HOME WORK #5

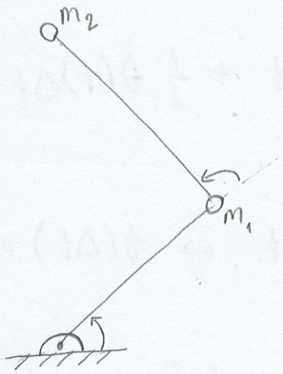
Pham Thi Ngoc Thao
20215435

Mass parameters

$m_1 = 10\ kg$

$m_2 = 5\ kg$

$l_1 = l_2 = 0,5\ m$



$\theta_1(0) = 30^\circ$; $\dot{\theta}_1(0) = 0$; $\theta_2(0) = 150^\circ$; $\dot{\theta}_2(0) = 0$; $t_f = 1\ sec$

When $\tau_1 = 10\,\&\sin(0,5\pi t)$
$\tau_2 = 10\,\&\sin(0,5\pi t)$ is applied

Find the trajectories of each point $\theta_1(t), \theta_2(t)$ $0 \le t \le 1$

Solve:

We have: $\theta(0) = \begin{bmatrix} \theta_1(0) \\ \theta_2(0) \end{bmatrix} = \begin{bmatrix} 30^\circ \\ 150^\circ \end{bmatrix}$; $\dot{\theta}(0) = \begin{bmatrix} \dot{\theta}_1(0) \\ \dot{\theta}_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Acceleration at $t = 0$ can be calculated by:

$$\ddot{\theta}(0) = M^{-1}\left[\tau - V - G\right]$$

$$M = \begin{bmatrix} l_2^2 m_2 + 2 l_1 l_2 m_2 c_2 + l_1^2 (m_1 + m_2) & l_2^2 m_2 + l_1 l_2 m_2 c_2 \\ l_2^2 m_2 + l_1 l_2 m_2 c_2 & l_2^2 m_2 \end{bmatrix} = \begin{bmatrix} 2,8349 & 0,1675 \\ 0,1675 & 1,25 \end{bmatrix}$$

$$\rightarrow M^{-1} = \begin{bmatrix} 0,3556 & -0,0476 \\ -0,0476 & 0,8064 \end{bmatrix}$$

$$\tau(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$V = \begin{bmatrix} -m_2 l_1 l_2 s_2 \dot{\theta}_2^2 - 2 m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 \\ m_2 l_1 l_2 s_2 \dot{\theta}_1^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$G = \begin{bmatrix} m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{bmatrix} = \begin{bmatrix} 39,1928 \\ -24,525 \end{bmatrix}$$

$$\Rightarrow \ddot{\theta}(0) = \begin{bmatrix} -15,1035 \\ 21,6435 \end{bmatrix}$$

Besides that,

$$\dot{\theta}(t+\Delta t) = \dot{\theta}(t) + \ddot{\theta}(t)\Delta t$$

$$\theta(t+\Delta t) = \theta(t) + \dot{\theta}(t)\Delta t + \frac{1}{2}\ddot{\theta}(t)\Delta t^2$$

When $t = 0$, we have:

$$\dot{\theta}(0+\Delta t) = \dot{\theta}(0) + \ddot{\theta}(0)\Delta t \rightsquigarrow \dot{\theta}(\Delta t) = \begin{bmatrix} -15,1035 \\ 21,6435 \end{bmatrix} \cdot \Delta t$$

$$\theta(0+\Delta t) = \theta(0) + \dot{\theta}(0)\Delta t + \frac{1}{2}\ddot{\theta}(0)\Delta t^2$$

$$\rightsquigarrow \theta(\Delta t) = \begin{bmatrix} 30 \\ 150 \end{bmatrix} + 0 + \frac{1}{2}\begin{bmatrix} -15,1035 \\ 21,6435 \end{bmatrix}\Delta t^2$$

Replace Δt to t we obtained the trajectories

$$\theta(t) = \frac{1}{2}\begin{bmatrix} -15,1035 \\ 21,6435 \end{bmatrix} t^2 + \begin{bmatrix} 30 \\ 150 \end{bmatrix}.$$

$$\Rightarrow \theta_1(t) = -7,55175\, t^2 + 30$$

$$\theta_2(t) = 10,82175\, t^2 + 150$$