CHUNG QUANG KHANH – 20245360 – Homework 01 – Professor. Chung, Jin-Ho

Using Gauss-Jordan elimination to calculate the solution of the systems of linear equations

- Using Gauss-Jordan elimination algorithm to solve a linear system:

Ex1: The augmented matrix of the system:

pivot

$$\begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & -2 \end{bmatrix}$$

Row2 = Row2 + Row1

Row3 = Row3 + (−1)Row1

pivot

$$\begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -3 & -9 & -3 \end{bmatrix}$$

Row3 = Row3 + (3)Row2

$$\begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

There is no equation such as $0 = 1$

$\Rightarrow$ At least one solution exists (or the syetem is consistent)

In addition, there are 2 basic variables (x_1 and x_3) and 2 free variables (x_2 and x_4) as follows:

$$\begin{cases} x_1 = -2x_2 - 2x_4 + 1 \\ x_3 = -3x_4 + 1 \\ x_2 \text{ is free} \\ x_4 \text{ is free} \end{cases}$$

Thus the system has infinitely many solutions

Ex 2: The augmented matrix of the system:

pivot   pivot

$$\begin{bmatrix} 1 & 3 & 3 & 2 & 1 \\ 2 & 6 & 9 & 7 & 5 \\ -1 & -3 & 3 & 4 & 5 \end{bmatrix} \xrightarrow[R3 \mathrel{+}= R1]{R2 \mathrel{+}= -2R1} \begin{bmatrix} 1 & 3 & 3 & 2 & 1 \\ 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 6 & 6 & 6 \end{bmatrix} \xrightarrow{R3 \mathrel{+}= -2R2}$$

$$\begin{bmatrix} 1 & 3 & 3 & 2 & 1 \\ 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

There is no equation such as $0 = 1$

$\Rightarrow$ At least one solution exists

(or the system is consistent)

Additionally, there are 2 basic variables (x_1 x_3) and 2 free variable (x_2, x_4)

$\Rightarrow$ The system has infinitely many solutions

Do backward elimination:

pivot   pivot

$$\begin{bmatrix} 1 & 3 & 3 & 2 & 1 \\ 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R2 /= 3} \begin{bmatrix} 1 & 3 & 3 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R1 \mathrel{+}= -3R2}$$

$$\begin{bmatrix} 1 & 3 & 0 & -1 & -2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ This is the reduced echelon form of the original matrix

$\therefore$ General Solution
$$\begin{cases} x_1 = -3x_2 + x_4 - 2 \\ x_2 \text{ is free} \\ x_3 = -x_4 + 1 \\ x_4 \text{ is free} \end{cases}$$