

Electrostatics Basics

Line Charge:	$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\vec{r}')}{ \vec{r}_{\text{sep}} ^2} \hat{r}_{\text{sep}} dl'$
Surface Charge:	$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\vec{r}')}{ \vec{r}_{\text{sep}} ^2} \hat{r}_{\text{sep}} da'$
Volume Charge:	$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{ \vec{r}_{\text{sep}} ^2} \hat{r}_{\text{sep}} d\tau'$
Potential Difference:	$V(\vec{b}) - V(\vec{a}) = - \int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l}$
Potential of a Volume Charge:	$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{ \vec{r}_{\text{sep}} } d\tau' t$
Potential of a Collection of Point Charges:	$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=0}^n \frac{q_i}{ \vec{r}_{\text{sep}} }$
Work to Move a Charge:	$W = \int_{\mathbf{a}}^{\mathbf{b}} \vec{F} \cdot d\vec{l} = -Q \int_{\mathbf{a}}^{\mathbf{b}} \vec{E} \cdot d\vec{l}$
Work to Move a Charge (Potential):	$W = Q[V(\vec{b}) - V(\vec{a})]$
Energy of a Collection of Point Charges:	$W = \frac{1}{2} \sum_{i=1}^n q_i V(\vec{r}_i)$
Energy of a Continuous Charge Distribution:	$W = \frac{\epsilon_0}{2} \int_{\text{univ}} E^2 d\tau = \frac{1}{2} \iiint \rho V d\tau$
Field at a Charged Surface:	$\vec{E}_{\text{surf}} = \frac{1}{2} (\vec{E}_{\text{above}} + \vec{E}_{\text{below}})$
Parallel Plate Voltage:	$V = \frac{Q}{A\epsilon_0} d$
Capacitance:	$C \equiv \frac{Q}{V}$
Energy of a Capacitor:	$W = \int_0^Q \left(\frac{q}{C}\right) dq = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2$

Electrostatic Multipoles

Generalized Multipole Expansion:	$V(r) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int_V \vec{r}' ^n P_n(\cos \alpha) \rho(\vec{r}') d\tau'$
Monopole Voltage:	$V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$
Dipole Voltage:	$V(r) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$
Dipole Moment (Continuous):	$\vec{p} \equiv \int_V \vec{r}' \rho(\vec{r}') d\tau'$
Dipole Moment (Discrete):	$\vec{p} = \sum_{i=1}^N q_i \vec{r}'_i$
Dipole Moment (Change of Origin):	$\vec{p}' = \vec{p} - Q\vec{a}$
Torque of a Dipole:	$\vec{N} = \vec{p} \times \vec{E}$
Energy of a Dipole:	$U = -\vec{p} \cdot \vec{E}$
Force on a Dipole:	$\vec{F} = (\vec{p} \cdot \nabla) \vec{E}$
Field of a Dipole:	$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta})$

Electrostatics in Matter

Electric Displacement:	$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$
Bound Surface Charge:	$\sigma_{\text{bound}} = \vec{P} \cdot \hat{n}$
Bound Volume Charge:	$\rho_{\text{bound}} \equiv -\nabla \cdot \vec{P}$
Linear Dielectrics – Polarization:	$\vec{P} = \epsilon_0 \chi_e \vec{E}$
Linear Dielectrics – E-Field:	$\vec{D} = \epsilon \vec{E}$
Electric Permittivity:	$\epsilon = \epsilon_0 (1 + \chi_e) = \epsilon_0 \epsilon_r$
Gauss's Law for \vec{D} (Derivative):	$\nabla \cdot \vec{D} = \rho_f$
Gauss's Law for \vec{D} (Integral):	$\oint \vec{D} \cdot d\vec{A} = Q_{f\text{encl}}$
Force on a Dielectric:	$F = -\nabla U$
Energy of a Dielectric:	$W = \frac{1}{2} \int \vec{D} \cdot \vec{E} d\tau$

Names of Stuff

ϵ_0 : Permittivity of Free Space
 ϵ_r : Dielectric Constant or Relative Permittivity
 ϵ : Permittivity of a Material

Techniques for Solving Problems

Cylindrical Laplacian Solution:

$$V(s, \phi) = A \ln s + B + \sum_{n=1}^{\infty} \left(A_n s^n + \frac{B_n}{s^n} \right) (C_n \cos(\phi n) + D_n \sin(\phi n))$$

Solution for Spherical Laplacians:

$$f(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

Method of Images General Principle:

$$\sum \frac{q_i}{||\vec{r}_{\text{sep } i}||} = 0$$

Method of Images for Two Points:

$$-\frac{q_1}{q_2} = \frac{||\vec{r}_{\text{sep } 1}||}{||\vec{r}_{\text{sep } 2}||}$$

Image Charge Surface Charge:

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial n}$$

Binomial Expansion:

$$(1+x)^k = \sum_{n=0}^{\infty} \left(\frac{k!}{n!(k-n)!} \right) x^n$$

Taylor Expansion:

$$f(x) \approx \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}$$

Because I can't remember:

$$f(x) \approx f(0) + f'(0) \cdot x + f''(0) \cdot \frac{x^2}{2} + \dots$$

Legendre Polynomials:

$$\begin{aligned}
 P_0(\cos \theta) &= 1 \\
 P_1(\cos \theta) &= \cos \theta \\
 P_2(\cos \theta) &= \frac{3}{2} \cos^2 \theta - \frac{1}{2} \\
 P_3(\cos \theta) &= \frac{5}{2} \cos^3 \theta - \frac{3}{2} \cos \theta
 \end{aligned}$$

Magnetostatics

Lorentz Force Law:	$\vec{F}_{\text{mag}} = Q(\vec{v} \times \vec{B})$
Surface Current Density:	$\vec{K} \equiv \frac{d\vec{I}}{dl_{\perp}}$ $\vec{K} = \sigma \vec{v}$
Volume Current Density:	$\vec{J} \equiv \frac{d\vec{I}}{da_{\perp}}$ $\vec{J} = \rho \vec{v}$
Magnetic Force (General):	$\vec{F}_{\text{mag}} = \int I(d\vec{l} \times \vec{B}) = I \int (d\vec{l} \times \vec{B})$
Magnetic Force (Surface):	$\vec{F}_{\text{mag}} = \int (\vec{K} \times \vec{B}) da$
Magnetic Force (Volume):	$\vec{F}_{\text{mag}} = \int (\vec{J} \times \vec{B}) d\tau$
Biot-Savart Law:	$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times \hat{r}_{\text{sep}}}{ \vec{r}_{\text{sep}} ^2} dl'$ $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{d\vec{I}' \times \hat{r}_{\text{sep}}}{ \vec{r}_{\text{sep}} ^2}$
Ampere's Law:	$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$
Magnetic Vector Potential:	$\vec{B} = \vec{\nabla} \times \vec{A}$ $\vec{\nabla} \cdot \vec{A} = 0$ $\nabla^2 \vec{A} = -\mu_0 \vec{J}$
Vector Potential – Line:	$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{I}}{ \vec{r}_{\text{sep}} } dl' = \frac{\mu_0 I}{4\pi} \int \frac{1}{ \vec{r}_{\text{sep}} } d\vec{l}'$
Vector Potential – Surface:	$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{K}}{ \vec{r}_{\text{sep}} } da'$
Vector Potential – Volume:	$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{ \vec{r}_{\text{sep}} } d\tau'$
Magnetic Dipole Moment:	$\vec{m} \equiv I \int d\vec{a} = I \vec{a}$
Magnetic Dipole Potential Expansion:	$\vec{A}_{\text{dip}}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$
Magnetic Torque:	$\vec{N} = \vec{m} \times \vec{B}$
Energy of a Dipole:	$U = -\vec{m} \cdot \vec{B}$
Force on a Magnetic Dipole:	$\vec{F} = \nabla(\vec{m} \cdot \vec{B})$
Force Between Two Current-Carrying Wires:	$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$
Linear Current:	$\vec{I} = \int \vec{K} \cdot d\vec{l} = \iint \vec{J} \cdot d\vec{a}$

Maxwell's Equations

Gauss's Law: $\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$

(unnamed) $\vec{\nabla} \cdot \vec{B} = 0$

Faraday's Law: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Ampere's Law with Maxwell's Correction: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Magnetostatics in Matter

Paramagnets: magnetization is **parallel** to \vec{B}

Diamagnets: magnetization is **opposite** to \vec{B}

Ferromagnets: magnetization holds outside the presence of an external magnetic field.

Magnetization: $\vec{M} \equiv$ magnetic dipole per unit volume

Bound Volume Current: $\vec{J}_{\text{bound}} = \vec{\nabla} \times \vec{M}$

Bound Surface Current: $\vec{K}_{\text{bound}} = \vec{M} \times \hat{n}$

Free Current: $\vec{J} = \vec{J}_b + \vec{J}_f$

Auxillary Field: $\vec{H} \equiv \frac{1}{\mu_0} \vec{B} - \vec{M}$

Ampere's Law for Auxillary Fields: $\vec{\nabla} \times \vec{H} = \vec{J}_f$
 $\oint \vec{H} \cdot d\vec{l} = I_{f\text{enc}}$

Linear Magnetics: $\vec{M} = \chi_m \vec{H}$

Permeability: $\mu \equiv \mu_0 (1 + \chi_m)$

Relative Permeability: $\mu_r 1 + \chi_m$

Linear Magnetics with E and H Fields: $\vec{B} = \mu \vec{H}$.

Dipole Field: $\vec{B} = \frac{\mu_0 \vec{m}}{4\pi r^3} (2 \cos(\theta) \hat{r} + \sin(\theta) \hat{\theta})$, for $\vec{m} = m\hat{z}$

Electrodynamics

Ohm's Law: $V = IR$

Power in a Circuit: $P = VI = I^2 R = \frac{V^2}{R}$

Electromotive Force: $\mathcal{E} = \frac{F_{\text{mag, tot}}}{Q} = \int (\vec{v} \times \vec{B}) d\vec{l}$
 $\mathcal{E} = \oint \vec{f} \cdot d\vec{l}$
 $\mathcal{E} = -\frac{d\Phi}{dt}$

Faraday's Law: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Inductance: $\Phi = LI$
 $\mathcal{E} = -L \frac{dI}{dt}$

Work of a Magnetic Field: $\frac{1}{2} LI^2$
 $W = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 d\tau$

Displacement Current: $\vec{J}_d \equiv \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Maxwell's Equations in Matter

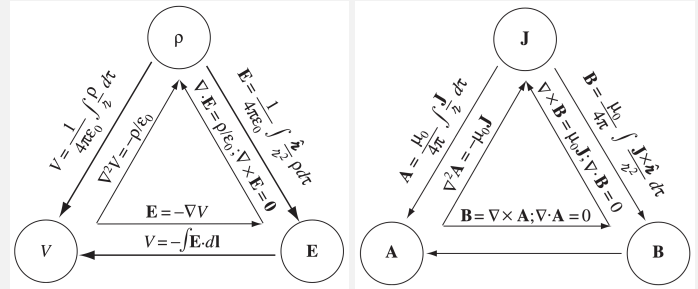
Gauss's Law: $\vec{\nabla} \cdot \vec{D} = \rho_f$

(unnamed) $\vec{\nabla} \cdot \vec{B} = 0$

Faraday's Law: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Ampere's Law with Maxwell's Correction: $\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$

Triangles



Takeaways from Practice

- Gauss's law inside a dielectric always includes both ρ_b and σ_b .
- Use the definition for displacement where ever possible: $\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$.
- The potential is constant below an image plane and the field is always zero.

Boundary Conditions in Electrostatics

$$\begin{aligned} \vec{D}_{\text{above}}^\perp - \vec{D}_{\text{below}}^\perp &= \sigma_f \\ \vec{D}_{\text{above}}^\parallel - \vec{D}_{\text{below}}^\parallel &= \vec{P}_{\text{above}}^\parallel - \vec{P}_{\text{below}}^\parallel \\ \vec{E}_{\text{above}}^\perp - \vec{E}_{\text{below}}^\perp &= \sigma_{\text{tot}}/\epsilon_0 \\ \vec{E}_{\text{above}}^\parallel - \vec{E}_{\text{below}}^\parallel &= 0 \\ V_{\text{above}} &= V_{\text{below}} \\ \epsilon_{\text{above}} \vec{E}_{\text{above}}^\perp - \epsilon_{\text{below}} \vec{E}_{\text{below}}^\perp &= \sigma_f \\ \epsilon_{\text{above}} \frac{\partial V_{\text{above}}}{\partial n} - \epsilon_{\text{below}} \frac{\partial V_{\text{below}}}{\partial n} &= -\sigma_f \end{aligned}$$

Boundary Conditions in Magnetostatics

$$\begin{aligned} \vec{B}_{\text{above}}^\parallel - \vec{B}_{\text{below}}^\parallel &= \mu_0 \vec{K} \\ \vec{B}_{\text{above}} - \vec{B}_{\text{below}} &= \mu_0 (\vec{K} \times \hat{n}) \\ \vec{A}_{\text{above}} &= \vec{A}_{\text{below}} \\ \frac{\partial A_{\text{above}}}{\partial n} - \frac{\partial A_{\text{below}}}{\partial n} &= -\mu_0 \vec{K} \\ \vec{H}_{\text{above}}^\perp - \vec{H}_{\text{below}}^\perp &= -(\vec{M}_{\text{above}}^\perp - \vec{M}_{\text{below}}^\perp) \\ \vec{H}_{\text{above}}^\parallel - \vec{H}_{\text{below}}^\parallel &= \vec{K}_f \times \hat{n} \end{aligned}$$

More Boundary Conditions

Equations 7.60-63 are the general boundary conditions for electrodynamics. In the case of *linear* media, they can be expressed in terms of \vec{E} and \vec{B} alone:

$$\left. \begin{aligned} \text{(i)} \quad \epsilon_1 E_1^\perp - \epsilon_2 E_2^\perp &= \sigma_f, & \text{(iii)} \quad \vec{E}_1^\parallel - \vec{E}_2^\parallel &= 0, \\ \text{(ii)} \quad B_1^\perp - B_2^\perp &= 0, & \text{(iv)} \quad \frac{1}{\mu_1} \vec{B}_1^\parallel - \frac{1}{\mu_2} \vec{B}_2^\parallel &= \vec{K}_f \times \hat{n}. \end{aligned} \right\} \quad (7.64)$$

In particular, if there is no free charge or free current at the interface, then

$$\left. \begin{aligned} \text{(i)} \quad \epsilon_1 E_1^\perp - \epsilon_2 E_2^\perp &= 0, & \text{(iii)} \quad \vec{E}_1^\parallel - \vec{E}_2^\parallel &= 0, \\ \text{(ii)} \quad B_1^\perp - B_2^\perp &= 0, & \text{(iv)} \quad \frac{1}{\mu_1} \vec{B}_1^\parallel - \frac{1}{\mu_2} \vec{B}_2^\parallel &= 0. \end{aligned} \right\} \quad (7.65)$$

As we shall see in Chapter 9, these equations are the basis for the theory of reflection and refraction.

Common Fields

Magnetic Finite Wire: $\frac{\mu_0}{4\pi} \frac{I}{R} (\sin(\theta_1) - \sin(\theta_2))$

Magnetic Field of an Arc: $\frac{\mu_0 I \theta}{4\pi R}$

Magnetic Field of an Infinite Wire: $\frac{\mu_0 I}{2\pi R}$

Magnetic Field of a Solenoid: $\mu_0 n I$