CSCI 132: Basic Data Structures and Algorithms

Growth Rates

Reese Pearsall Spring 2023

Announcements

Program 2 due 3/10

Midterm Exam one week from today

No lab next week

English Teachers: You will almost never use the semicolon.

Java programmers:



As computer scientists, we write many algorithms

We want to be able to describe how well our algorithms perform on a variety of inputs

As computer scientists, we write many algorithms

We want to be able to describe how well our algorithms perform on a variety of inputs



Consider an algorithm that will make a cake

How could measure the effectiveness and performance of our cake making algorithm?



What are some ways we could measure the performance and effectiveness of our algorithm?

What is the **total time needed** to make the cake?

- Prep time
- Combining ingredients
- Baking
- Cooling
- Icing

The **time** an algorithm takes to finish is important. We generally want our algorithms to run as fast as possible



What are some ways we could measure the performance and effectiveness of our algorithm?

How well does our algorithm work on a variety of problems?







Suppose we needed to make different types of cake, How well would our algorithm do?

Generalizability and Reliability



What are some ways we could measure the performance and effectiveness of our algorithm?

How well does our cake taste?



We want our algorithm to output the best cake possible

Does our algorithm always yield the **optimal** result?



What are some ways we could measure the performance and effectiveness of our algorithm?

Does our algorithm actually make a cake?



Does our algorithm *actually* do what we say it does?

Our algorithm needs to be valid



What are some ways we could measure the performance and effectiveness of our algorithm?



Suppose we needed to make a lot of the same cake

How well does our algorithm scale?



What are some ways we could measure the performance and effectiveness of our algorithm?



How much kitchen **space** is needed to make the cake?

What if the cake needed is really big?



Algorithm Analysis

Performance & Efficiency

- 1. Running time of Algorithm
- 2. Space needed to run the algorithm

Algorithm Correctness

- Validity
- Optimality
- Generalizability



Algorithm Analysis

Performance & Efficiency

- 1. Running time of Algorithm
- 2. Space needed to run the algorithm



Important: How well does the algorithm do as the problem gets bigger? (Scalability)

Algorithm Correctness

- Validity
- Optimality
- Generalizability

The **growth rate** of the algorithm looks at how much more resource an algorithm needs (time or space) as the input size increases

(In this class, we will be focusing on time)

The easiest way I could prove the running time of an algorithm is by starting a stopwatch when the algorithm starts, and stop when algorithm finishes



This is not a very good way to accurately show the running time because ...

The easiest way I could prove the running time of an algorithm is by starting a stopwatch when the algorithm starts, and stop when algorithm finishes



This is not a very good way to accurately show the running time because ...



What if this is my computer?

Issue: The time needed to run an algorithm varies depending on the <u>hardware of computer</u> that is running the algorithm











Issue: The time needed to run an algorithm varies depending on the <u>hardware of computer</u> that is running the algorithm

Instead of focusing on the actual time needed to run an algorithm (seconds), we will look at the **number of steps/instructions in the algorithm** that need be executed as the input grows!

The **growth rate** of the algorithm looks at how much more resource an algorithm needs (time or space) as the input size increases



```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {</pre>
       if ( array[i] > largest so far ) {
           largest_so_far = array[i];
   return largest so far;
```

This algorithm will find the largest value in some **N** sized array

```
public int find max value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {</pre>
       if ( array[i] > largest so far ) {
           largest so far = array[i];
   return largest so far;
```

This algorithm will find the largest value in some **N** sized array

This code checks each spot in the array

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {</pre>
       if ( array[i] > largest so far ) {
           largest so far = array[i];
   return largest so far;
```

This algorithm will find the largest value in some **N** sized array

This code checks each spot in the array

Let's look at how many times this instruction is executed as the array size grows

10	

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if ( array[i] > largest_so_far ) {
            largest_so_far = array[i];
        }
   }
   return largest_so_far;
}
```

10	10

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if (array[i] > largest_so_far ) {
            largest_so_far = array[i];
        }
   }
   return largest_so_far;
}
```

10

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if (array[i] > largest_so_far ) {
            largest_so_far = array[i];
        }
   }
   return largest_so_far;
}
```

10	10
100	100

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if (array[i] > largest_so_far) {
          largest_so_far = array[i];
      }
   }
   return largest_so_far;
}
```

10	10
100	100
533	533

```
public int find_max_value(int[] array) {
    int largest_so_far = -1;
    for(int i = 0; i < array.length; i++) {
        if (array[i] > largest_so_far ) {
            largest_so_far = array[i];
        }
    }
    return largest_so_far;
}
```

10	10
100	100
533	533
1000	1000

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if (array[i] > largest_so_far ) {
          largest_so_far = array[i];
      }
   }
   return largest_so_far;
}
```

10	10
100	100
533	533
1000	1000
9999	

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if (array[i] > largest_so_far ) {
            largest_so_far = array[i];
        }
   }
   return largest_so_far;
}
```

10	10
100	100
533	533
1000	1000
9999	9999

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if (array[i] > largest_so_far ) {
            largest_so_far = array[i];
        }
   }
   return largest_so_far;
}
```

10	10
100	100
533	533
1000	1000
9999	9999
1	

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if (array[i] > largest_so_far) {
            largest_so_far = array[i];
        }
   }
   return largest_so_far;
}
```

10	10
100	100
533	533
1000	1000
9999	9999
1	1

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if (array[i] > largest_so_far) {
          largest_so_far = array[i];
      }
   }
  return largest_so_far;
}
```

10	10
100	100
533	533
1000	1000
9999	9999
1	1
1000000000	

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if (array[i] > largest_so_far ) {
            largest_so_far = array[i];
        }
   }
   return largest_so_far;
}
```

10	10
100	100
533	533
1000	1000
9999	9999
1	1
100000000	100000000

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if (array[i] > largest_so_far) {
            largest_so_far = array[i];
        }
   }
   return largest_so_far;
}
```

Array Size Number of Spots Checked

10	10
100	100
533	533
1000	1000
9999	9999
1	1
100000000	100000000

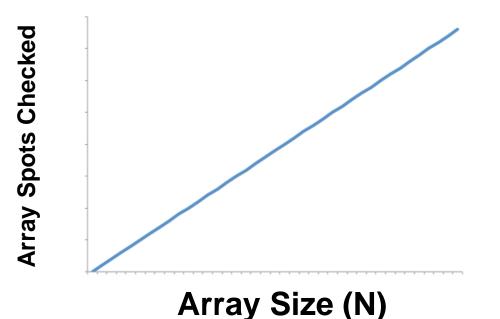
```
public int find_max_value(int[] array) {
    int largest_so_far = -1;
    for(int i = 0; i < array.length; i++) {
        if (array[i] > largest_so_far ) {
            largest_so_far = array[i];
        }
    }
    return largest_so_far;
}
```

What if we graphed these points?

```
(10,10), (100,100), (533,533), (1000,100) ....
```

10	10
100	100
533	533
1000	1000
9999	9999
1	1
100000000	100000000

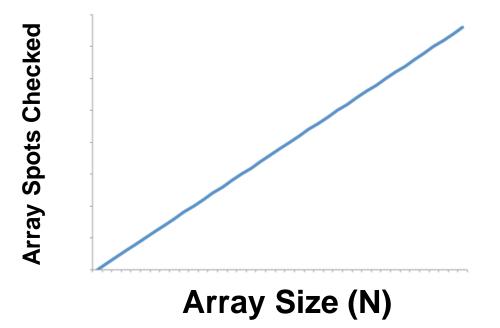
```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if (array[i] > largest_so_far ) {
            largest_so_far = array[i];
        }
   }
   return largest_so_far;
}
```



The growth rate of this algorithm is linear

A linear growth rate is a growth rate where the resource needs and the amount of data is directly proportional to each other.

```
public int find_max_value(int[] array) {
   int largest_so_far = -1;
   for(int i = 0; i < array.length; i++) {
      if (array[i] > largest_so_far ) {
          largest_so_far = array[i];
      }
   }
  return largest_so_far;
}
```



Algorithm #1: Finding the Maximum Value in an Array

The growth rate of this algorithm is linear

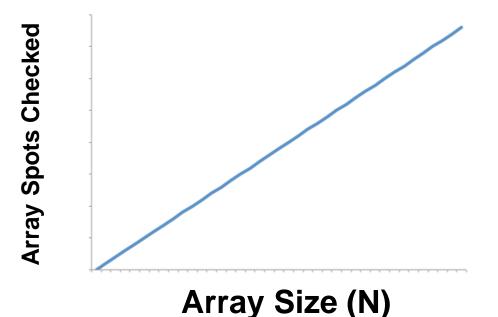
A linear growth rate is a growth rate where the resource needs and the amount of data is directly proportional to each other.

The driving factor of this algorithm is the size of the array

As N increases, the number of steps executed in this algorithm linearly increases

$$F(x) = N$$

```
public int find_max_value(int[] array) {
    int largest_so_far = -1;
    for(int i = 0; i < array.length; i++) {
        if (array[i] > largest_so_far ) {
            largest_so_far = array[i];
        }
    }
    return largest_so_far;
}
```

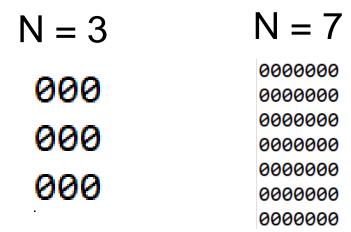


```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

Given an N x N 2D array, print out its contents

```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

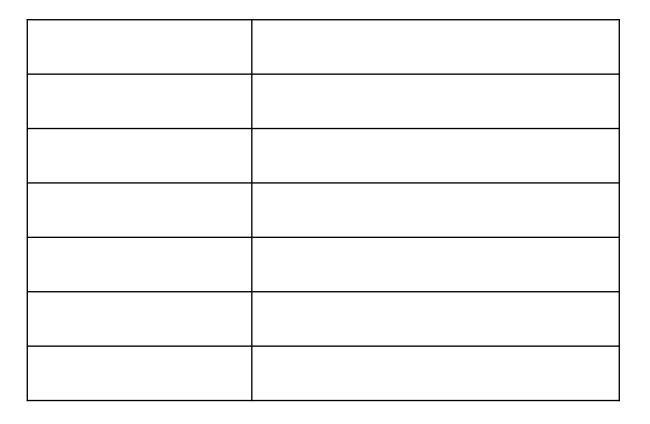
Given an N x N 2D array, print out its contents



```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

Given an N x N 2D array, print out its contents

Let's look at how many times this operation is executed as N increases



```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

1	

```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

1

```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

1	1
2	4

```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

1	1
2	4
3	

```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

1	1
2	4
3	9

```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

1	1
2	4
3	9
4	

```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

1	1
2	4
3	9
4	16

```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

1	1
2	4
3	9
4	16
5	25

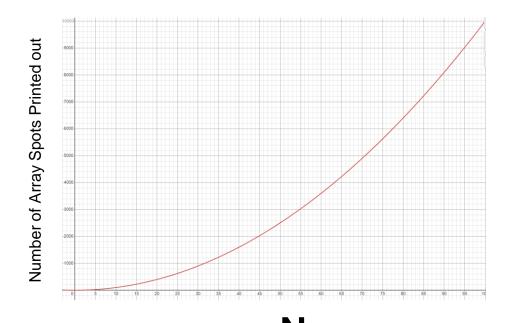
```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

1	1
2	4
3	9
4	16
5	25
•••	•••
100	10000

```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```

1	1
2	4
3	9
4	16
5	25
•••	•••
100	10000

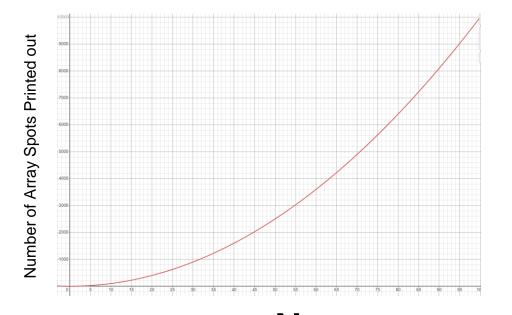
```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```



The growth rate of this algorithm is quadratic

A quadratic growth rate is a growth rate where the resource needs and the amount of data is proportional to the *square* of a function

```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```



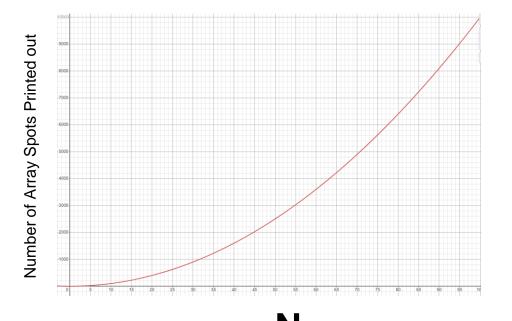
The growth rate of this algorithm is quadratic

A quadratic growth rate is a growth rate where the resource needs and the amount of data is proportional to the *square* of a function

$$F(x) = X ^2$$

We have a for loop inside of a for loop, so as N increases, the number of times the inside for loop executes = N * N

```
public void print2Darray(int[][] array) {
    for(int[] x: array) {
        for(int y: x) {
            System.out.print(y);
        }
        System.out.println();
    }
}
```



```
public void addToFront(Node newNode) {
    newNode.setNext(this.head);
    this.head = newNode;
}
```

```
public void addToFront(Node newNode) {
    newNode.setNext(this.head);
    this.head = newNode;
}
```

Given a singly linked list (with at least one node), this algorithm adds a new node to the front of the LL

```
public void addToFront(Node newNode) {
    newNode.setNext(this.head);
    this.head = newNode;
}
```

Given a singly linked list (with at least one node), this algorithm adds a new node to the front of the LL

```
public void addToFront(Node newNode) {
    newNode.setNext(this.head);
    this.head = newNode;
}
```

Given a singly linked list (with at least one node), this algorithm adds a new node to the front of the LL

This algorithm consists of two operations. Let's look at how many times these operations are executed as the Linked List size increases

of operations executed

N # of operations executed

1	

N # of operations executed

1	2

of operations executed

1	2
2	

N # of operations executed

2
2

of operations executed

1	2
2	2
3	

of operations executed

1	2
2	2
3	2

of operations executed

1	2
2	2
3	2
4	

of operations executed

1	2
2	2
3	2
4	2

of operations executed

1	2
2	2
3	2
4	2
100	

of operations executed

1	2
2	2
3	2
4	2
100	2

N # of 0

of operations executed

1	2
2	2
3	2
4	2
100	2
777	

of operations executed

1	2
2	2
3	2
4	2
100	2
777	2

N # of operations executed

1	2
2	2
3	2
4	2
100	2
777	2
100000000	

of operations executed

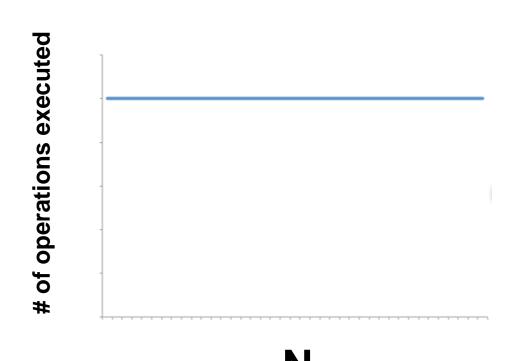
1	2
2	2
3	2
4	2
100	2
777	2
100000000	2

```
public void addToFront(Node newNode) {
    newNode.setNext(this.head);
    this.head = newNode;
}
```

N

of operations executed

1	2
2	2
3	2
4	2
100	2
777	2
100000000	2



N= # of nodes in LL

Algorithm #3: Adding a node to front of linked list

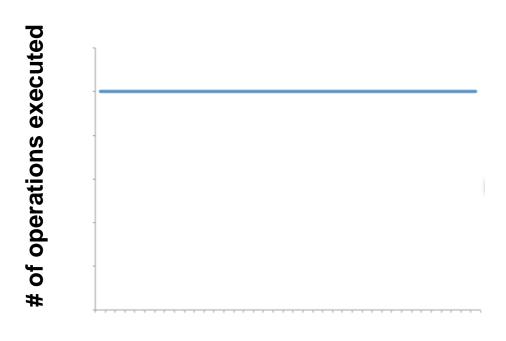
The growth rate of this algorithm is constant

A **constant growth rate** is one where the resource need does not grow as N increases.

$$F(x) = 1$$

ie. As N increases, the number of steps our algorithm performs is constant

```
public void addToFront(Node newNode) {
    newNode.setNext(this.head);
    this.head = newNode;
}
```



Input: 2 Output: 0 0 0 1 1 0 1 1

```
Input: 2
             Input: 3
Output:
             Output:
0 0
             0 0 0
0 1
             0 0 1
1 0
             0 1 0
1 1
             0 1 1
             1 0 0
             1 0 1
             1 1 0
```

1 1 1

Input: 2	Input: 3	Input: 4
Output:	Output:	Output
0 0	000	2 3 4 5 5
0 1	0 0 1	0000
1 0		0001
1 1	0 1 0	0010
	0 1 1	0011
	1 0 0	0100
		0101
	1 0 1	0110
	1 1 0	0111
	1 1 1	
		1111

```
static void generateAllBinaryStrings(int n, int arr[], int i) {
       if (i == n) {
           printTheArray(arr, n);
           return;
       arr[i] = 0;
       generateAllBinaryStrings(n, arr, i + 1);
       arr[i] = 1;
       generateAllBinaryStrings(n, arr, i + 1);
```

Don't worry too much about the specifics of the algorithm. Let's look at the amount a string that get generated as N increases

N # Of binary digits generated

1	

N # Of binary digits generated

1	2

N = length of binary digits

N # Of binary digits generated

1	2
2	

N # Of binary digits generated

1	2
2	4

N # Of binary digits generated

1	2
2	4
3	

N # Of binary digits generated

1	2
2	4
3	8

N = length of binary digits

N # Of binary digits generated

1	2
2	4
3	8
4	

N # Of binary digits generated

1	2
2	4
3	8
4	16

N # Of binary digits generated

1	2
2	4
3	8
4	16
5	

N = length of binary digits

N # Of binary digits generated

1	2
2	4
3	8
4	16
5	32

N # Of binary digits generated

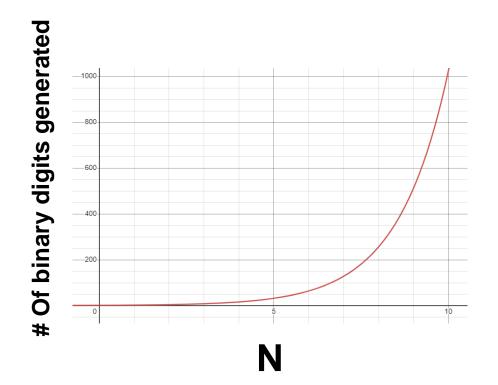
1	2
2	4
3	8
4	16
5	32
8	256

N # Of binary digits generated

1	2
2	4
3	8
4	16
5	32
8	256
16	65536

N # Of binary digits generated

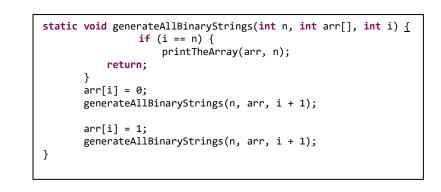
1	2
2	4
3	8
4	16
5	32
8	256
16	65536

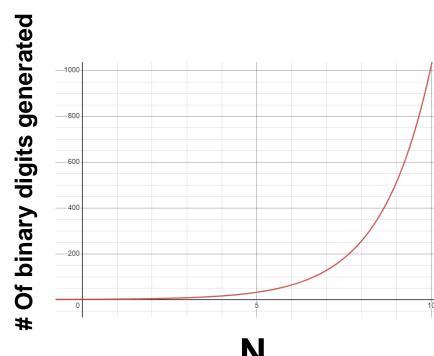


The growth rate of this algorithm is exponential

An **exponential growth rate** is one where the resource needed begins to double or increase very drastically as N increases

$$F(x) = B^x$$



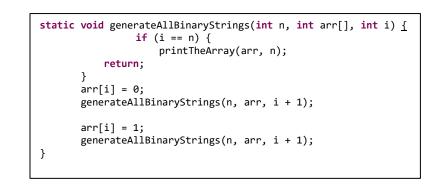


The growth rate of this algorithm is exponential

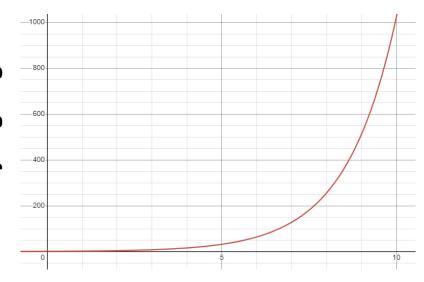
An **exponential growth rate** is one where the resource needed begins to double or increase very drastically as N increases

$$F(x) = B \wedge x$$

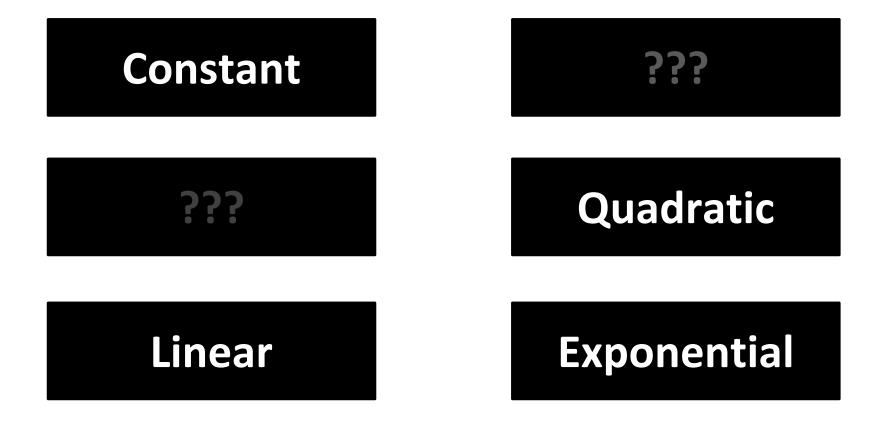
An algorithm that has an exponential growth rate is generally perceived as inefficient. When N gets big, sometimes the algorithm won't finish for years





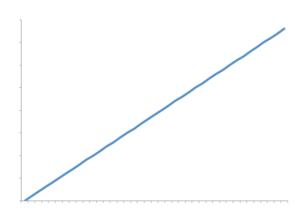


The **growth rate** of the algorithm looks at how much more resource an algorithm needs (time or space) as the input size increases

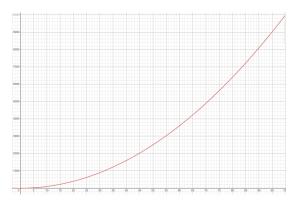


Constant

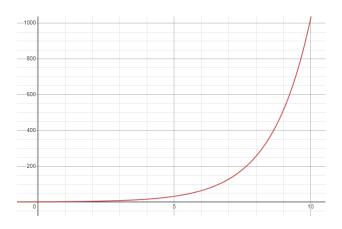
Linear

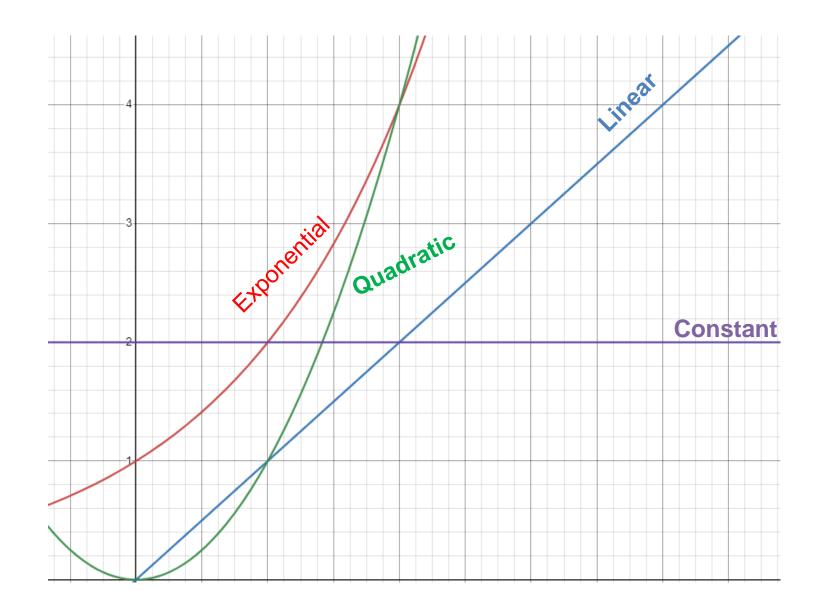


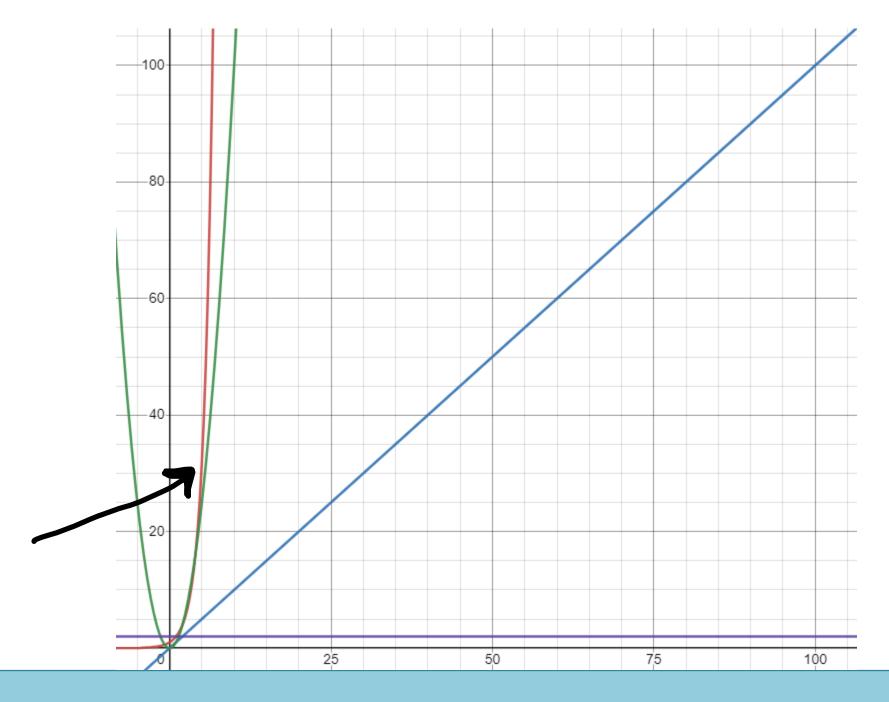
Quadratic



Exponential







After a certain point, exponential growth rates goes crazy and beings to grow much faster than the other growth rates