Final Examination

December 12, 2018

1. (30pts) Let X and Y be independent Poisson random variables with respective means λ_1 and λ_2.

   a. (5pt) Find the probability that $X = x$

   b. (5pt) Find the moment generation function of random variable X.

   c. (5pt) Find the expectation of X, $E[X]$ using the result of (b).

   d. (5pt) Find the second moment of X, $E[X^2]$ using the result of (b).

   e. (5pt) Find the variance of X using the results of (c) and (d).

   f. (5pt) Find the distribution of $X + Y$ using the result of (b).

2. (10pts) If X is a random variable with mean μ and variance σ^2, then, for any value $k > 0$,
$$\Pr\{|X - \mu|\} \leq \frac{\sigma^2}{k^2}.$$

3. (20pts) Independent trials, each of which is a success with probability p, are performed until there are 3 consecutive successes. What is the mean number of necessary trials?

4. (20pts) Let $p_0 = PX = 0$ and suppose that $0 < p_0 < 1$. Let $\mu = E[X]$ and $\sigma^2 = VarX$.

   a. (10pt) Find $E[X|X \neq 0]$.

   b. (10pt) Find $Var(X|X \neq 0)$.

5. (40pts) Coin 1 comes up heads with probability 0.6 and coin 2 with probability 0.5. A coin is continually flipped until it comes up tails, at which time that coin is put aside and we start flipping the other one.

   a. (10pt) Find a transition probability matrix.

   b. (10pt) Specify the classes of the Markov chins and determine whether they are transient or recurrent.

   c. (10pt) What proportion of flips use coin 1.

   d. (10pt) If we start the process with coin 1 what is the probability that coin 2 is used on the fifth flip?

6. (20pts) A and B play a series of games with A winning each game with probability p. The overall winner is the first player to have won two more games than the other.

   a. (10pt) Find the probability that A is the overall winner.

   b. (10pt) Find the expected number of games played.