Final Exam

December 15, 2011

1. (20pts) Show that for any two events A and B in a probability space (S,F, P) the following relationship holds:

   $$P(A)P(B)-P(A\bigcap B) = P(A\bigcap B)-P(A)P(B) = P(A\bigcap B)-P(A)P(B).$$

2. (20pts) Let X have exponential distribution $f_X(x) = \frac{1}{\mu}e^{-x/\mu}1_{[0,)}(x)$.

   a. (10pt) Find $P\{\mu < X \le 2\mu\}$.

   b. (10pt) Find the conditional density $f_X(x|\mu < X \le 2\mu)$.

3. (20pts) Consider a random variable X with men η_x and standard deviation σ_x. Find the mean and standard deviation of random variable Y if $Y = (X - \eta_x)/\sigma_x$.

4. (20pts) Let X and Y be independent random variables with variances with variances σ_x^2 and σ_y^2, respectively. Consider the sum

   $$Z = aX + (1-a)Y \text{ for } 0 \le a \le 1.$$

   Then, find a that minimize the variance of Z.

5. (20pts) Consider independent and identically distributed random variables $X_1, X_2, \cdots, X_n$ with mean η and standard deviation σ. Find the mean and standard deviation of random variable Z if $Z = \frac{1}{n}\sum_{i=1}^{n} X_i$.

6. (20pts) Express $F_{zw}(z,w)$ in terms of $F_{xy}(x,y)$ if $z = \max(x,y)$ and $w = \min(x,y)$.

7. (20pts) Consider a random variable X with mean η and standard deviation σ. For any $\epsilon > 0$, prove

   $$P\{|X \ge \epsilon|\} \le \frac{\sigma^2}{\epsilon^2}.$$