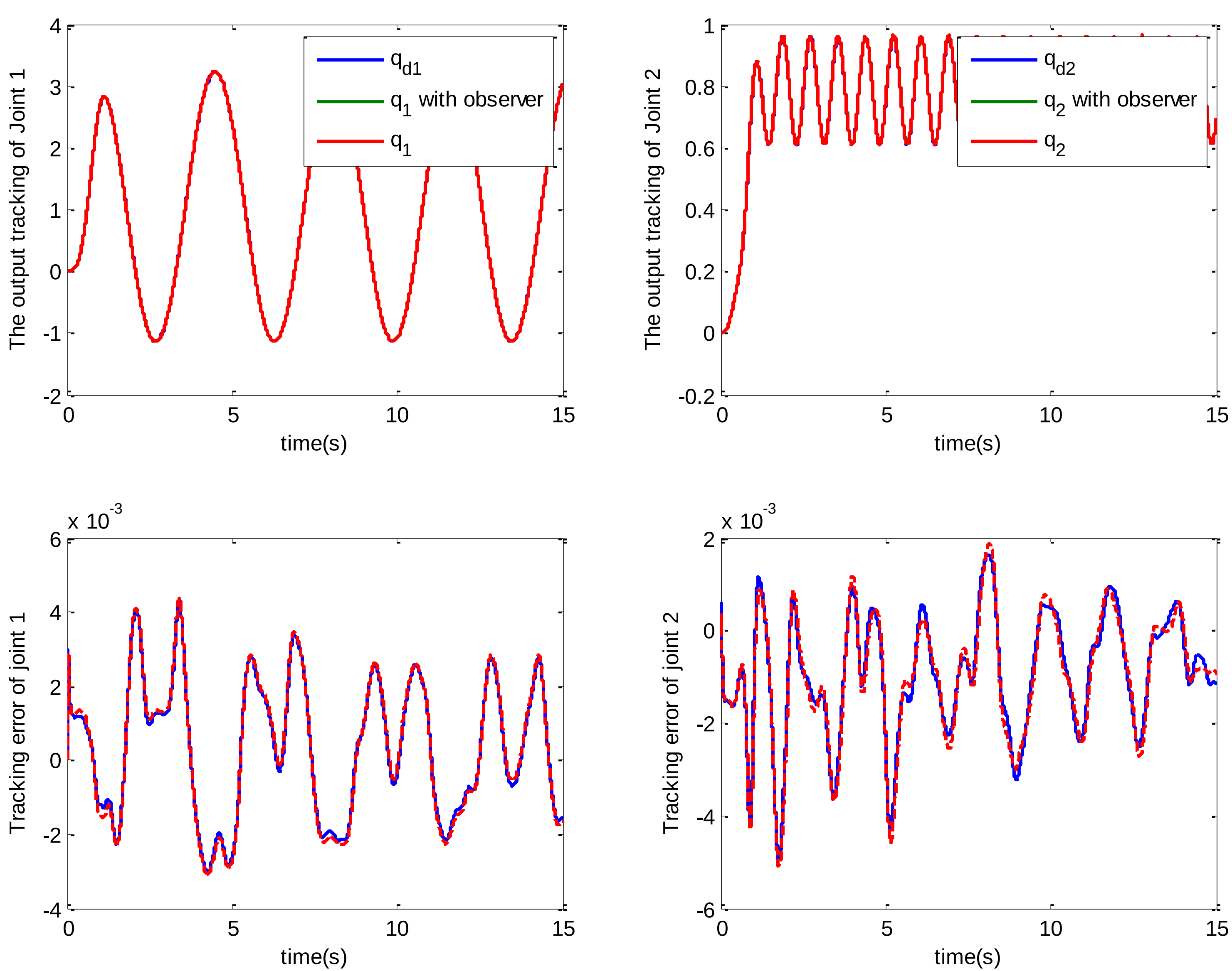
Simulation results



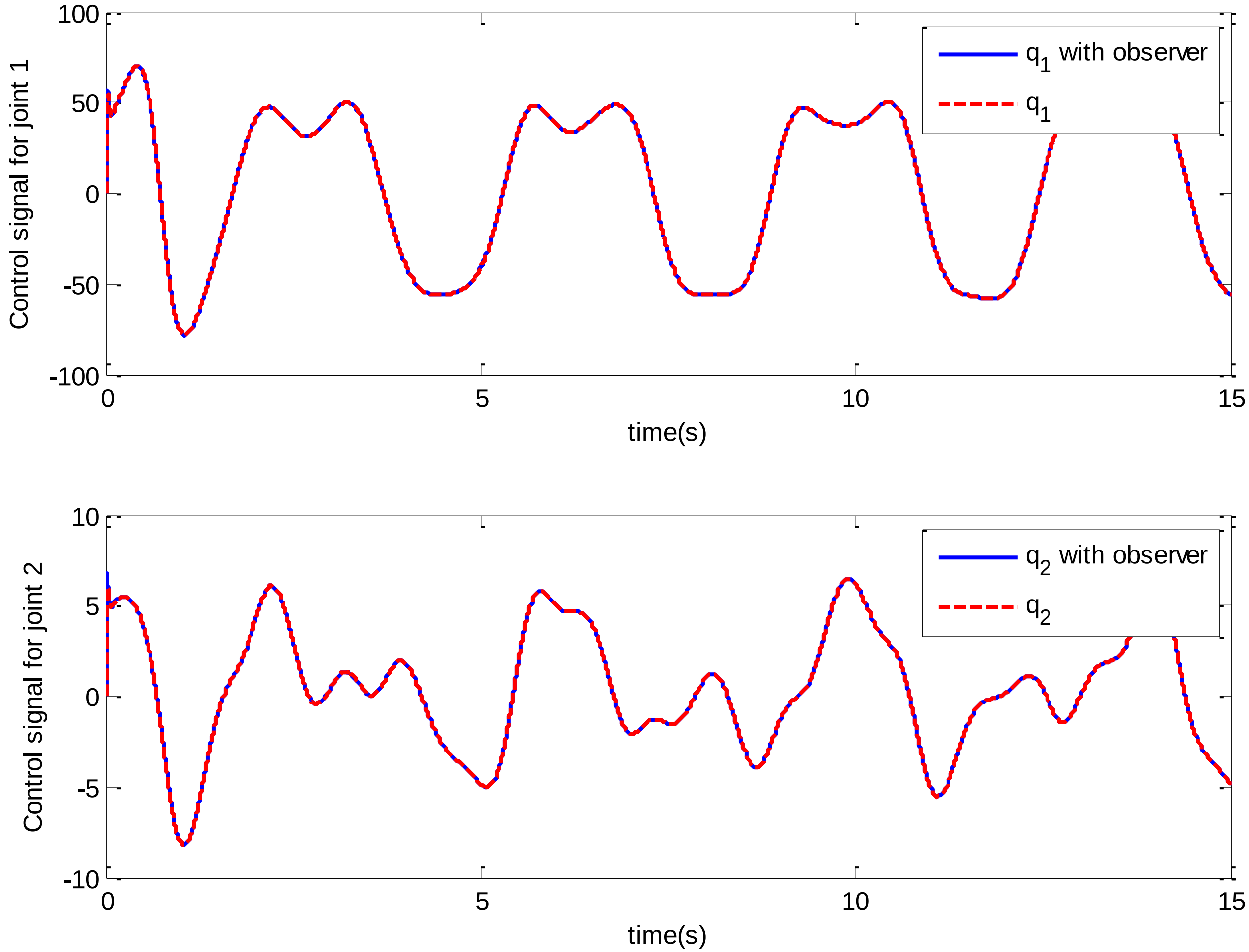
References

- F. Reyes and R. Kelly, “ Experimental Evaluation of Identification Schemes on a Direct Drive Robot”, *Robotica (1997),* volume 15, pp 563-571.
- Dawson DM, Carroll JJ, Schneider M. “Integrator backstepping control of a brush dc motor turning a robotic load”. *IEEE Transactions on Control System Technology* 1994;2(3):233–44

Simulation results



Simulation results



Simulation results