PHYS 401 Radiation Notes

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We can separate the calculations for radiation in three main ways:

Ideal Electric Dipole

For an ideal electric dipole in the $\hat{z}$ direction, $\vec{p}(t) = p_0 \cos(\omega t)\hat{\boldsymbol{z}}$ we get that:

$$V(r,\theta,t) \approx -\frac{p_0\omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin\left[\omega(t-r/c)\right] \qquad \mathbf{A}(r,\theta,t) \approx -\frac{\mu_0 p_0 \omega}{4\pi r}\sin\left[\omega(t-r/c)\right]\hat{\boldsymbol{z}}$$

$$\mathbf{E}(\vec{r},t) \approx -\frac{\mu_0 p_0 \omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega(t-r/c)\right]\hat{\boldsymbol{\theta}} \qquad \mathbf{B}(\vec{r},t) = -\frac{\mu_0 p_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega(t-r/c)\right]\hat{\boldsymbol{\phi}}$$

$$\mathbf{S}(\vec{r},t) \approx \frac{\mu_0}{c}\left\{\frac{p_0\omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t-r/c)]\right\}^2\hat{\boldsymbol{r}}$$

We can attempt to generalize this in more "general coordinates":

$$V(\vec{r},t) \approx \frac{1}{4\pi\epsilon}\frac{\vec{n}\cdot\dot{\vec{p}}(t_0)}{rc} \qquad \vec{A}(\vec{r},t) \approx \frac{\mu_0}{4\pi}\frac{\dot{\vec{p}}(t_0)}{r}$$

$$\vec{E}(\vec{r},t) \approx \frac{\mu_0}{4\pi r}\left[\vec{n}\times(\vec{n}\times\ddot{\vec{p}})\right] \qquad \vec{B}(\vec{r},t) \approx -\frac{\mu_0}{4\pi rc}\left(\vec{n}\times\ddot{\vec{p}}\right)$$

$$\vec{S}(\vec{r},t) \approx \frac{\mathbf{E}\times\mathbf{B}}{\mu_0} = \frac{\mu_0\vec{n}}{16\pi^2 r^2 c^2}\left[\ddot{\vec{p}}^2 - (\vec{n}\cdot\ddot{\vec{p}})^2\right]$$

For all dipole orientations, one can also use the following:

$$\langle \mathbf{S} \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c}\right)\frac{\sin^2\theta}{r^2}\hat{\boldsymbol{r}}$$

$$\langle P \rangle = \int \langle \mathbf{S} \rangle \cdot d\mathbf{a} = \frac{\mu_0 p_0^2 \omega^4}{12\pi c^3}$$

General Radiation Formulae

Recall the formula for generalized dipole moment:

$$\mathbf{p}(t) = \int \mathbf{r}'\rho(\mathbf{r}', t_0)d^3\mathbf{r}'$$

Given this, we can use the generalized coordinates formula from the prior section.

This also extends to the total radiated power:

$$P_{\text{rad}}(t_0) \approxeq \frac{\mu_0}{6\pi c}\left[\ddot{p}(t_0)\right]^2$$

Note that this power P is energy per unit time, so we can also write $P = \frac{dW}{dt}$, where W is energy.

We also introduce the Larmor Formula, which gives the power of an arbitrary moving charge with acceleration $a(t)$:

$$P = \frac{\mu_0 q^2}{6\pi c}[a(t)]^2$$

Magnetic Dipole Radiation

Given some magnetic dipole formed from some oscillating current $I(t) = I_0 \cos(\omega t)$ in a wire loop fo radius b, the dipole takes the form:

$$\mathbf{m}(t) = \pi b^2 I(t)\hat{\boldsymbol{z}} = m_0 \cos(\omega t)\hat{\boldsymbol{z}}$$

Thus, we get some similar equations:

$$\mathbf{A}(r,\theta,t) \approx -\frac{\mu_0 m_0 \omega}{4\pi c}\left(\frac{\sin\theta}{r}\right)\sin\left[\omega(t - r/c)\right]\hat{\boldsymbol{\phi}}$$

$$\mathbf{E}(\mathbf{r},t) \approx \frac{\mu_0 m_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega(t - r/c)\right]\hat{\boldsymbol{\phi}} \qquad \mathbf{B}(\mathbf{r},t) \approx \frac{-\mu_0 m_0 \omega^2}{4\pi c^2}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega(t - r/c)\right]\hat{\boldsymbol{\theta}}$$

We can actually generalize this as in the same way to the electric case:

$$\begin{aligned} V &= 0 \\ \mathbf{A}(\mathbf{r},t) &\approx \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{n}}{r^2} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3} \\ \mathbf{E}(\mathbf{r},t) &\approx -\frac{\mu_0}{4\pi r c}(\vec{n}\times\ddot{\vec{m}}) \\ \mathbf{B}(\mathbf{r},t) &\approx \frac{\mu_0}{4\pi r c^2}\left[\vec{n}\times(\vec{n}\times\ddot{\vec{m}})\right] \\ \mathbf{S}(\mathbf{r},t) &\approx \frac{\mu_0 \vec{n}}{16\pi^2 r^2 c^3}\left[\ddot{\vec{m}}^{\,2} - (\vec{n}\cdot\ddot{\vec{m}}\,)^2\right] \end{aligned}$$

$$\omega = kv$$