Final Examination

December 20, 2013

1. (20pts) Let X have exponential distribution $f_X(x) = \frac{1}{\mu}e^{-x/\mu}(x)$ for $0 \leq x < \infty$. Find the conditional density $f_X(x|\mu < X \leq 2\mu)$.

2. (30pts) Compute the expected value and the variance of a exponential random variable with parameter μ, of which probability density function is defined as in Prob. 1.

3. (20pts) Prove that

$$Var(X) = E[Var(X|Y)] + Var(E[X|Y]),$$

   where X and Y are random variables.

4. (30pts) Consider independent and identically distributed random variables $X_1, X_2, \cdots, X_n$ with mean η and standard deviation σ. Find the mean and standard deviation of random variable Z if $Z = \frac{1}{n}\sum_{i=1}^{n} X_i$.

5. (20pts) Consider a random variable X with mean η and standard deviation σ. For any $\epsilon > 0$, prove

$$P\{|X \geq \epsilon|\} \leq \frac{\sigma^2}{\epsilon^2}.$$

6. (30pts) Independent trials, each of which is a success with probability p, are performed until there are three consecutive successes. What is the mean number of necessary trials?