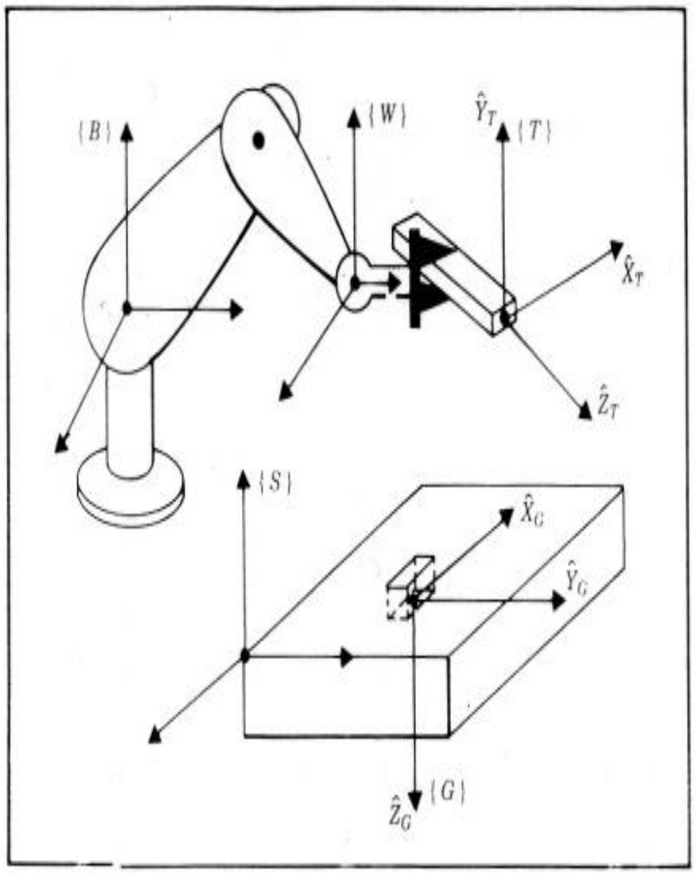
Ch. 4



Determination of the tool frame

Generally, {W} = {T}

$^B_TT = {}^B_WT = {}^0_6T$ for 6 dof manipulator

Inverse Kinematics problem?

Desired position and Orientation of the tool $\Rightarrow$ Find the set of joint angles which will achieve this desired result

$^0_6T = \begin{bmatrix} ^0_6R & P \\ \underline{0} & 1 \end{bmatrix}$: known 12 numeric values

$\downarrow$

R: 9 eqs. → 3 ind.eqs.
P: 3 eqs.
$\}$ 6eqs. 6 unknowns

6 unknown joint angle set

• 6 eqs. 6 unknown 의 Inverse Kinematics Problem은 highly nonlinear, involves transcendental eqs. 이므로 α, a, d 가 $(0°, \pm 90°, 0)$ 등으로 간단하지 않으면 일반적으로 풀기 어렵다.

⊙ 이 Eqs.을 풀기 위하여 고려하여야 할 사항들

① Existence of Solutions

② Multiple Solutions

③ Method of solution

1) Existence of solution

Workspace : volume of space which the end-effector of manipulator can reach.

① dextrous work space : reach with all orientations

② reachable work space : reach in at least one orientation

1) Existence of solution

Workspace: Volume of space which the end-effector of manipulator can reach

(1) Dextrouswork space: reach with all orientations

(2) Reachable work space: reach in at least one orientation

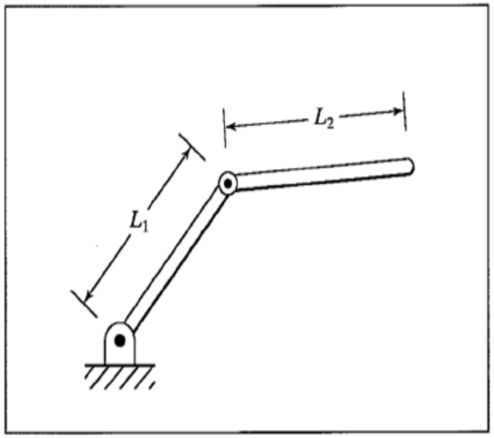
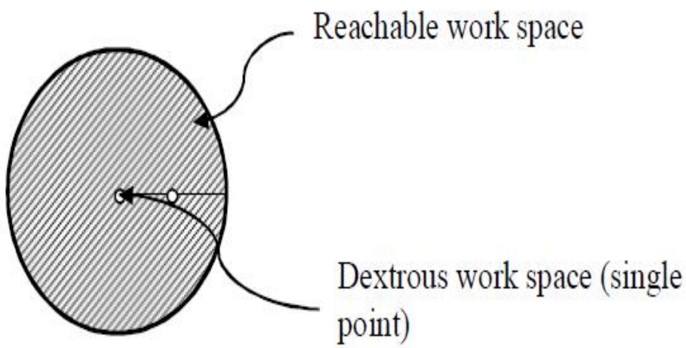
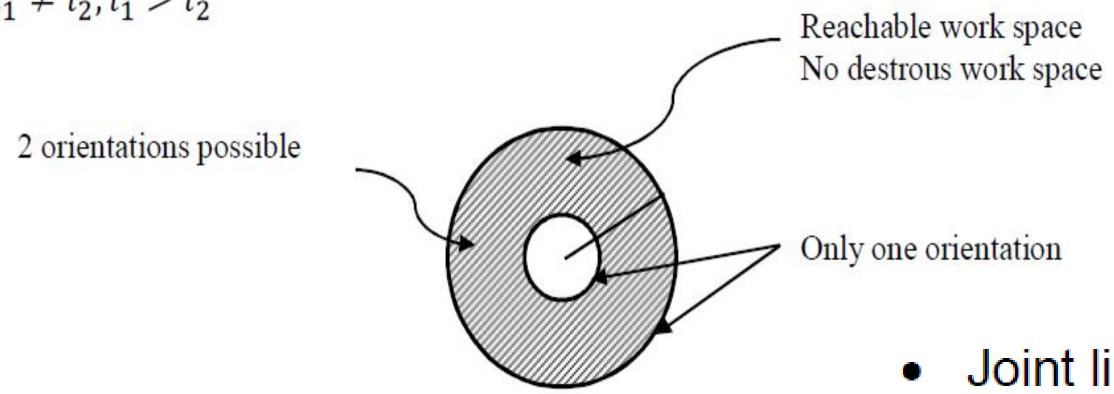


FIGURE 4.1: Two-link manipulator with link lengths l_1 and l_2.

If $l_1 = l_2$



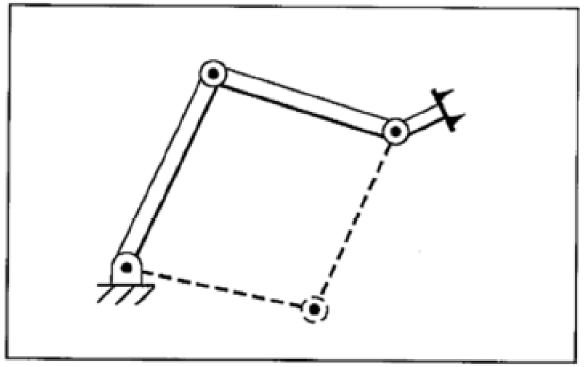
If $l_1 \neq l_2, l_1 > l_2$



- Joint limit
- Manipulators of less than 6dof

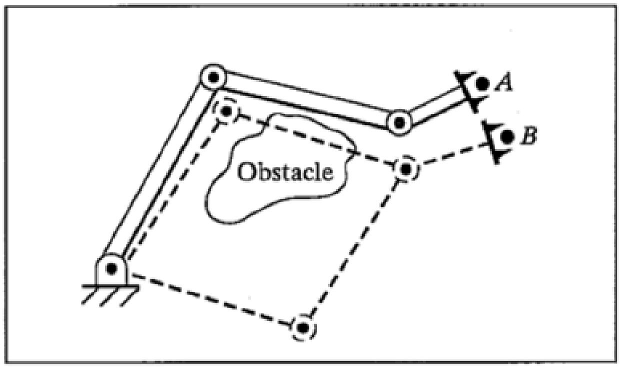
Specified orientation and position
➔ Existence of solution

2) Multiple Solutions



3 links: 2 possible solutions

FIGURE 4.2: Three-link manipulator. Dashed lines indicate a second solution.



Choice:
"Closest solution" in the absence of obstacle

Need define "Closest solution"

FIGURE 4.3: One of the two possible solutions to reach point B causes a collision.

- Number of solution depends on
  - (1) Number of joints
  - (2) Function of link parameters
  - (3)Allowable range of motion of the joints

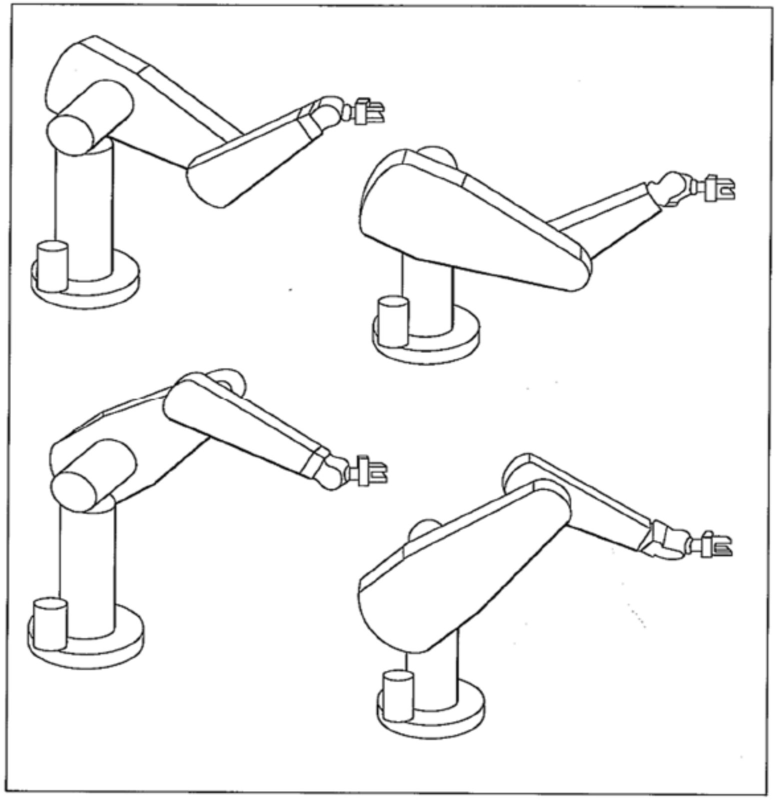


FIGURE 4.4 Four solution of the PUMA 560

$$\theta_4' = \theta_4 + 180^\circ,$$

$$\theta_5' = -\theta_5,$$

$$\theta_6' = \theta_6 + 180^\circ.$$

- Generally, 6 revolute robot with nonzero link parameters has 16 possible solutions

a_i	Number of solutions
$a_1 = a_3 = a_5 = 0$	$\leqslant 4$
$a_3 = a_5 = 0$	$\leqslant 8$
$a_3 = 0$	$\leqslant 16$
All $a_i \neq 0$	$\leqslant 16$

FIGURE 4.5: Number of solutions vs. nonzero a_i.

3) Method of Solution:

- Solvable:

A given Position and orientation $\longrightarrow$ All sets of Joint variables

Cf) numerical algorithm $\rightarrow$ get one set of solutions

Closed form solution. VS. Numerical Solution

Closed form solution. $\downarrow$ Analytic expression

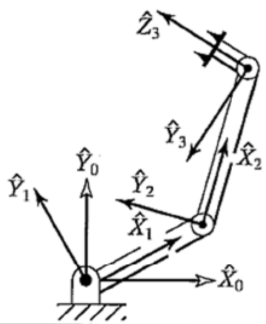
Analytic expression $\rightarrow$ Algebraic, Geometric

Numerical Solution $\downarrow$ Iterative approach, time consuming

- All system with revolute and prismatic joints having a total of 6 degrees of freedom in a single series chain are now solvable
  $\longrightarrow$ Numerial approach.

- Sufficient condition for a closed-form soluability in case of 6R manipulator:
  " Joint axes of three consecutive revolute joints interest at a single point for all arm configuration".

- **Algebraic Solution**



i	$\alpha_i - 1$	$\alpha_i - 1$	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

FIGURE 4.7: Three-link planar manipulator and its link parameters.

$$ {}^{B}_{W}T = {}^{0}_{3}T = \begin{bmatrix} c_{123} & -s_{123} & 0.0 & l_1c_1 + l_2c_{12} \\ s_{123} & c_{123} & 0.0 & l_1s_1 + l_2s_{12} \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (1)$$

3R planar Manipulator $\longrightarrow$ specification of the goal point with (x,y,Ø)

$$ {}^{B}_{W}T = \begin{bmatrix} c_{\emptyset} & -s_{\emptyset} & 0.0 & x \\ s_{\emptyset} & c_{\emptyset} & 0.0 & y \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (2)$$

(1)=(2),

$c_{\emptyset} = c_{123},$ $\qquad x = l_1c_1 + l_2c_{12},$

$s_{\emptyset} = s_{123},$ $\qquad y = l_1s_1 + l_2s_{12},$

$$x^2 + y^2 = l_1^2 + l_2^2 + 2l_1l_2c_2 \qquad (3)$$

Where we have made use of

$c_{12} = c_1c_2 - s_1s_2,$

$s_{12} = c_1s_2 - s_1c_2.$

Solving (3) for c_2 we obtain

$$c_2 = \frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2} \qquad (4)$$

$-1 \leq \frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2} \leq 1$ $\Rightarrow$ check the existence of solution

$s_2 = \pm\sqrt{1-c_2^2}$.

Finally, we compute θ_2 using the two-argument arctangent routine

$$\theta_2 = Atan2(s_2, c_2)$$

For θ_1,

$x = k_1c_1 - k_2s_1,$

$y = k_1s_1 + k_2c_1.$

Where

$k_1 = l_1 + l_2c_2,$

$k_2 = l_2s_2$.

If

$r = +\sqrt{k_2^1 + k_2^2}$

And

$\gamma = Atan2(k_2, k_1),$

Then

$k_1 = rcos\gamma$,

$k_2 = rsin\gamma.$

Equation (4.17) and (4.18) can now be written

$\frac{x}{r} = cos\gamma cos\theta_1 - sin\gamma sin\theta_1,$

$\frac{y}{r} = cos\gamma sin\theta_1 + sin\gamma cos\theta_1,$

or

$\cos(\gamma + \theta_1) = \frac{x}{r},$

$\sin(\gamma + \theta_1) = \frac{y}{r}.$

Using the two-argument arctangent we get

$\gamma + \theta_1 = Atan2\left(\frac{y}{r}, \frac{x}{r}\right) = Atan2(y, x).$

And so

$\theta_1 = Atan2(y, x) - Atan2(k_2, k_1).$

for θ_2,

$\theta_1 + \theta_2 + \theta_3 = Atan2(s_\emptyset, c_\emptyset) = \emptyset.$

- **Geometric Solution**

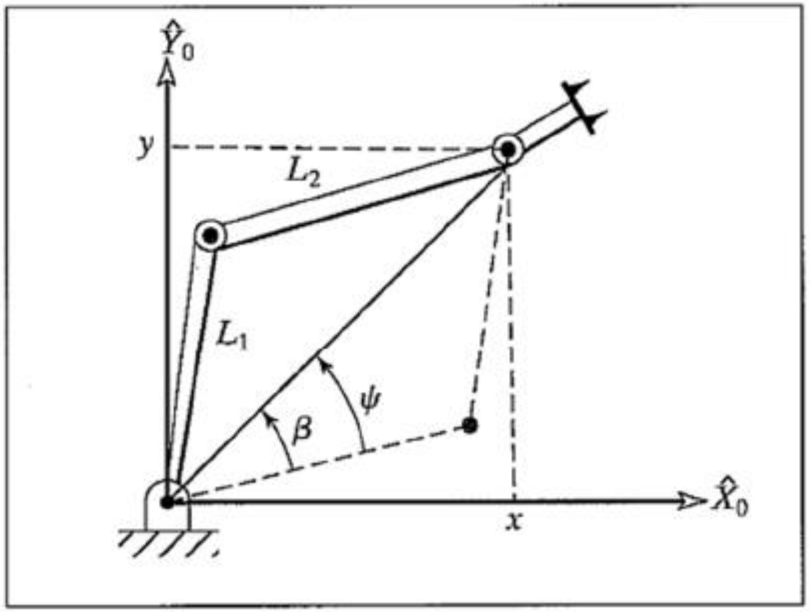


FIGURE 4.8: Plane geometry associated with a three-link planar robot.

We can apply the " law of cosines" to solve for θ_2:

$$x^2 + y^2 = l_1^2 + l_2^2 + 2l_1 l_2 \cos(180 + \theta_2).$$

Since $\cos(180 + \theta_2) = -\cos(\theta_2)$, we have

$$c_2 = \frac{x^2+y^2-l_2^1-l_2^2}{2l_1 l_2}$$

$\sqrt{x^2 + y^2} \leq l_1 + l_2$: Existence of solve

$-180 \leq \theta_2 \leq 0^{\circ}$

$\theta_2^{'} = -\theta_2$ by symmetry

For θ_1,

$\beta = Atan2(y, x)$.

$Atan2$: table loop-up for computational solving

We again apply the law of cosines to find ψ

$$l_2^2 = x^2 + y^2 + l_1^2 - 2l_1\sqrt{x^2+y^2}cos\psi$$

$$cos\psi = \frac{x^2 + y^2 + l_1^2 - l_2^2}{2l_1\sqrt{x^2+y^2}}$$

$$\theta_1 = \beta \pm \psi,$$

$$\theta_3 = \emptyset - \theta_1 - \theta_2.$$

- **Algebraic solution by reduction to polynomial**

Transcendental eqs. (cos, cosine) $\longrightarrow$ Polynomial in forms of single variable

$$u = tan\frac{\theta}{2},$$

$$cos\theta = \frac{1-u^2}{1+u^2},$$

$$sin\theta = \frac{2u}{1+u^2},$$

$$acos\theta + bsin\theta = c$$

$$a(1-u^2) + 2bu = c(1+u^2).$$

$$(a+c)u^2 + 2bu + (c-a) = 0$$

$$u = \frac{b \pm \sqrt{b^2-a^2-c^2}}{a+c}$$

$$\theta = 2tan^{-1}(\frac{b \pm \sqrt{b^2-a^2-c^2}}{a+c})$$

- **Pieper's solution when three axes intersect**

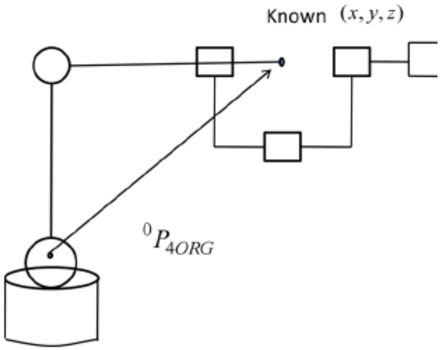
$$
{}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c_{\alpha_{i-1}} & c\theta_i c_{\alpha_{i-1}} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c_{\alpha_{i-1}} & c_{\alpha_{i-1}} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

When the last three axes intersect, the origins of link frames {4}, {5}, and {6} are all located at this point of intersection. This point is given in base coordinates as

$$
{}^{0}P_{4ORG} = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}P_{4ORG} \qquad (4.41)
$$

or, using the fourth column of (3.6) for i=4.

$$
{}^{0}P_{4ORG} = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T \begin{bmatrix} a_3 \\ -d_4 s\alpha_3 \\ d_4 c\alpha_3 \\ 1 \end{bmatrix}, \qquad (4.42)
$$



or as

$$
{}^{0}P_{4ORG} = {}^{0}_{1}T\,{}^{1}_{2}T \begin{bmatrix} f_1(\theta_3) \\ f_2(\theta_3) \\ f_3(\theta_3) \\ 1 \end{bmatrix}, \qquad (4.43)
$$

Where

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ 1 \end{bmatrix} = {}^2_3T \begin{bmatrix} a_3 \\ -d_4 s\alpha_3 \\ d_4 c\alpha_3 \\ 1 \end{bmatrix} \quad (4.44)$$

Using (3.6) for ${}^2_3T$ in (4.44) yields the following expressions for f_i

Function of θ_3

$$\begin{aligned} f_1 &= a_3 c_3 + d_4 s\alpha_3 + a_2, \\ f_2 &= a_3 c\alpha_2 s_3 - d_4 s\alpha_3 c\alpha_2 c_3 - d_4 s\alpha_2 c\alpha_3 - d_3 s\alpha_2, \quad (4.45) \\ f_3 &= a_3 s\alpha_2 s_3 - d_4 s\alpha_3 s\alpha_2 c_3 + d_4 c\alpha_2 c\alpha_3 + d_3 c\alpha_2. \end{aligned}$$

Using (3.6) for ${}^0_1T$ and ${}^1_2T$ in (4.43) we obtain

$${}^0P_{4ORG} = \begin{bmatrix} c_1 g_1 - s_1 g_2 \\ s_1 g_1 + c_1 g_2 \\ g_3 \\ 1 \end{bmatrix} = {}^0_1T \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ 1 \end{bmatrix}, \quad (4.46)$$

$${}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $g_1 = c_2 f_1 - s_2 f_2 + a_1,$

$$\begin{aligned} g_2 &= s_2 c\alpha_1 f_1 + c_2 c\alpha_1 f_2 - s\alpha_1 f_3 - d_2 s\alpha_1, \quad (4.47) \\ g_3 &= s_2 s\alpha_1 f_1 + c_2 s\alpha_1 f_2 + c\alpha_1 f_3 + d_2 c\alpha_1, \end{aligned}$$

$\alpha_0 = a_0 = d_1 = 0$

We now write an expression for the magnitude squared of ${}^0P_{4ORG}$, which is seen from (4.46) to be

$$r = g_1^2 + g_2^2 + g_3^2, \qquad (4.48)$$

$|{}^0P_{4ORG}| = |{}^1P_{4ORG}|$

or, using (4.47) for the g_i,we have

$$r = g_1^2 + g_2^2 + g_3^2 + a_1^2 + d_2^2 + 2d_2f_3 + 2a_1(c_2f_1 - s_2f_2) \qquad (4.49)$$

We now write this equation, along with the Z component equation from (4.46), as a system of two equations in the form

$$r = (k_1c_2 + k_2s_2)2a_1 + k_3$$
$$z = (k_1s_2 - k_2c_2)s\alpha_1 + k_4 \qquad (4.50)$$

where

$$k_1 = f_1$$
$$k_2 = -f_2$$
$$k_3 = f_1^2 + f_2^2 + f_3^2 + a_1^2 + d_2^2 + 2d_2f_3 \qquad (4.51)$$
$$k_4 = f_3c\alpha_1 + d_2c\alpha_1$$

Equation (4.50) is useful because dependence on $\theta_1$hase been eliminated and dependence on θ_2 takes a simple form.

Check 3 cases:

1. If $a_1 = 0$ then we have $r = k_3$ where r is known. The right-hand side$k_3$is a function of θ_3 only. After making the substitution (4.35), a quadratic equation in $tan\frac{\theta_3}{2}$ may be solved for θ_3.

2. If $sa_1 = 0$ then we have $z = k_4$ where z is known. Again, after substituting (4.35) a quadratic equation arises which may be solved for θ_3.

3. Otherwise, eliminate $s_2$and c_2 from(4.50) to obtain

$$\frac{(r-k_3)^2}{4a_1^2}+\frac{(z-k_4)^2}{s^2\alpha_1}=k_1^2+k_2^2.$$

Where

$$u = tan\frac{\theta}{2}$$

$$cos\theta = \frac{1-u^2}{1+u^2}$$

$$sin\theta = \frac{2u}{1+u^2}$$

$\theta_3 \longrightarrow \theta_2 \longrightarrow \theta_1$

For θ_4, θ_5 and θ_6 , given

$$ {}^{0}_{6}R = {}^{0}_{4'}R_{(\theta_4=0)}\, {}^{4'}_{4}R_{(\theta_4)}\, {}^{4}_{5}R\, {}^{5}_{6}R $$

$$ {}^{4'}_{6}R = {}^{0}_{4'}R'_{(\theta_4=0)}\, {}^{0}_{6}R $$

- ${}^{4'}_{6}R$ can be solved for by using exactly *Z-Y-Z* Euler angle solution.
- Euler angle solution, 2 solution set
- Position solution, 8 solution set
- General 6 DOF manipulator inverse kinematic solution set 16

1) Pay load
2) Speed
3) Resolution
4) Accuracy
5) Repeatability
6) Workspace
7) Cost

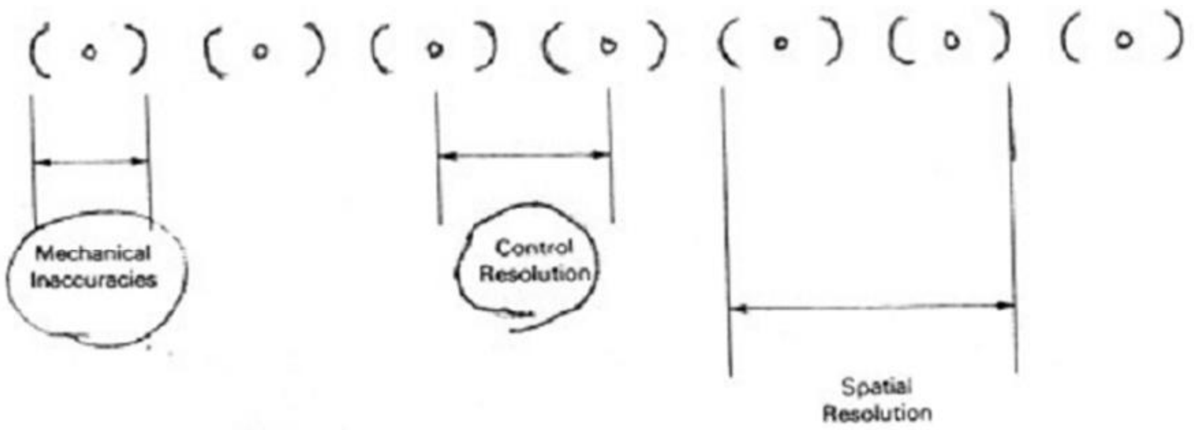


Figure 3.8.1 One-dimensional representation of control and spatial resolution.

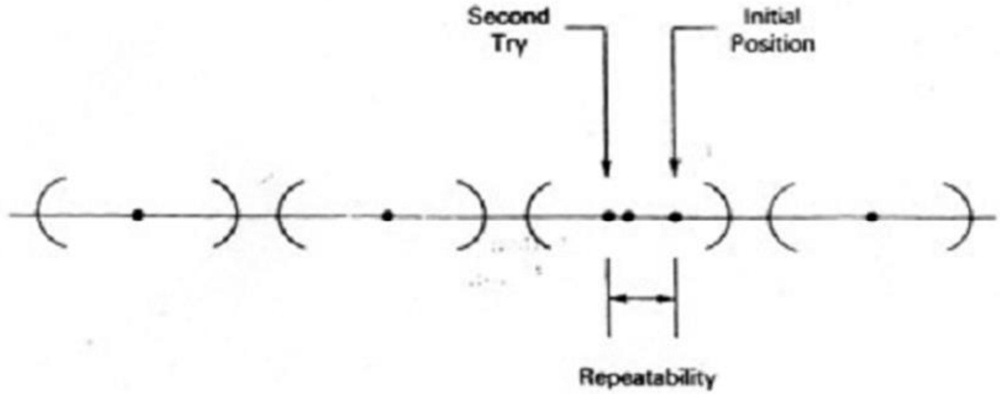


Figure 3.8.3 One-dimensional representation of repeatability.

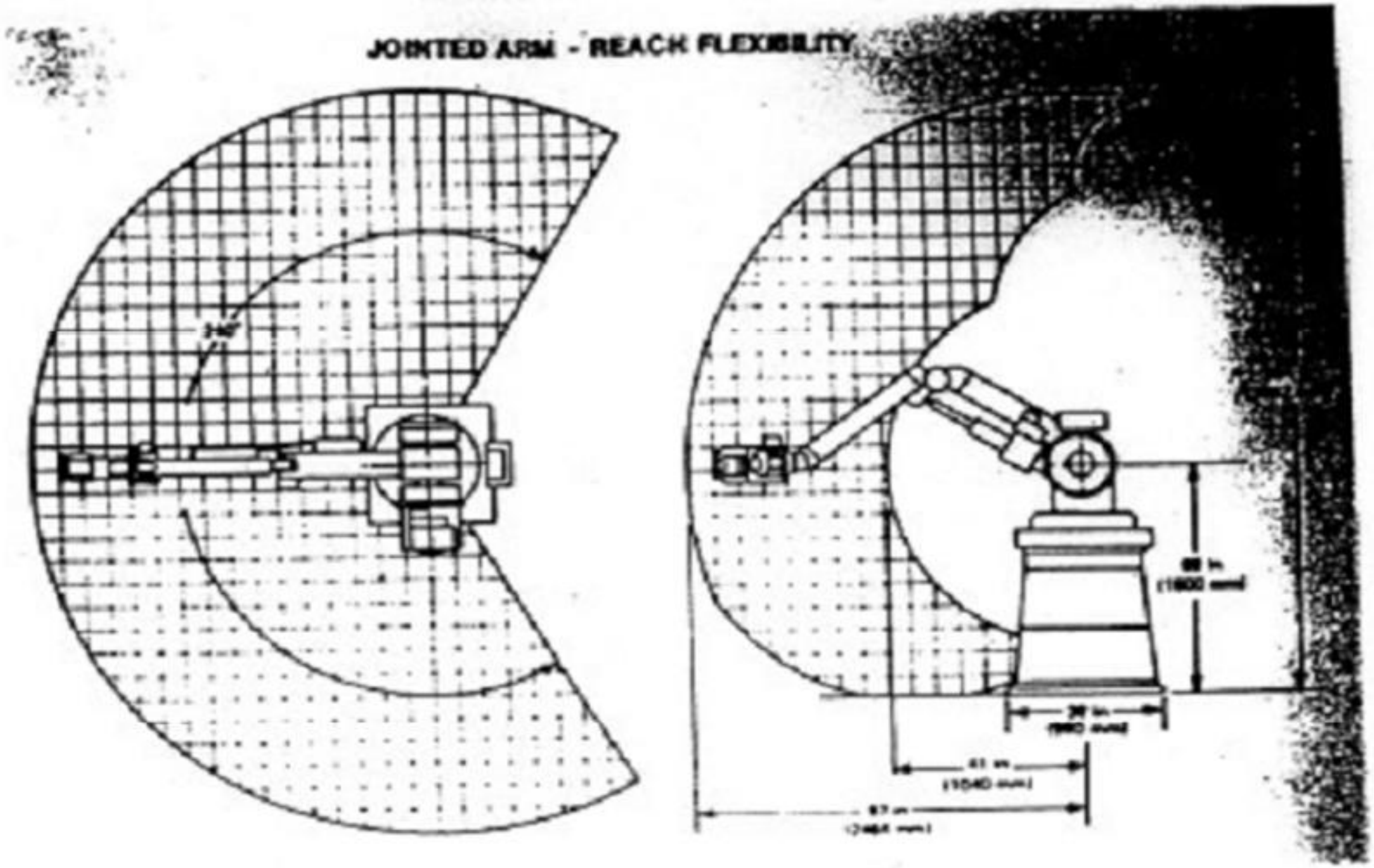


Figure 1.3.8. Geometry of a pure spherical jointed robot. (Courtesy of J. Coshnitzke, Cincinnati Milacron, Cincinnati, OH.)

4.6) Describe a simple algorithm for choosing the nearest solution from a set of possible solution

Solution:

To derive the nearest solution, we would like to minimize the rotation of each joint. Denote the starting angle of each joint as θ_{jo} (for i-th joint), and the final position of each joint as θ_{jF}. For each propose solution compute

$$S = \sum_{j=1}^{N} |\theta_{jF} - \theta_{jo}|$$

And choose the nearest as the one which minimize S. Sometimes a weighting factor is used (to penalize motion of large joints, for example) and so the score for each propose solution is

$$S = \sum_{j=1}^{N} W_j |\theta_{jF} - \theta_{jo}|$$

4.7) Make a list of factors which might affect the repeatability of a manipulator. Make a second list of additional factors which affect the accuracy of a manipulator.

Soluyion:

Repeatability us affected by:

Steady state error is

Flexibility of links

Backlash in gears

Looseness in bearings

Noise in sensing readings

Thermals effects

Accuracy is affected by all of the above, plus

Imprecise knowledge of DH parameters