Sliding Mode Controller for a Manipulator

Introduction is from

Sliding Mode Control

Theory and Applications

Christopher Edwards

and

Sarah K. Spurgeon

Variable structure control systems (VSCS)

Double Integrator Control example

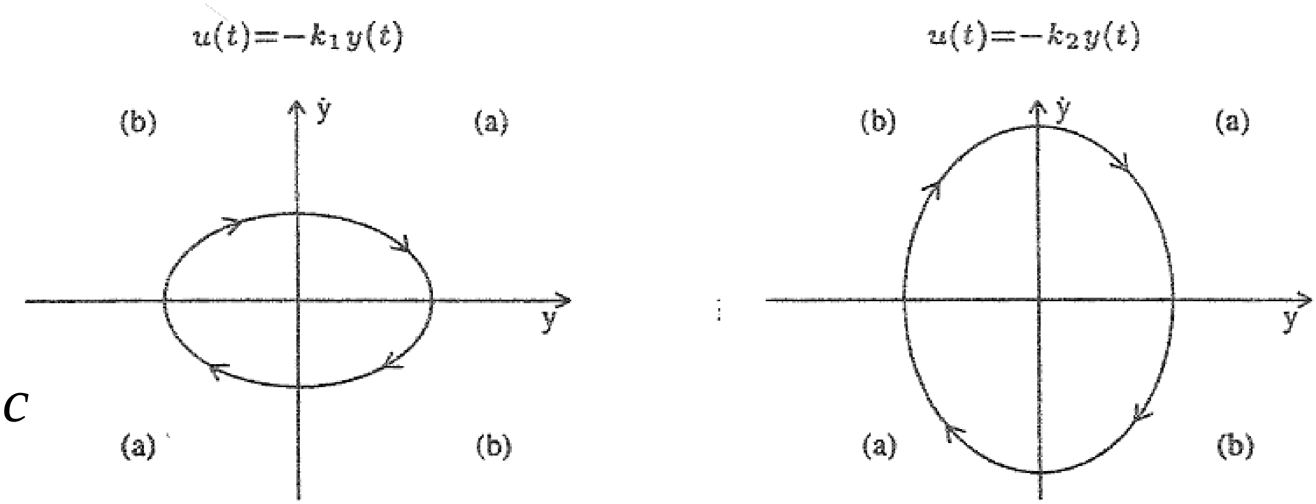
$$\ddot{y}(t) = u(t) = -ky(t) \quad k: strictly\ \ positive\ scalar$$

$$\dot{y}\ddot{y}(t) = -ky\dot{y}$$

Integrating this

$$\dot{y}^2 + ky^2 = c$$

$$\dot{y}^2 + y^2/(\sqrt{1/k})^2 = c$$



$$0 < k_1 < 1 < k_2.$$

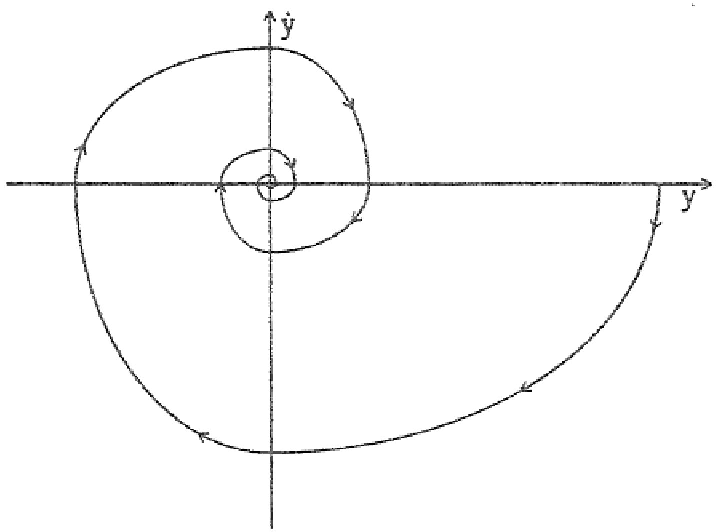
Figure 1.1: Phase portraits of simple harmonic motion

Consider instead the control law

$$u(t) = \begin{cases} -k_1 y(t) & \text{if } y\dot{y} < 0 \\ -k_2 y(t) & \text{otherwise} \end{cases} \qquad 0 < k_1 < 1 < k_2.$$

$$V(y, \dot{y}) = y^2 + \dot{y}^2$$

$$\begin{aligned} \dot{V} &= 2\dot{y}y + 2\ddot{y}\dot{y} \\ &= 2\dot{y}(y+u) = \begin{cases} 2y\dot{y}(1-k_1) & \text{if } y\dot{y} < 0 \\ 2y\dot{y}(1-k_2) & \text{if } y\dot{y} > 0 \end{cases} \end{aligned}$$



Phase portrait of the system under VSCS

Sliding Mode Controller for a Manipulator

A more significant example results from using the variable structure law

$$u(t) = \begin{cases} -1 & \text{if } s(y, \dot{y}) > 0 \\ 1 & \text{if } s(y, \dot{y}) < 0 \end{cases}$$

where the *switching function* is defined by

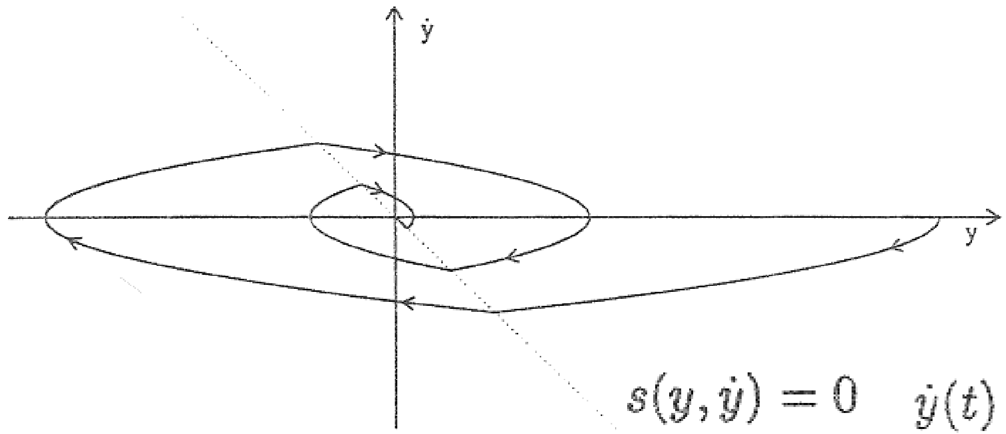
$$s(y, \dot{y}) = my + \dot{y}$$

m is a positive design scalar

written more concisely as $\quad u(t) = -\text{sgn}\,(s(t))$

where $\text{sgn}(\cdot)$ is the *signum*, or more colloquially, the sign function.

$$s\,\text{sgn}(s) = |s|$$



$s(y,\dot{y}) = 0 \quad \dot{y}(t) = -my(t)$

Phase portrait of the system for large $\dot{y}$

for values of $\dot{y}$ satisfying the inequality $m|\dot{y}| < 1$

$$s\dot{s} = s\left(m\dot{y} + \ddot{y}\right) = s\left(m\dot{y} - \text{sgn}(s)\right) < |s|\left(m|\dot{y}| - 1\right) < 0$$

or equivalently

$$\lim_{s \to 0^+} \dot{s} < 0 \qquad \text{and} \qquad \lim_{s \to 0^-} \dot{s} > 0$$

Consequently, when $m|\dot{y}| < 1$ the system trajectories on either side of the line

$$\mathcal{L}_s = \{(y,\dot{y}) \ : \ s(y,\dot{y}) = 0\}$$

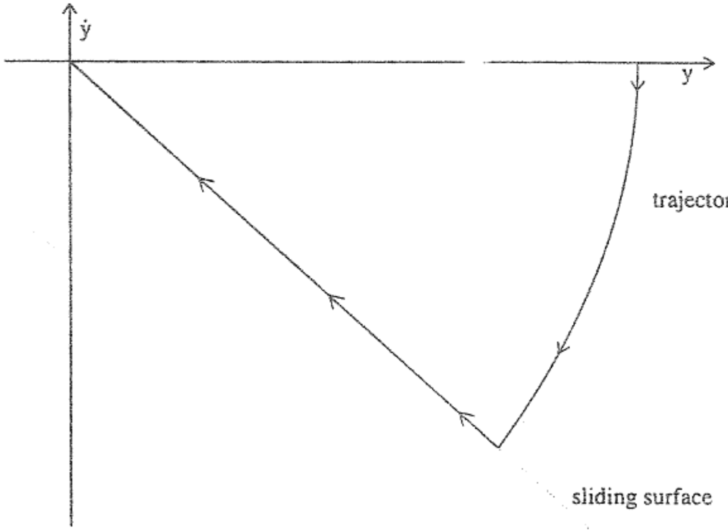
Intuitively, high frequency switching between the two different control structures will take place as the system trajectories repeatedly cross the line $\mathcal{L}_s$. This high frequency motion is described as *chattering*. If infinite frequency switching were possible, the motion would be trapped or constrained to remain on the line $\mathcal{L}_s$. The motion when confined to the line $\mathcal{L}_s$ satisfies the differential equation obtained from rearranging $s(y, \dot{y}) = 0$, namely

$$\dot{y}(t) = -my(t)$$

This represents a first-order decay and the trajectories will 'slide' along the line $\mathcal{L}_s$ to the origin (Figure 1.5).

This dynamic behavior:
Ideal sliding mode or Ideal Sliding Motion

the line $\mathcal{L}_s$ is termed the *sliding surface*.

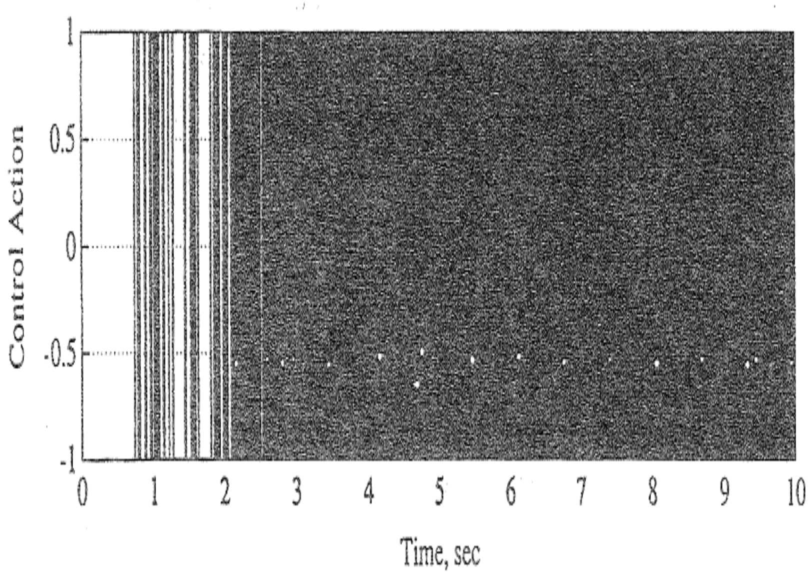


Phase portrait of a sliding motion

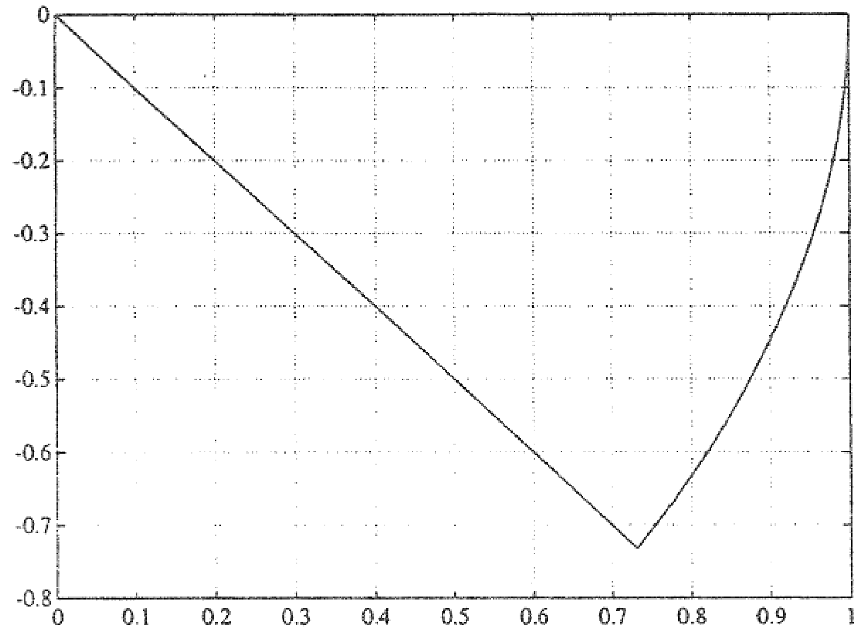
$s\dot{s} < 0$ the *reachability condition*.

a simulation of the closed-loop behaviour when $m = 1$,

the initial conditions are given by $y = 1$ and $\dot{y} = 0$.



Discontinuous control action



Phase portrait of a sliding motion

sliding takes place after 0.732 second

when high frequency switching takes place.

Sliding Mode Controller for a Manipulator

Suppose at time t_s the switching surface is reached

an ideal sliding motion takes place.

the switching function satisfies $s(t) = 0$ for all $t > t_s$

in turn implies that $\dot{s}(t) = 0$ for all $t \geq t_s$

$$\dot{s}(t) = m\dot{y}(t) + u(t)$$

$$u(t) = -m\dot{y}(t) \qquad (t \geq t_s)$$

This control law is referred to as the *equivalent control* action.

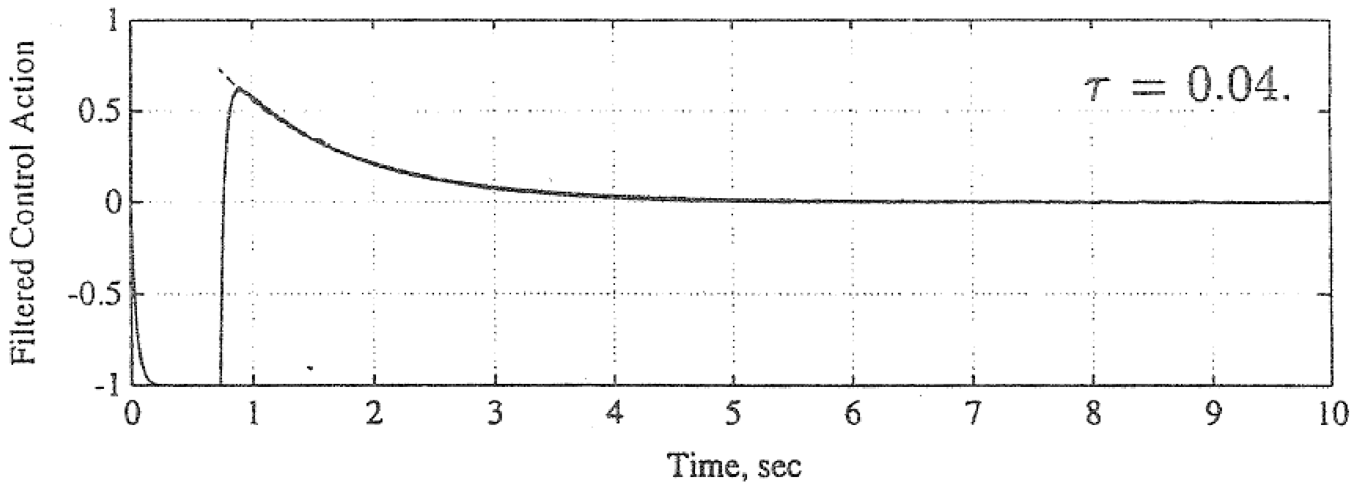
may be thought of as the control signal which is applied 'on average'.

passing the discontinuous control signal through the low pass filter

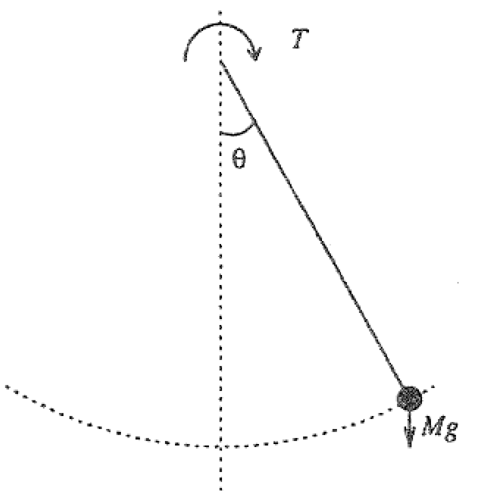
$$\tau\dot{u}_a(t) + u_a(t) = u(t)$$

$$u(t) = \underbrace{u_a(t)}_{low\ frequency} + \underbrace{(u(t) - u_a(t))}_{high\ frequency}$$

Sliding Mode Controller for a Manipulator



Equivalent control



Schematic of a pendulum

$$\ddot{\theta}(t) = -\frac{l}{g}\sin\theta(t) + \frac{1}{Ml^2}u(t)$$

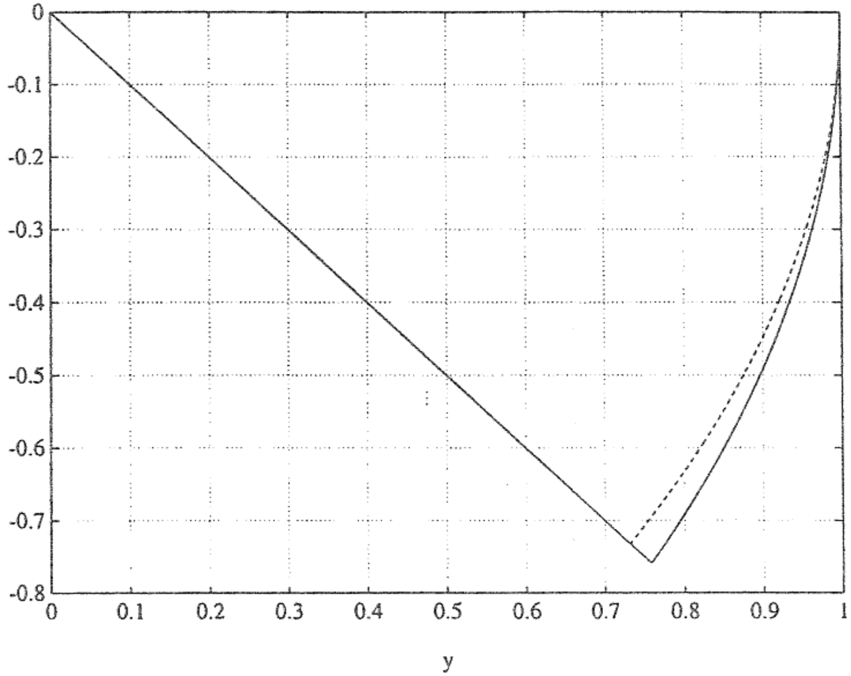
the *normalised pendulum equation* or *pendulum system.*

$$\ddot{y}(t) = -a_1 \sin y(t) + u(t)$$

Sliding Mode Controller for a Manipulator

$a_1 = 0.25$ $\quad \dot{y} = 0$ and $y = 1$

$$u(t) = -\text{sgn}(s(t))$$



Controlled pendulum

the dotted line when $a_1 = 0$

$$\ddot{y}(t) = -a_1 \sin y(t) + u(t)$$

The significance of this is that, once ideal sliding is established the double-integrator system and the normalised pendulum behave in an identical fashion, namely

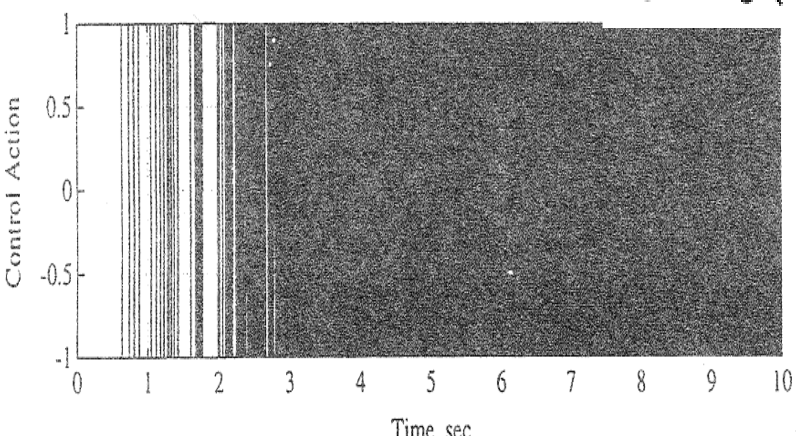
$$\dot{y}(t) = -my(t)$$

An alternative interpretation is that the effect of the nonlinear term $a_1 \sin y(t)$, which may be construed as a disturbance or uncertainty in the nominal double-integrator system, has been completely rejected. As such, the closed-loop system is said to be *robust*, i.e. it is insensitive to mismatches between the model used for control law design and the plant on which it will be implemented.

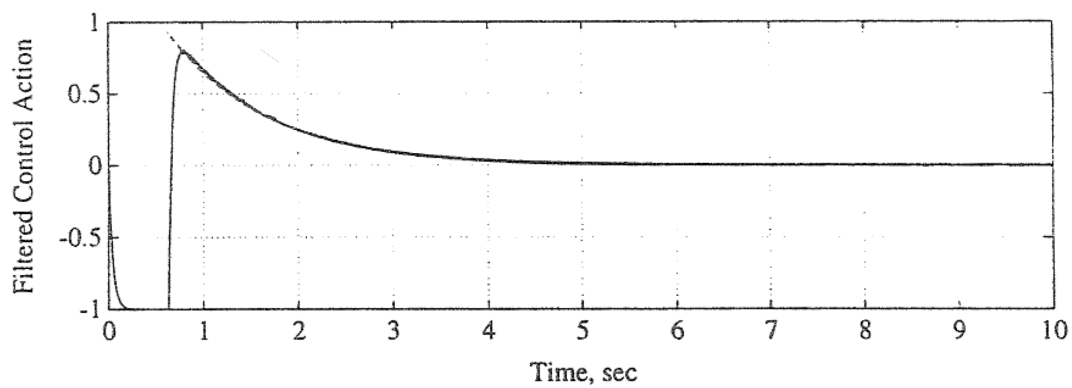
$$\dot{s}(t) = m\dot{y}(t) - a_1 \sin y(t) + u(t)$$

The equivalent control $\quad u_{eq}(t) = -m\dot{y}(t) + a_1 \sin y(t)$

The discontinuity control actions cancels the uncertainty and disturbances such as $a_1 \sin y(t)$



Applied control action



Filtered control action compared to the equivalent control

Sliding Mode Controller for a Manipulator

A candidate control structure

$$u(t) = l_1 y(t) + l_2 \dot{y}(t) - \rho \operatorname{sgn}(s(t))$$

l_1, l_2 and ρ represent scalars yet to be designed.

Choose the three parameters so that the inequality $s\dot{s} < 0$ is always satisfied

$$s\dot{s} = s(m\dot{y} + \ddot{y}) = s(m\dot{y} - a_1 \sin(y) + u)$$

$$s\dot{s} = s(m\dot{y} - a_1 \sin(y) + l_1 y + l_2 \dot{y} - \rho \operatorname{sgn}(s))$$

By choosing $l_1 = 0$ and $l_2 = -m$

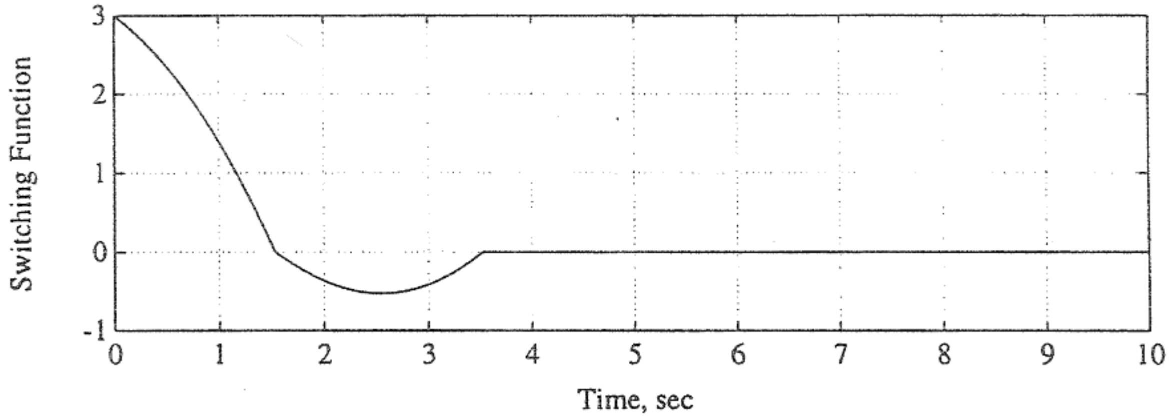
$$s\dot{s} = -s a_1 \sin(y) - \rho|s| < |s|(a_1 - \rho)$$

by choosing $\rho > a_1 + \eta$ where η is a positive design scalar

$$s\dot{s} < -\eta|s| \qquad \eta\text{-}\textit{reachability condition.}$$

Control law of $u(t) = l_1 y(t) + l_2 \dot{y}(t) - \rho \operatorname{sgn}(s(t))$ **guarantees that, whenever the sliding surface is reached, an ideal sliding motion takes place**

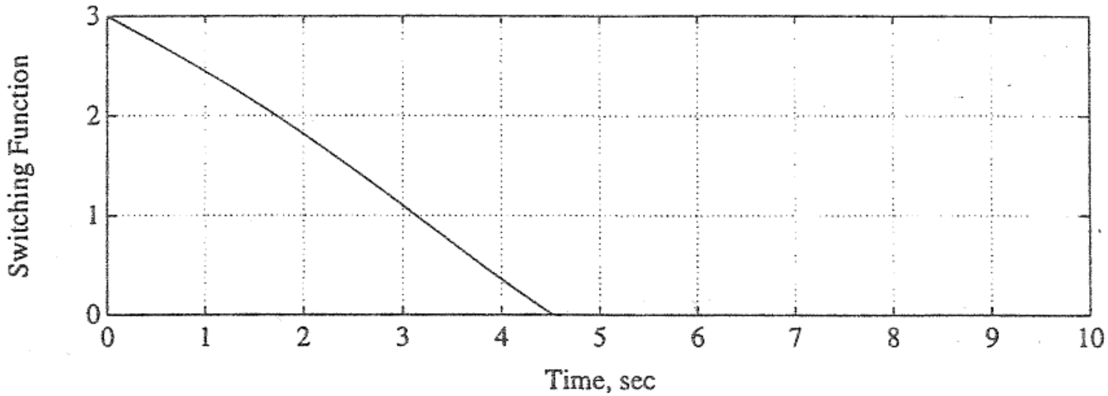
$a_1 = 0.25$ $y = 3$ and $\dot{y} = 0.$ $u(t) = -\mathrm{sgn}\,(s(t))$



The sign of the control law switches, the trajectory pierces the switching line and moves away before intercepting the line again at approximately 3.5 seconds, at which point sliding takes place.

$a_1 = 0.25$ $y = 3$ and $\dot{y} = 0$. $u(t) = -m\dot{y}(t) - \rho\,\mathrm{sgn}(s(t))$

with $m = 1$ and $\rho = 1$



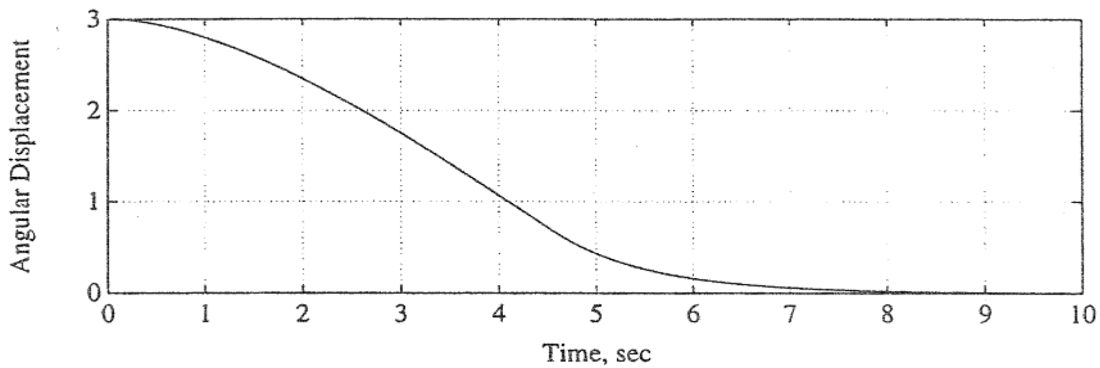
Switching function with respect to time

no overshoot can occur

the closed-loop performance is unduly sluggish.

$\rho = 1$ is rather conservative

it need only be greater than 0.25 for the η-reachability condition



Angular displacement with respect to time

A lower value of ρ reduce the amplitude of the high frequency switching.

To overcome this difficulty, add the term $-\Phi s$

Φ is a positive design scalar.

Sliding Mode Controller for a Manipulator

$$u(t) = -m\dot{y}(t) - \rho\,\mathrm{sgn}(s(t)) - \Phi s$$

$$l_1 = -\Phi m \qquad l_2 = -(m + \Phi)$$

$$u(t) = -(m + \Phi)\,\dot{y}(t) - \Phi m\, y(t) - \rho\,\mathrm{sgn}(s(t))$$

$$s\dot{s} = s(m\dot{y} + u) = s(m\dot{y} - (m+\Phi)\dot{y} - \Phi m y - \rho\,\mathrm{sgn}(s)) = -\Phi s^2 - \rho s\,\mathrm{sgn}(s)$$

$$s\dot{s} \le -\Phi s^2 - \eta|s|$$

since $\Phi s^2 \ge 0$, an η-reachability condition has been established
a sliding motion will take place.

By ignoring the nonlinear term

$$\frac{d}{dt}|s(t)| \le -\Phi|s(t)| \qquad \text{implies} \qquad |s(t)| \le |s(0)|e^{-\Phi t}$$

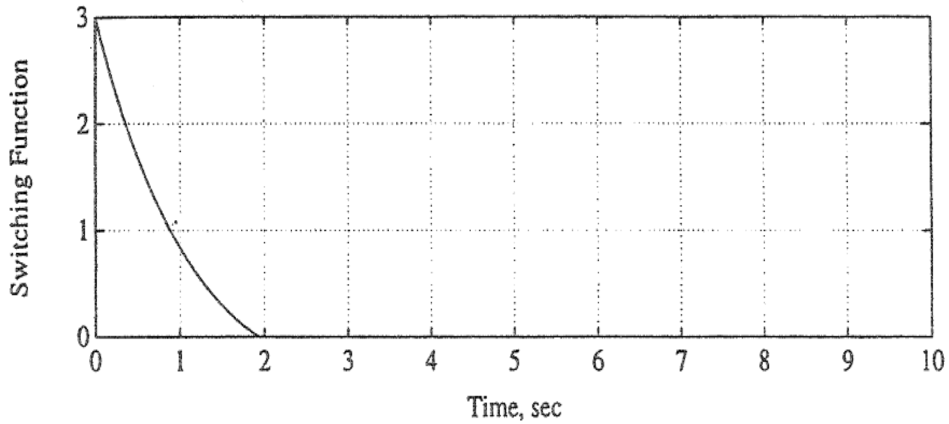
$|s(0)|$ represents the initial distance away from the sliding surface

Φ can thus be seen to affect the rate at which the sliding surface is attained.

ρ can be chosen as small as possible (to reduce the amplitude of the switching)

Φ can be chosen to determine the time taken to attain sliding.

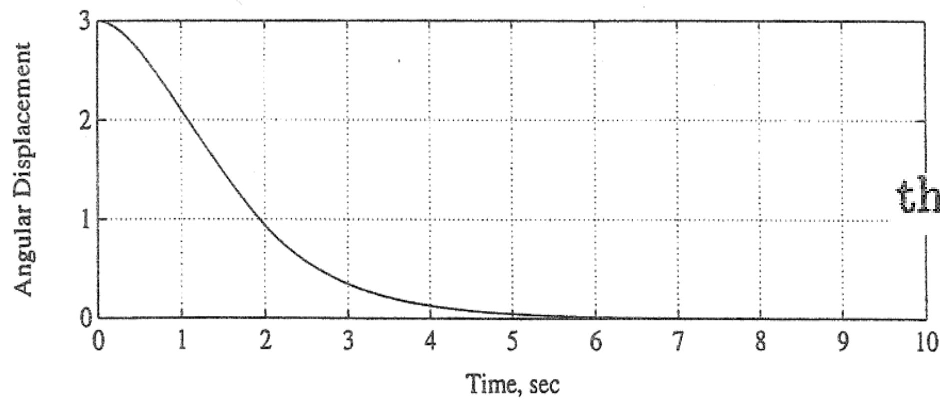
$a_1 = 0.25$ $y = 3$ and $\dot{y} = 0.$ $\Phi = 1$ and $\rho = 0.3$



Switching function with respect to time

To obtain a faster response, the value of Φ can be increased

the settling time is much improved



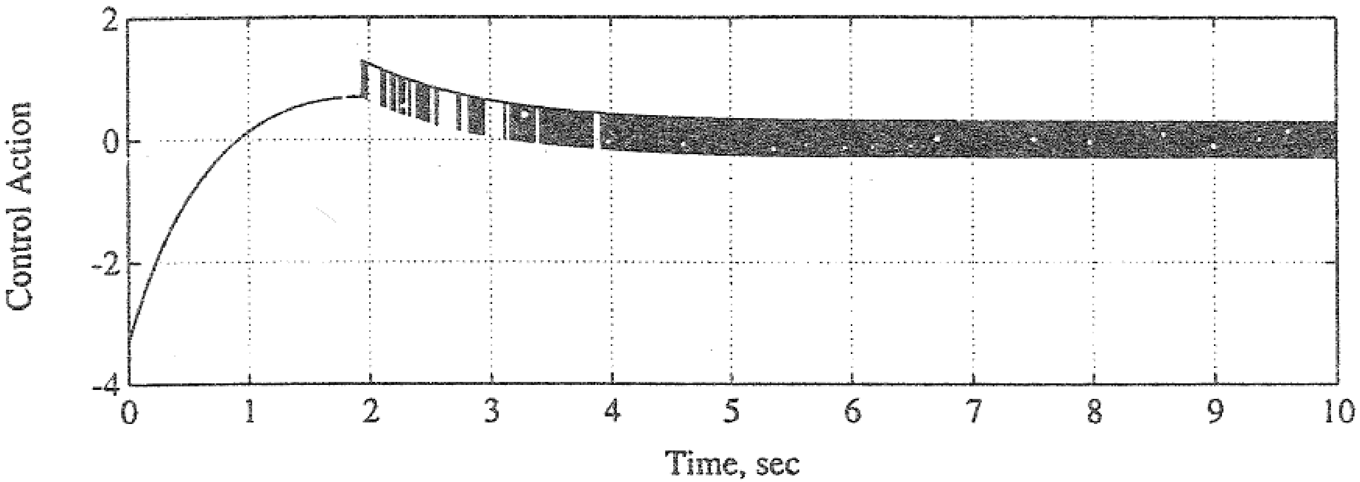
Angular displacement with respect to time

The big advantage of the controller

the control signal is much less aggressive

the amplitude of the switching now ± 0.3

Sliding Mode Controller for a Manipulator



Evolution of control action with respect to time

In the nominal double-integrator case with $\rho = 0$

$$\ddot{y}(t) + (m + \Phi)\,\dot{y}(t) + \Phi m\, y(t) = 0$$ a stable motion with poles at $\{-\Phi, -m\}$

the pole at $-\Phi$ corresponds to the rate at which the sliding surface is attained.

The other pole, located at $-m$, corresponds to the pole of the sliding motion.

the linear part of the control action establishes a sliding mode for the nominal system

the discontinuous component counteracts the effects of the uncertainty or nonlinearity.

PSEUDO-SLIDING WITH A SMOOTH CONTROL ACTION

its chattering behaviour would still not be considered acceptable.

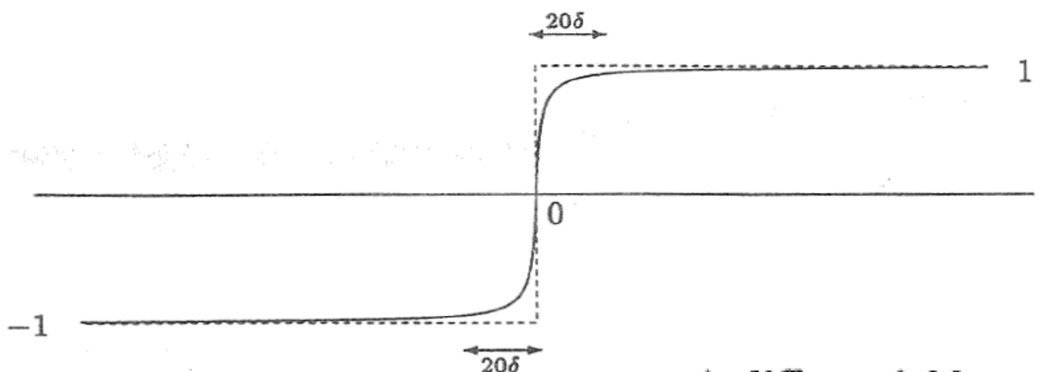
to attempt to smooth the discontinuity in the signum function

the sigmoid-like function $\nu_\delta(s) = \dfrac{s}{(|s| + \delta)}$ δ is a small positive scalar

as $\delta \to 0$, the function $\nu_\delta(\cdot)$ tends pointwise to the signum function.

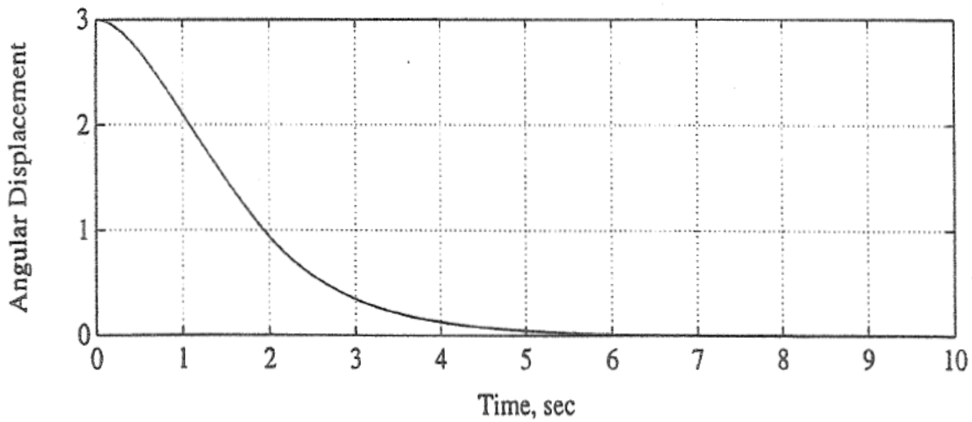
$u(t) = -m\dot{y}(t) - \rho\,\mathrm{sgn}(s(t)) - \Phi s$ $\nu_\delta(s)$ replacing $\mathrm{sgn}(s)$ with $\delta = 0.005$

$a_1 = 0.25$ $y = 3$ and $\dot{y} = 0$. $\Phi = 1$ and $\rho = 0.3$



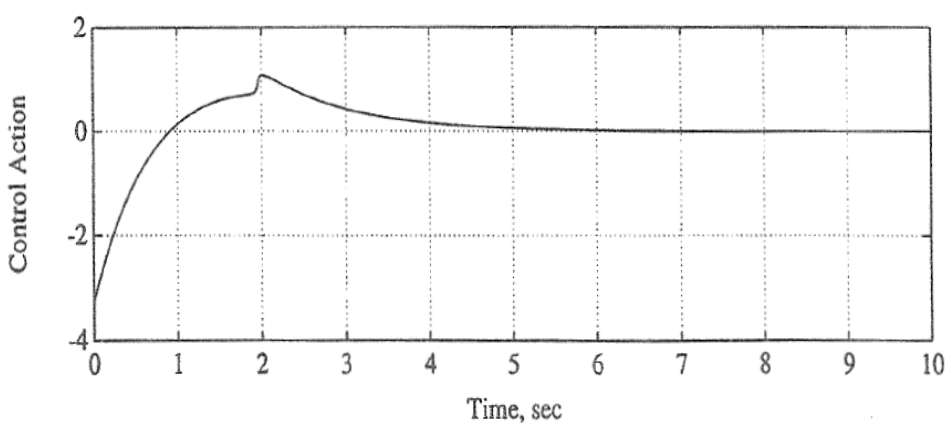
A differentiable approximation of the signum function

Sliding Mode Controller for a Manipulator



Angular displacement with respect to time

Indistinguishable from the previous response



Evolution of control action with respect to time

the control action is smooth

ideal sliding no longer takes place:

Control action only drives the states
to a neighborhood of the switching surface

Arbitrarily close approximation to ideal sliding
Can be obtained by making δ small

often referred to as *pseudo-sliding*.

Manipulator Dynamics

➢ **Robot dynamic equation**:

$$\tau = M(\theta)\ddot{\theta} + V\left(\theta, \dot{\theta}\right) + G(\theta) + \Delta(t) \qquad (1)$$

$$\Rightarrow \quad \ddot{\theta} = M(\theta)^{-1}\left[\tau - V\left(\theta, \dot{\theta}\right) - G(\theta) - \Delta(t)\right] \qquad (2)$$

Where:

$\theta \in R^n$ is the state vector

τ is the torque produced by actuators

$M(q) \in R^{n\times n}$ is the mass matrix

$V_m(q, \dot{q}) \in R^n$ is the vector of centrifugal and Coriolis terms

$G(q) \in R^n$ is the vector of gravity terms

$\Delta(t) \in R^n$ is the unmodelled error of system

Model manipulator

➢ From Eq. (2), we can rewrite in state space form as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + g(x)\boldsymbol{\tau} - d(x) \\ y &= x_1 \end{aligned} \qquad (3)$$

Where

$x_1 = \theta \in R^n$

$x_2 = \dot{\theta} \in R^n$

$f(x) = M(\theta)^{-1}[-V(\theta, \dot{\theta}) - G(\theta)]$

$g(x) = M^{-1}(\theta)$

$d(x) = M^{-1}(\theta)[\Delta(t)]$ is the uncertainty of the system.

The uncertainty is bounded: $|M^{-1}(\theta)[\Delta(t)]| \leq$ D

Design of Sliding mode control

- The design procedure of the sliding mode control includes two main steps:

  - The first step involves the construction of the desired sliding surface, which is chosen such that when it converges to zero, the desired control is achieved.

  - The next step is to select a control law that forces the system state to reach the sliding surface in a finite time.

Design of Sliding mode control

- The first step is to choose a proper switching surface:

$$s = \dot{e} + \lambda e \qquad (4)$$

Where

$e = x_d - x_1$

x_d is the desired trajectory

λ is a strictly positive constant.

- The second step, to ensure the trajectories of the system approach the sliding surface, the derivative of the sliding surface $\dot{s} = 0$ should be satisfied such that

$$\begin{aligned}\dot{s} &= \ddot{e} + \lambda\dot{e} \\ &= \ddot{x}_d - \dot{x}_2 + \lambda\dot{e} \\ &= \ddot{x}_d + \lambda\dot{e} - f(x) - g(x)\boldsymbol{\tau} + d(x)\end{aligned}$$

Design of Sliding mode control

- According to the sliding mode design procedure, we choose:

$$\tau = \tau_{eq} + \tau_{SMC} \qquad (5)$$

  - The equivalent control signal τ_{eq} is obtained by equation $\dot{s} = 0$ without considering the presence of the system uncertainties. :

$$\tau_{eq} = g(x)^{-1}[\ddot{x}_d + \lambda\dot{e} - f(x)] \qquad (6)$$

  - τ_{SMC} is the term that compensates for the effect of the uncertainties:

$$\tau_{SMC} = g(x)^{-1}\rho sign(s) \qquad (7)$$

where ρ is a constant chosen based on the upper bound of the modeling uncertainties in the system.

- So that:

$$\tau = g(x)^{-1}[\ddot{x}_d + \lambda\dot{e} - f(x) + \rho sign(s)] \qquad (8)$$

$$\tau = M(\theta)[\ddot{x}_d + \lambda\dot{e} + M(\theta)^{-1}\left[V\left(\theta,\dot{\theta}\right) + G(\theta)\right] + \rho sign(s)] \; (9)$$

$$\tau = M(\theta)(\ddot{x}_d + \lambda\dot{e}) + V\left(\theta,\dot{\theta}\right) + G(\theta) + M(\theta)\rho sign(s)$$

Lyapunov function

- Define a Lyapunov function candidate as $V = \frac{1}{2}s^2$, its time derivative given by

$$\dot{V} = s\dot{s} = s[\ddot{x}_d + \lambda\dot{e} - f(x) - g(x)\tau + d(x)]$$

$$= s(\ddot{x}_d + \lambda\dot{e} - f(x) - g(x)g(x)^{-1}[\ddot{x}_d + \lambda\dot{e} - f(x) + \rho sign(s)] + d(x))$$

$$= s[-\rho sign(s) + d(x)]$$

$$\leq s[-\rho sign(s) + D]$$

- If $\rho \geq D$ is satisfied, then $\dot{V} \leq 0$ is sufficiently ensured. This means that the system is stable.

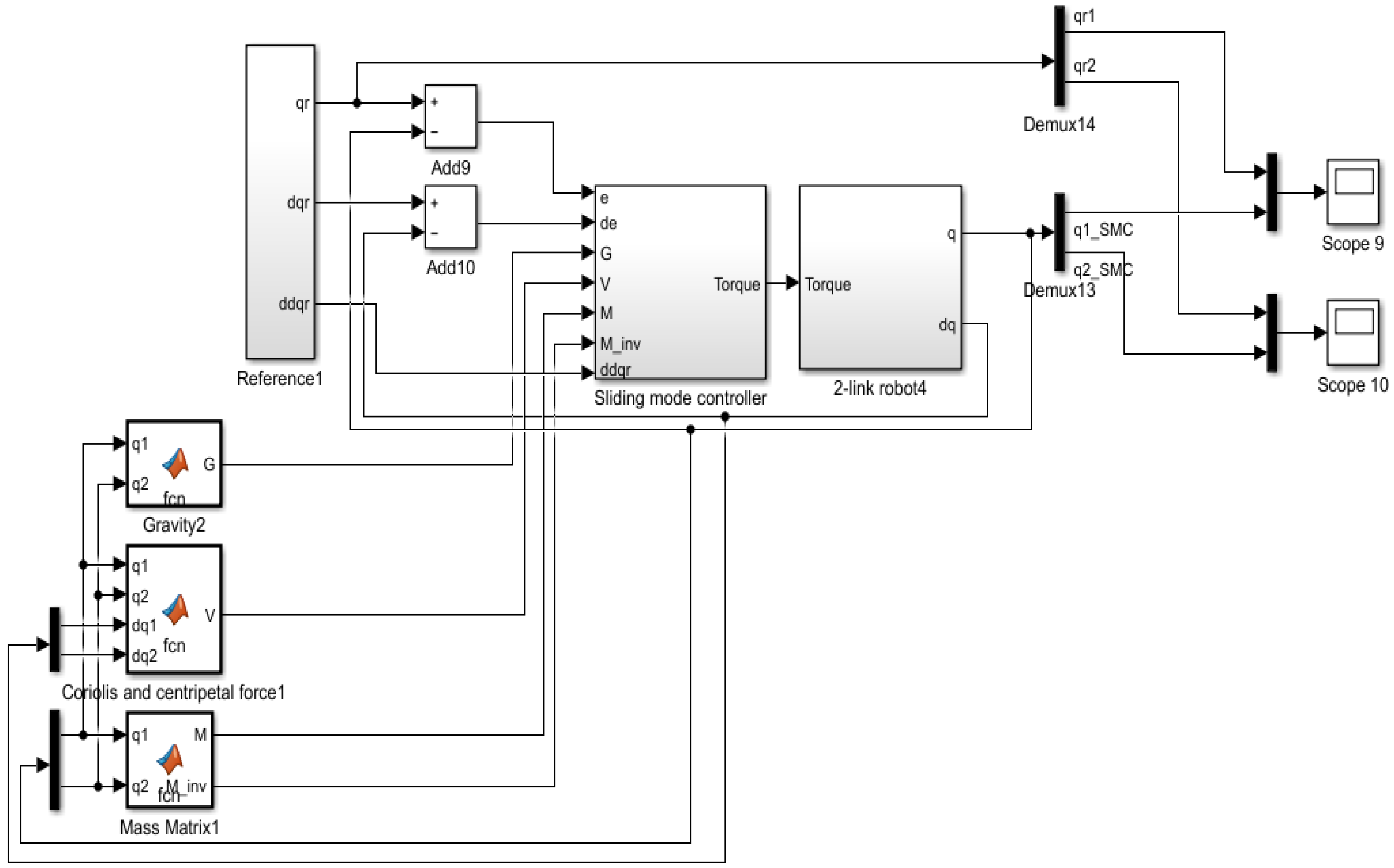
Lyapunov function

- The major drawback in the practical realization of SMC is chattering. To avoid chattering, various methods have been proposed to “soften” the chattering.

- For example, the continuous approximation method in which the $\text{sign}(s)$ function is replaced by a continuous approximation $\frac{s}{|s|+\epsilon}$ where ϵ is a small positive number.

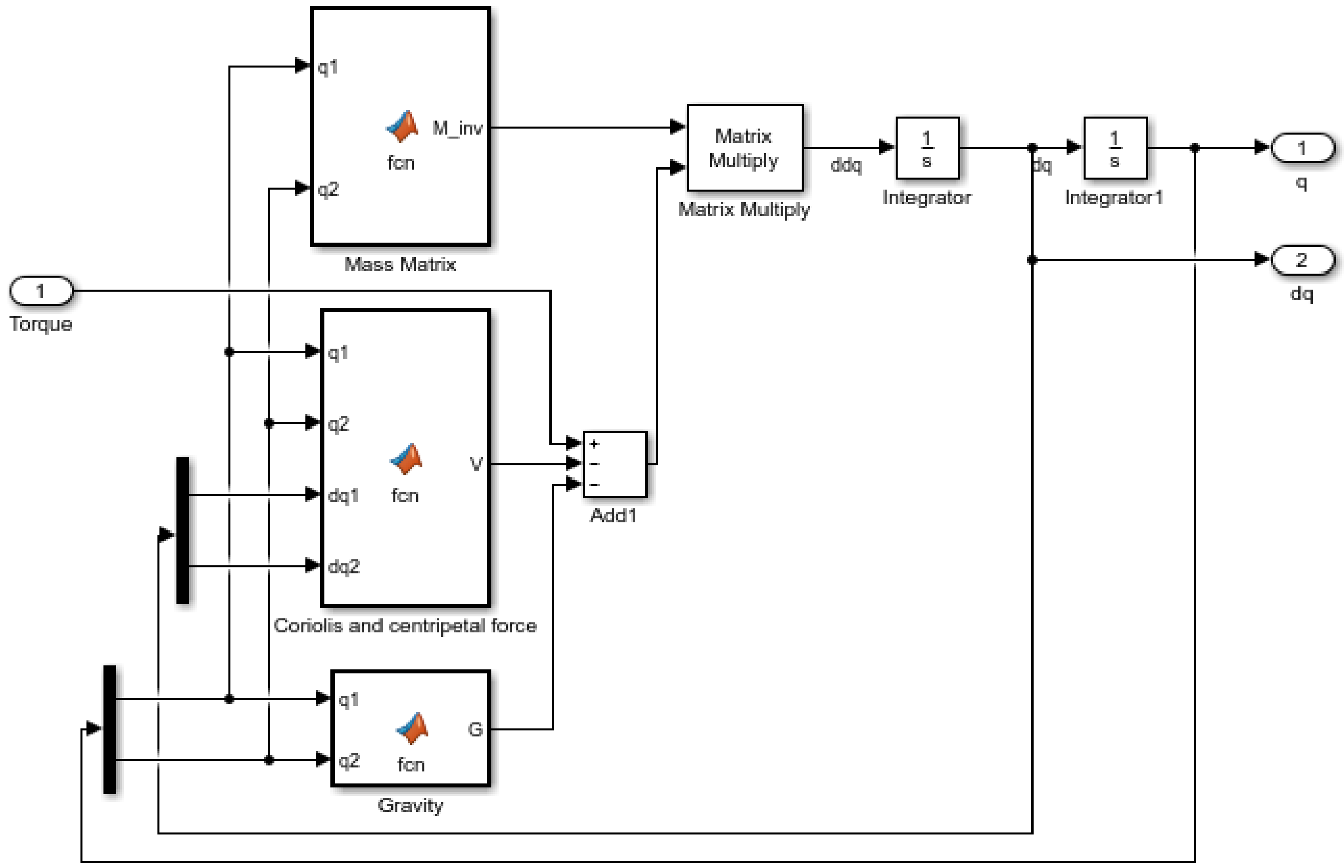
Simulation Results

Choose the constant λ=20 and sliding gain ρ=50.

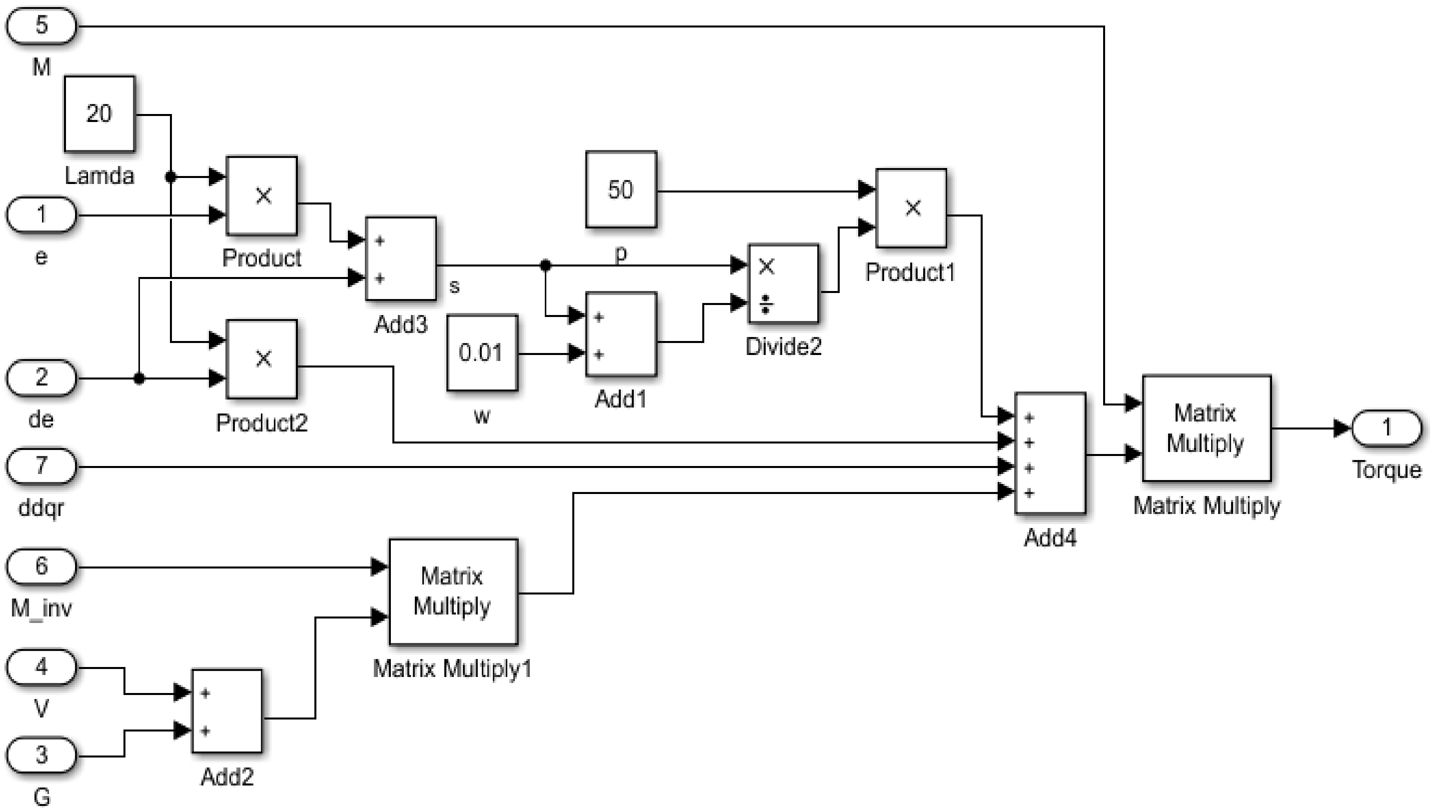


Simulation Results

2-link robot:

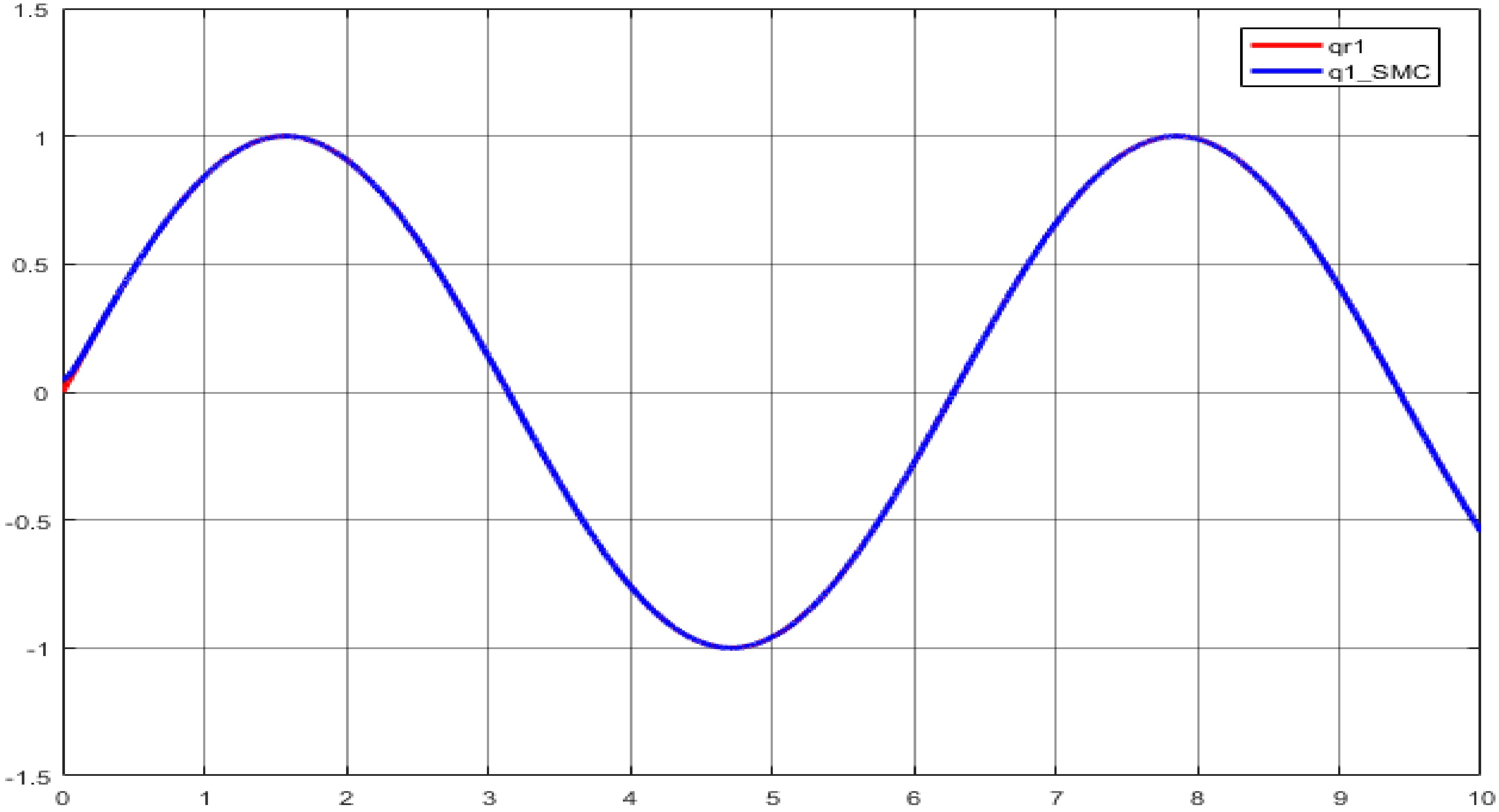


Sliding mode controller



Simulation Results

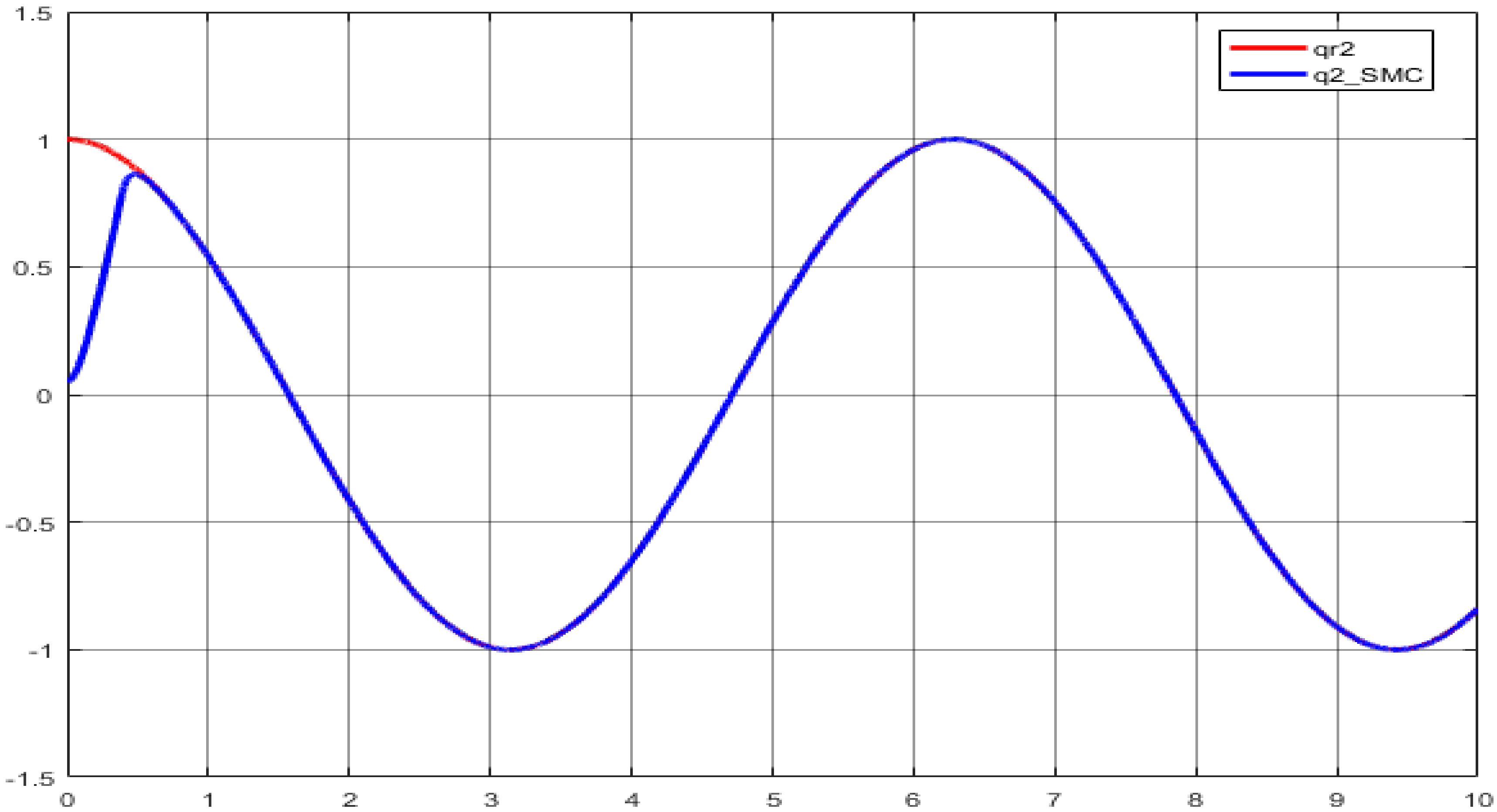
Result:



Joint 1

Simulation Results

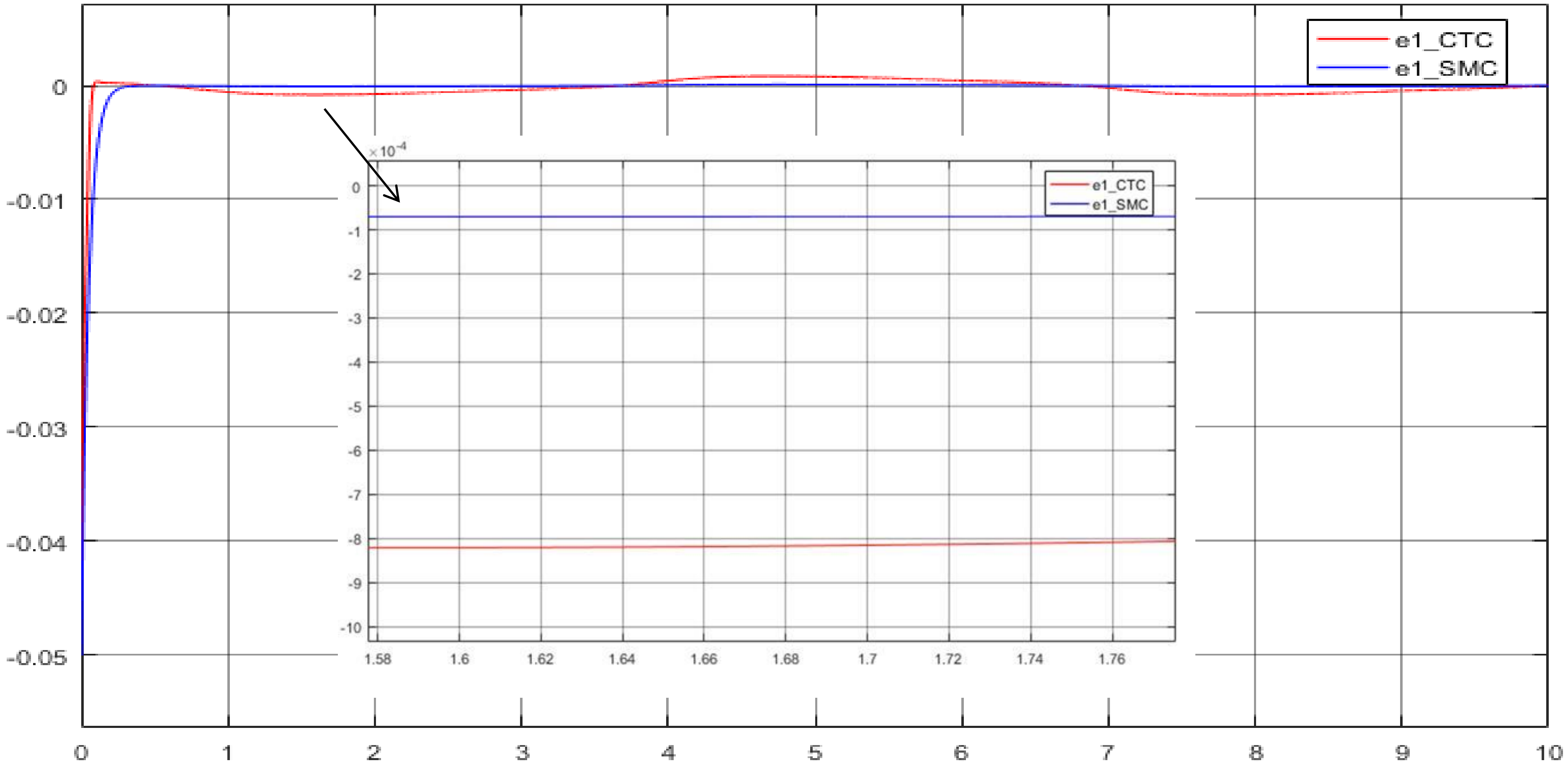
Result:



Joint 2

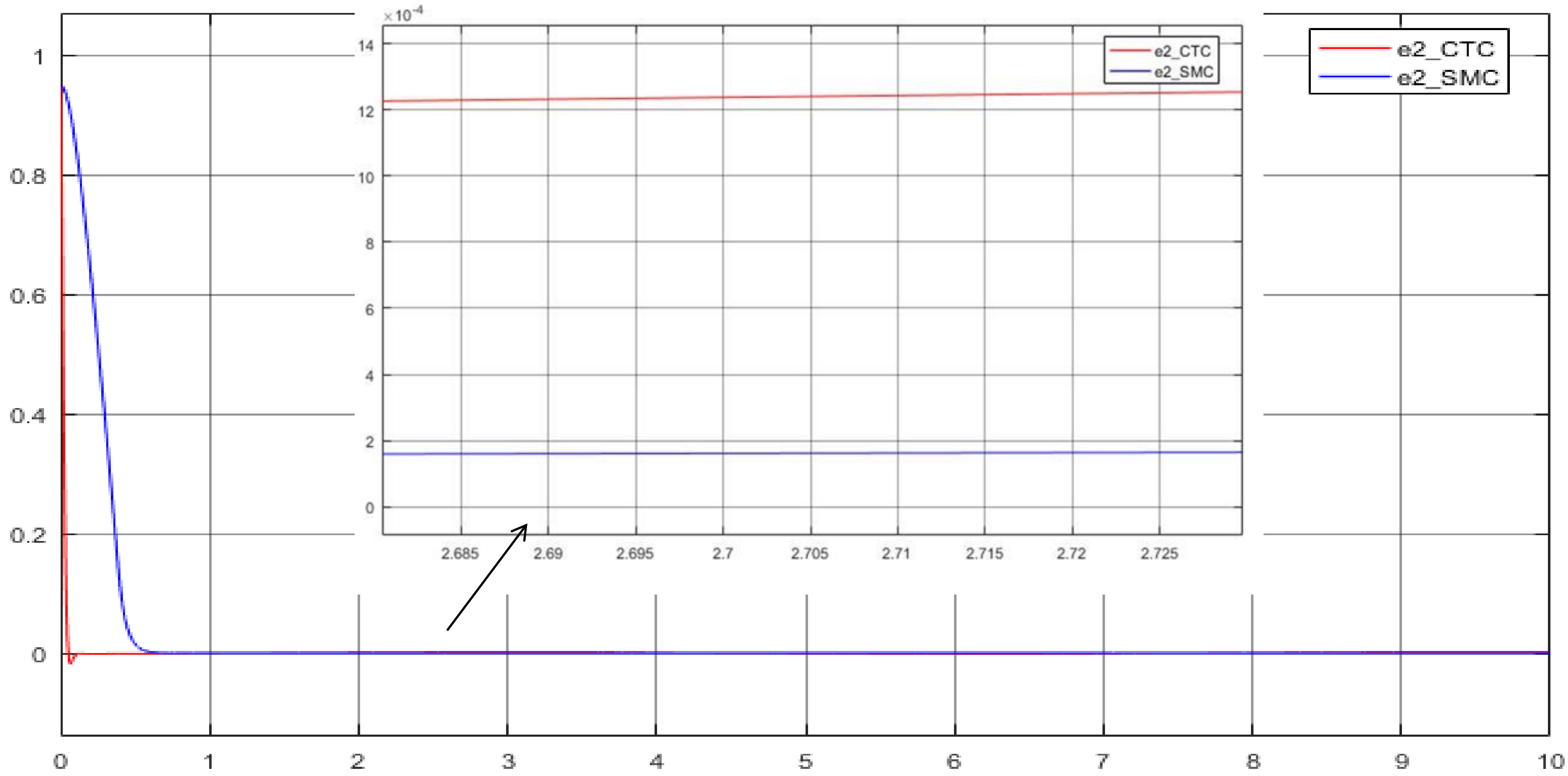
Simulation Results

Compare with Computed torque control



Joint 1

Compare with Computed torque control



Joint 2

Sliding mode controller

- The second order system: $\ddot{\theta} = f(\theta, \dot{\theta}) + bu$
- Let : $x_1 = \theta$, $x_2 = \dot{\theta}$, the system is described

$$\dot{x_2} = f(x_1, x_2) + bu$$

- The trajectory command is denoted as : x_d , then the error: $e = x_d - x_1$
- The sliding surface: $s = ce + \dot{e}$, and we easily get:

$$\begin{aligned} \dot{s} &= c\dot{e} + \ddot{e} \\ &= c\dot{e} + \ddot{x}_d - \ddot{x}_1 \\ &= c\dot{e} + \ddot{x}_d - (f(x_1, x_2) + bu) \end{aligned}$$

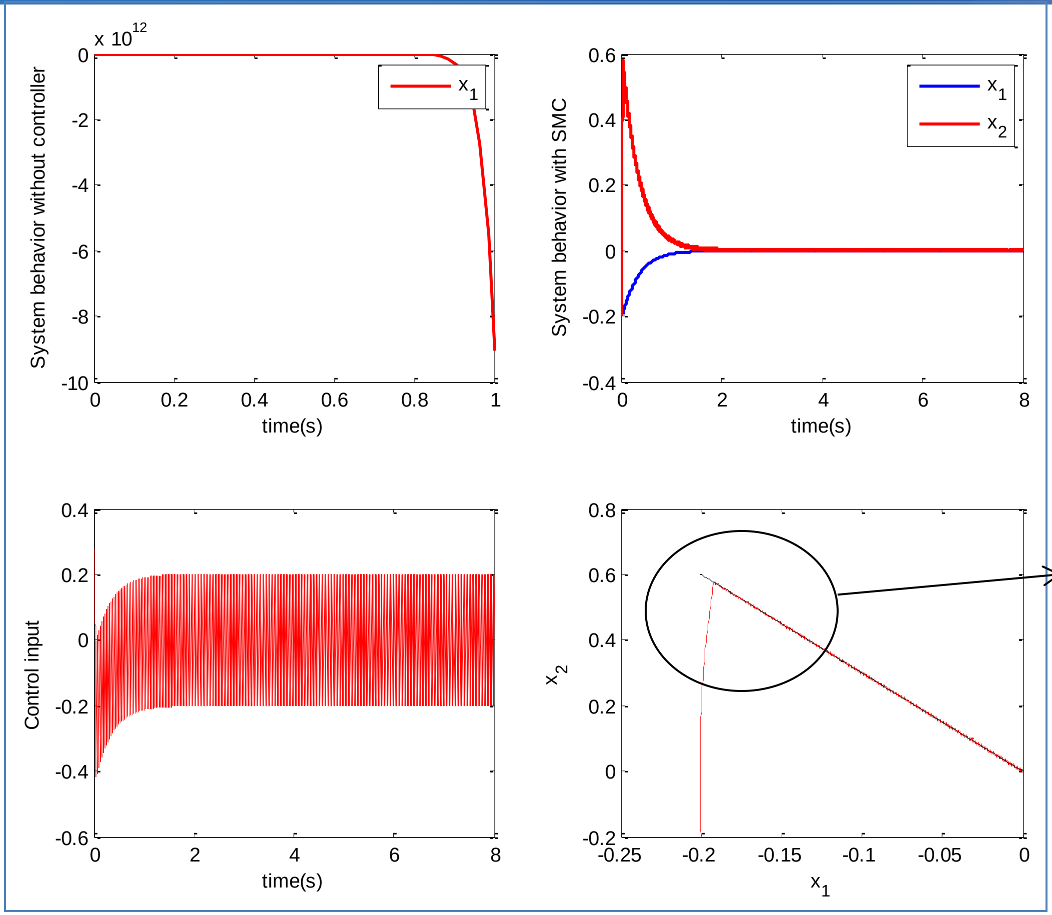
- Following Utkin's theory, we have the equivalent control part and the switching part as follows:

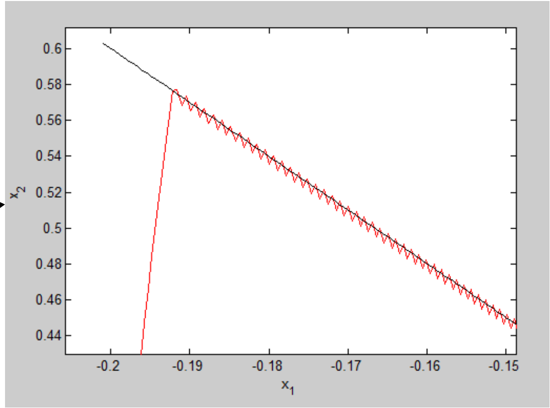
$$u_{eq} = \frac{1}{b}[c\dot{e} + \ddot{x}_d - f(x_1, x_2)]$$

$$u_{sw} = \frac{1}{b}\eta . sgn(s)$$

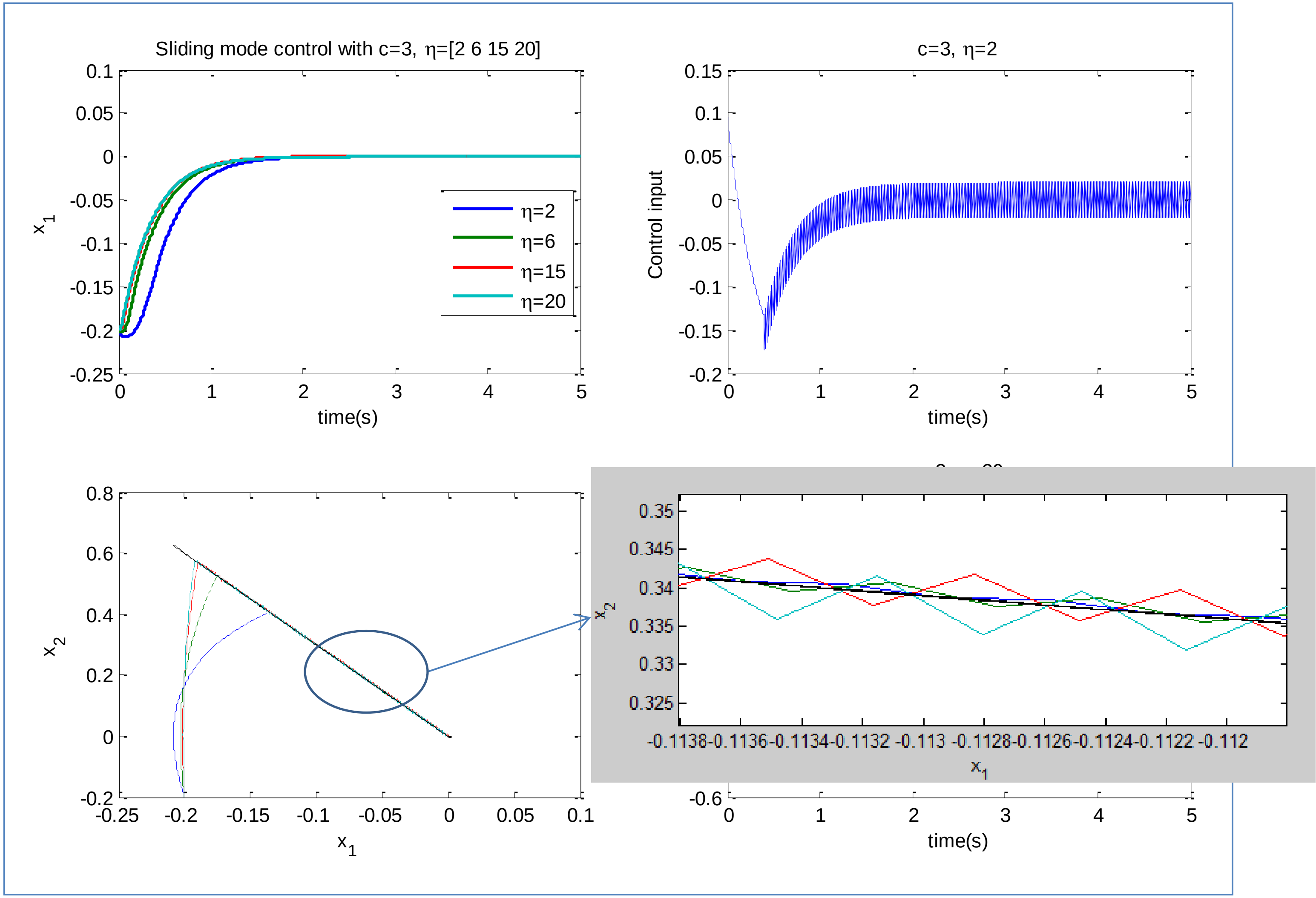
- The total control signal: $u_{SMC} = u_{eq} + u_{sw}$ that guarantees : $s\dot{s} = -\eta|s| < 0$

The system: $\ddot{\theta} = 35\dot{\theta} + 100u$ with initial condition [x1,x2]=[-0.2,-0.2]

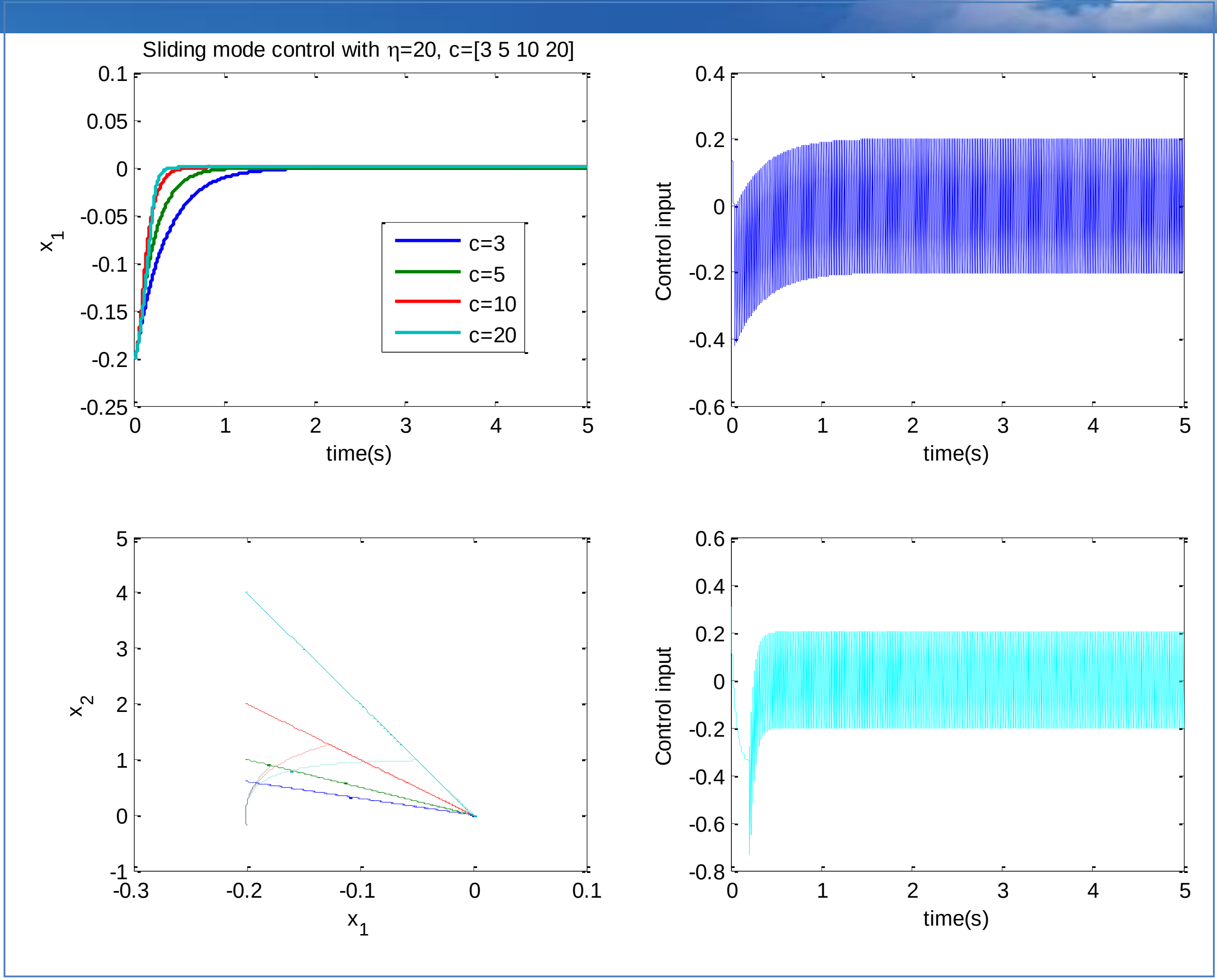




1. The original system is unstable at equilibrium point x1=0
2. The sliding mode controller makes system stable at its equilibrium point
3. Because the control signal includes the high switching part, so it causes chattering phenomenon

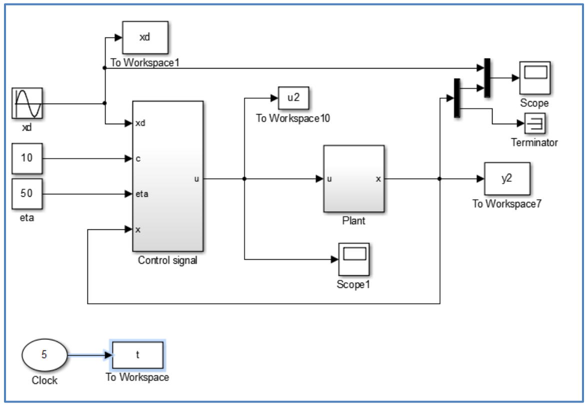


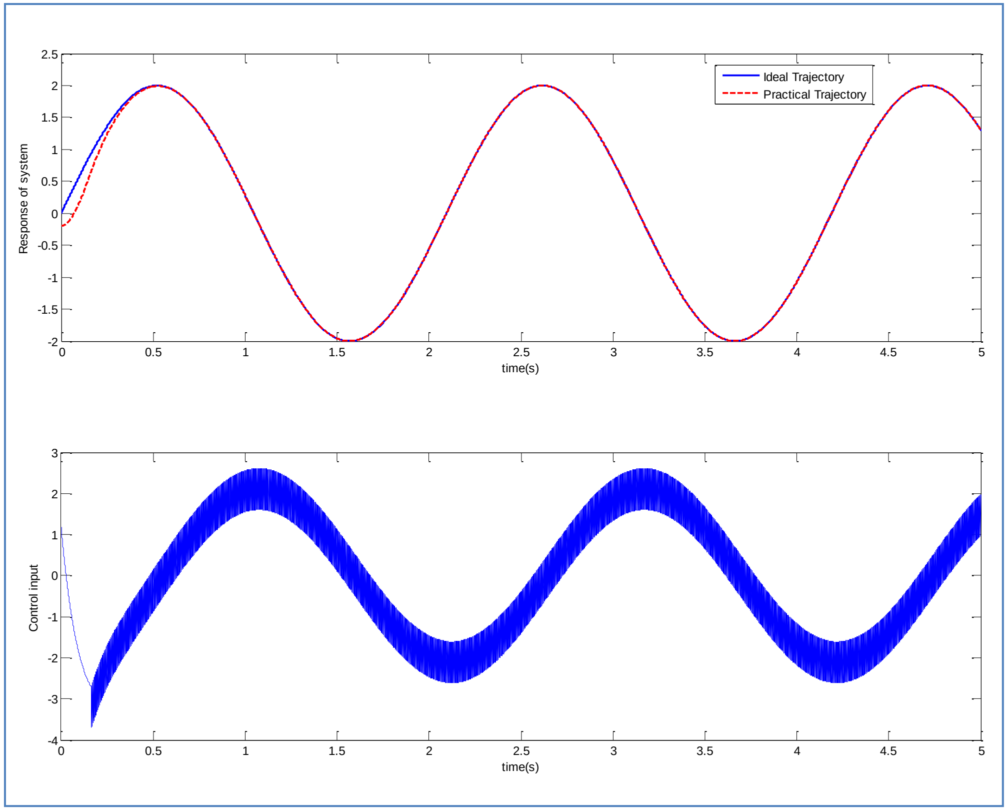
1. The effect of eta parameter to system behavior. An fundamental trade-off between the speed of convergence and control signal, chattering level



1. The effect of ‘c’(λ) parameter to system behavior. An fundamental trade-off between the speed of convergence and control signal, chattering level

A simulation results for trajectory tracking control problem





High gain observer based Adaptive Sliding Mode Control for robot manipulators

Student: **Tran Xuan Toa**

Professor: **Hee-Jun Kang**

Contents

- Preliminaries
- High-gain observer
- Proposed control scheme
- Stability analysis
- Simulation and results.

Preliminaries

- The dynamics of an n-joint robotic manipulator can be described by the following equation:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \tau_d = \tau \qquad (1)$$

- The manipulator dynamics has the following properties:
  - The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric matrix.
  - The dynamics of robotic manipulator can be linearly parameterized as follows:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{F}(\dot{\boldsymbol{q}}) = \boldsymbol{\tau} - \boldsymbol{\tau}_d = \boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\theta} \qquad (2)$$

Where $Y(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n\times p}$ is the regression matrix, and $\theta \in \mathbb{R}^p$ is the constant vector of system parameters

- Assumption 1: The disturbance torques τ_d are bounded

$$|\tau_{di}| \leq D_i \qquad i = 1,2,\dots,n \qquad (3)$$

Preliminaries

- The dynamic equation of robot manipulator can be written in the state space as follow:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}x_2\\ f(x_1,x_2)+\Delta f\end{bmatrix} + \begin{bmatrix}0\\ g(x_1)\end{bmatrix}\tau$$

$$y = x_1 \qquad (4)$$

where $x_1 = q \,\epsilon \mathbb{R}^n$ is the vector of joint position, and

$$f(x_1,x_2) = M(x_1)^{-1}\big(-C(x_1,x_2) - G(x_1) - F(x_2)\big)$$

$$g(x_1) = M(x_1)^{-1}.$$

- Problem: Trajectory tracking control for robot manipulator :
  - Unknown system parameters
  - External disturbance torques
  - Without velocity measurements

Output feedback control ->
Velocity observer design

High-gain Observer

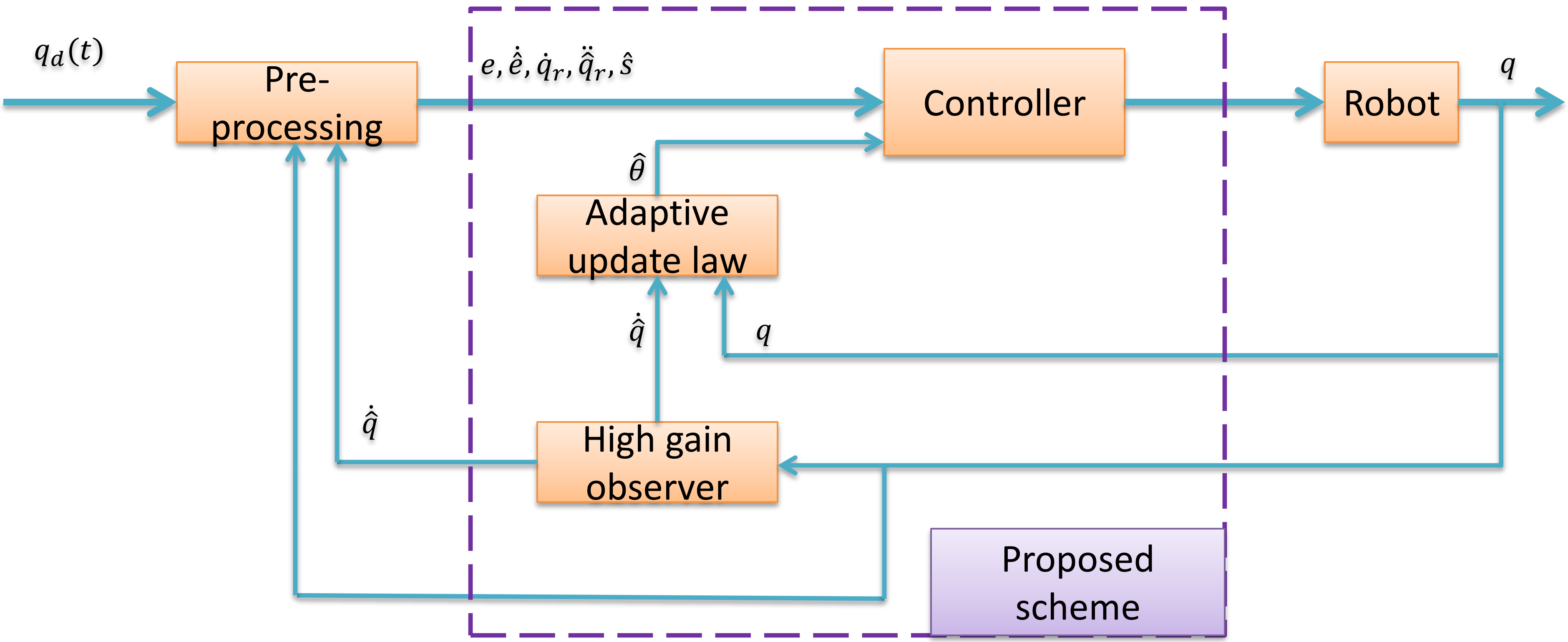
- High-gain observer robustly estimates velocity with fast convergence in the absence of measurement noise.
- Consider the robot manipulator (1), $x_1 = q$ is the measurement output. The high-gain observer is designed as:

$$\dot{\hat{x}}_1 = \hat{x}_2 - \frac{1}{\epsilon} L_p(\hat{x}_1 - x_1)$$

$$\dot{\hat{x}}_2 = -\frac{1}{\epsilon^2} L_v(\hat{x}_1 - x_1) \tag{5}$$

where $\hat{x}_1, \hat{x}_2$ denote the estimated values of x_1, x_2 respectively, ϵ is a small positive parameter, $L_p = diag(l_{pi}), Lv = diag(l_{vi})$ are positive definite matrices chosen such that $H = \begin{bmatrix} -L_p & I \\ -L_v & 0_{n\times n} \end{bmatrix}$ is a Hurwitz matrix.

Proposed control Scheme



Proposed control scheme

- For the system (1), The desired trajectories denote: $q_d(t)$, so the tracking error $e = q - q_d$. We also denote:

$$\dot{q}_r = \dot{q}_d - \Lambda e, \text{ where } \Lambda = diag([\lambda_1, \lambda_2, \dots, \lambda_n]), \lambda_i > 0$$

- Define the sliding surface as: $s = \dot{e} + \Lambda e = \dot{q} - \dot{q}_r$

- The observing sliding mode variables: $\hat{s} = \dot{\hat{e}} + \Lambda e$

- For eq (2), we denote $\hat{\theta}$ is the estimation of θ, and from (2) we have

$$\widetilde{M}(q)\ddot{q}_d + \tilde{C}(q,\dot{q})\dot{q}_d + \tilde{G}(q) + \tilde{F}(\dot{q}) = Y(q,\dot{q},\dot{q}_d,\ddot{q}_d)\tilde{\theta}$$

where

$$\widetilde{\mathrm{M}}(q) = \widehat{M}(q) - M(q)$$
$$\tilde{C}(q,\dot{q}) = \hat{C}(q,\dot{q}) - C(q,\dot{q})$$
$$\tilde{G}(q) = \hat{G}(q) - G(q)$$
$$\tilde{F}(\dot{q}\ ) = \hat{F}(\dot{q}) - F(\dot{q})$$
$$\tilde{\theta} = \hat{\theta} - \theta$$

Proposed control scheme

- We also have

$$\tilde{M}(q)\ddot{q}_r + \tilde{C}(q,\dot{q})\dot{q}_r + \tilde{G}(q) + \tilde{F}(\dot{q}) = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\theta}$$

- The proposed controller is given by:

$$\boldsymbol{\tau} = \boldsymbol{\hat{M}(q)\ddot{\hat{q}}_r + \hat{C}(q,\dot{\hat{q}})\dot{q}_r + \hat{G}(q) + \hat{F}(\dot{\hat{q}}) - K\hat{s} - \eta sign(\hat{s})} \quad (6)$$

cf) Inertia related adaptive control $\tau = \hat{M}(\theta)\ddot{\theta}_r + \hat{V}_m(\theta,\dot{\theta})\dot{\theta}_r + \hat{G}(\theta) + K_D r$

with : $K = diag([k_1, k_2, \ldots, k_n]), k_i > 0,$

the update law : $\dot{\hat{\theta}} = -\Gamma Y(q,\dot{\hat{q}},\dot{q}_r,\ddot{\hat{q}}_r)^T\hat{s}, \Gamma^{-1} = diag([\gamma_1,\gamma_2,\ldots,\gamma_p]), \gamma_j > 0$

$\eta_i = D_i + \xi_i, \xi_i > 0$

Stability analysis

- We consider the Lyapunov function candidate as:

$$V = \tfrac{1}{2}s^T M s + \tfrac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \qquad \text{cf)} \quad V = ½ r^T M(q) r + ½ \tilde{\varphi}^T \Gamma^{-1} \tilde{\varphi}$$

- Therefore, we have:

$$\dot{V}(t) = s^T M \dot{s} + \frac{1}{2} s^T \dot{M} s + \tilde{\theta} \Gamma^{-1} \dot{\tilde{\theta}}$$

$$= s^T (M(q)\ddot{q} - M(q)\ddot{q}_r) + \frac{1}{2} s^T \dot{M} s + \tilde{\theta} \Gamma^{-1} \dot{\tilde{\theta}}$$

$$= s^T (\tau - C(q,\dot{q})\dot{q} - G(q) - F(\dot{q}) - \tau_d - M(q)\ddot{q}_r) + \frac{1}{2} s^T \dot{M} s + \tilde{\theta} \Gamma^{-1} \dot{\tilde{\theta}} \tag{7}$$

$$= s^T (\tau - C(q,\dot{q})(s + \dot{q}_r) - G(q) - F(\dot{q}) - \tau_d - M(q)\ddot{q}_r) + \frac{1}{2} s^T \dot{M} s + \tilde{\theta} \Gamma^{-1} \dot{\tilde{\theta}}$$

$$= s^T (\tau - C(q,\dot{q})\dot{q}_r - G(q) - F(\dot{q}) - \tau_d - M(q)\ddot{q}_r) + \frac{1}{2} s^T (\dot{M} - 2C(q,\dot{q})) s + \tilde{\theta} \Gamma^{-1} \dot{\tilde{\theta}}$$

- Substituting (6) into (7) and base on the convergence of the high-gain observer, we have:

$$\dot{V}(t) \le -s^T K s - \sum \xi_i |s_i| \le 0$$

Thus, The system stability is guaranteed in the sense of Lyapunov theorem.

Simulation results

- Consider a direct drive vertical robot manipulator with 2DOF (Fernando Reyes and Rafael Kelly (Robotica-1997)that has the parameters and the entries of robot dynamics as follows :

$$M(\boldsymbol{q}) = \begin{bmatrix} \theta_1 + 2\theta_2 \cos(q_2) & \theta_3 + \theta_2 \cos(q_2) \\ \theta_3 + \theta_2 \cos(q_2) & \theta_3 \end{bmatrix},$$

$$C(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{bmatrix} -2\theta_2 \sin(q_2)\dot{q}_2 & -\theta_2 \sin(q_2)\dot{q}_2 \\ \theta_2 \sin(q_2)\dot{q}_1 & 0 \end{bmatrix},$$

$$\boldsymbol{g}(\boldsymbol{q}) = \begin{bmatrix} \theta_4 \sin(q_1) + \theta_5 \sin(q_1 + q_2) \\ \theta_5 \sin(q_1 + q_2) \end{bmatrix},$$

$$\theta_1 = m_1 l_{c1}^2 + m_2 l_1^2 + m_2 l_{c2}^2 + I_1 + I_2$$

$$\theta_2 = l_1 m_2 l_{c2}$$

$$\theta_3 = m_2 l_{c2}^2 + I_2$$

$$\theta_4 = g(l_{c1} m_1 + m_2 l_1)$$

$$\theta_5 = g m_2 l_{c2}$$

$$\tau_d = \begin{bmatrix} 2sin(2t) \\ 3sin(\pi t) \end{bmatrix}$$

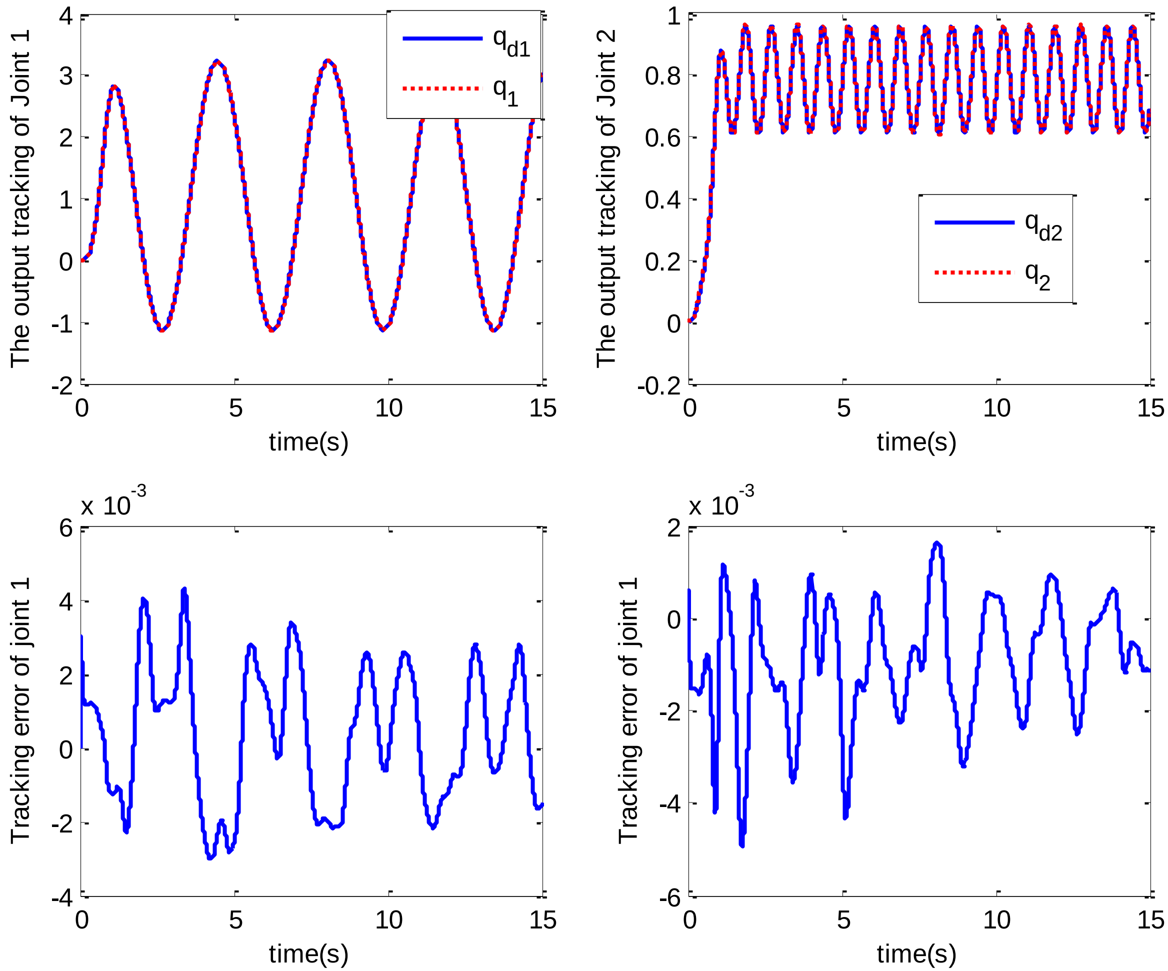
Table I. Parameter values.

Parameter	Notation	Value	Unit
Length link 1	l_1	0.45	m
Mass link 1	m_1	23.902	Kg
Mass link 2	m_2	3.880	Kg
Link (1) center of mass	l_{c1}	0.091	m
Link (2) center of mass	l_{c2}	0.048	m
Inertia link 1	I_1	1.266	Kg m^2
Inertia link 2	I_2	0.093	Kg m^2

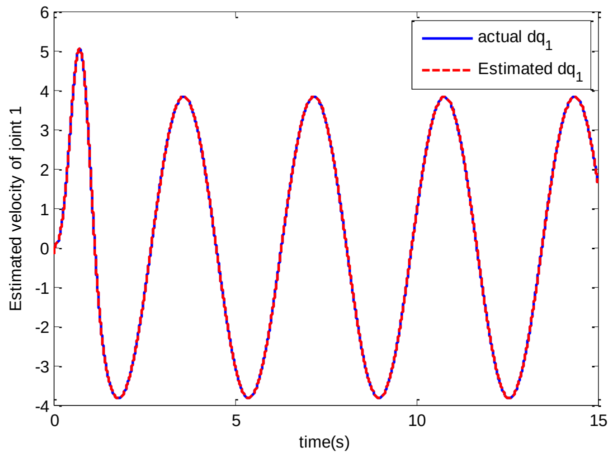
- The desired trajectory used in our simulation (Dawson DM, Carroll JJ, IEEE Transaction on control system 1994)

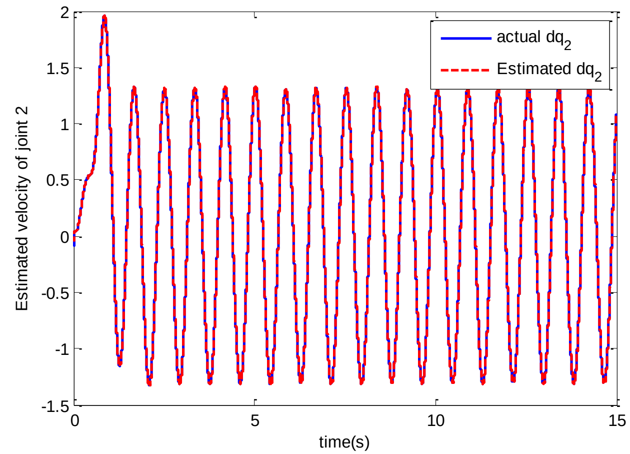
$$q_d(t) = \begin{bmatrix} 1.0472\left(1 - e^{-1.8t^3}\right) + 2.1816\left(1 - e^{-1.8t^3}\right)\sin(1.75t) \\ 0.7854\left(1 - e^{-2.0t^3}\right) + 2.1816\left(1 - e^{-2.0t^3}\right)\sin(7.5t) \end{bmatrix}(rad)$$

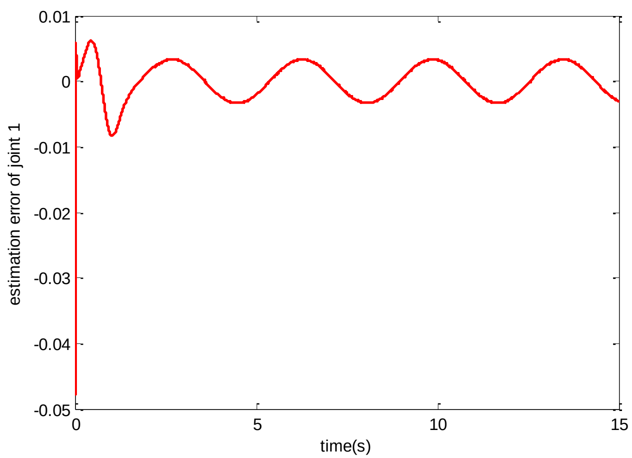
Simulation results

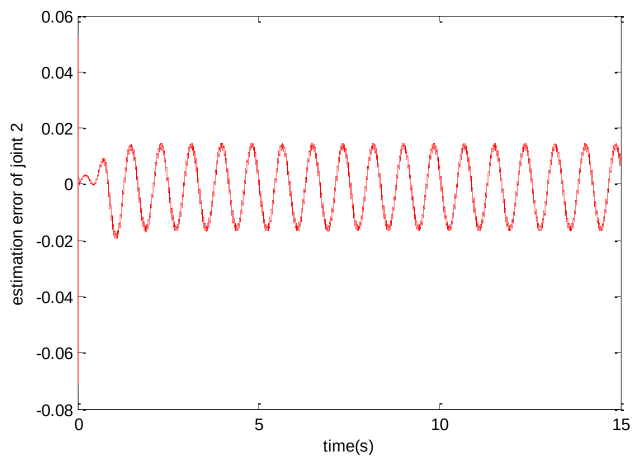


Simulation results

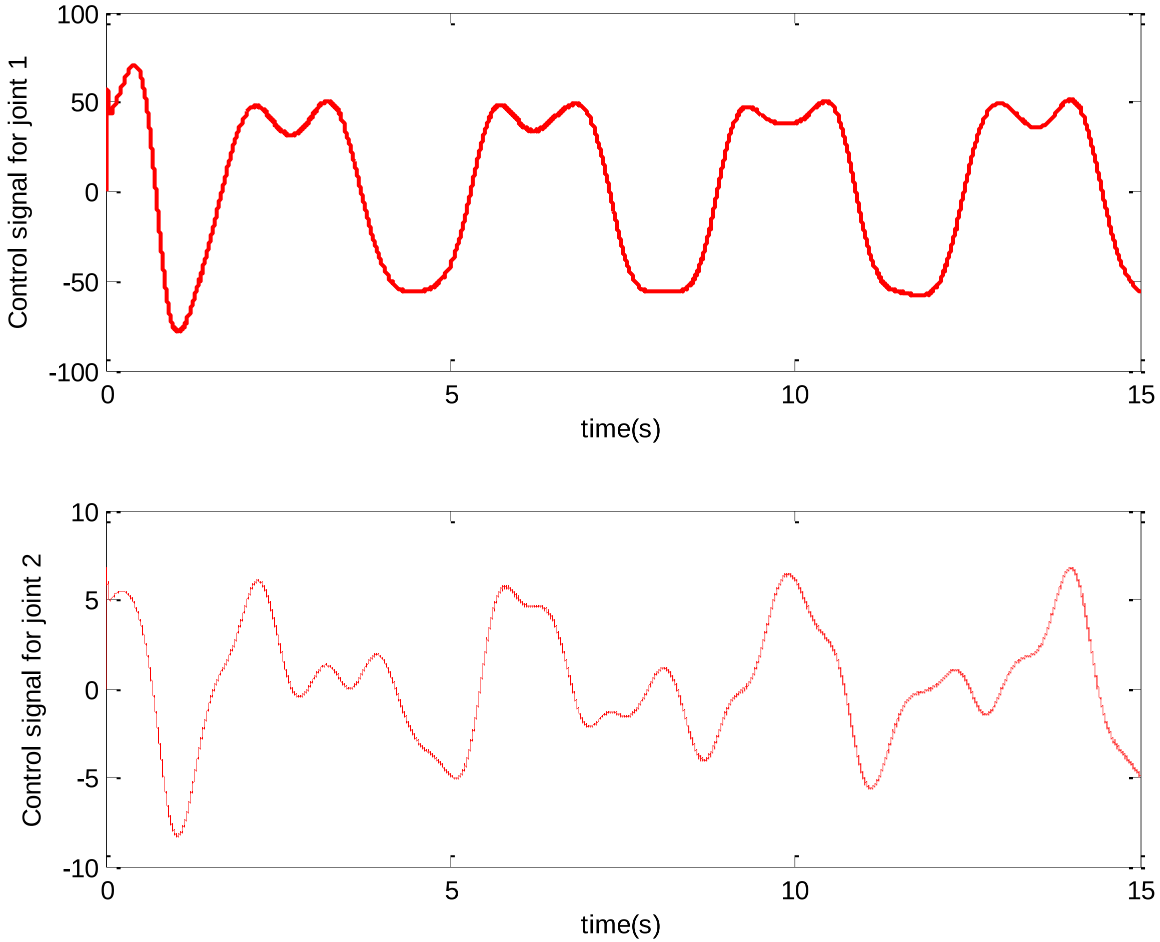




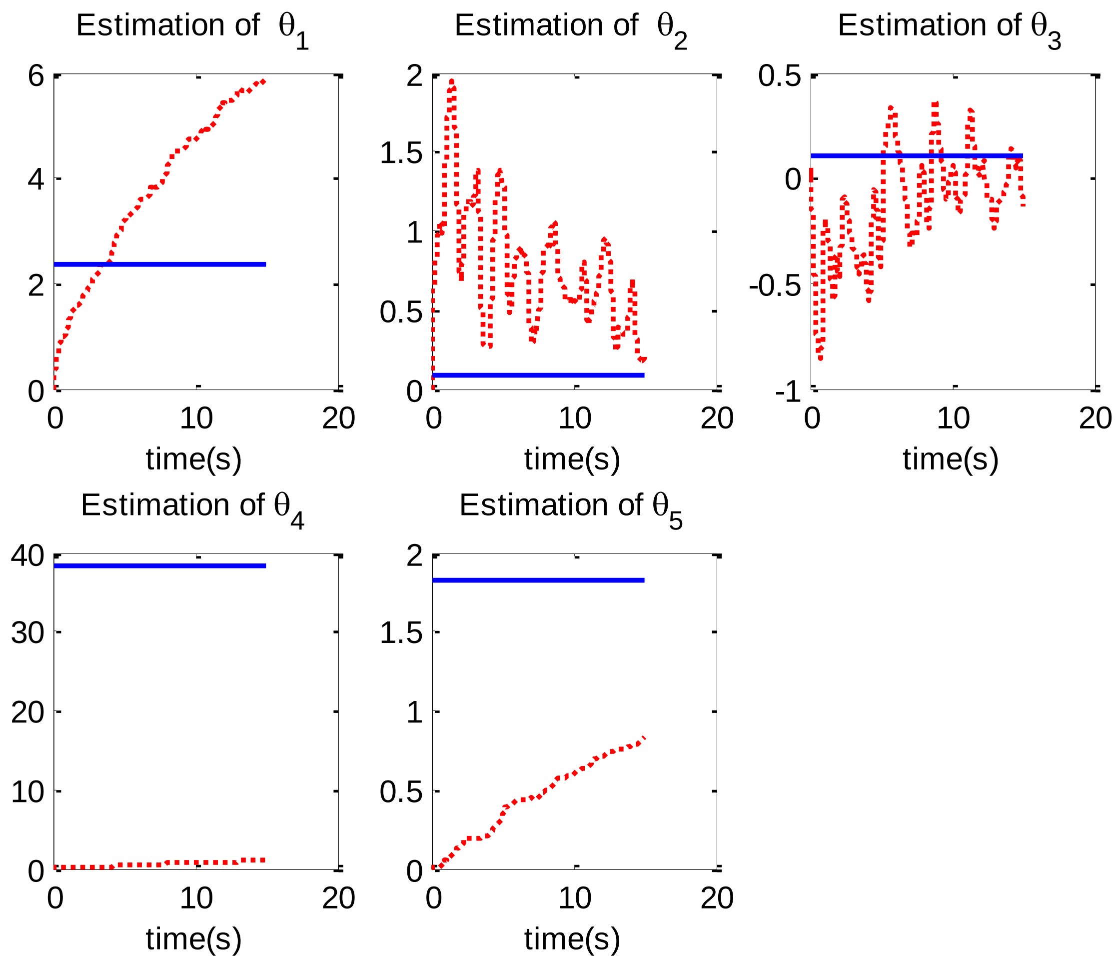




Simulation results



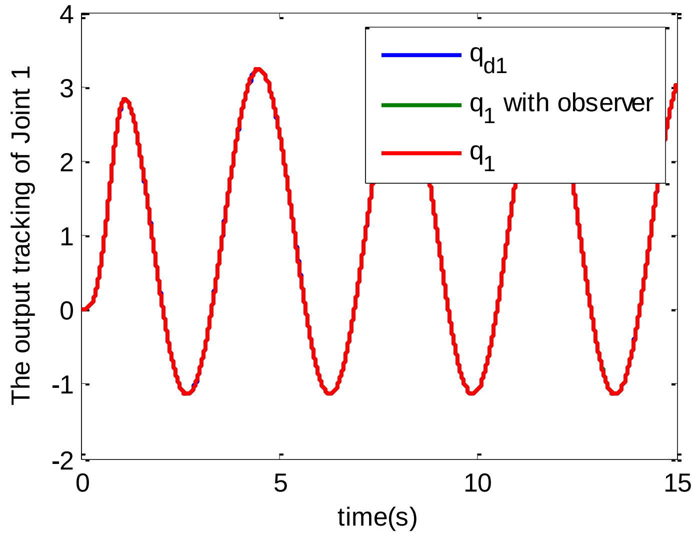
Simulation results

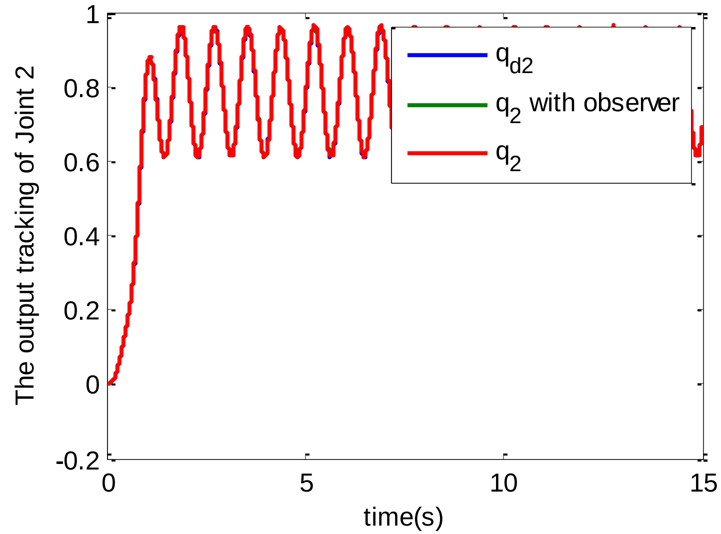


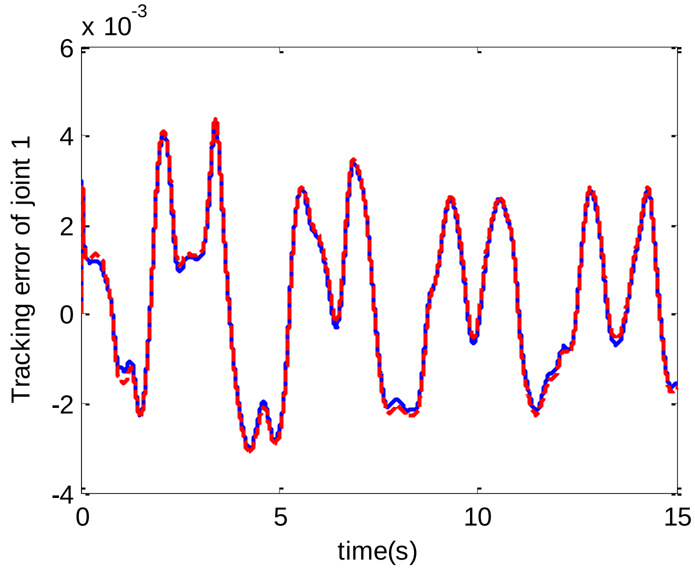
References

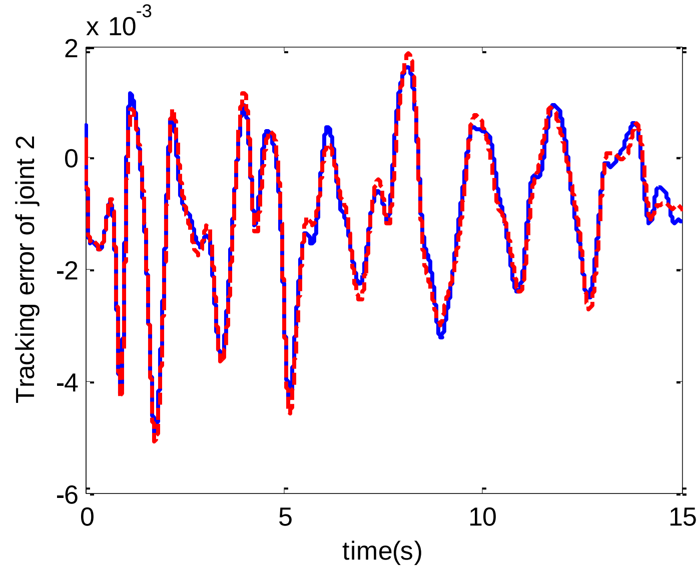
- F. Reyes and R. Kelly, “ Experimental Evaluation of Identification Schemes on a Direct Drive Robot”, *Robotica (1997),* volume 15, pp 563-571.
- Dawson DM, Carroll JJ, Schneider M. “Integrator backstepping control of a brush dc motor turning a robotic load”. *IEEE Transactions on Control System Technology* 1994;2(3):233–44

Simulation results

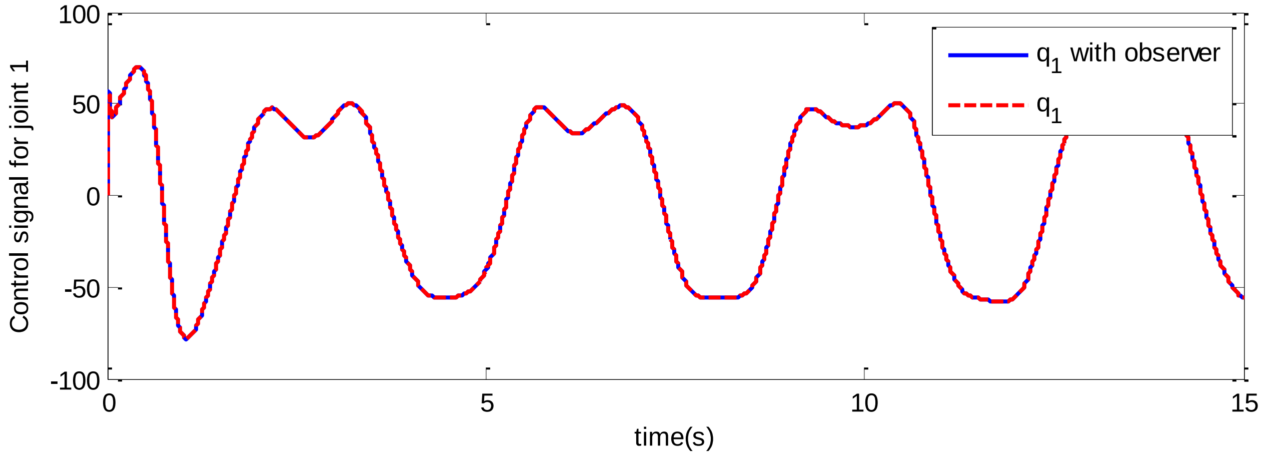


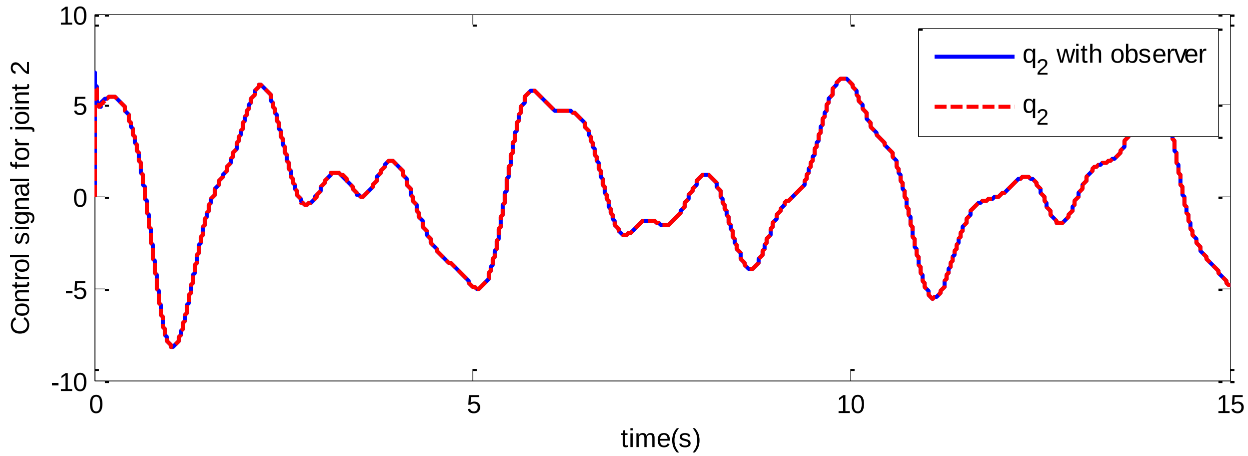






Simulation results





Simulation results