

Generalized Multipole Expansion

$$V(r) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int_V ||\vec{r}'|| P_n(\cos \alpha) \rho(\vec{r}') d\tau'$$

Monopole Voltage

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

Dipole Moment – Continuous Charge Distribution

$$\vec{p} \equiv \int_V \vec{r}' \rho(\vec{r}') d\tau'$$

Dipole Voltage

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$$

Dipole Moment – Point Charge Distribution

$$\vec{p} = \sum_{i=1}^n q_i \vec{r}'_i$$

Dipole Moment – Change of Origin

$$\vec{p}' = \vec{p} - Q\vec{a}$$

Force on a Dipole

$$\vec{F} = (\vec{p} \cdot \nabla) \vec{E}$$

Torque of a Dipole

$$\vec{N} = \vec{p} \times \vec{E}$$

Energy of a Dipole in an Electric Field

$$U = -\vec{p} \cdot \vec{E}$$

Bound Surface Charge Density

$$\sigma_{\text{bound}} = \vec{P} \cdot \hat{n}$$

Bound Volume Charge Density

$$\rho_{\text{bound}} \equiv -\nabla \cdot \vec{P}$$

Linear Dielectrics

$$\begin{aligned}\vec{P} &= \epsilon_0 \chi_e \vec{E} \\ \vec{D} &= \epsilon \vec{E} \\ \epsilon &= \epsilon_0 (1 + \chi_e) = \epsilon_0 \epsilon_r\end{aligned}$$

Gauss's Law for Electric Displacement

$$\begin{aligned}\nabla \cdot \vec{D} &= \rho_f \\ \oint \vec{D} \cdot d\vec{A} &= Q_{f_{\text{encl}}}\end{aligned}$$

Displacement Boundary Condition

$$D_{\text{above}}^{\perp} - D_{\text{below}}^{\perp} = \sigma_{\text{free}}$$

Electric Displacement

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$

Solution for For Spherical Laplacians

$$f(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

Legendre Polynomials

$$\begin{aligned}P_0(\cos \theta) &= 1 \\ P_1(\cos \theta) &= \cos \theta \\ P_2(\cos \theta) &= \frac{3}{2} \cos^2 \theta - \frac{1}{2} \\ P_3(\cos \theta) &= \frac{5}{2} \cos^3 \theta - \frac{3}{2} \cos \theta\end{aligned}$$

Common Boundary Conditions

- $\vec{D}_{\text{above}}^{\perp} - \vec{D}_{\text{below}}^{\perp} = \sigma_f$
- $\vec{D}_{\text{above}}^{\parallel} - \vec{D}_{\text{below}}^{\parallel} = \vec{P}_{\text{above}}^{\parallel} - \vec{P}_{\text{below}}^{\parallel}$
- $\vec{E}_{\text{above}}^{\perp} - \vec{E}_{\text{below}}^{\perp} = \frac{\sigma_{\text{surf}}}{\epsilon_0}$
- $V_{\text{above}} = V_{\text{below}}$
- $\epsilon_{\text{above}} E_{\text{above}}^{\perp} - \epsilon_{\text{below}} E_{\text{below}}^{\perp} = \sigma_f$
- $\epsilon_{\text{above}} \frac{\partial V_a}{\partial n}^{\perp} - \epsilon_{\text{below}} \frac{\partial V_b}{\partial n}^{\perp} = \sigma_f$
- $V(r = 0) = \text{finite}$
- $V(r \rightarrow \infty) = \text{finite or zero.}$