Midterm Examination

October 23, 2013

1. (20pts) Let X have exponential distribution $f_X(x) = \frac{1}{\mu}e^{-x/\mu}(x)$ for $0 \leq x < \infty$. Find the conditional density $f_X(x|\mu < X \leq 2\mu)$.

2. (30pts) A call occurs at time T where T is a random point in the interval $(0, 10)$ and uniformly distributed over the interval.

   a. (5pt) Find $P\{6 \leq T \leq 8\}$.

   b. (5pt) Find $P\{6 \leq T \leq 8|T > 5\}$.

   c. (5pt) Find the probability density function $f_T(t)$.

   d. (5pt) Find the expectation of T $E[T]$.

   e. (5pt) Find the expectation of T^2 $E[T^2]$.

   f. (5pt) Find the variance of T $VarT$.

3. (20pts) If $f(x) = 0$ for $x < 0$, then, for any $\alpha > 0$, prove

$$P\{X \geq \alpha\} \leq \frac{E[X]}{\alpha}.$$

4. (30pts) Consider a random variable X with moment generating function $\Phi_X(\omega)$.

   a. (10pt) Show that if X is a Gaussian RV with mean μ and variance σ^2, then $\Phi_X(\omega) = e^{i\omega\mu - \omega^2\sigma^2/2}$.

   b. (10pt) Let X_1 and X_2 be independent and identically distributed Gaussian RVs with mean μ and variance σ^2 and $Z = X_1 + X_2$, then find characteristic function $\Phi_Z(\omega)$.

   c. (10pt) Using the result of (a), find the mean and standard deviation of Z in (b).