PHYS 304 Review Notes

Reese Critchlow

General Forms for Solving Schrodinger Equations in Bound States

---

For a Schrodinger equation of the form:

$$-\frac{\hbar}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi = i\hbar\frac{\partial\psi}{\partial t},$$

we can define a general solution for bound states to be of the following form:

$$\Psi(x,t) = \sum_{n=0}^{\infty} c_n\psi_n(x)\cdot\varphi_n(t),$$

where $\psi_n(x)$ is a stationery state for the given potential $V(x)$, and $\varphi_n(t)$ is the time-dependance of the solution, given by:

$$\varphi_n(t) = e^{-iE_nt/\hbar},$$

where E_n is the energy corresponding to the state.

For the potentials that have been covered thus far in the course, there are two different stationery states corresponding to each potential. We can also define their energies:

Infinite Square Well

$$\psi_n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$$

$$E_n = \frac{\hbar^2k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

where $k_n = \frac{n\pi}{a}$

Harmonic Oscillator

$$\psi_n = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}\frac{1}{\sqrt{2^nn!}}H_n(\xi)e^{-\xi^2/2}$$

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega$$

where $\xi = \sqrt{\frac{m\omega}{\hbar}}x$

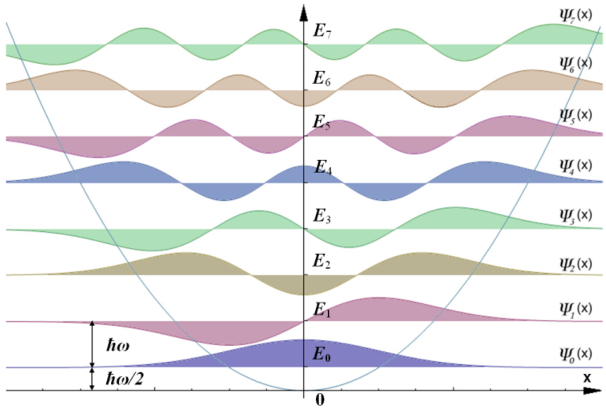
The Hermite Polynomials are also important to note:

$$H_n(\xi) = \begin{cases} H_0(\xi) = 1 \\ H_1(\xi) = 2\xi \\ H_2(\xi) = 4\xi^2 - 2 \\ H_3(\xi) = 8\xi^3 - 12\xi \end{cases}$$

Stationery States: Stationery states are states in which:

1. All expectation values are independent of time.
2. Total energy is definite.
3. The general solution is a linear combination of stationery states.

It is also useful to know what the bound states look like for the harmonic oscillator.



General Forms for Solving Free Particle Problems

For many potentials, particles appear in scattered states, instead of bound states. In these cases, energies are not quantized and general forms are computed over integrals, not sums.

For the case of the free particle, where the potential is zero everywhere, one can find the solution using the following procedure.

1. Identify $\phi(k)$, which is the distribution of states over the variable k, using a Fourier transform:

$$\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi(x, 0) e^{-ikx} dx.$$

2. Transform the function out of the frequency domain using another Fourier transform:

$$\Psi(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(k) e^{i\left(kx - \frac{\hbar k^2}{2m} t\right)} dk.$$

The distribution of states $\phi(k)$ in the general solution is known as the wavepacket. There are two velocities that are important in this case.

- Phase Velocity: $v_{\text{phase}} = \dfrac{\omega}{k} = \dfrac{\hbar k}{2m} = \sqrt{\dfrac{E}{2m}}$
- Group Velocity: $v_{\text{group}} = 2v_{\text{phase}} = \dfrac{d\omega}{dk}$

In all of these forms, k is k_0, which is the fundamental frequency of the group.

Probabilities and Expectation Values

Heisenberg Uncertainty Principle:

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$