Midterm Examination

April 15, 2015

1. (30pts) Let X have the exponential distribution $f_X(x) = \frac{1}{\mu}e^{-x/\mu}(x)$ for $0 \leq x < \infty$.

   a. (5pt) Find the probability that $\mu < X \leq 2\mu$

   b. (5pt) Find the conditional density $f_X(x|\mu < X \leq 2\mu)$.

   c. (5pt) Find the moment generating function of X.

   d. (5pt) Find the expectation of X, $E[X]$.

   e. (5pt) Find the second moment of X, $E[X^2]$.

   f. (5pt) Find the variance of X.

2. (20pts) You know that a certain letter is equally likely to be in any one of three different folders. Let α_i be the probability that you will find your letter upon making a quick examination of folder i if the letter is, in fact, in folder i, i = 1, 2, 3. (We may have α_i ¡ 1.) Suppose you look in folder 1 and do not find the letter. Let F_i, i = 1, 2, 3 be the event that the letter is in folder i; and let E be the event that a search of folder 1 does not come up with the letter.

   a. (10pt) Find the probability that a search of folder 1 does not come up with the letter.

   c. (10pt) What is the probability that the letter is in folder 1, $P[F_1|E]$?

3. (10pts) If $f(x) = 0$ for $x < 0$, then, for any $\alpha > 0$, prove

$$P\{X \geq \alpha\} \leq \frac{E[X]}{\alpha}.$$

4. (15pts) Let X_is be independent and identically distributed random variables. A new random variable Y is equal to the sum of a random number N of X_is that are also independent of N, i.e., $Y = \sum_{i=1}^{N} X_i$. Find the expected value of a Y.

5. (25pts) Independent trials, each of which is a success with probability p, are performed until there are 2 consecutive successes. What is the mean number of necessary trials?