Graphs

CSCI 232

How could we visualize: The US Road Network?

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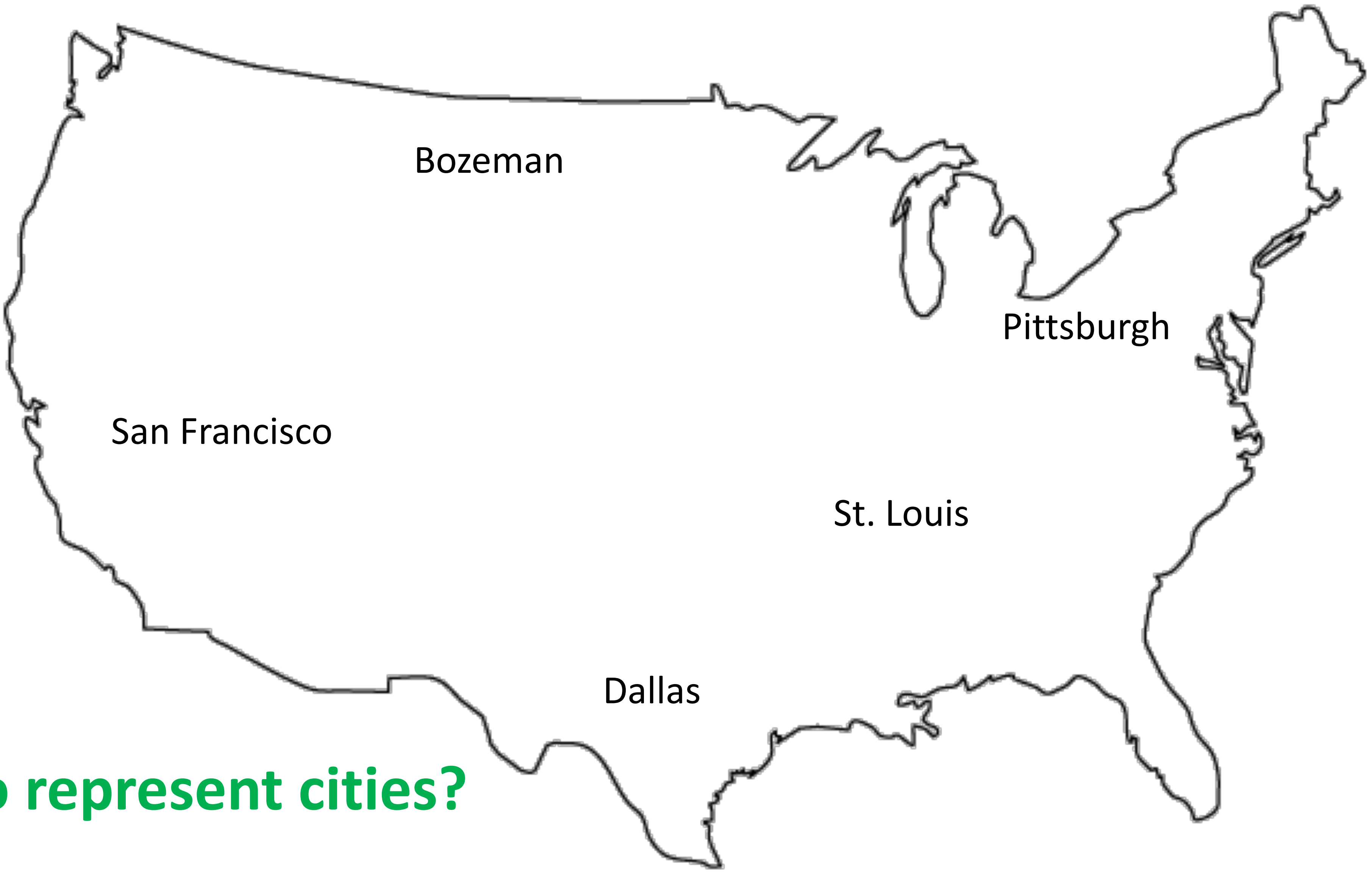
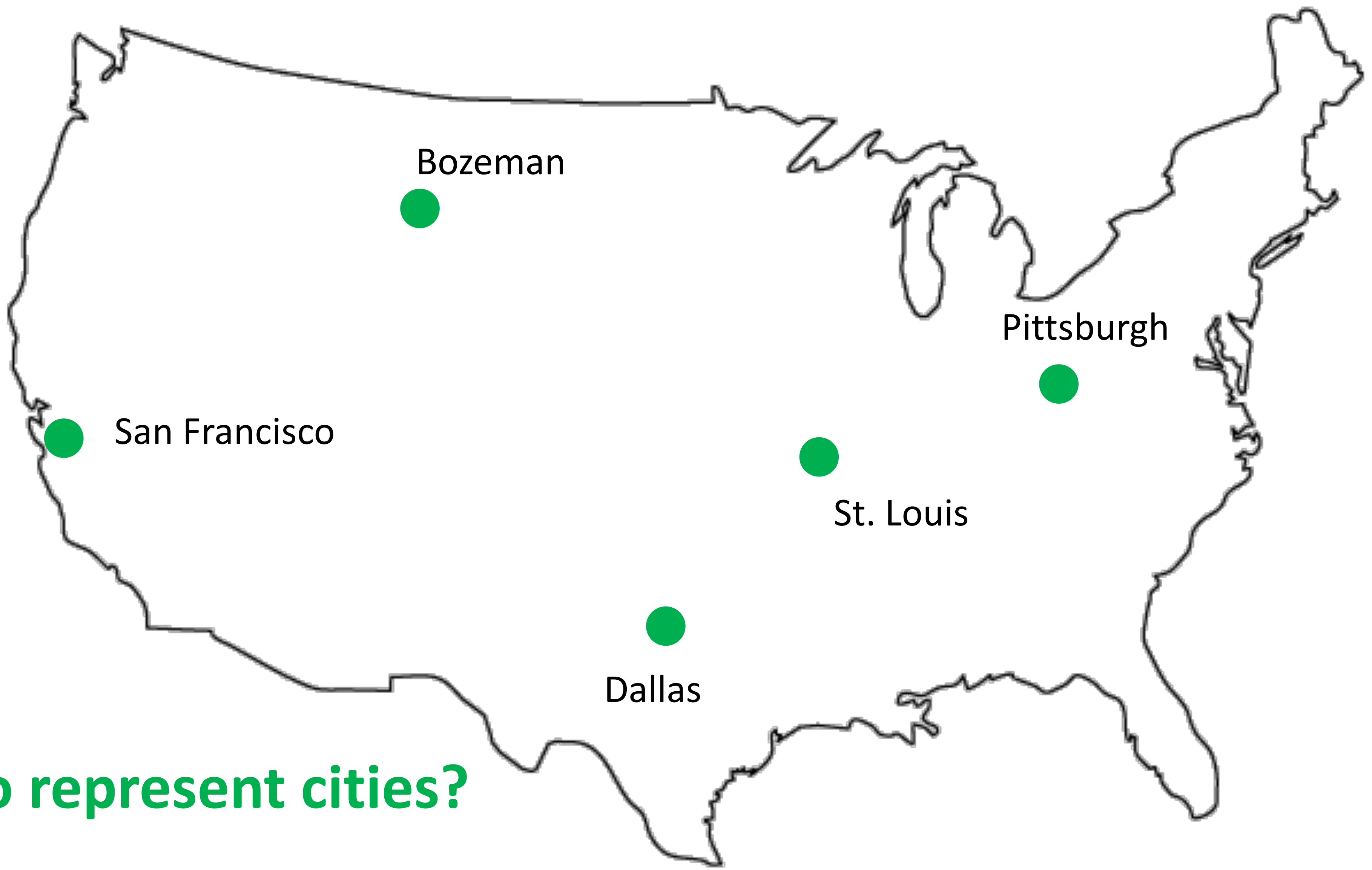


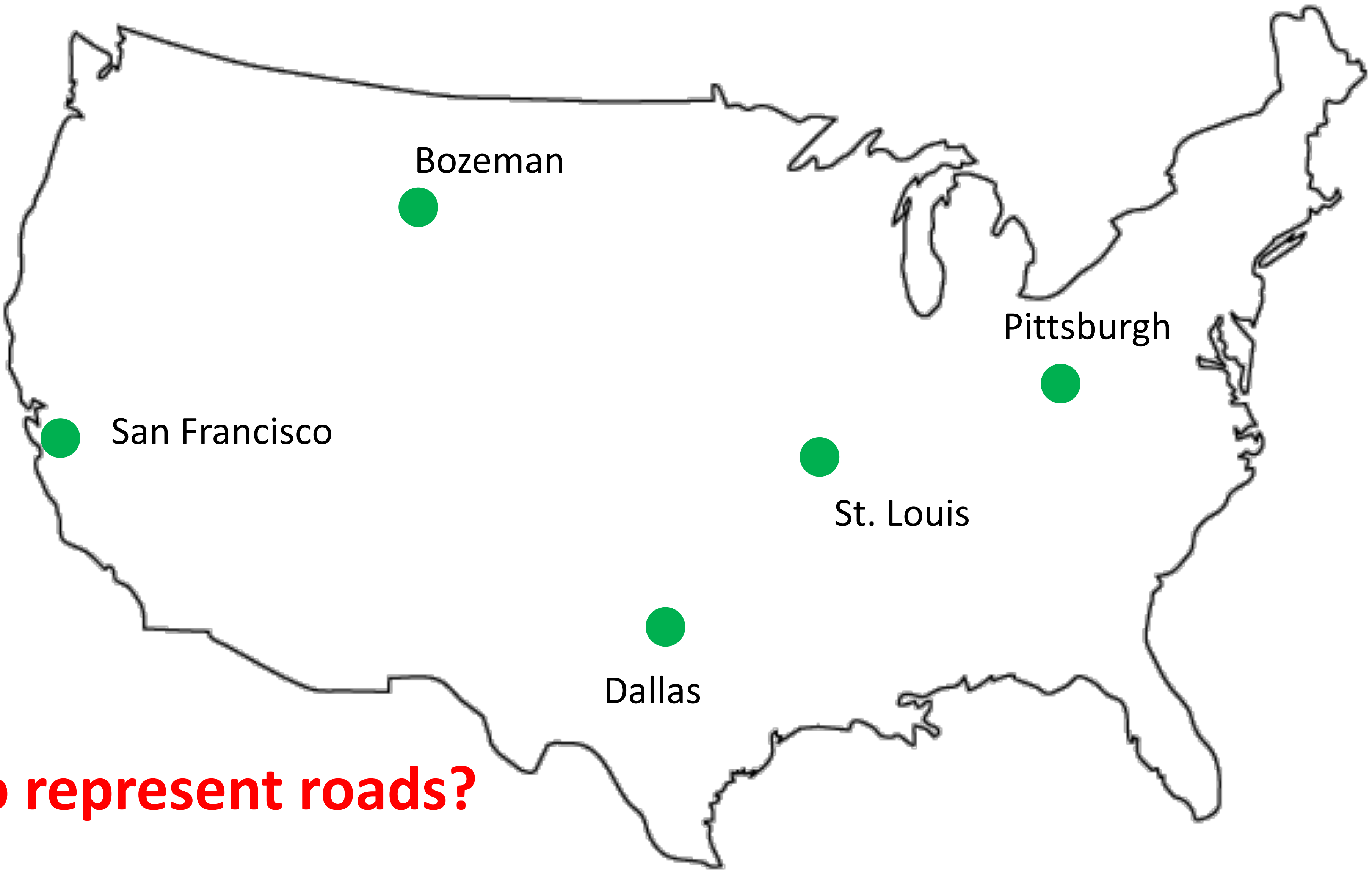
How to represent cities?

How could we visualize: The US Road Network?



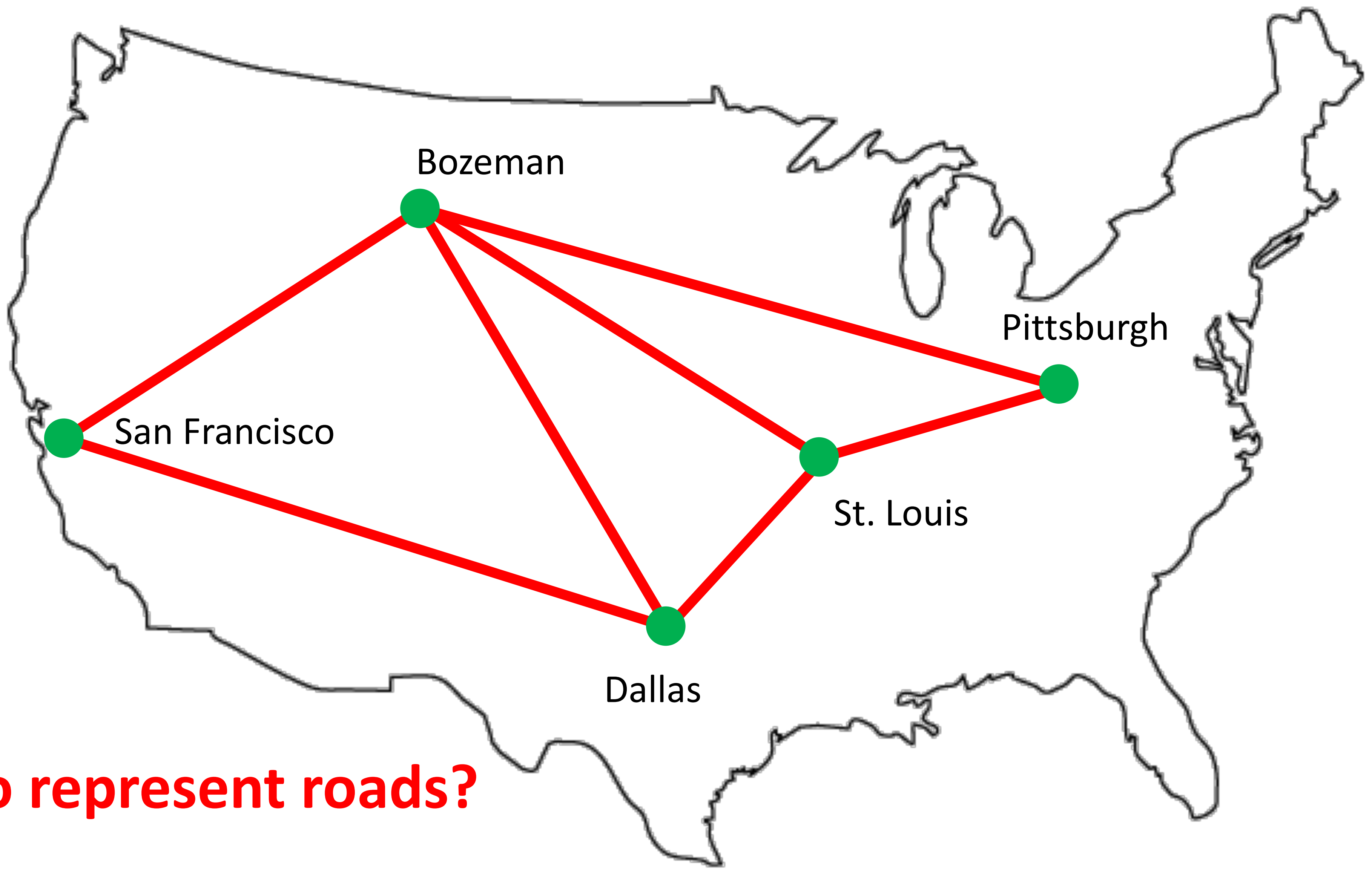
How to represent cities?

How could we visualize: The US Road Network?



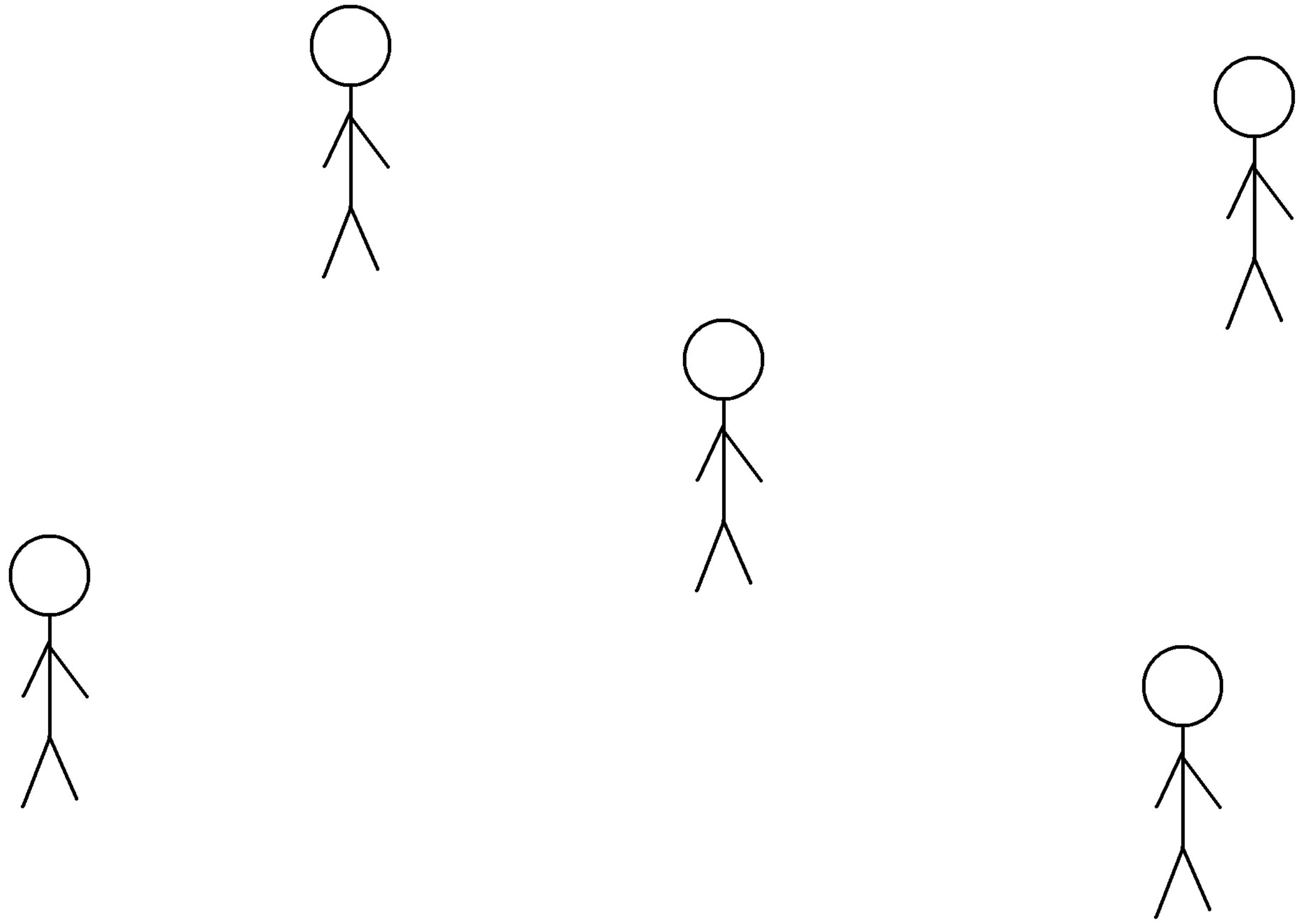
How to represent roads?

How could we visualize: The US Road Network?

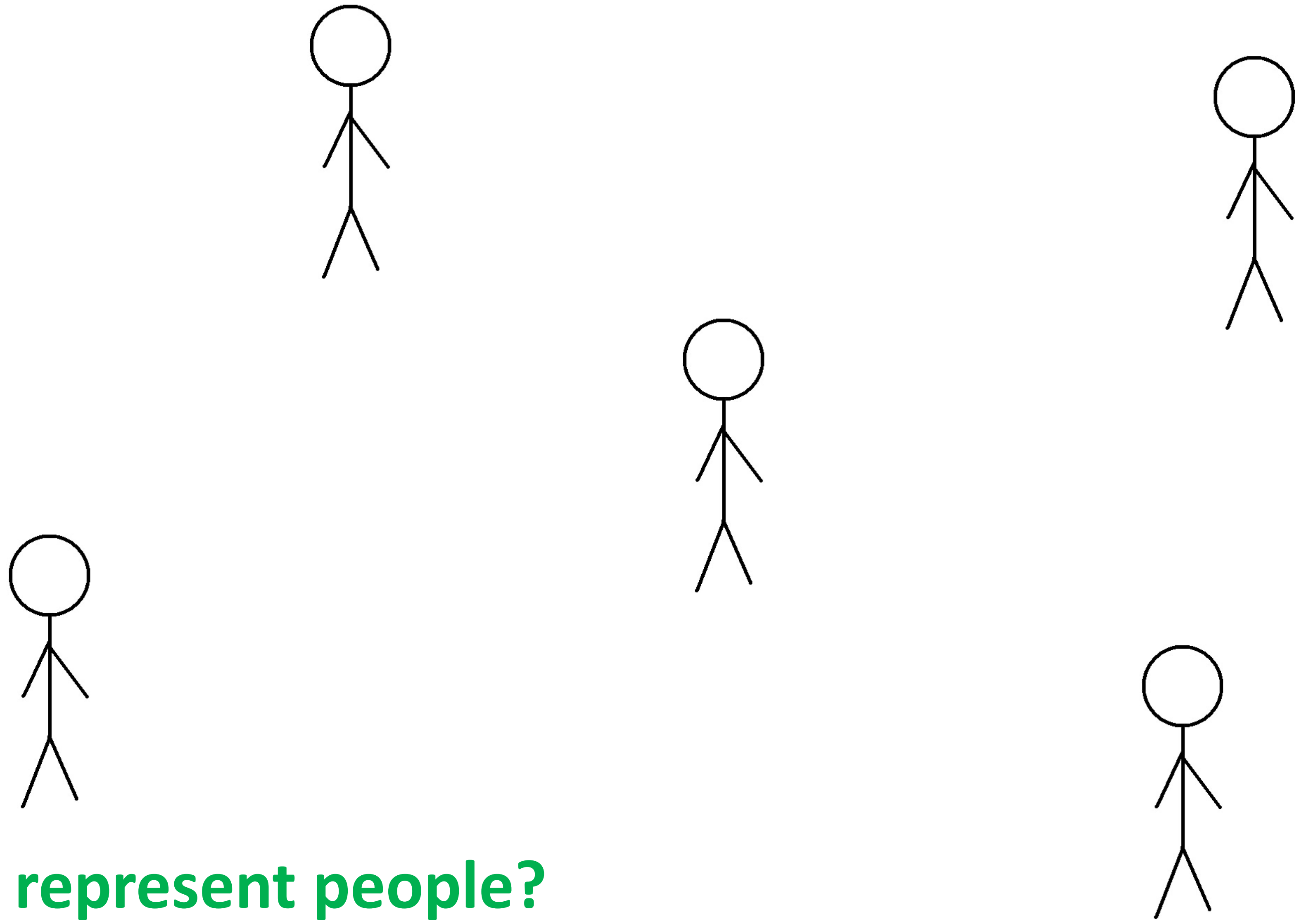


How to represent roads?

How could we visualize: A Social Network?



How could we visualize: A Social Network?



How to represent people?

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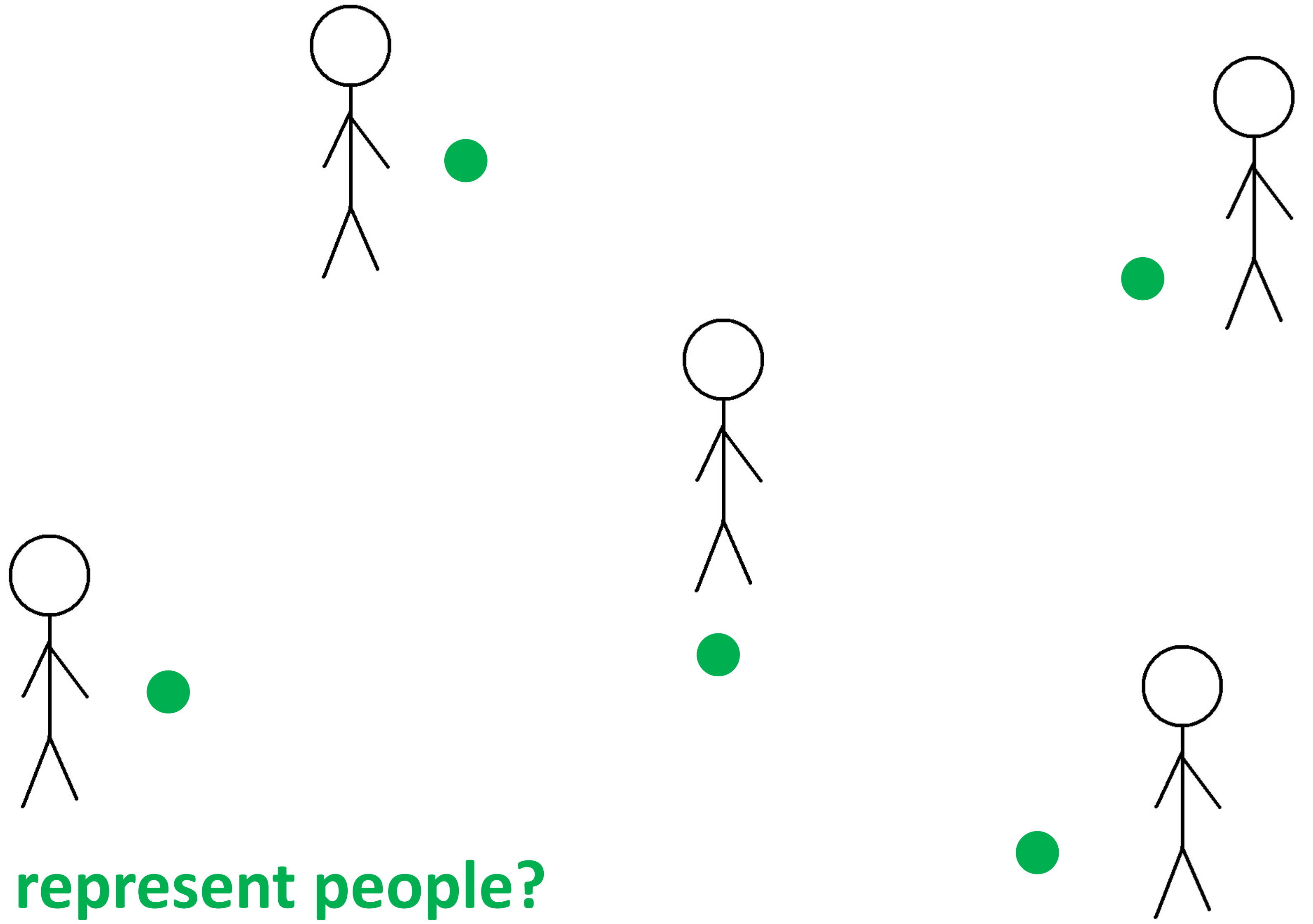

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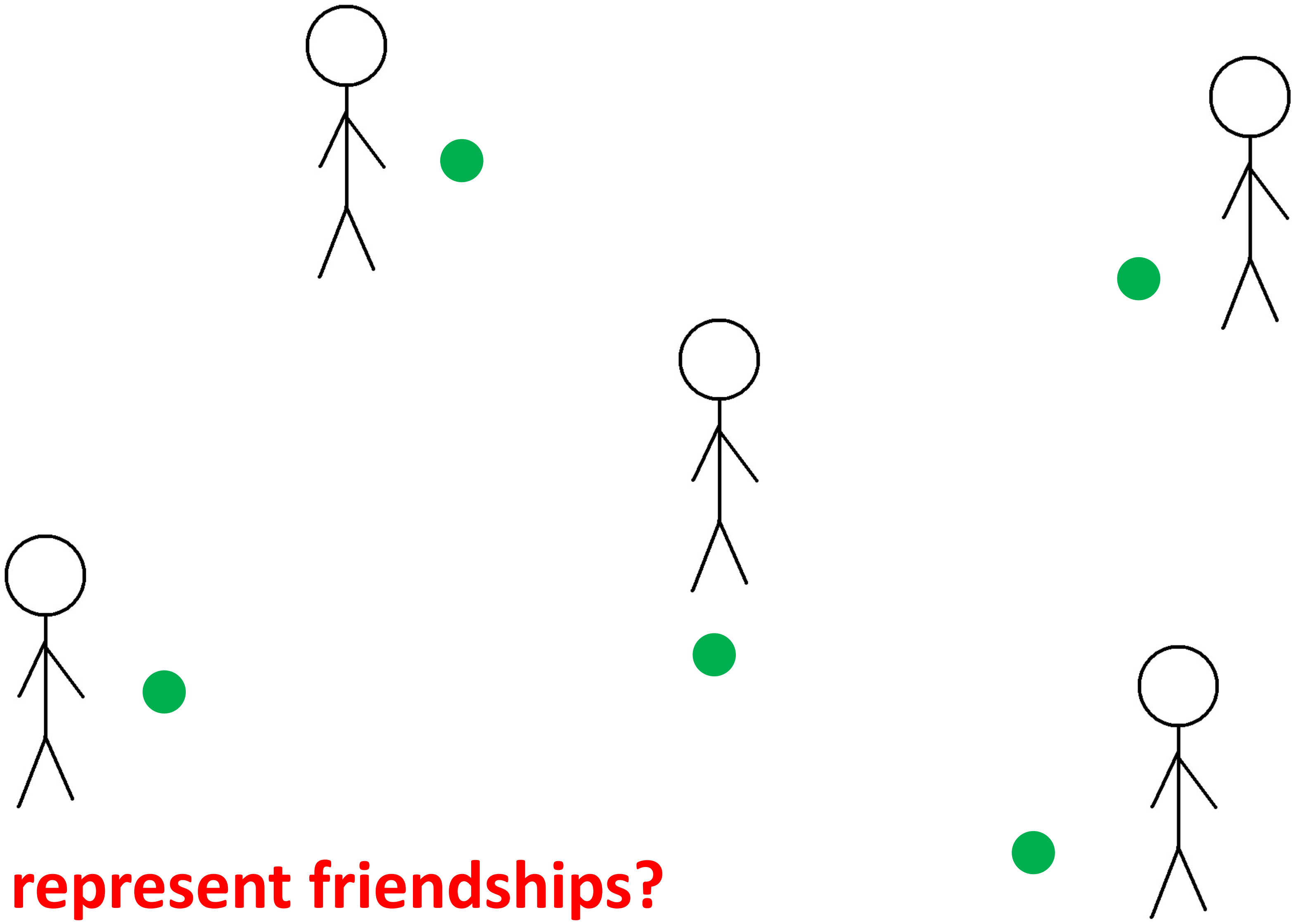

How to represent friendships?

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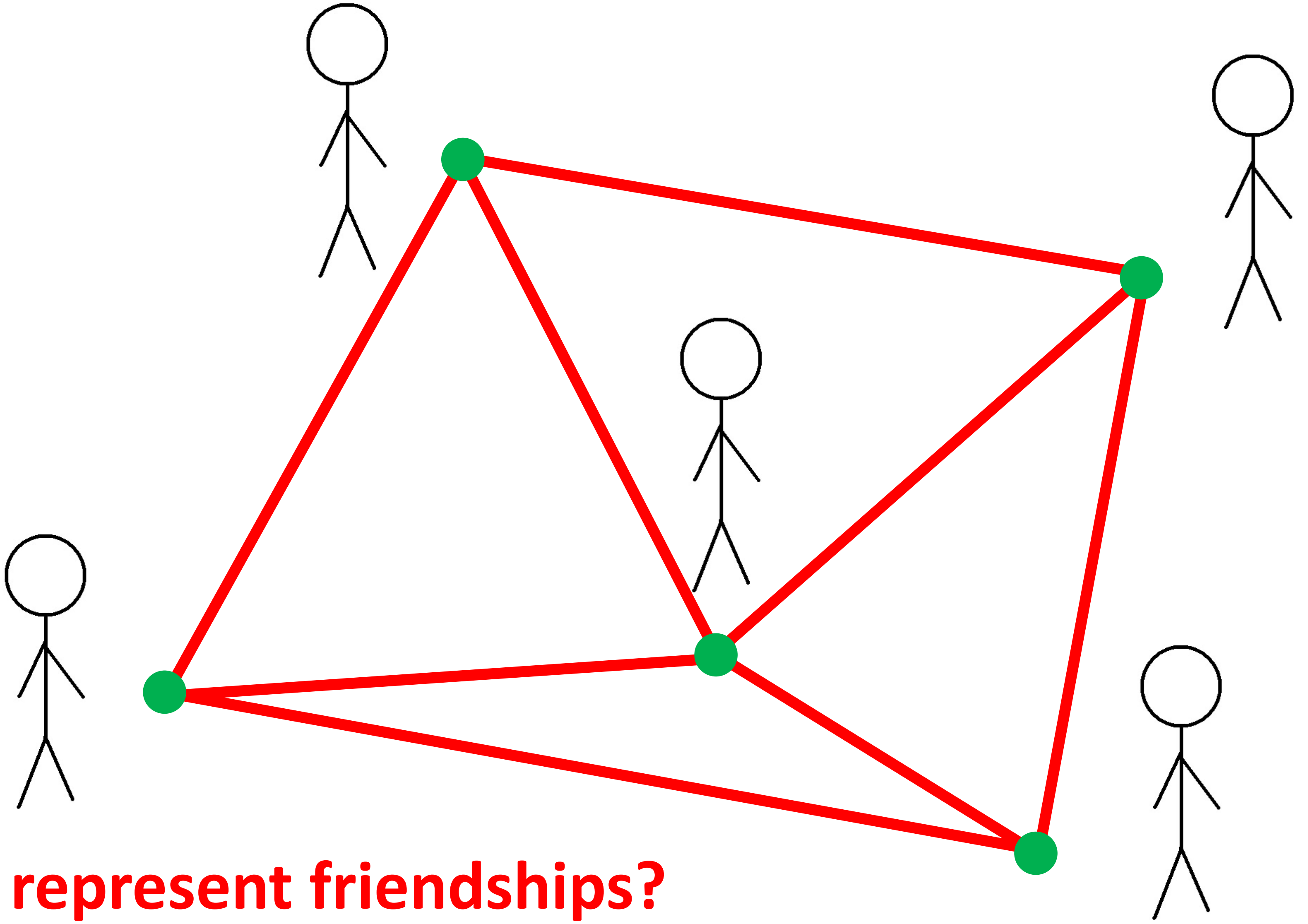
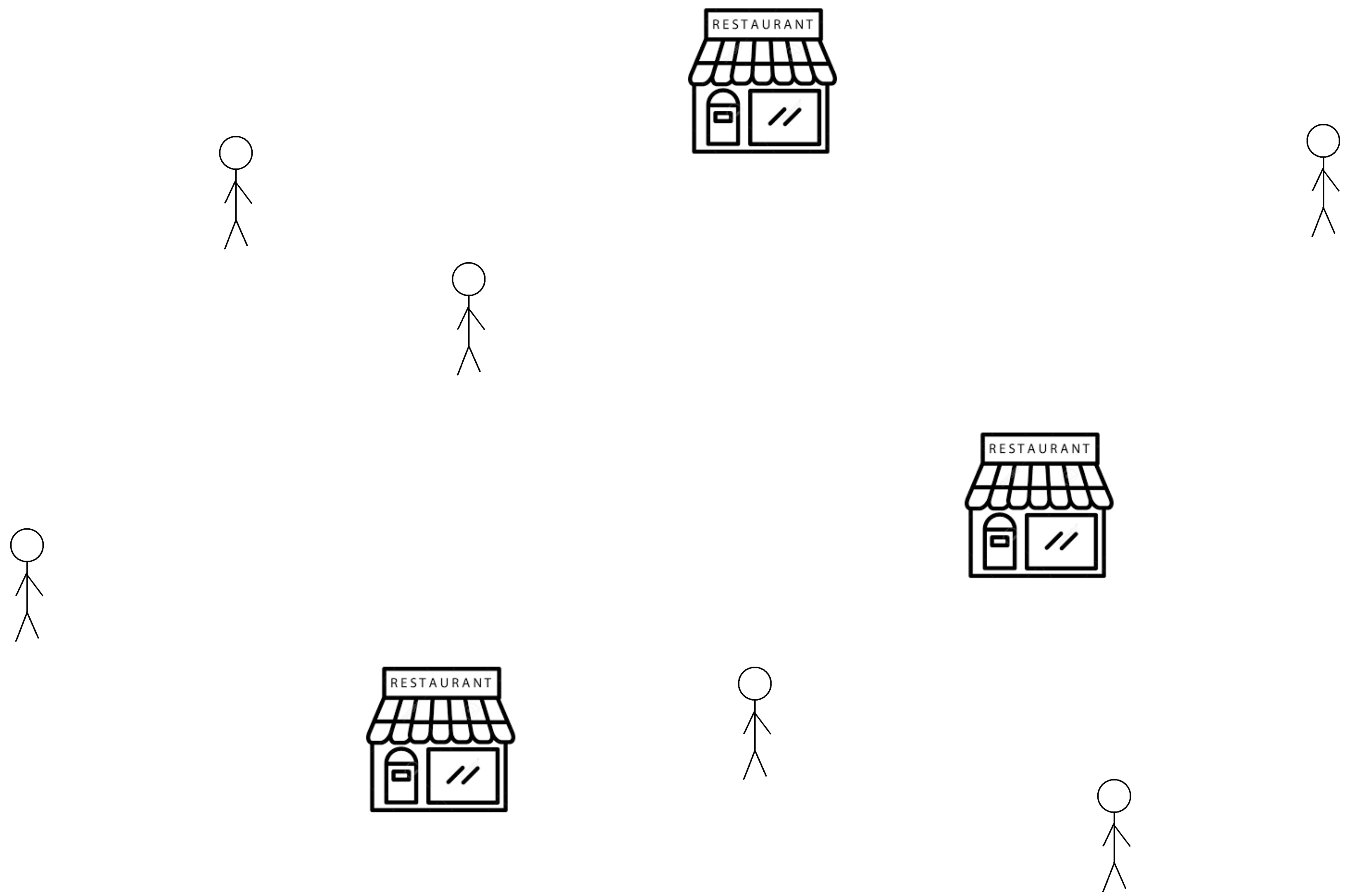


How to represent friendships?

How could we visualize: Restaurants and Potential Customers?



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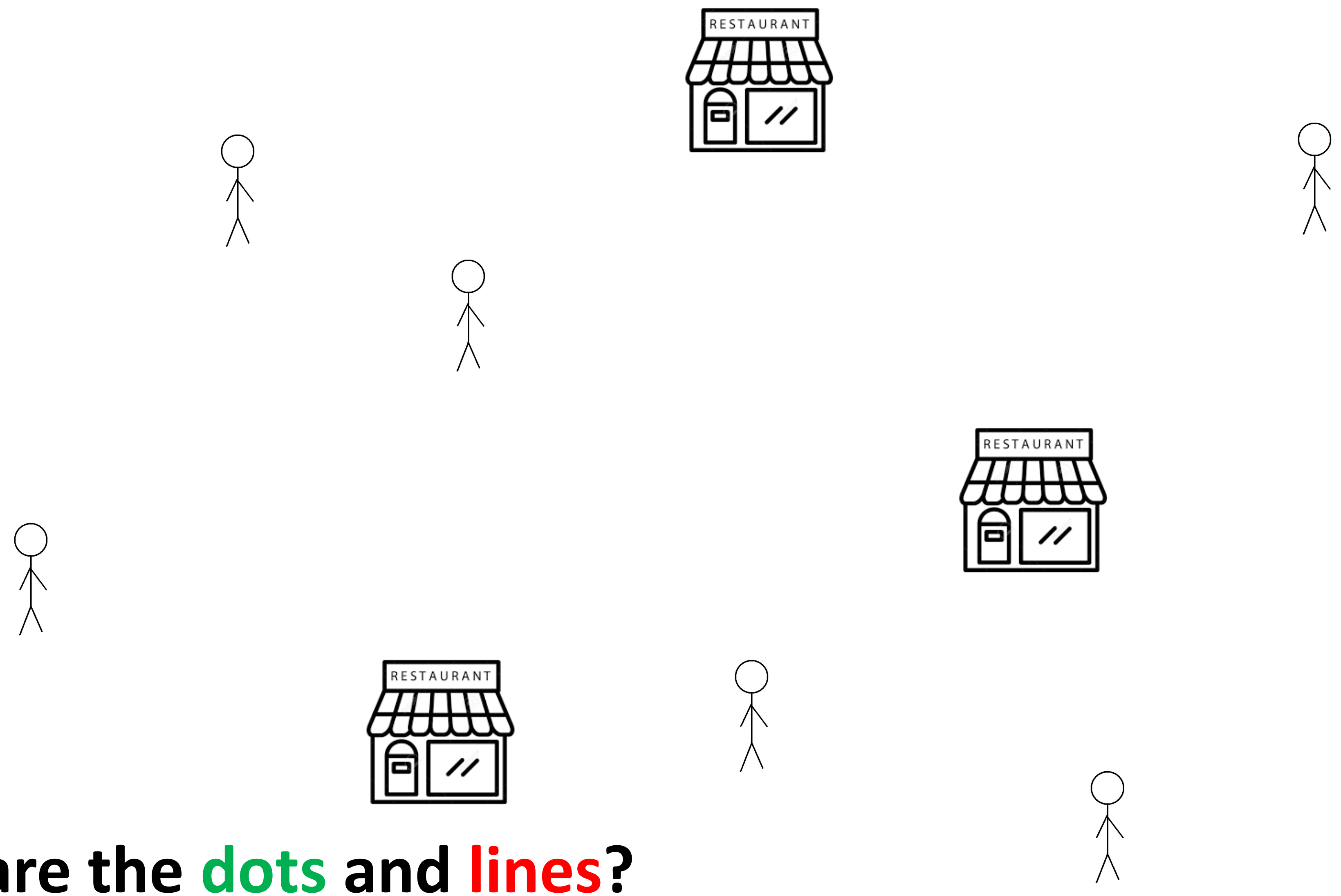

What are the dots and lines?

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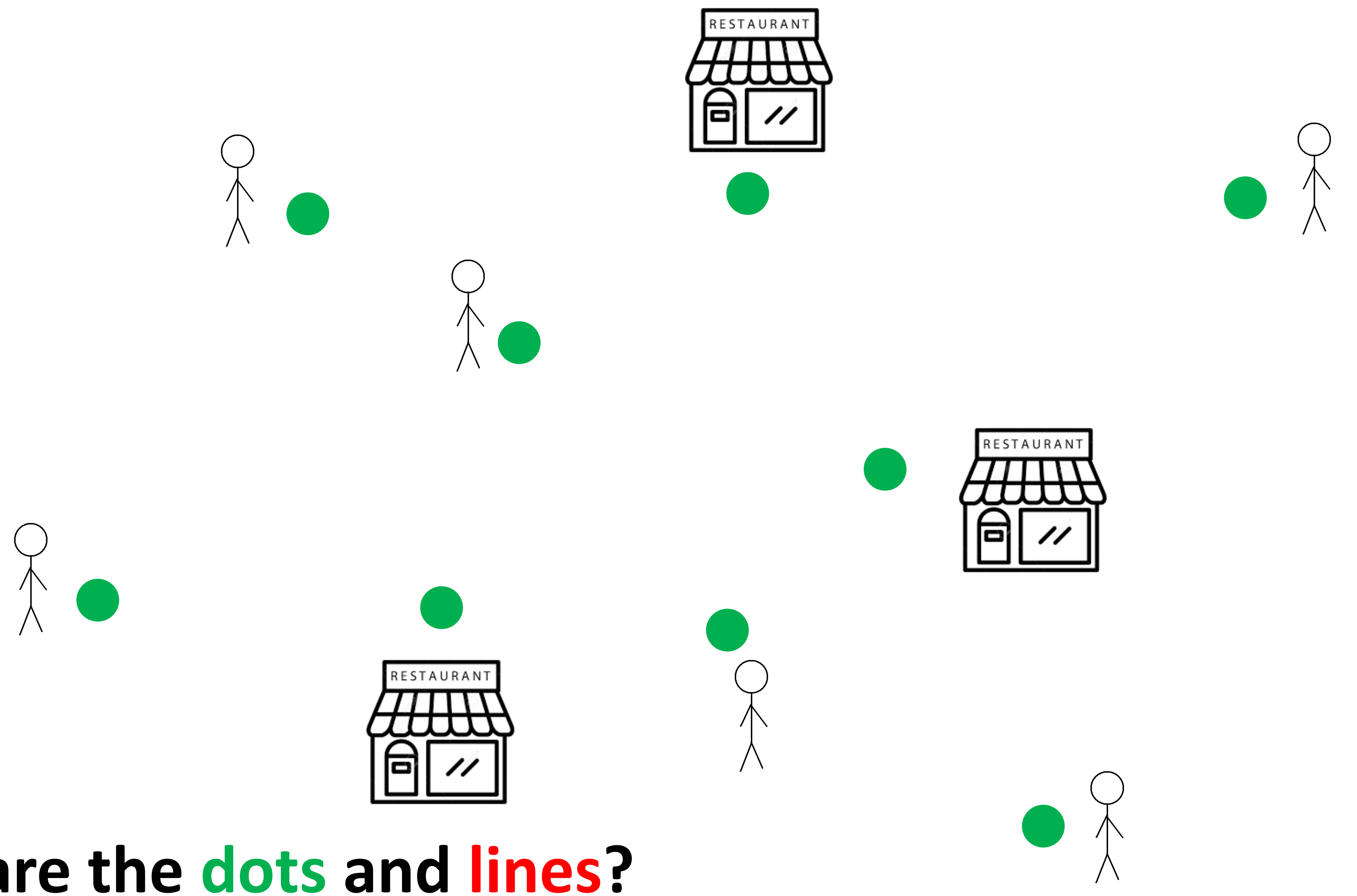

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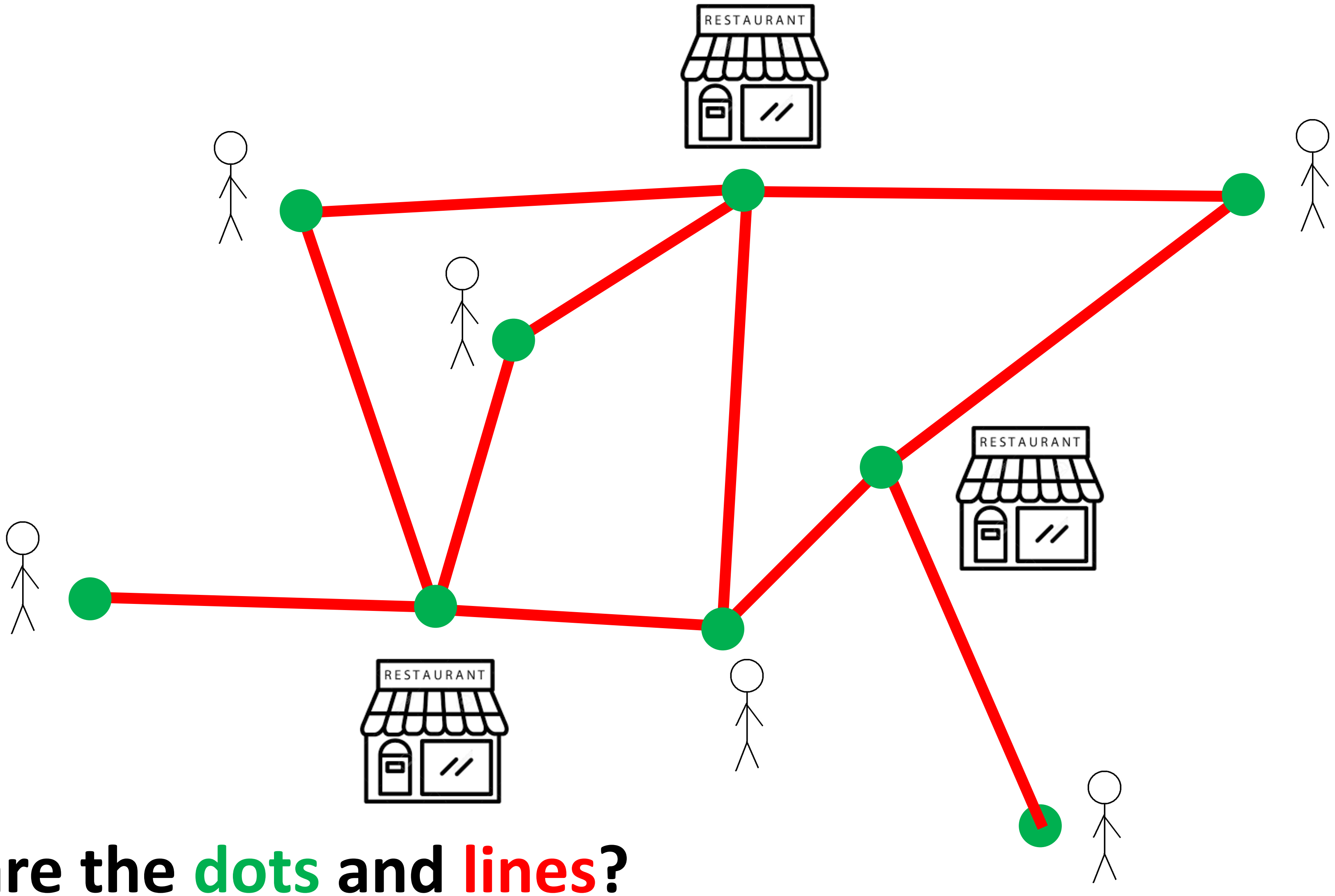

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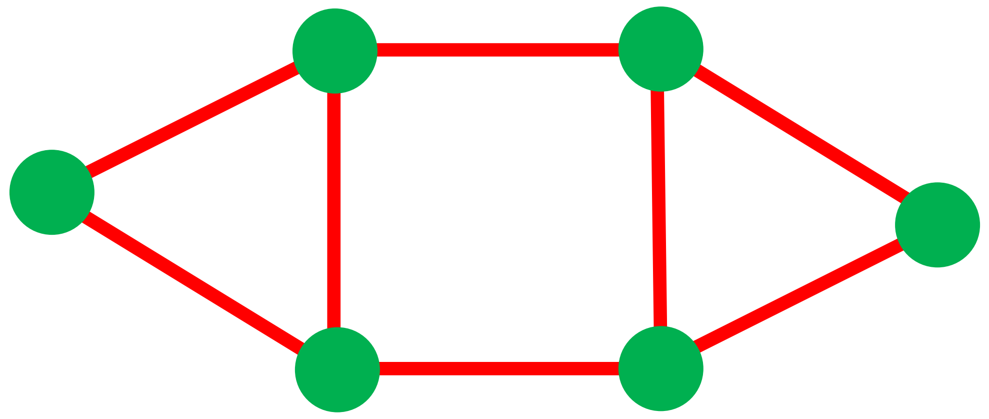
Graphs



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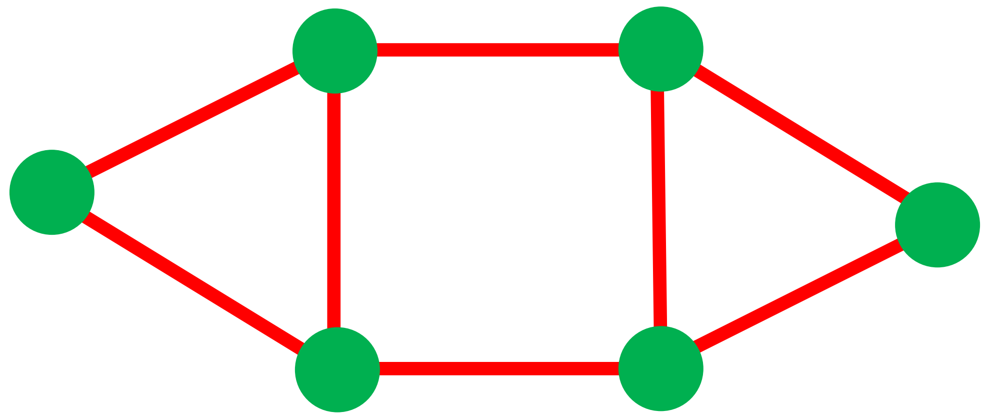
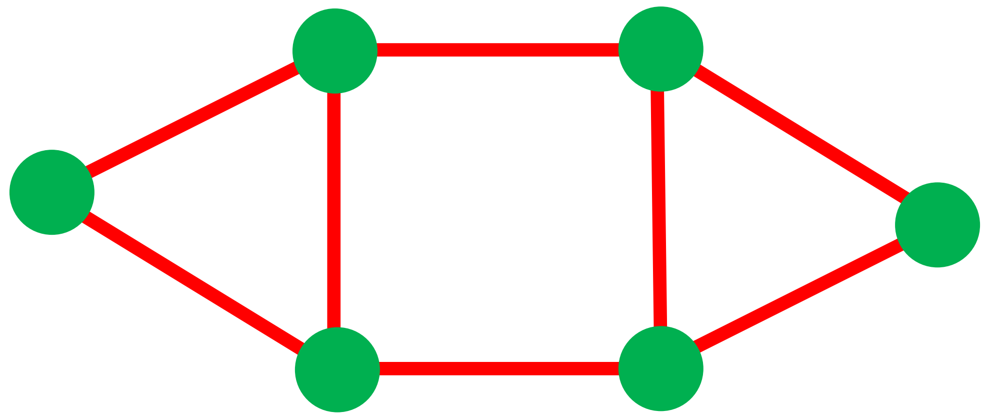


Vertices (or Nodes)

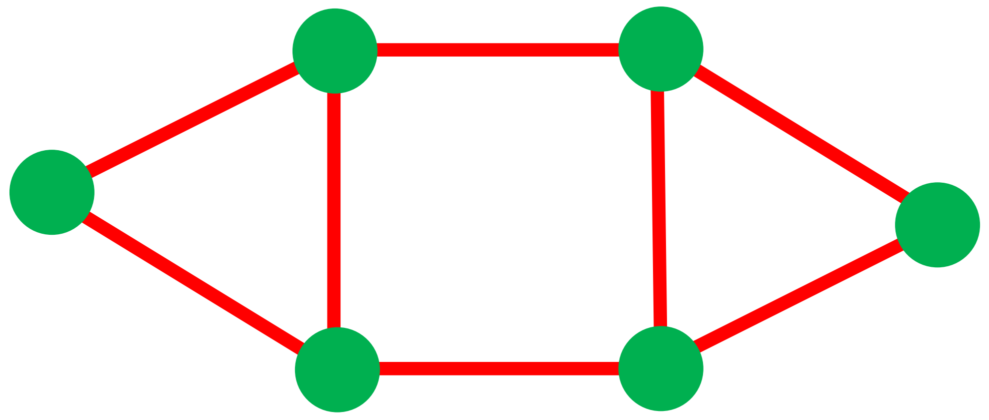
Graphs



Vertices (or Nodes)

Edges

Graphs



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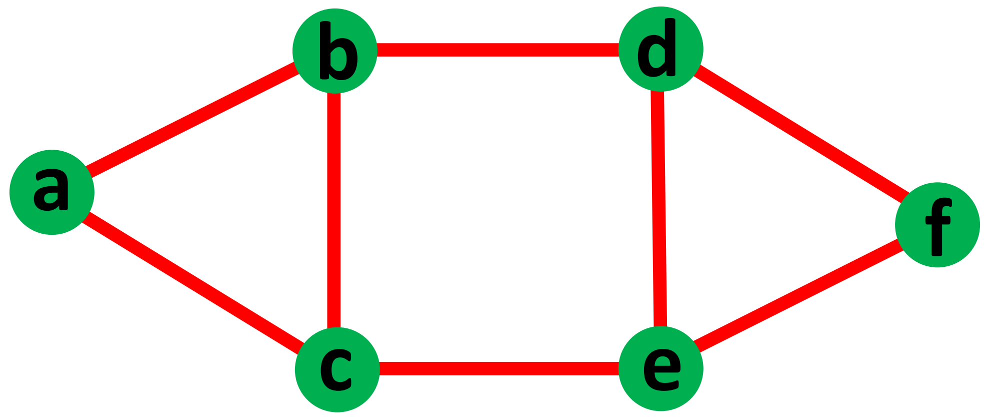
Graphs



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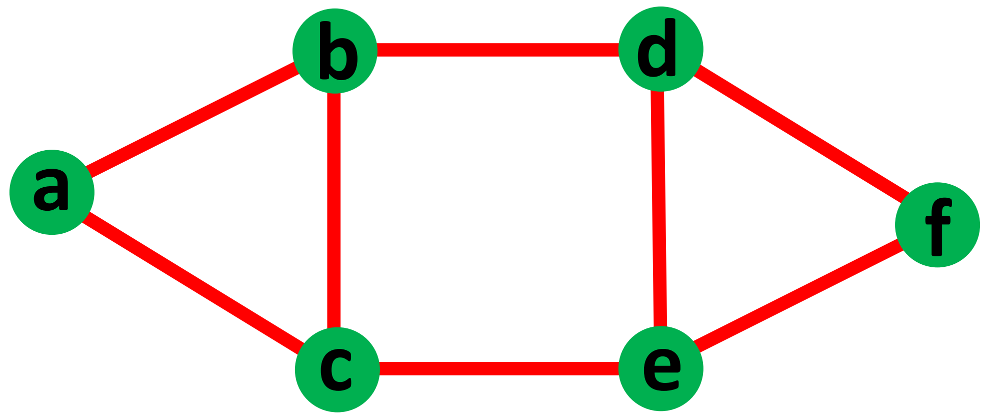
Graphs



Vertices (or Nodes)
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$\Big\}\ \boldsymbol{G = (V, E)}$

$\boldsymbol{G_1 = G_2}$
If and only if $V_1 = V_2$ and $E_1 = E_2$.

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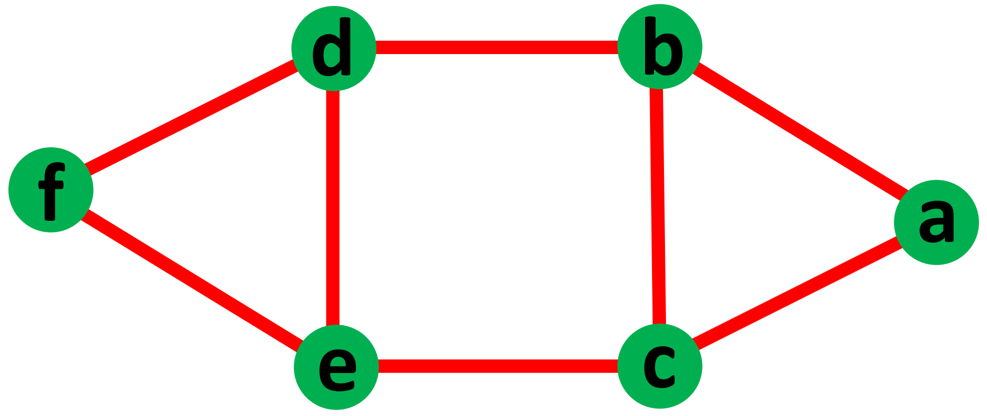
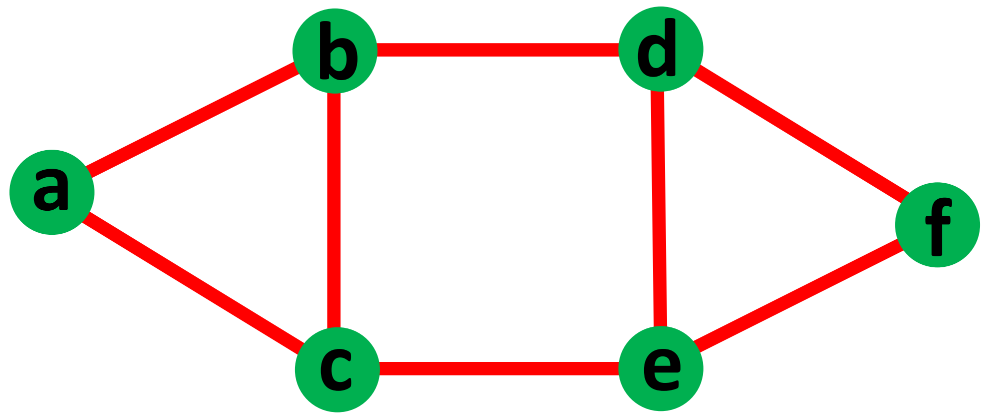


Same graph? ✓

Graphs



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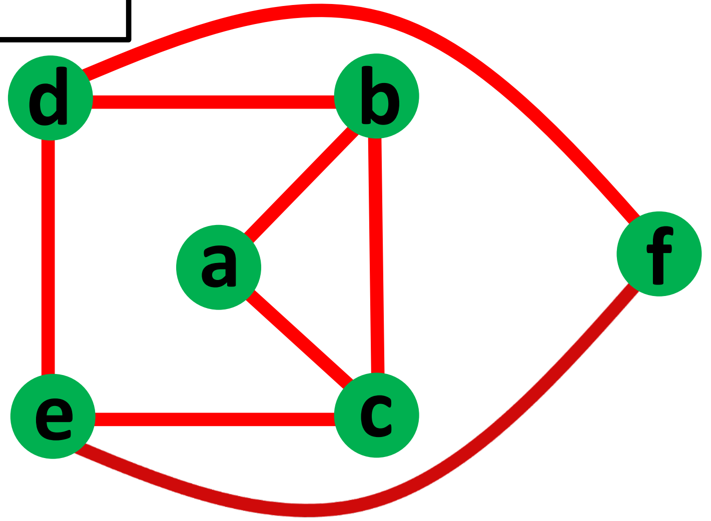

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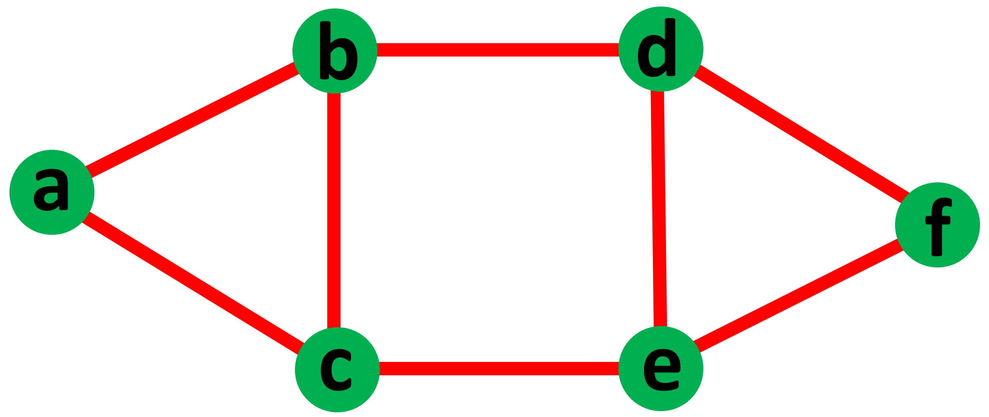

Vertices (or Nodes)
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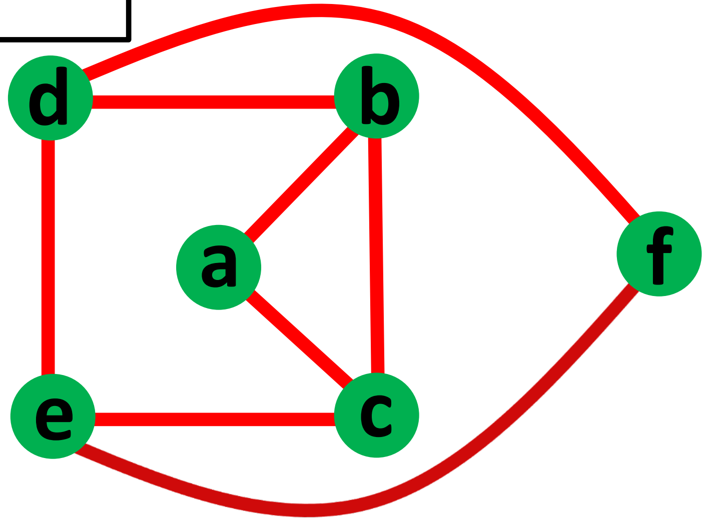

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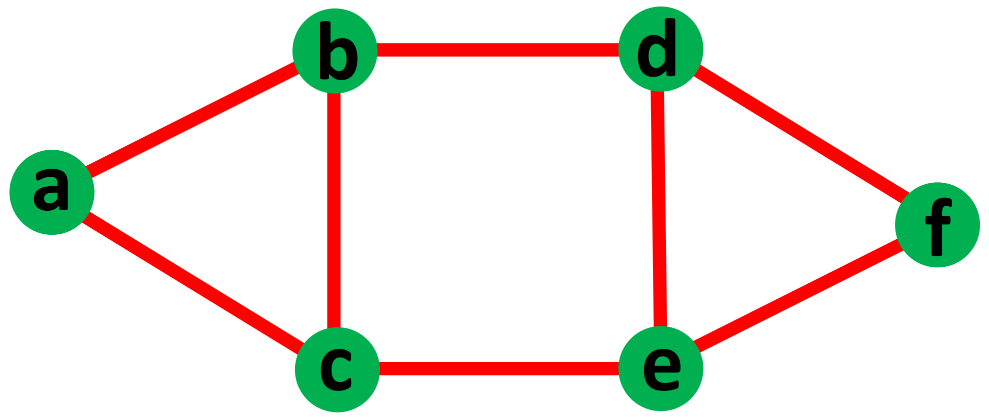

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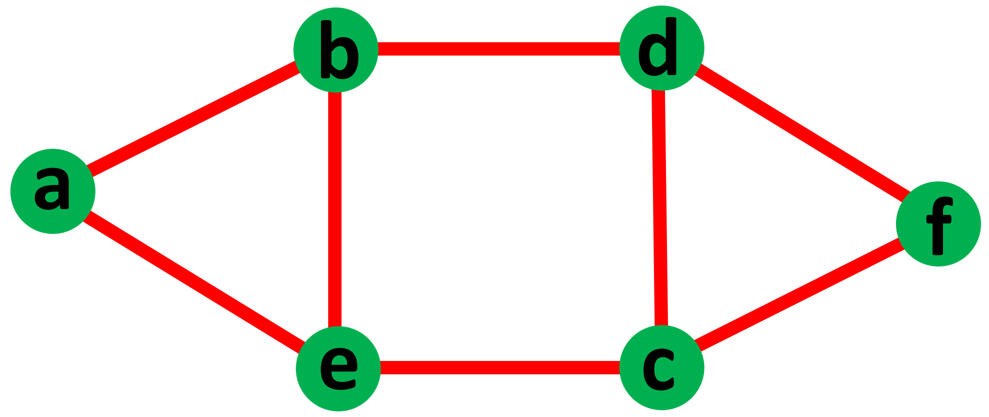

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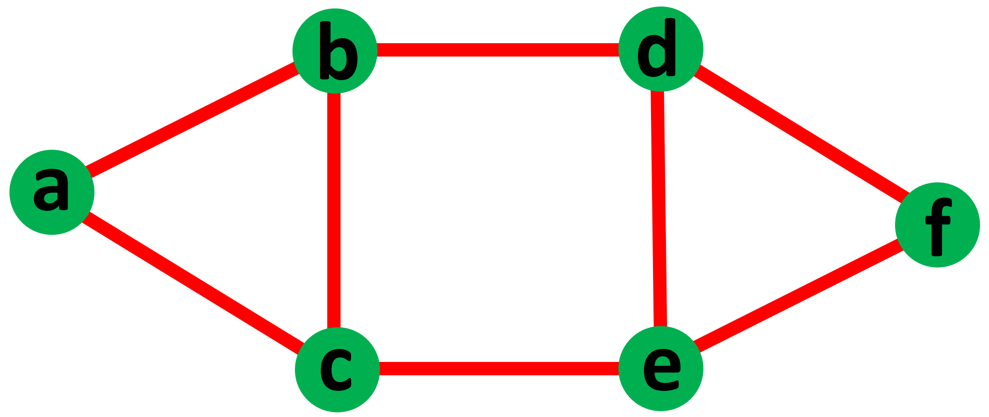

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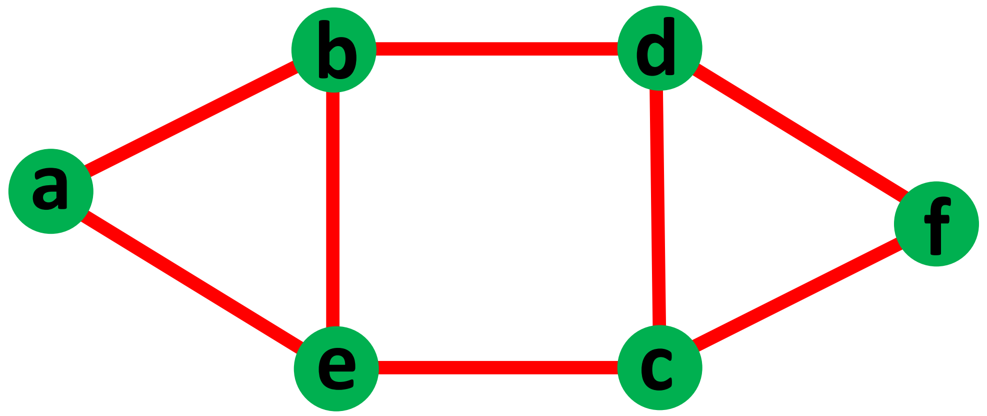
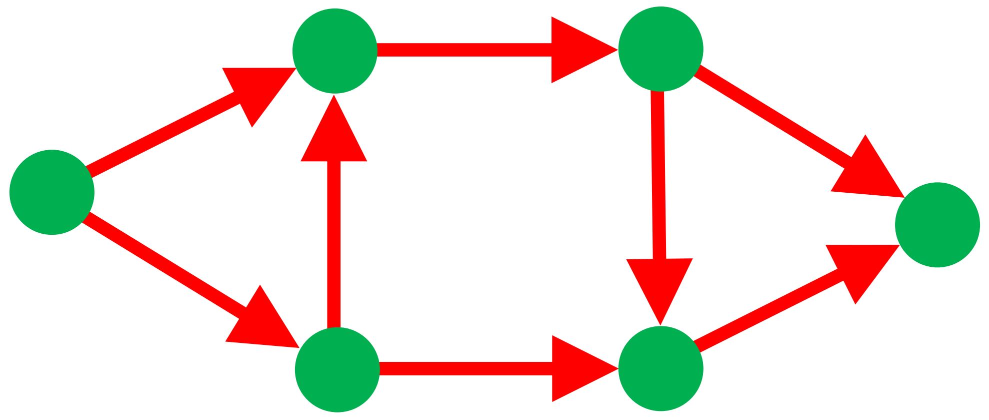


Same graph? ✗

Graphs



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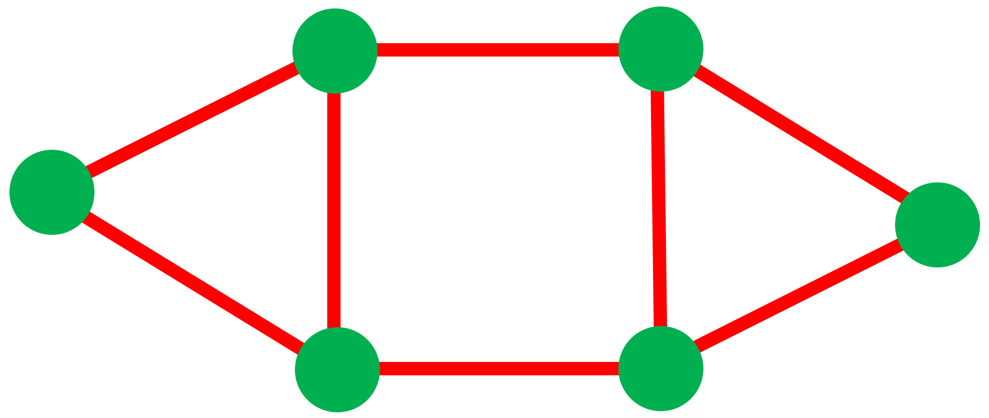
- Edges can be directed…

Graphs



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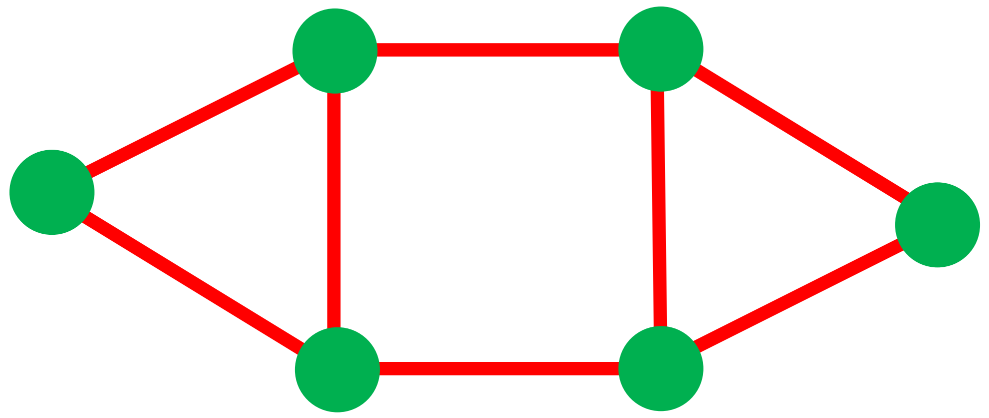
- Edges can be directed or undirected.

Graphs



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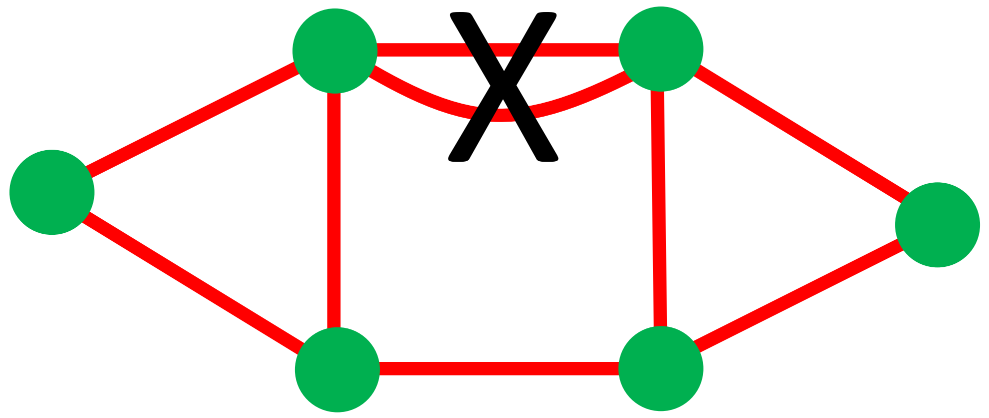
- <u>Simple graph</u> = At most one edge between pair of vertices and no edges that start and end at same vertex.

Graphs



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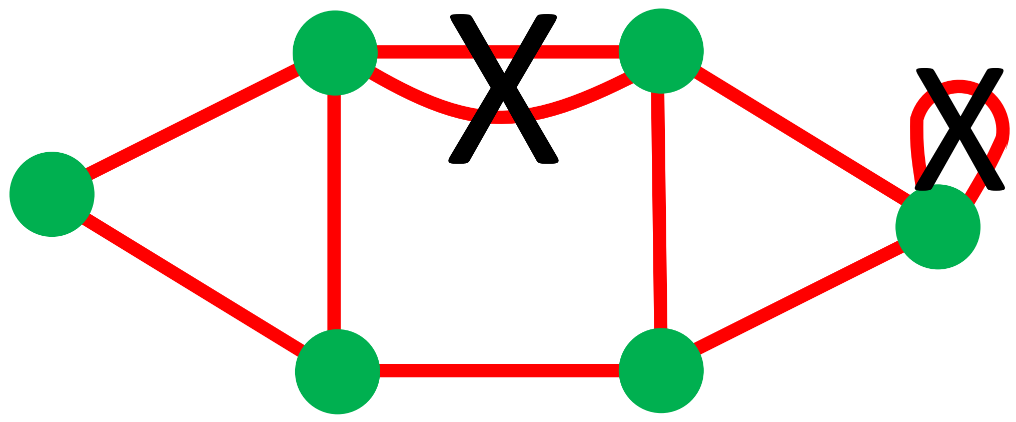

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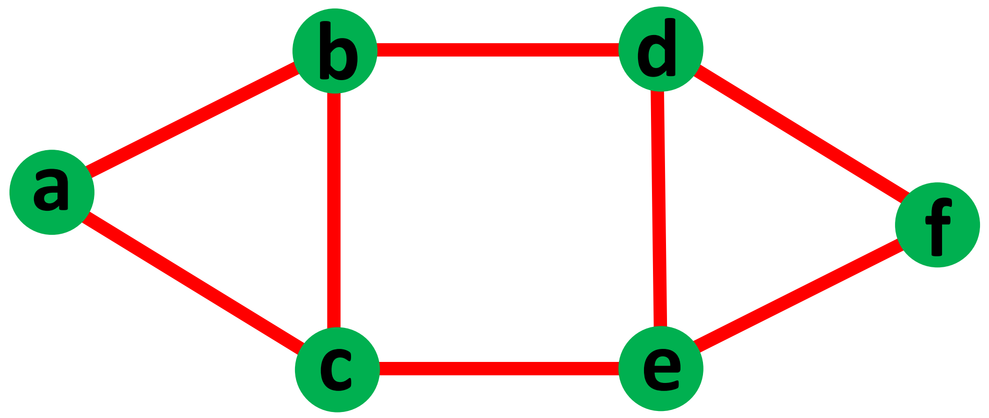

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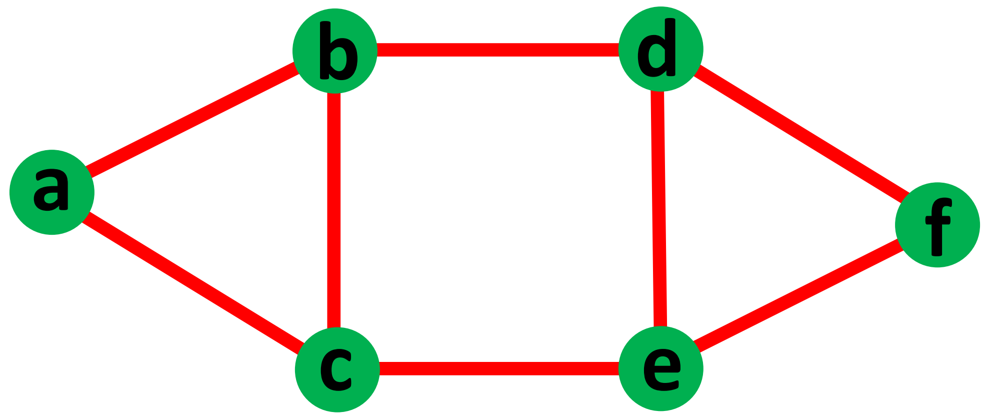

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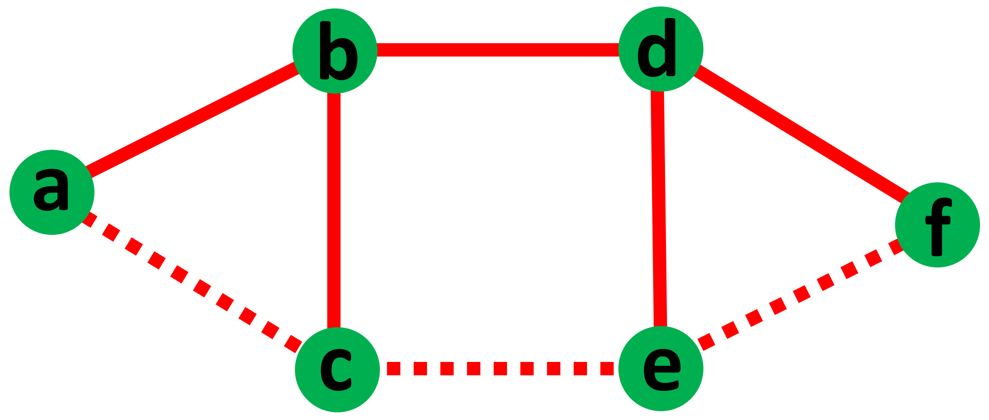
a,c,e,f

Graphs



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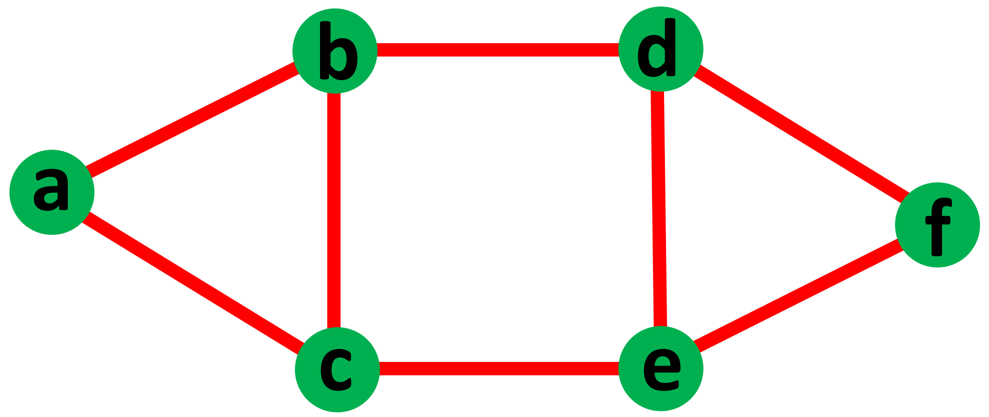

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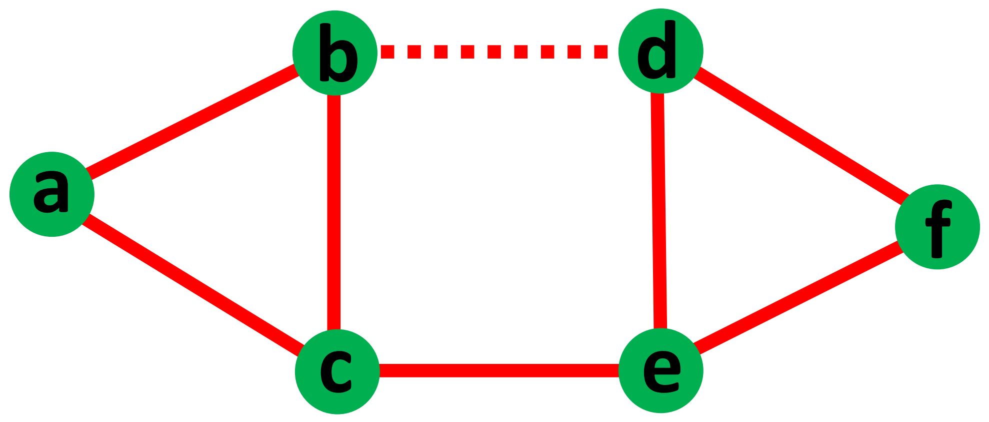
b,d

Graphs



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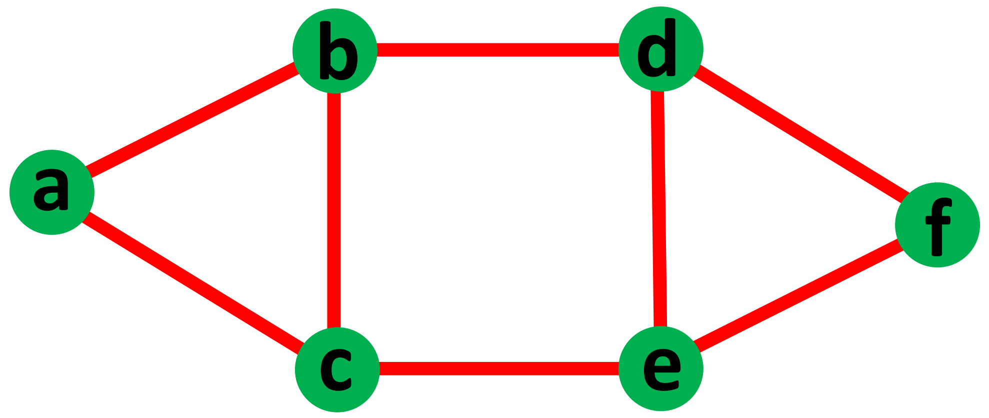
Graphs



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a,c,e,f ✓ **a,c,d,f**

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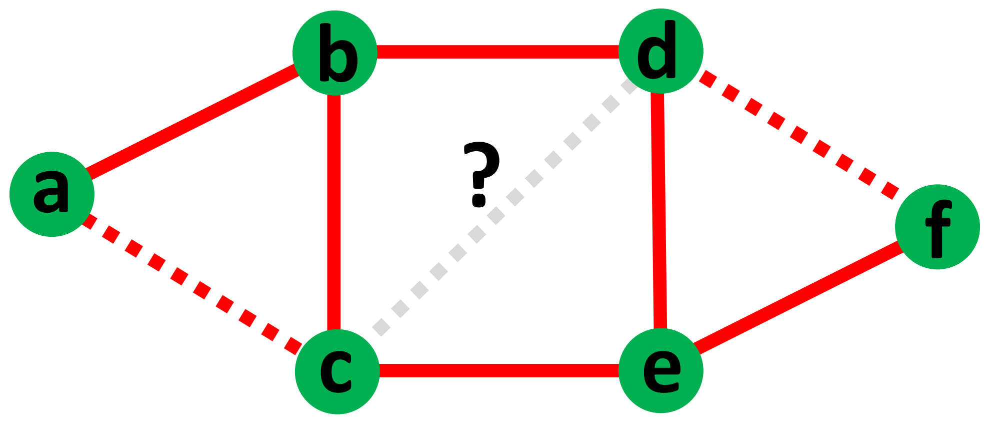
Graphs



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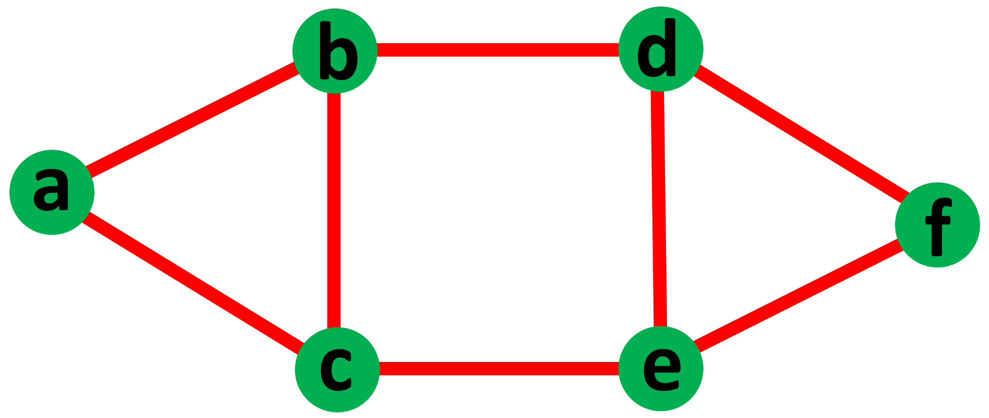

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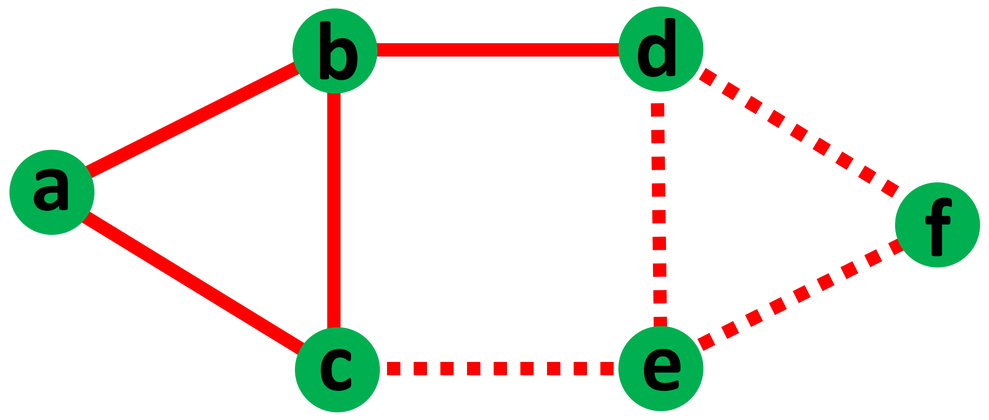
a,c,e,f ✓	**a,c,d,f** ✗
b,d ✓	**c,e,d,f,e**

Graphs



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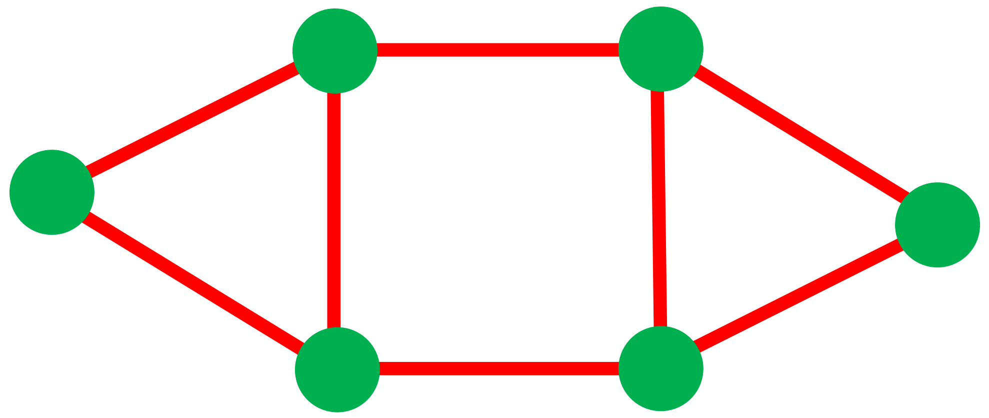
Graphs



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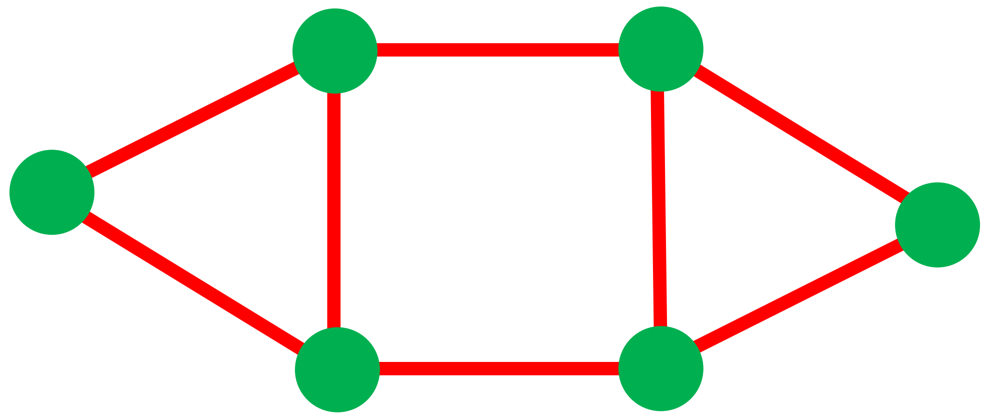
- Cycle = Sequence of vertices connected by edge with loop(s).

Graphs



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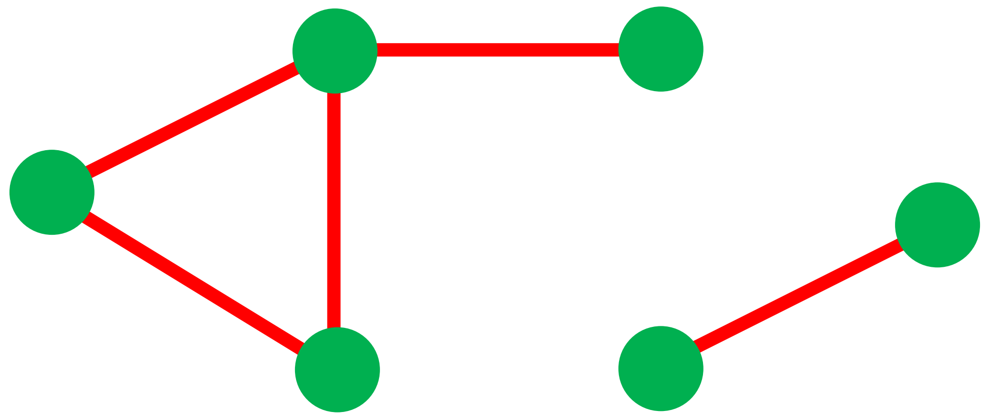
- Connected Graph = Graph that has a path between every vertex pair.

Graphs



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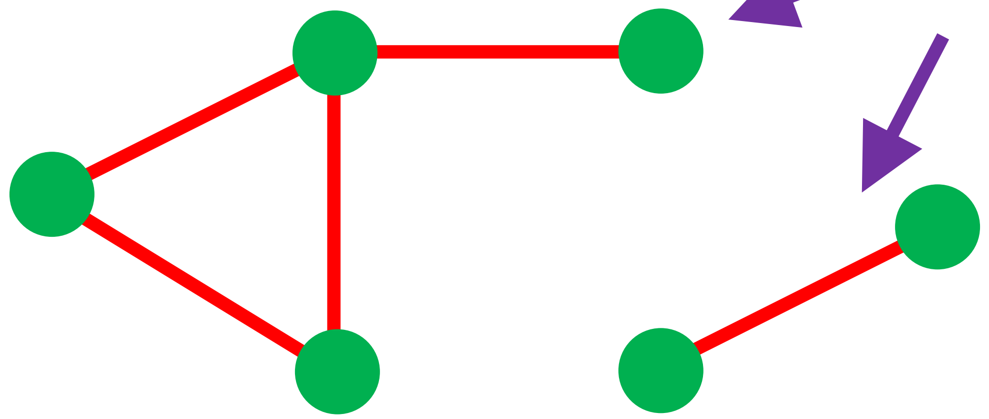


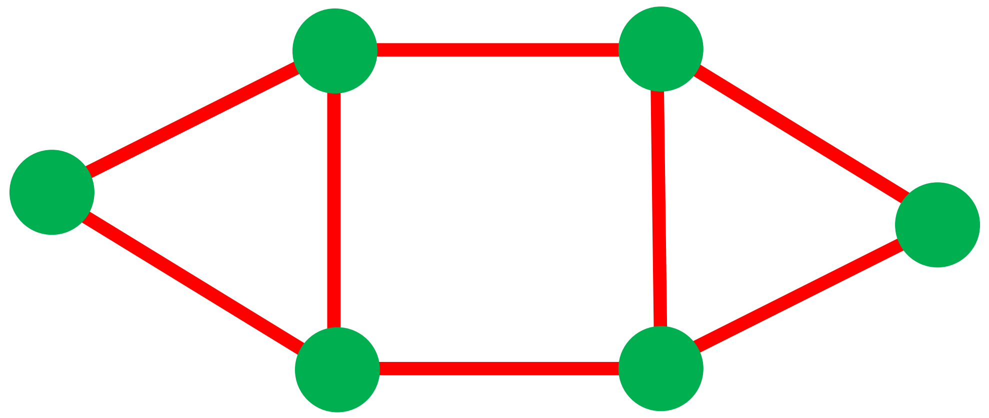
Two Connected Components

Vertices (or Nodes)
Edges

$$\left.\begin{array}{l}\text{Vertices (or Nodes)}\\ \text{Edges}\end{array}\right\} \boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$$

- Edges can be directed or undirected.
- Simple graph = At most one edge between pair of vertices and no edges that start and end at same vertex.
- Path = Sequence of vertices connected by edges without loops.
- Cycle = Sequence of vertices connected by edge with loop(s).
- Connected Graph = Graph that has a path between every vertex pair.

Graphs



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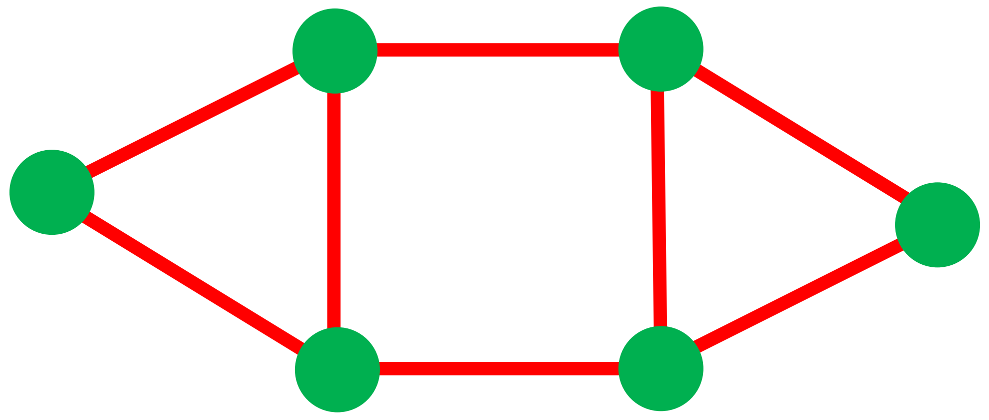
- Connected Graph = Graph that has a path between every vertex pair.
- Degree of a vertex = $\deg(v)$ = # of edges touching it (undirected).

Graphs



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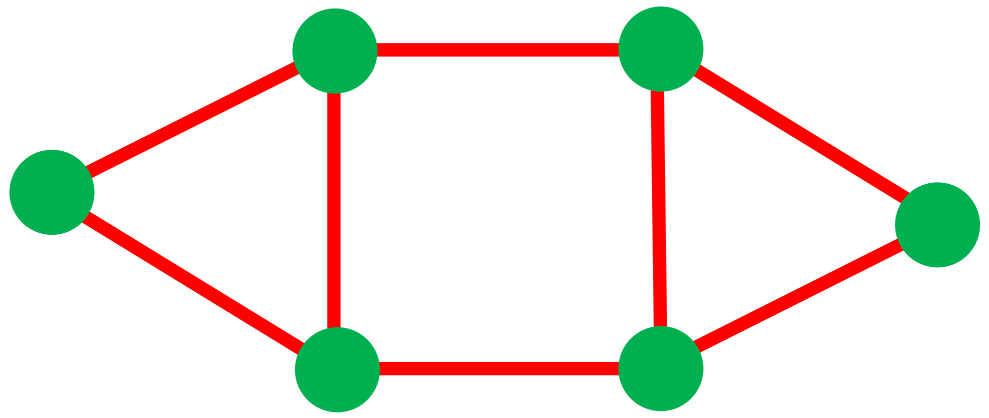
What are some operations we may want to perform on a graph?

Graphs

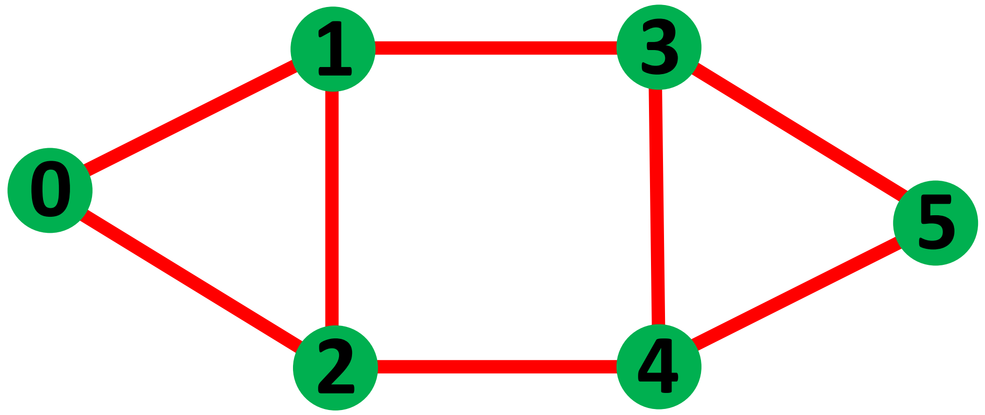


$$\left.\begin{array}{l}\textbf{Vertices (or Nodes)}\\ \textbf{Edges}\end{array}\right\} \boldsymbol{G = (V, E)}$$

What are some operations we may want to perform on a graph?

- Add vertices/edges.
- Find path between vertex pair.
- Is graph connected?
- Find degree of vertex.
- Is the graph simple?
- Get number of vertices/edges.
- Get neighbors of vertex.
- Is there a cycle?
- Find max degree of graph.

How can we represent a graph in a computer?



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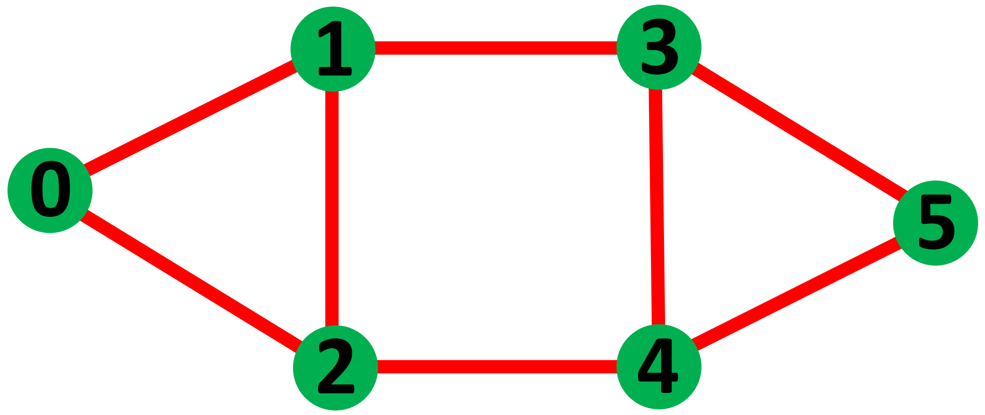

1. Adjacency List

0	→	{1,2}
1	→	{0,2,3}
2	→	{0,1,4}
3	→	{1,4,5}
4	→	{2,3,5}
5	→	{3,4}

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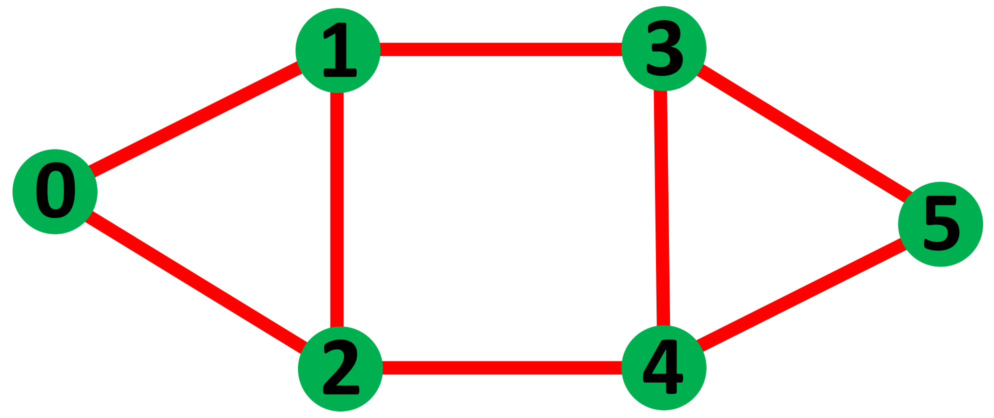

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2. Adjacency Matrix

	0	1	2	3	4	5
0	F	T	T	F	F	F
1	T	F	T	T	F	F
2	T	T	F	F	T	F
3	F	T	F	F	T	T
4	F	F	T	T	F	T
5	F	F	F	T	T	F

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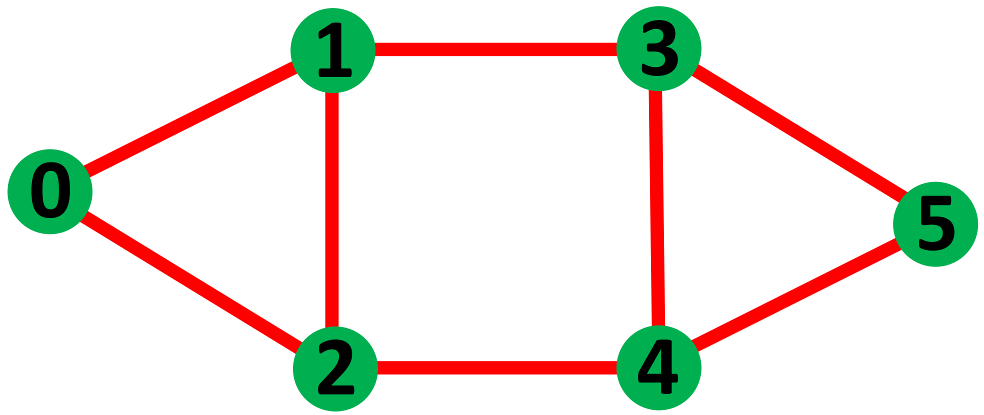

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3. Objects

```
public class Node {
   private Set<Node> neighbors;

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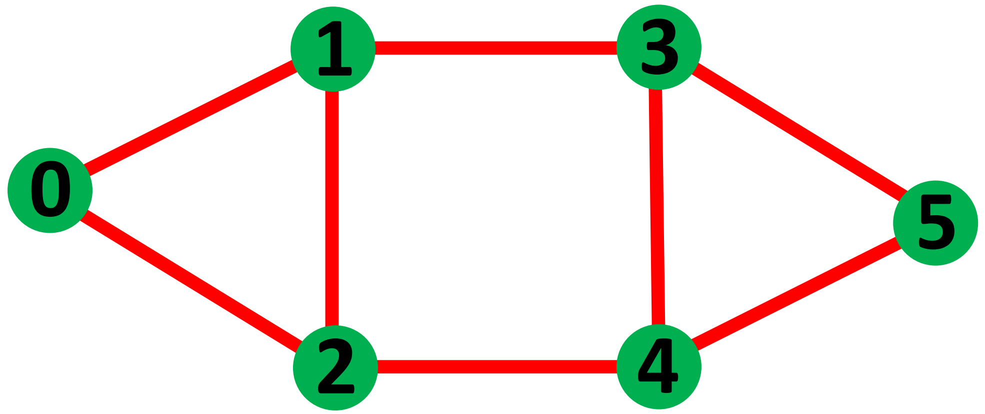

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