Midterm Examination

October 17, 2018

1. (20pts) If the density of X equals

$$f(x) = \begin{cases} ce^{-2x}, & \text{if } 0 < x < \infty. \\ 0, & \text{otherwise.} \end{cases}$$

   a. (10pt) Find c.

   c. (10pt) Find $\Pr\{X > 2\}$.

2. (20pts) You know that a certain letter is equally likely to be in any one of three different folders. Let α_i be the probability that you will find your letter upon making a quick examination of folder i if the letter is, in fact, in folder i, i = 1, 2, 3. (We may have $\alpha_i < 1$.) Suppose you look in folder 1 and do not find the letter. Let F_i, i = 1, 2, 3 be the event that the letter is in folder i; and let E be the event that a search of folder 1 does not come up with the letter.

   a. (10pt) Find the probability that a search of folder 1 does not come up with the letter.

   c. (10pt) What is the probability that the letter is in folder 1, $P[F_1|E]$?

3. (30pts) Let X and Y be independent Poisson random variables with respective means λ_1 and λ_2.

   a. (5pt) Find the probability that $X = x$

   b. (5pt) Find the moment generation function of random variable X.

   c. (5pt) Find the expectation of X, $E[X]$.

   d. (5pt) Find the second moment of X, $E[X^2]$.

   e. (5pt) Find the variance of X.

   f. (5pt) Find the distribution of $X + Y$.

4. (15pts) If $f(x) = 0$ for $x < 0$, then, for any $\alpha > 0$, prove

$$P\{X \geq \alpha\} \leq \frac{E[X]}{\alpha}.$$

5. (15pts) Prove that $E[X^2] \geq (E[X])^2$. When do we have equality?