Generalized Multipole Expansion

$$V(r) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int_V ||\vec{r'}||^n P_n(\cos\alpha)\rho(\vec{r'})d\tau'$$

Monopole Voltage

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

Dipole Voltage

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

Dipole Moment – Continuous Charge Distribution

$$\vec{p} \equiv \int_V \vec{r'}\rho(\vec{r'})d\tau'$$

Dipole Moment – Point Charge Distribution

$$\vec{p} = \sum_{i=1}^{n} q_i \vec{r'}_i$$

Dipole Moment – Change of Origin

$$\vec{p'} = \vec{p} - Q\vec{a}$$

Force on a Dipole

$$\vec{F} = (\vec{p}\cdot\nabla)\vec{E}$$

Torque of a Dipole

$$\vec{N} = \vec{p}\times\vec{E}$$

Energy of a Dipole in an Electric Field

$$U = -\vec{p}\cdot\vec{E}$$

Bound Surface Charge Density

$$\sigma_{\text{bound}} = \vec{P}\cdot\hat{n}$$

Image Charge Surface Charge

$$\sigma = -\epsilon_0\frac{\partial V}{\partial n}$$

Linear Dielectrics

$$\vec{P} = \epsilon_0\chi_e\vec{E}$$
$$\vec{D} = \epsilon\vec{E}$$
$$\epsilon = \epsilon_0(1+\chi_e) = \epsilon_0\epsilon_r$$

Bound Volume Charge Density

$$\rho_{\text{bound}} \equiv -\nabla\cdot\vec{P}$$

Electric Displacement

$$\vec{D} \equiv \epsilon_0\vec{E} + \vec{P}$$

Gauss's Law for Electric Displacement

$$\nabla\cdot\vec{D} = \rho_f$$

$$\oint \vec{D}\cdot d\vec{A} = Q_{f_{\text{encl}}}$$

Displacement Boundary Condition

$$D^{\perp}_{\text{above}} - D^{\perp}_{\text{below}} = \sigma_{\text{free}}$$

Force on a Dielectric

$$F = -\nabla U$$

Solution for For Spherical Laplacians

$$f(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Legendre Polynomials

$$P_0(\cos\theta) = 1$$
$$P_1(\cos\theta) = \cos\theta$$
$$P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}$$
$$P_3(\cos\theta) = \frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta$$

Field of a Dipole

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^2}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

Common Boundary Conditions

- $\vec{D}^{\perp}_{\text{above}} - \vec{D}^{\perp}_{\text{below}} = \sigma_f$
- $\vec{D}^{\parallel}_{\text{above}} - \vec{D}^{\parallel}_{\text{below}} = \vec{P}^{\parallel}_{\text{above}} - \vec{P}^{\parallel}_{\text{below}}$
- $\vec{E}^{\perp}_{\text{above}} - \vec{E}^{\perp}_{\text{below}} = \frac{\sigma_{\text{surf}}}{\epsilon_0}$
- $V_{\text{above}} = V_{\text{below}}$
- $\epsilon_{\text{above}}E^{\perp}_{\text{above}} - \epsilon_{\text{below}}E^{\perp}_{\text{below}} = -\sigma_f$
- $\epsilon_{\text{above}}\frac{\partial V_a}{\partial n}^{\perp} - \epsilon_{\text{below}}\frac{\partial V_b}{\partial n}^{\perp} = \sigma_f$
- $V(r=0) =$ finite
- $V(r\to\infty) =$ finite or zero.

Method of Images Voltage

$$V(r,\theta) = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{||\vec{r}_{\text{sep}_1}||} + \frac{q_2}{||\vec{r}_{\text{sep}_2}||}\right)$$

$$\frac{-q_1}{q_2} = \frac{||\vec{r}_{\text{sep}_1}||}{||\vec{r}_{\text{sep}_2}||}$$

Binomial Expansion

$$(1+x)^k = \sum_{n=0}^{\infty}\left(\frac{k!}{n!(k-n)!}\right)x^n$$