Electrostatics Basics

Line Charge:	$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(\vec{r'})}{\|\|\vec{r_{\text{sep}}}\|\|^2}\hat{r}_{\text{sep}}\, dl'$
Surface Charge:	$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\vec{r'})}{\|\|\vec{r_{\text{sep}}}\|\|^2}\hat{r}_{\text{sep}}\, da'$
Volume Charge:	$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r'})}{\|\|\vec{r_{\text{sep}}}\|\|^2}\hat{r}_{\text{sep}}\, d\tau'$
Potential Difference:	$V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{b}\vec{E}\cdot d\vec{l}$
Potential of a Volume Charge:	$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r'})}{\|\|\vec{r_{\text{sep}}}\|\|}d\tau' t$
Potential of a Collection of Point Charges:	$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=0}^{n}\frac{q_i}{\|\|\vec{r_{\text{sep}}}\|\|}$
Work to Move a Charge:	$W = \int_{\mathbf{a}}^{\mathbf{b}}\vec{F}\cdot d\vec{l} = -Q\int_{\mathbf{a}}^{\mathbf{b}}\vec{E}\cdot d\vec{l}$
Work to Move a Charge (Potential):	$W = Q[V(\vec{b}) - Q(\vec{a})]$
Energy of a Collection of Point Charges:	$W = \frac{1}{2}\sum_{i=1}^{n}q_i V(\vec{r_i})$
Energy of a Continuous Charge Distribution:	$W = \frac{\epsilon_0}{2}\int_{\text{univ}} E^2 d\tau$
Field at a Charged Surface:	$\vec{E}_{\text{surf}} = \frac{1}{2}\left(\vec{E}_{\text{above}} + \vec{E}_{\text{below}}\right)$
Parallel Plate Voltage:	$V = \frac{Q}{A\epsilon_0}d$
Capacitance:	$C \equiv \frac{Q}{V}$
Energy of a Capacitor:	$W = \int_0^Q \left(\frac{q}{C}\right)dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$

Techniques for Solving Problems

Cylindrical Laplacian Solution:

$$V(s,\phi) = A\ln s + B + \sum_{n=1}^{\infty}\left(A_n s^n + \frac{B_n}{s^n}\right)(C_n\cos(\phi n) + D_n\sin(\phi n))$$

Solution for Spherical Laplacians:	$f(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$
Method of Images General Principle:	$\sum \frac{q_i}{\|\|\vec{r_{\text{sep}}}_i\|\|} = 0$
Method of Images for Two Points:	$-\frac{q_1}{q_2} = \frac{\|\|\vec{r_{\text{sep}}}_1\|\|}{\|\|\vec{r_{\text{sep}}}_2\|\|}$
Image Charge Surface Charge:	$\sigma = -\epsilon_0\frac{\partial V}{\partial n}$
Binomial Expansion:	$(1+x)^k = \sum_{n=0}^{\infty}\left(\frac{k!}{n!(k-n)!}\right)x^n$
Taylor Expansion:	$f(x) \approx \sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}$
Because I can't remember:	$f(x) \approx f(0) + f'(0)\cdot x + f''(0)\cdot\frac{x^2}{2} + \cdots$
Legendre Polynomials:	$P_0(\cos\theta) = 1$ $P_1(\cos\theta) = \cos\theta$ $P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}$ $P_3(\cos\theta) = \frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta$

Electrostatic Multipoles

Generalized Multipole Expansion:

$$V(r) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int_V \|\|\vec{r'}\|\|^n P_n(\cos\alpha)\rho(\vec{r'})d\tau'$$

Monopole Voltage:	$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$
Dipole Voltage:	$V(r) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$
Dipole Moment (Continuous):	$\vec{p} \equiv \int_V \vec{r'}\rho(\vec{r'})d\tau'$
Dipole Moment (Discrete):	$\vec{p} = \sum_{i=1}^{n} q_i\vec{r'}_i$
Dipole Moment (Change of Origin):	$\vec{p'} = \vec{p} - Q\vec{a}$
Torque of a Dipole:	$\vec{N} = \vec{p}\times\vec{E}$
Energy of a Dipole:	$U = -\vec{p}\cdot\vec{E}$
Force on a Dipole:	$\vec{F} = (\vec{p}\cdot\nabla)\vec{E}$
Field of a Dipole:	$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^2}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$

Magnetostatics

Lorentz Force Law:	$\vec{F}_{\text{mag}} = Q(\vec{v}\times\vec{B})$
Surface Current Density:	$\vec{K} \equiv \frac{d\vec{I}}{dl_\perp}$ $\vec{K} = \sigma\vec{v}$
Volume Current Density:	$J \equiv \frac{d\vec{I}}{da_\perp}$ $\vec{J} = \rho\vec{v}$
Magnetic Force (General):	$\vec{F}_{\text{mag}} = \int I(d\vec{I}\times\vec{B}) = I\int(d\vec{I}\times\vec{B})$
Magnetic Force (Surface):	$\vec{F}_{\text{mag}} = \int(\vec{K}\times\vec{B})da$
Magnetic Force (Volume):	$\vec{F}_{\text{mag}} = \int(\vec{J}\times\vec{B})d\tau$
Biot-Savart Law:	$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{\vec{I}\times\hat{r}_{\text{sep}}}{\|\|\vec{r_{\text{sep}}}\|\|^2}dl' = \frac{\mu_0}{4\pi}I\int\frac{d\vec{I'}\times\hat{r}_{\text{sep}}}{\|\|\vec{r_{\text{sep}}}\|\|^2}$
Ampere's Law:	$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ $\oint\vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$
Magnetic Vector Potential:	$\vec{B} = \vec{\nabla}\times\vec{A}$ $\vec{\nabla}\cdot\vec{A} = 0$ $\nabla^2\vec{A} = -\mu_0\vec{J}$
Vector Potential – Line:	$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{I}}{\|\|\vec{r_{\text{sep}}}\|\|}dl' = \frac{\mu_0 I}{4\pi}\int\frac{1}{\|\|\vec{r_{\text{sep}}}\|\|}d\vec{l'}$
Vector Potential – Surface:	$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{K}}{\|\|\vec{r_{\text{sep}}}\|\|}da'$
Vector Potential – Volume:	$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r'})}{\|\|\vec{r_{\text{sep}}}\|\|}d\tau'$
Magnetic Dipole Moment:	$\vec{m} \equiv I\int d\vec{a} = I\vec{a}$
Magnetic Dipole Potential Expansion:	$\vec{A}_{\text{dip}}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$
Magnetic Torque:	$\vec{N} = \vec{m}\times\vec{B}$
Force on a Magnetic Dipole:	$\vec{F} = \nabla(\vec{m}\cdot\vec{B})$
Force Between Two Current-Carrying Wires:	$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$

Electrostatics in Matter

Electric Displacement:	$\vec{D} \equiv \epsilon_0\vec{E} + \vec{P}$
Bound Surface Charge:	$\sigma_{\text{bound}} = \vec{P}\cdot\hat{n}$
Bound Volume Charge:	$\rho_{\text{bound}} \equiv -\nabla\cdot\vec{P}$
Linear Dielectrics – Polarization:	$\vec{P} = \epsilon_0\chi_e\vec{E}$
Linear Dielectrics – E-Field:	$\vec{D} = \epsilon\vec{E}$
Electric Permittivity:	$\epsilon = \epsilon_0(1+\chi_e) = \epsilon_0\epsilon_r$
Gauss's Law for $\vec{D}$ (Derivative):	$\nabla\cdot\vec{D} = \rho_f$
Gauss's Law for $\vec{D}$ (Integral):	$\oint\vec{D}\cdot d\vec{A} = Q_{f_{\text{encl}}}$
Force on a Dielectric:	$F = -\nabla U$
Energy of a Dielectric:	$W = \frac{1}{2}\int\vec{D}\cdot\vec{E}d\tau$

Magnetostatics in Matter

Paramagnets:	magnetization is **parallel** to $\vec{B}$
Diamagnets:	magnetization is **opposite** to $\vec{B}$
Ferromagnets:	magnetization holds outside the presence of an external magnetic field.
Magnetization:	$\vec{M} \equiv$ magnetic dipole per unit volume
Bound Volume Current:	$\vec{J}_{\text{bound}} = \vec{\nabla} \times \vec{M}$
Bound Surface Current:	$\vec{K}_{\text{bound}} = \vec{M} \times \hat{n}$
Free Current:	$\vec{J} = \vec{J}_b + \vec{J}_f$
Auxillary Field:	$\vec{H} \equiv \frac{1}{\mu_0}\vec{B} - \vec{M}$
Ampere's Law for Auxillary Fields:	$\vec{\nabla} \times \vec{H} = \vec{J}_f$ $\oint \vec{H} \cdot d\vec{l} = I_{f_{\text{encl}}}$
Linear Magnetics:	$\vec{M} = \chi_m \vec{H}$
Permeability:	$\mu \equiv \mu_0(1 + \chi_m)$
Linear Magnetics with E and H Fields:	$\vec{B} = \mu \vec{H}$.

Electrodynamics

Ohm's Law:	$V = IR$
Power in a Circuit:	$P = VI = I^2 R = \frac{V^2}{R}$
Electromotive Force:	$\mathcal{E} = \frac{F_{\text{mag, tot}}}{Q} = \int (\vec{v} \times \vec{B}) d\vec{l}$ $\mathcal{E} = \oint \vec{f} \cdot d\vec{l}$ $\mathcal{E} = -\frac{d\Phi}{dt}$
Faraday's Law:	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Inductance:	$\Phi = LI$ $\mathcal{E} = -L\frac{dI}{dt}$
Work of a Magnetic Field:	$\frac{1}{2}LI^2$ $W = \frac{1}{2\mu_0}\int_{\text{all space}} B^2 d\tau$
Displacement Current:	$\vec{J}_d \equiv \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Maxwell's Equations

Gauss's Law:	$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0}\rho$
(unnamed)	$\vec{\nabla} \cdot \vec{B} = 0$
Faraday's Law:	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Ampere's Law with Maxwell's Correction:	$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0\ \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Maxwell's Equations in Matter

Gauss's Law:	$\vec{\nabla} \cdot \vec{D} = \rho_f$
(unnamed)	$\vec{\nabla} \cdot \vec{B} = 0$
Faraday's Law:	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Ampere's Law with Maxwell's Correction:	$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$

Boundary Conditions in Electrostatics

$$\vec{D}^{\perp}_{\text{above}} - \vec{D}^{\perp}_{\text{below}} = \sigma_f$$

$$\vec{D}^{\parallel}_{\text{above}} - \vec{D}^{\parallel}_{\text{below}} = \vec{P}^{\parallel}_{\text{above}} - \vec{P}^{\parallel}_{\text{below}}$$

$$\vec{E}^{\perp}_{\text{above}} - \vec{E}^{\perp}_{\text{below}} = \sigma_{\text{tot}}/\epsilon_0$$

$$\vec{E}^{\parallel}_{\text{above}} - \vec{E}^{\parallel}_{\text{below}} = \vec{0}$$

$$V_{\text{above}} = V_{\text{below}}$$

$$\epsilon_{\text{above}}\vec{E}^{\perp}_{\text{above}} - \epsilon_{\text{below}}\vec{E}^{\perp}_{\text{below}} = \sigma_f$$

$$\epsilon_{\text{above}}\frac{\partial V_{\text{above}}}{\partial n} - \epsilon_{\text{below}}\frac{\partial V_{\text{below}}}{\partial n} = -\sigma_f$$

Boundary Conditions in Magnetostatics

$$\vec{B}^{\parallel}_{\text{above}} - \vec{B}^{\parallel}_{\text{below}} = \mu_0 \vec{K}$$

$$\vec{B}_{\text{above}} - \vec{B}_{\text{below}} = \mu_0(\vec{K} \times \hat{\mathbf{n}})$$

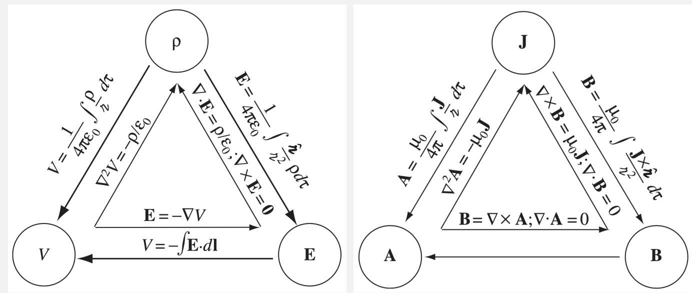
$$\vec{A}_{\text{above}} = \vec{A}_{\text{below}}$$

$$\frac{\partial A_{\text{above}}}{\partial n} - \frac{\partial A_{\text{below}}}{\partial n} = -\mu_0 \vec{K}$$

$$\vec{H}^{\perp}_{\text{above}} - \vec{H}^{\perp}_{\text{below}} = -(\vec{M}^{\perp}_{\text{above}} - \vec{M}^{\perp}_{\text{below}})$$

$$\vec{H}^{\parallel}_{\text{above}} - \vec{H}^{\parallel}_{\text{below}} = \vec{K}_f \times \hat{\mathbf{n}}$$

Triangles



More Boundary Conditions

Equations 7.60-63 are the general boundary conditions for electrodynamics. In the case of *linear* media, they can be expressed in terms of **E** and **B** alone:

$$\left.\begin{array}{ll} \text{(i)}\ \epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma_f, & \text{(iii)}\ \mathbf{E}_1^{\parallel} - \mathbf{E}_2^{\parallel} = \mathbf{0}, \\ \text{(ii)}\ B_1^{\perp} - B_2^{\perp} = 0, & \text{(iv)}\ \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} - \frac{1}{\mu_2}\mathbf{B}_2^{\parallel} = \mathbf{K}_f \times \hat{\mathbf{n}}. \end{array}\right\} \tag{7.64}$$

In particular, if there is no free charge or free current at the interface, then

$$\begin{array}{ll} \text{(i)}\ \epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = 0, & \text{(iii)}\ \mathbf{E}_1^{\parallel} - \mathbf{E}_2^{\parallel} = \mathbf{0}, \\ \text{(ii)}\ B_1^{\perp} - B_2^{\perp} = 0, & \text{(iv)}\ \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} - \frac{1}{\mu_2}\mathbf{B}_2^{\parallel} = \mathbf{0}. \end{array} \tag{7.65}$$

As we shall see in Chapter 9, these equations are the basis for the theory of reflection and refraction.

Takeaways from Practice

- Gauss's law inside a dielectric always includes both ρ_b and σ_b.
- Use the definition for displacement where ever possible: $\vec{D} \equiv \epsilon_0\vec{E} + \vec{P}$.
- The potential is constant below an image plane and the field is always zero.

Names of Stuff

ϵ_0: Permittivity of Free Space
ϵ_r: Dielectric Constant or Relative Permittivity
ϵ: Permittivity of a Material