

Electrostatics Basics

Line Charge:	$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\vec{r}')}{ \vec{r}_{\text{sep}} ^2} \hat{r}_{\text{sep}} d\ell'$
Surface Charge:	$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\vec{r}')}{ \vec{r}_{\text{sep}} ^2} \hat{r}_{\text{sep}} da'$
Volume Charge:	$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{ \vec{r}_{\text{sep}} ^2} \hat{r}_{\text{sep}} d\tau'$
Potential Difference:	$V(\vec{b}) - V(\vec{a}) = - \int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l}$
Potential of a Volume Charge:	$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{ \vec{r}_{\text{sep}} } d\tau'$
Potential of a Collection of Point Charges:	$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=0}^n \frac{q_i}{ \vec{r}_{\text{sep}} }$
Work to Move a Charge:	$W = \int_{\vec{a}}^{\vec{b}} \vec{F} \cdot d\vec{l} = -Q \int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l}$
Work to Move a Charge (Potential):	$W = Q[V(\vec{b}) - V(\vec{a})]$
Energy of a Collection of Point Charges:	$W = \frac{1}{2} \sum_{i=1}^n q_i V(\vec{r}_i)$
Energy of a Continuous Charge Distribution:	$W = \frac{\epsilon_0}{2} \int_{\text{univ}} E^2 d\tau = \frac{1}{2} \iiint \rho V d\tau$
Field at a Charged Surface:	$\vec{E}_{\text{surf}} = \frac{1}{2} (\vec{E}_{\text{above}} + \vec{E}_{\text{below}})$
Parallel Plate Voltage:	$V = \frac{Q}{A\epsilon_0} d$
Capacitance:	$C \equiv \frac{Q}{V}$
Energy of a Capacitor:	$W = \int_0^Q \left(\frac{q}{C} \right) dq = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2$

Electrostatic Multipoles

Generalized Multipole Expansion:	$V(r) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int_V \vec{r}' ^n P_n(\cos \alpha) \rho(\vec{r}') d\tau'$
Monopole Voltage:	$V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$
Dipole Voltage:	$V(r) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$
Dipole Moment (Continuous):	$\vec{p} \equiv \int_V \vec{r}' \rho(\vec{r}') d\tau'$
Dipole Moment (Discrete):	$\vec{p} = \sum_{i=1}^N q_i \vec{r}'_i$
Dipole Moment (Change of Origin):	$\vec{p}' = \vec{p} - Q\vec{a}$
Torque of a Dipole:	$\vec{N} = \vec{p} \times \vec{E}$
Energy of a Dipole:	$U = -\vec{p} \cdot \vec{E}$
Force on a Dipole:	$\vec{F} = (\vec{p} \cdot \nabla) \vec{E}$
Field of a Dipole:	$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta})$

Electrostatics in Matter

Electric Displacement:	$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$
Bound Surface Charge:	$\sigma_{\text{bound}} = \vec{P} \cdot \hat{n}$
Bound Volume Charge:	$\rho_{\text{bound}} \equiv -\nabla \cdot \vec{P}$
Linear Dielectrics – Polarization:	$\vec{P} = \epsilon_0 \chi_e \vec{E}$
Linear Dielectrics – E-Field:	$\vec{D} = \epsilon \vec{E}$
Electric Permittivity:	$\epsilon = \epsilon_0 (1 + \chi_e) = \epsilon_0 \epsilon_r$
Gauss's Law for \vec{D} (Derivative):	$\nabla \cdot \vec{D} = \rho_f$
Gauss's Law for \vec{D} (Integral):	$\oint \vec{D} \cdot d\vec{A} = Q_{f_{\text{encl}}}$
Force on a Dielectric:	$F = -\nabla U$
Energy of a Dielectric:	$W = \frac{1}{2} \int \vec{D} \cdot \vec{E} d\tau$

Names of Stuff

ϵ_0 : Permittivity of Free Space
 ϵ_r : Dielectric Constant or Relative Permittivity
 ϵ : Permittivity of a Material

Techniques for Solving Problems

Cylindrical Laplacian Solution:

$$V(s, \phi) = A \ln s + B + \sum_{n=1}^{\infty} \left(A_n s^n + \frac{B_n}{s^n} \right) (C_n \cos(\phi n) + D_n \sin(\phi n))$$

Solution for Spherical Laplacians:

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

Method of Images General Principle:

$$\sum \frac{q_i}{||\vec{r}_{\text{sep } i}||} = 0$$

Method of Images for Two Points:

$$-\frac{q_1}{q_2} = \frac{||\vec{r}_{\text{sep } 1}||}{||\vec{r}_{\text{sep } 2}||}$$

Image Charge Surface Charge:

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial n}$$

Binomial Expansion:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots + nx^{n-1} + x^n$$

Taylor Expansion:

$$f(x) \approx \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}$$

Because I can't remember:

$$f(x) \approx f(0) + f'(0) \cdot x + \frac{f''(0)}{2} x^2 + \dots$$

Legendre Polynomials:

$$\begin{aligned}
 P_0(\cos \theta) &= 1 \\
 P_1(\cos \theta) &= \cos \theta \\
 P_2(\cos \theta) &= \frac{3}{2} \cos^2 \theta - \frac{1}{2} \\
 P_3(\cos \theta) &= \frac{5}{2} \cos^3 \theta - \frac{3}{2} \cos \theta
 \end{aligned}$$

Magnetostatics

Lorentz Force Law:	$\vec{F}_{\text{mag}} = Q(\vec{v} \times \vec{B})$
Surface Current Density:	$\vec{K} \equiv \frac{d\vec{I}}{dl_{\perp}}$ $\vec{K} = \sigma \vec{v}$
Volume Current Density:	$\vec{J} \equiv \frac{d\vec{I}}{da_{\perp}}$ $\vec{J} = \rho \vec{v}$
Magnetic Force (General):	$\vec{F}_{\text{mag}} = \int I(d\vec{l} \times \vec{B}) = I \int (d\vec{l} \times \vec{B})$
Magnetic Force (Surface):	$\vec{F}_{\text{mag}} = \int (\vec{K} \times \vec{B}) da$
Magnetic Force (Volume):	$\vec{F}_{\text{mag}} = \int (\vec{J} \times \vec{B}) d\tau$
Biot-Savart Law:	$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times \hat{r}_{\text{sep}}}{ \vec{r}_{\text{sep}} ^2} d\ell'$ $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}' \times \hat{r}_{\text{sep}}}{ \vec{r}_{\text{sep}} ^2}$
Biot-Savart – Surface:	$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{r}') \times \hat{r}_{\text{sep}}}{ \vec{r}_{\text{sep}} ^2} da'$
Biot-Savart – Volume:	$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times \hat{r}_{\text{sep}}}{ \vec{r}_{\text{sep}} ^2} d\tau'$
Ampere's Law:	$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$
Magnetic Vector Potential:	$\vec{B} = \vec{\nabla} \times \vec{A}$ $\vec{\nabla} \cdot \vec{A} = 0$ $\nabla^2 \vec{A} = -\mu_0 \vec{J}$
Vector Potential – Line:	$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{I}}{ \vec{r}_{\text{sep}} } d\ell' = \frac{\mu_0 I}{4\pi} \int \frac{1}{ \vec{r}_{\text{sep}} } d\ell'$
Vector Potential – Surface:	$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{K}}{ \vec{r}_{\text{sep}} } da'$
Vector Potential – Volume:	$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{ \vec{r}_{\text{sep}} } d\tau'$
Magnetic Dipole Moment:	$\vec{m} \equiv I \int d\vec{a} = I \vec{a}$
Magnetic Dipole Potential Expansion:	$\vec{A}_{\text{dip}}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$
Magnetic Torque:	$\vec{N} = \vec{m} \times \vec{B}$
Energy of a Dipole:	$U = -\vec{m} \cdot \vec{B}$
Force on a Magnetic Dipole:	$\vec{F} = \nabla(\vec{m} \cdot \vec{B})$
Force Between Two Current-Carrying Wires:	$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$
Linear Current:	$\vec{I} = \int \vec{K} \cdot d\vec{l} = \iint \vec{J} \cdot d\vec{a}$

Maxwell's Equations

Gauss's Law:	$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$	$\oiint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$
No monopoles:	$\vec{\nabla} \cdot \vec{B} = 0$	$\oiint \vec{B} \cdot d\vec{a} = 0$
Faraday's Law:	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t}$
Ampere's Law with Maxwell's Correction:	$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{\partial \Phi_E}{\partial t}$

Magnetostatics in Matter

Paramagnets:	magnetization is parallel to \vec{B}
Diamagnets:	magnetization is opposite to \vec{B}
Ferromagnets:	magnetization holds outside the presence of an external magnetic field.
Magnetization:	$\vec{M} \equiv$ magnetic dipole per unit volume
Bound Volume Current:	$\vec{J}_{bound} = \vec{\nabla} \times \vec{M}$
Bound Surface Current:	$\vec{K}_{bound} = \vec{M} \times \hat{n}$
Free Current:	$\vec{J} = \vec{J}_b + \vec{J}_f$
Auxillary Field:	$\vec{H} \equiv \frac{1}{\mu_0} \vec{B} - \vec{M}$
Ampere's Law for Auxillary Fields:	$\vec{\nabla} \times \vec{H} = \vec{J}_f$ $\oint \vec{H} \cdot d\vec{l} = I_{f,enc}$
Linear Magnetics:	$\vec{M} = \chi_m \vec{H}$
Permeability:	$\mu \equiv \mu_0(1 + \chi_m)$
Relative Permeability:	$\mu_r = 1 + \chi_m$
Linear Magnetics with E and H Fields:	$\vec{B} = \mu \vec{H}$
Dipole Field:	$\vec{B} = \frac{\mu_0 m}{4\pi r^3} (2 \cos(\theta) \hat{r} + \sin(\theta) \hat{\theta})$, for $\vec{m} = m \hat{z}$
Field Inside a Uniformly Magnetized Sphere:	$\vec{B} = \frac{2}{3} \mu_0 \vec{M}$
Really?:	$\vec{J}_b = \chi_m \vec{J}_f$
Force on a dipole moment:	$\vec{F} = \nabla(\vec{m} \cdot \vec{B})$

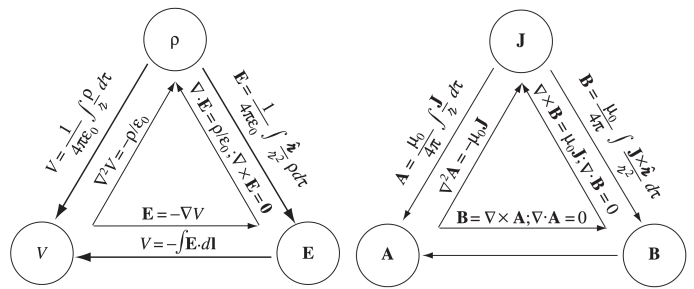
Electrodynamics

Ohm's Law:	$V = IR$
Power in a Circuit:	$P = VI = I^2 R = \frac{V^2}{R}$
Electromotive Force:	$\mathcal{E} = \frac{F_{mag, tot}}{Q} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$ $\mathcal{E} = \oint \vec{f} \cdot d\vec{l}$ $\mathcal{E} = -\frac{d\Phi}{dt}$
Faraday's Law:	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Inductance:	$\Phi = LI$ $\mathcal{E} = -L \frac{dI}{dt}$
Work of a Magnetic Field:	$\frac{1}{2} LI^2$ $W = \frac{1}{2\mu_0} \int_{all\ space} B^2 d\tau$
Displacement Current:	$\vec{J}_d \equiv \epsilon_0 \frac{\partial \vec{E}}{\partial t}$
Faraday's Law in Integral Form:	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$

Maxwell's Equations in Matter

Gauss's Law:	$\vec{\nabla} \cdot \vec{D} = \rho_f$
(unnamed)	$\vec{\nabla} \cdot \vec{B} = 0$
Faraday's Law:	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Ampere's Law with Maxwell's Correction:	$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$

Triangles



Boundary Conditions in Electrostatics

$\vec{D}_{above}^\perp - \vec{D}_{below}^\perp = \sigma_f$	$V_{above} = V_{below}$
$\vec{E}_{above}^\perp - \vec{E}_{below}^\perp = \sigma_{tot}/\epsilon_0$	$\vec{E}_{above}^\parallel - \vec{E}_{below}^\parallel = 0$
$\vec{D}_{above}^\parallel - \vec{D}_{below}^\parallel = \vec{P}_{above}^\parallel - \vec{P}_{below}^\parallel$	$\epsilon_{above} \vec{E}_{above}^\perp - \epsilon_{below} \vec{E}_{below}^\perp = \sigma_f$
$\epsilon_{above} \frac{\partial V_{above}}{\partial n} - \epsilon_{below} \frac{\partial V_{below}}{\partial n} = -\sigma_f$	

Boundary Conditions in Magnetostatics

$\vec{B}_{above}^\parallel - \vec{B}_{below}^\parallel = \mu_0 \vec{K}$	$\vec{B}_{above} - \vec{B}_{below} = \mu_0 (\vec{K} \times \hat{n})$
$\vec{A}_{above} = \vec{A}_{below}$	$\frac{\partial A_{above}}{\partial n} - \frac{\partial A_{below}}{\partial n} = -\mu_0 \vec{K}$
$\vec{H}_{above}^\perp - \vec{H}_{below}^\perp = -(\vec{M}_{above}^\perp - \vec{M}_{below}^\perp)$	$\vec{H}_{above}^\parallel - \vec{H}_{below}^\parallel = \vec{K}_f \times \hat{n}$

More Boundary Conditions

Equations 7.60-63 are the general boundary conditions for electrodynamics. In the case of *linear* media, they can be expressed in terms of \vec{E} and \vec{B} alone:

(i) $\epsilon_1 E_1^\perp - \epsilon_2 E_2^\perp = \sigma_f$,	(iii) $\vec{E}_1^\parallel - \vec{E}_2^\parallel = 0$,	} (7.64)
(ii) $B_1^\perp - B_2^\perp = 0$,	(iv) $\frac{1}{\mu_1} \vec{B}_1^\parallel - \frac{1}{\mu_2} \vec{B}_2^\parallel = \vec{K}_f \times \hat{n}$.	

In particular, if there is no free charge or free current at the interface, then

(i) $\epsilon_1 E_1^\perp - \epsilon_2 E_2^\perp = 0$,	(iii) $\vec{E}_1^\parallel - \vec{E}_2^\parallel = 0$,	} (7.65)
(ii) $B_1^\perp - B_2^\perp = 0$,	(iv) $\frac{1}{\mu_1} \vec{B}_1^\parallel - \frac{1}{\mu_2} \vec{B}_2^\parallel = 0$.	

As we shall see in Chapter 9, these equations are the basis for the theory of reflection and refraction.

Common Fields

E and V for inf line:	$E(s) = \frac{\lambda}{2\pi\epsilon_0 s}, V(s) = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{s_0}{s} $
E and V for inf plane:	$E = \frac{ \sigma }{2\epsilon_0}, V = \mp \frac{\sigma z}{2\epsilon_0}$
E and V at (0, 0, z) of ring:	$E(z) = \frac{\lambda z R}{2\epsilon_0 (R^2 + z^2)^{\frac{3}{2}}}, V(z) = \frac{\lambda \cdot R}{2\epsilon_0 \sqrt{R^2 + z^2}}$
E and V at (0, 0, z) of disk:	$E(z) = \frac{\sigma}{2\epsilon_0} [1 - \frac{z}{\sqrt{z^2 + R^2}}]$ $V(z) = \frac{\sigma}{2\epsilon_0} [\sqrt{R^2 + z^2} - z]$
E and V for uniform hollow sphere:	$E(r) = \frac{\sigma R^2}{\epsilon_0 r^2} u(r - R)$ $V(r) = \frac{\sigma R}{\epsilon_0} u(R - r) + \frac{\sigma R^2}{\epsilon_0 r} u(r - R)$
Magnetic Wire:	$\frac{\mu_0}{4\pi} \frac{I}{r} (\sin(\theta_1) - \sin(\theta_2))$
Magnetic Sheet:	$B = \frac{\mu_0 K}{2}$
Magnetic Field of an Arc Along z:	$\frac{\mu_0 I R^2 \theta}{4\pi (z^2 + R^2)^{3/2}}$
Magnetic Field of a Solenoid:	$\mu_0 n I$
B-field in thin magnetized cylinder:	$B = \mu_0 M$
B-field in magnetized sphere:	$B = \frac{2}{3} \mu_0 M = \frac{2}{3} \mu_0 \sigma R \omega$ for spinning sphere