

## Midterm Examination 1

October 6, 2011

1. (15pts) If  $A = \{2 \leq x \leq 5\}$  and  $B = \{3 \leq x \leq 6\}$ , find  $A \cup B$ ,  $A \cap B$ ,  $(A \cup B) \cap (\overline{A \cap B})$ , where  $\overline{A}$  implies the complementary set of  $A$
2. (15pts) Show that for any two events  $A$  and  $B$  in a probability space  $(S, \mathcal{F}, P)$  the following relationship holds:

$$P(A)P(B) - P(A \cap B) = P(A \cap B) - P(A)P(B) = P(A \cap B) - P(A)P(B).$$

3. (15pts) A call occurs at time  $t$  where  $t$  is a random point in the interval  $(0, 10)$ .
  - a. (5pt) Find  $P\{6 \leq t \leq 8\}$ .
  - b. (10pt) Find  $P\{6 \leq t \leq 8 | t > 5\}$ .
4. (15pts) A pair of dice is rolled 10 times. Find the probability that their sum will equal 7 on at least one roll.
5. (25pts) In the die experiment, we assign to the six outcomes  $f_i$  the numbers  $\mathbf{x}(f_i) = 10i$ . Thus,  $\mathbf{x}(f_1) = 10, \dots, \mathbf{x}(f_6) = 60$ . In the case that the die is fair, then,
  - a. (5pt) Find  $P\{\mathbf{x} \leq 100\}$ .
  - b. (5pt) Find  $P\{\mathbf{x} \leq 5\}$ .
  - c. (5pt) Find  $P\{15 \leq \mathbf{x} \leq 35\}$ .
  - d. (10pt) Draw the cumulative distribution function  $F_{\mathbf{x}}(x)$ .
6. (15pts) The probability density function (pdf)  $f(x)$  is defined as  $F_{\mathbf{x}}(x) = \int_{-\infty}^x f(x)dx$ , where  $F_{\mathbf{x}}(x)$  is the cumulative distribution function (cdf) of a random variable  $\mathbf{x}$ . Suppose that  $\mathbf{x}_u$  is the  $u$  percentile of the random variable  $\mathbf{x}$ , that is,  $F(\mathbf{x}_u) = u$ . Show that if  $f(-x) = f(x)$ , the  $\mathbf{x}_{1-u} = \mathbf{x}_u$