Midterm Examination 1

November 17, 2011

1. (20pts) Let X have exponential distribution $f_X(x) = \frac{1}{\mu}e^{-x/\mu}1_{[0,)}(x)$. Find the conditional density $f_X(x|\mu < X \leq 2\mu)$.

2. (20pts) Compute the mean and variance of the uniformly distributed random variable X having pdf $f_X(x) = \frac{1}{b-a}1_{[a,b]}(x)$.

3. (20pts) If $f(x) = 0$ for $x < 0$, then, for any $\alpha > 0$, prove

$$P\{X \geq \alpha\} \leq \frac{E[X]}{\alpha}.$$

4. (40pts) Consider a random variable X with characteristic function $\Phi_X(\omega)$.

   a. (10pt) Show that if $Y = aX + b$, then $\Phi_Y(\omega) = e^{i\omega}\Phi_X(\omega)$.

   b. (10pt) Show that if Y is a Gaussian RV with mean μ and variance σ^2, then $\Phi_Y(\omega) = e^{i\omega\mu - \omega^2\sigma^2/2}$.

   c. (10pt) Let X_1 and X_2 be independent and identically distributed Gaussian RVs with mean μ and variance σ^2 and $Z = X_1 + X_2$, then find characteristic function $\Phi_Z(\omega)$.

   d. (10pt) Using the result of (b), find the mean and standard deviation of Z in (c).