PHYS 301 Notes

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Midterm 1

Calculus Review

Spherical Coordinates:

Sometimes, spherical coordinates are easier to work with than cartesian coordinates. When using spherical coordinates, there are some key things to note:

1. The coordinates themselves::
   ϕ: Equitorial Azimuth $[0, 2\pi]$ (from the "x" axis)
   θ: Axial Azimuth $[0, \pi]$ (from the "z" axis)
   r: Radial Distance
2. The infinitesimal displacement is different:
   $d\vec{l} = dr\,\hat{r} + rd\theta\,\hat{\theta} + r\sin\theta d\phi\,\hat{\phi}$
3. The volume element is different:
   $d\tau = r^2 \sin\theta\, dr\, d\theta\, d\phi$

Cylindrical Coordinates:: See textbook, this would be too redundant.

Rules for Irrotational Fields/Conservative Fields:

1. $\vec{\nabla} \times \vec{F} = \vec{0}$ everywhere.
2. $\int_a^b \vec{F} \cdot d\vec{l}$ is independent of path, for any given end points.
3. $\oint \vec{F} \cdot d\vec{l} = 0$ for any closed loop.
4. $\vec{F}$ is the gradient of some scalar function: $\vec{F} = -\nabla V$.

It is important to note that since $\vec{E} = -\nabla V$, then all electrostatic fields are irrotational.

Rules for Divergence-less fields.:

1. $\vec{\nabla} \cdot \vec{F} = 0$ everwhere.
2. $\int \vec{F} \cdot d\vec{a}$ is independent of surface, for any given boundary line.
3. $\oint \vec{F} \cdot d\vec{a} = 0$ for any closed surface.
4. $\vec{F}$ is the curl of some vector function.

Electric Fields

It is important to recognize the notation in the Griffith's textbook, which is used primarily for this course when working with Fields and directions:

$\vec{r}$: distance from the origin to a "field point".
$\vec{r'}$: distance from the origin to the charge
$\vec{r_{\text{sep}}}$: distance from the charge to the "field point"
Thus, it is given that $\vec{r_{\text{sep}}} = \vec{r} - \vec{r'}$.

Now, while working with Coulomb's law, we can define the electric field due to a point charge to be:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{||\vec{r_{\text{sep}}}||^2} \vec{r_{\text{sep}}}$$

Types of Field Integrations: There exist three main types of field integrations:

Line Charge

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\vec{r'})}{||\vec{r_{\text{sep}}}||^2} \vec{r_{\text{sep}}}\, dl'$$

Surface Charge

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\vec{r'})}{||\vec{r_{\text{sep}}}||^2} \vec{r_{\text{sep}}}\, da'$$

Volume Charge

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r'})}{||\vec{r_{\text{sep}}}||^2} \vec{r_{\text{sep}}}\, d\tau'$$

Gauss's Law: Gauss's law is as follows:

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad\qquad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

By knowing the geometry and symmetry of a situation however, Gauss's law can require no integration whatsoever. This is when we know that field is the same for every point on a surface.

Electric Potential

The potential of a point in a field relative to another is defined as:

$$V(\vec{r}) \equiv -\int_{\mathcal{O}}^{r} \vec{E} \cdot d\vec{l}$$

Which is often referred to as the potential difference between two points:

$$V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l}$$

It is also to be noted the relation between field and potential:

$$\vec{E} = -\nabla V$$

Remark: it is important to note that unlike electric field, electric potential is a scalar. It has no direction, and should be handled accordingly.

Potential can also be obtained for a volume charge:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r'})}{||\vec{r_{\text{sep}}}||} d\tau'$$

Similar formulae can be extrapilated for line and surface charge distributions, analogous to those for electric fields.

Work and Energy in Electrostatics

To calculate the work that it takes to get from one point to another in a field, we can use a work integral. We can also employ the potential difference.

$$W = \int_{\mathbf{a}}^{\mathbf{b}} \vec{F} \cdot d\vec{l} = -Q \int_{\mathbf{a}}^{\mathbf{b}} \vec{E} \cdot d\vec{l} = Q[V(\vec{b}) - Q(\vec{a})]$$

It is important to note that for the second form of this integral uses **negative** Q, not positive. This is because the integral by default describes the amount of work that the field does, not the work that is required. One can use a positive value of Q if trying to find out how much work the field does.

It is also to be noted that the potential of a system is the work that is required to create the system per unit charge.

Work and Point Charges: We can also describe the amount of work that it takes to assemble a collection of point charges:

$$W = \frac{1}{2} \sum_{i=1}^{n} q_i V(\vec{r_i})$$