Taylor Expansion

$$f(x) \approx \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}$$

$$f(x) \approx f(0) + f'(0)\cdot x + f''(0)\cdot \frac{x^2}{2} + \cdots$$

Field Integrals

Line Charge

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(\vec{r'})}{||\vec{r_{\text{sep}}}||^2} r_{\text{sep}}\, dl'$$

Surface Charge

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\vec{r'})}{||\vec{r_{\text{sep}}}||^2} r_{\text{sep}}\, da'$$

Volume Charge

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r'})}{||\vec{r_{\text{sep}}}||^2} r_{\text{sep}}\, d\tau'$$

Potential Difference

$$V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}} \vec{E}\cdot d\vec{l}$$

Potentials

Volume Charge

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r'})}{||\vec{r_{\text{sep}}}||} d\tau'$$

Collection of Point Charges

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=0}^{n} \frac{q_i}{||\vec{r_{\text{sep}}}||}$$

Work to Move a Charge

$$W = \int_{\mathbf{a}}^{\mathbf{b}} \vec{F}\cdot d\vec{l} = -Q\int_{\mathbf{a}}^{\mathbf{b}} \vec{E}\cdot d\vec{l}$$

$$W = Q[V(\vec{b}) - Q(\vec{a})]$$

Energy of a Charge Distributions

Collection of Point Charges

$$W = \frac{1}{2}\sum_{i=1}^{n} q_i V(\vec{r_i})$$

Continuous Charge Distribution

$$W = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 d\tau$$

Parallel Plate Voltage

$$V = \frac{Q}{A\epsilon_0} d$$

Capacitance

$$C \equiv \frac{Q}{V}$$

Energy of a Capacitor

$$W = \int_0^Q \left(\frac{q}{C}\right) dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

Common Boundary Conditions

Continuity of Potential

$$V_{\text{above}}(a) = V_{\text{below}}(a)$$

Preservation of Field
(Symmetry Required)

$$\vec{E}_{\text{above}} - \vec{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{n}$$

Bounding of Origin
(Generally Spherical Distributions)

$V(0)$ is bounded.

Bounding of Infinity
(Generally Spherical Distributions)

$V(\infty) = 0.$

Uniqueness of Solutions

For a charge distribution with $\nabla^2 V = 0$:

1. V has no local maxima nor minima inside. The maxima and minima are located on the surrounding boundaries.
2. V is smooth and continuous, everywhere.
3. $V(\vec{r})$ is the average of V over surface of any surrounding sphere: $V(\vec{r}) = \frac{1}{4\pi R^2}\oint V dA$.
4. V is unique, as the solution of the Laplace equation is uniquely determined if V is specified on the boundary surface around the volume.

Earnshaw's Theorem

A charged particle cannot be held in a stable equilibrium by electrostatic forces alone. This can be analyzed using divergence amongst other methods of analysis.

Properties of Conductors

1. $\vec{E} = 0$ inside a conductor. In short, charges move to oppose any external $\vec{E}$ fields. This can also be interpreted under the principle that if there was any field inside a conductor, the free electrons would be moving, and hence not *electrostatic*.
2. Any net charges reside on the surface of a conductor.
3. $\rho = 0$ inside a conductor. Since there is no field inside a conductor, Gauss's law requires that there is no enclosed charge, thus, there is no charge density. One may make the argument for the surface charges, however, since they are equal in magnitude, they cancel.
4. A conductor is an equipotential. Since there is no field inside a conductor, given the relationship $\nabla V = -\vec{E}$, the potential V must be a constant.
5. $\vec{E}$ is $\perp$ to the surface of a conductor.

Rules for Irrotational Fields

Since all electrostatic fields are conservative and therefore irrotational:

1. $\vec{\nabla}\times\vec{F} = \vec{0}$ everywhere.
2. $\int_a^b \vec{F}\cdot d\vec{l}$ is independent of path, for any given end points.
3. $\oint \vec{F}\cdot d\vec{l} = 0$ for any closed loop.
4. $\vec{F}$ is the gradient of some scalar function: $\vec{F} = -\nabla V$.

Problem Solving Stragegy – Coulomb Integrals

1. Choose a coordinate system.
2. Identify $\vec{r}$, the vector from the origin to the point of interest.
3. Identify $\vec{r'}$, the vector from the origin to the charge distribution.
4. Identify $\vec{r_{\text{sep}}}$, the vector separating the charge distribution and the point of interest: ($\vec{r_{\text{sep}}} = \vec{r} - \vec{r'}$)
5. If possible, identify any symmetries to try to solve the problem with.
6. Integrate.

The Electricity and Magnetism Triangle