CSCI 232: Data Structures and Algorithms

Binary Search Trees (BST)

Reese Pearsall Spring 2024

Announcements

Lab 3 due **tomorrow** at 11:59

Program 1 will be posted very soon

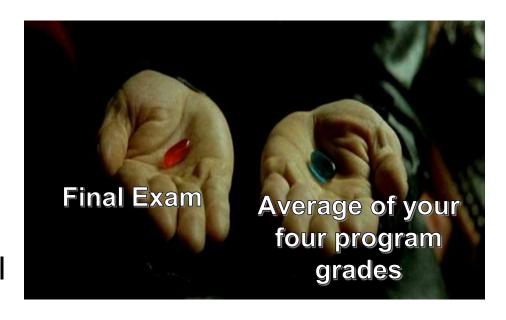
Final Exam

The final exam for 232 will be optional.

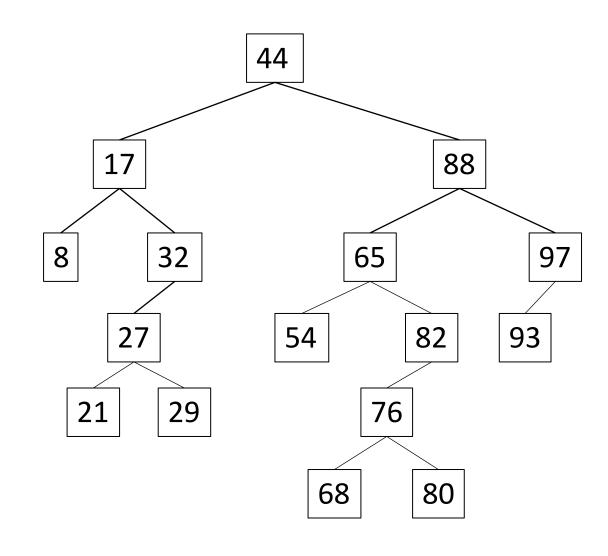
You can either:

1. Show up and take the final exam normally

2. Don't show up, and your final exam grade will be the average of your four program grades

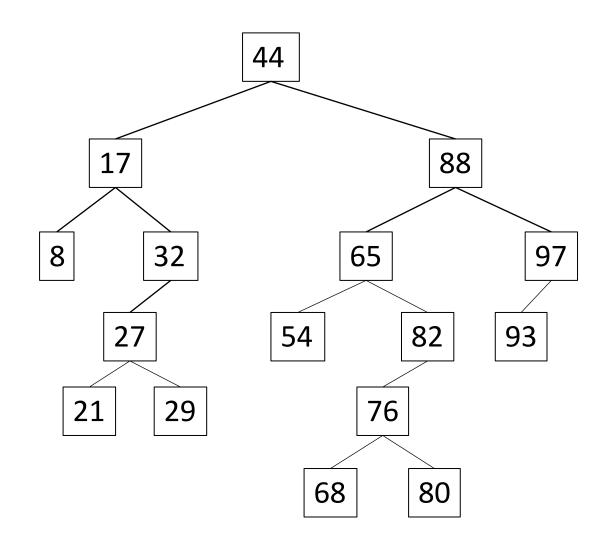


(If you show up, and you still do worse than the average of your four exam grades, I will give you the average of your program grades)

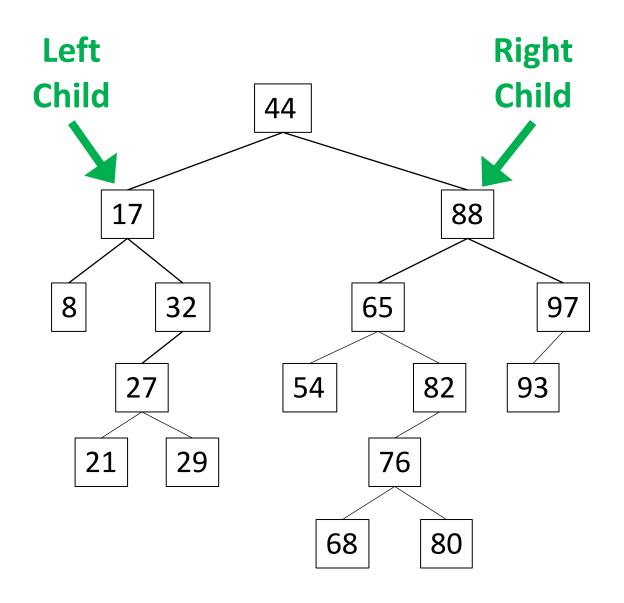


Binary Search Tree (BST) properties:

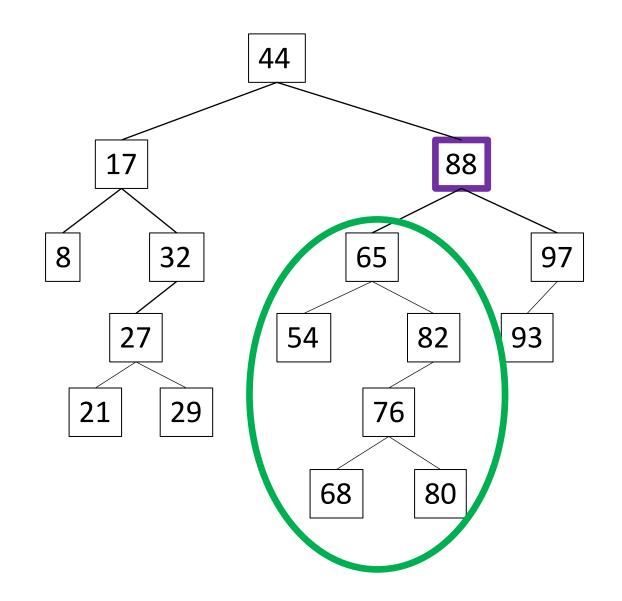
• A BST is composed of Comparable data elements.



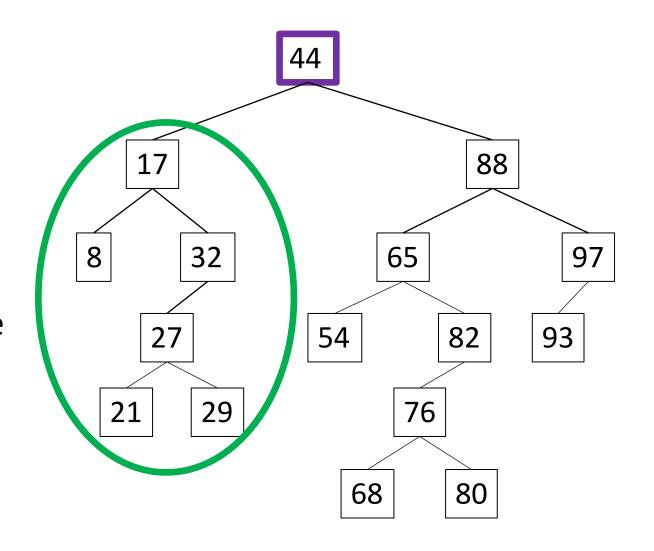
- A BST is composed of Comparable data elements.
- A BST is a binary tree (each node has at most two children).



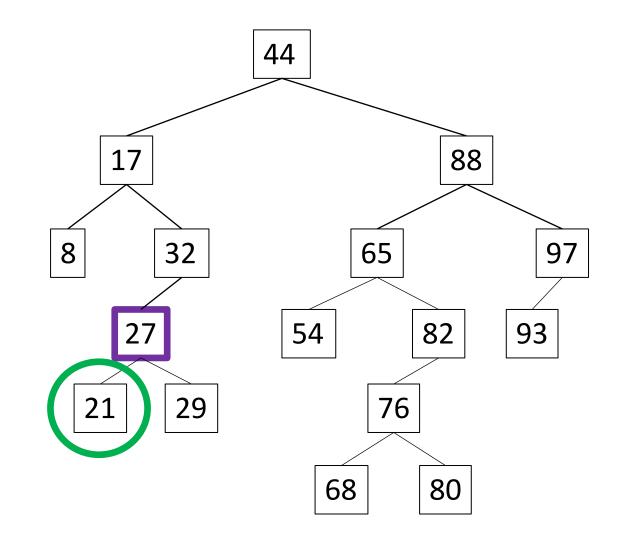
- A BST is composed of Comparable data elements.
- A BST is a binary tree (each node has at most two children).
- For each node, all left-hand descendants have values that are less that the node.



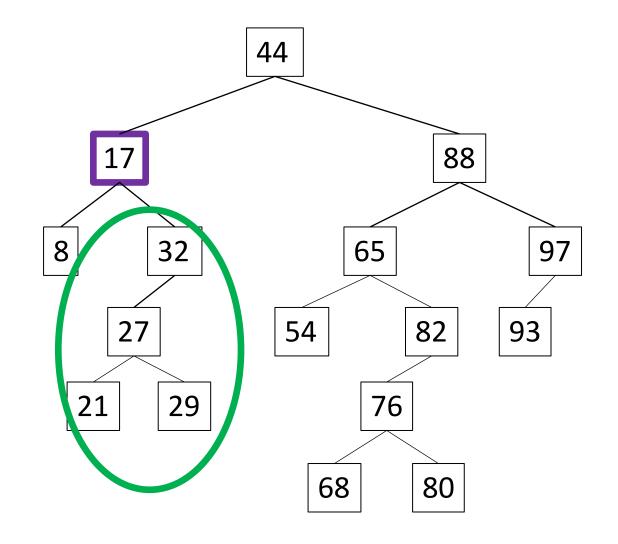
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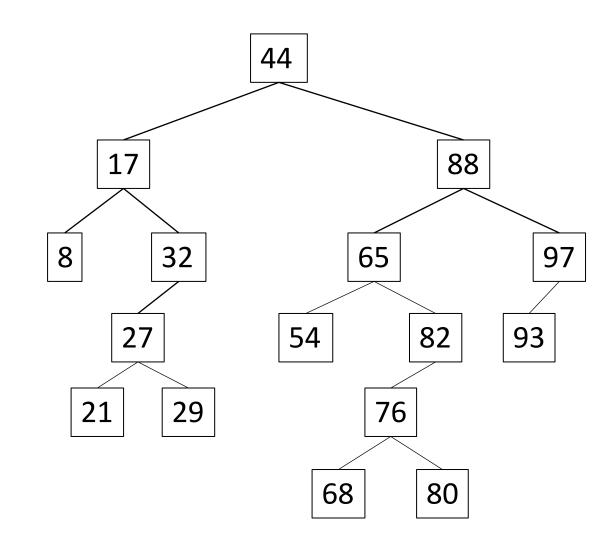
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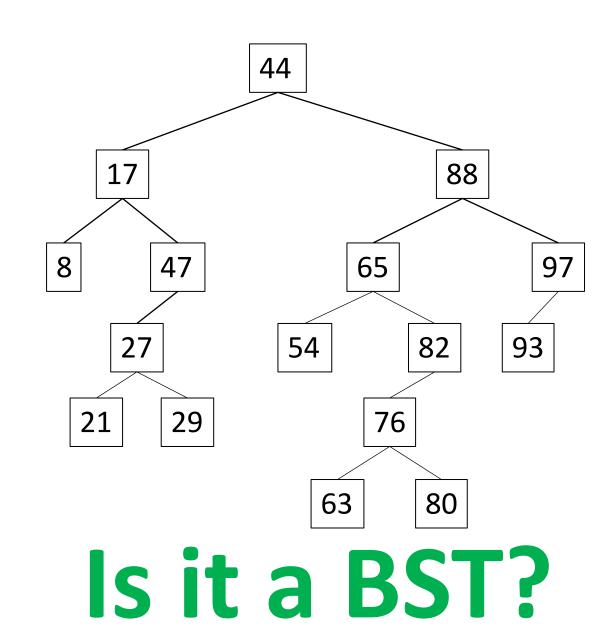
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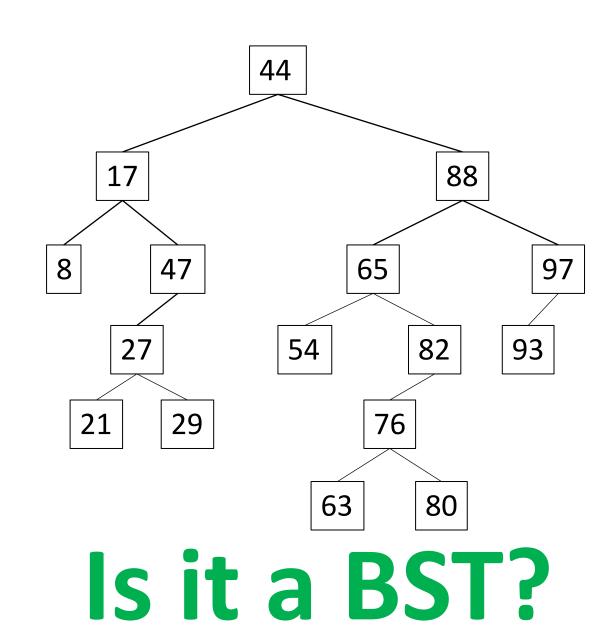
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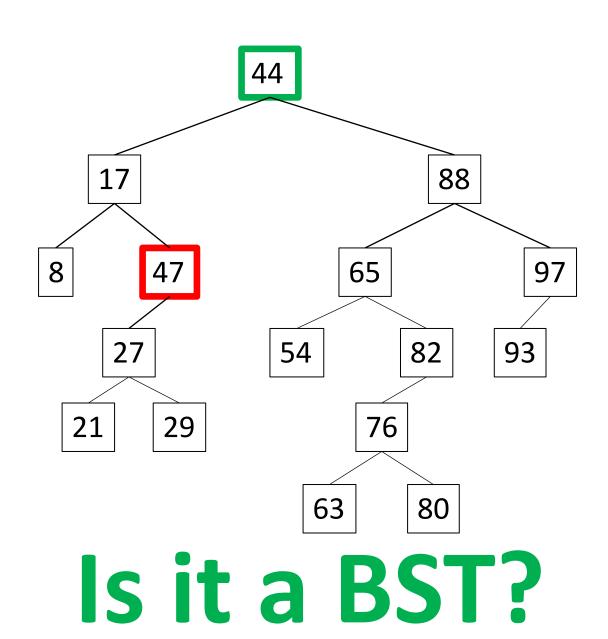
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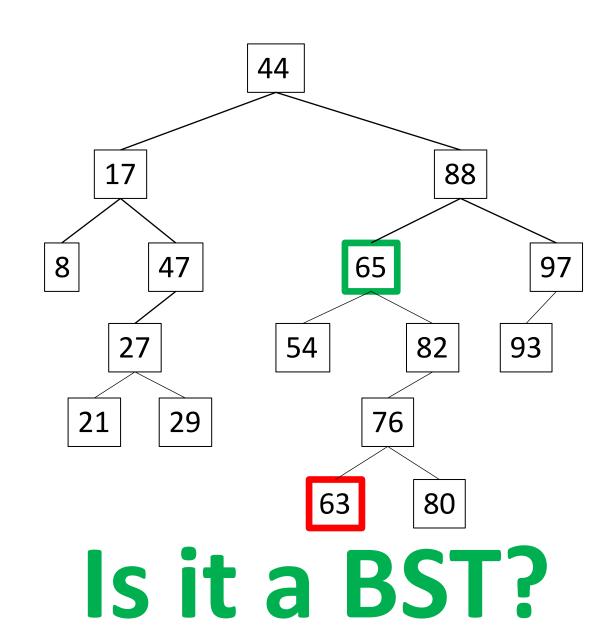
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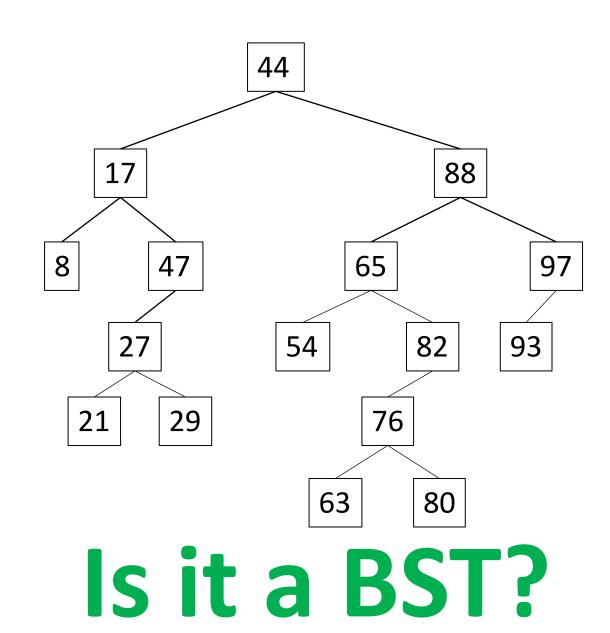
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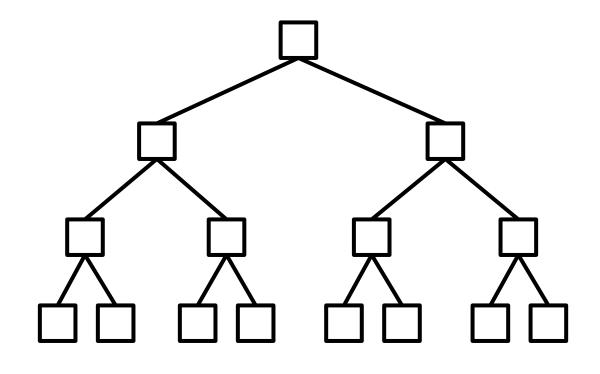


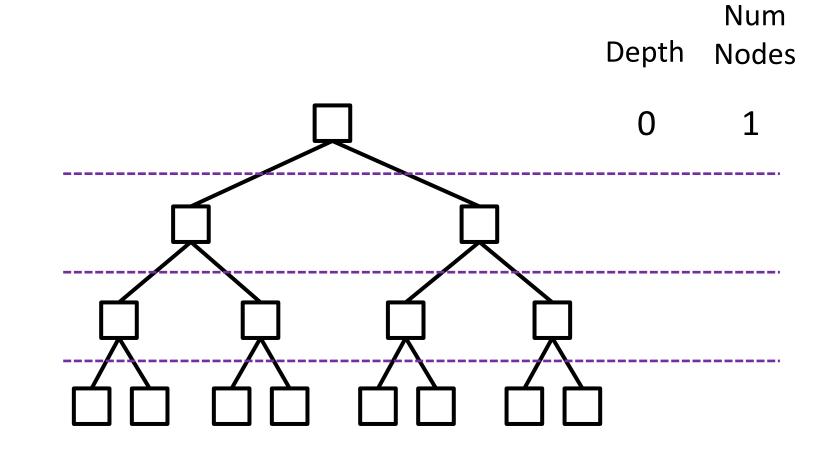
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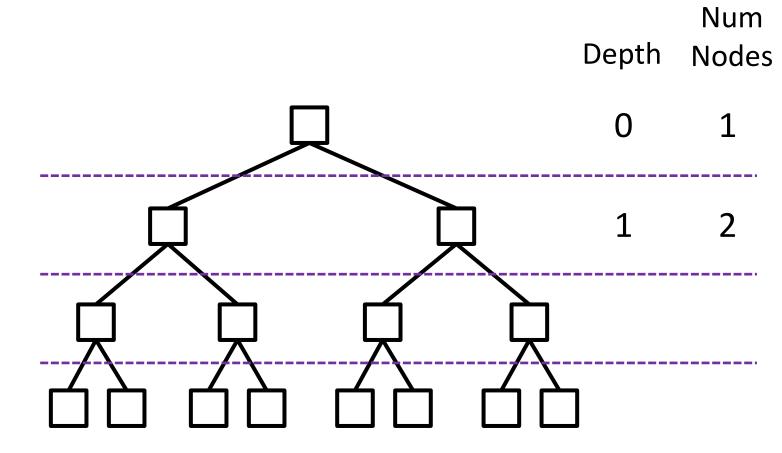


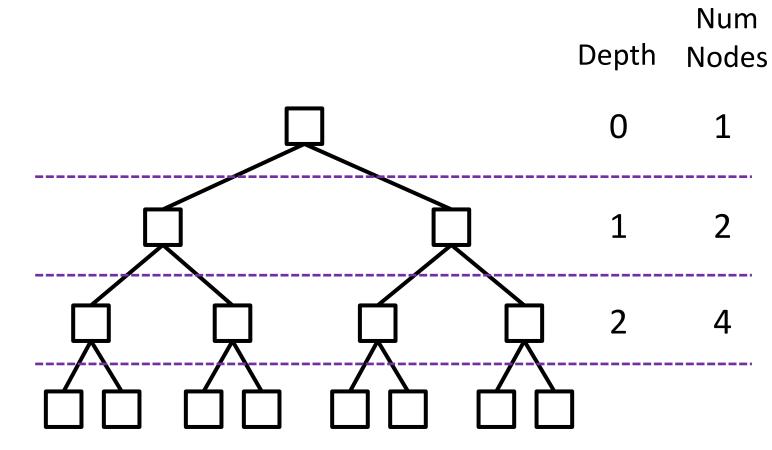
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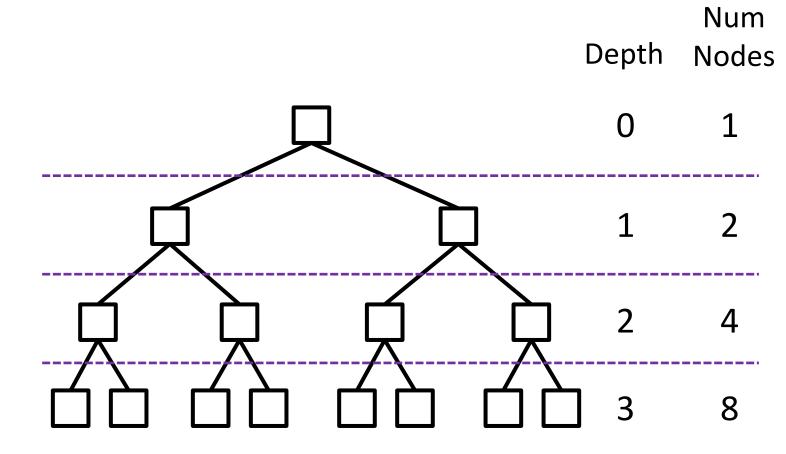






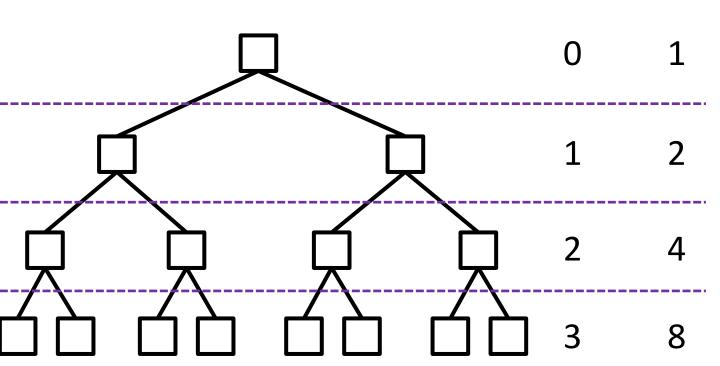






What is the point? Why use a BST?

In general, at depth d, there are at most ?? nodes.

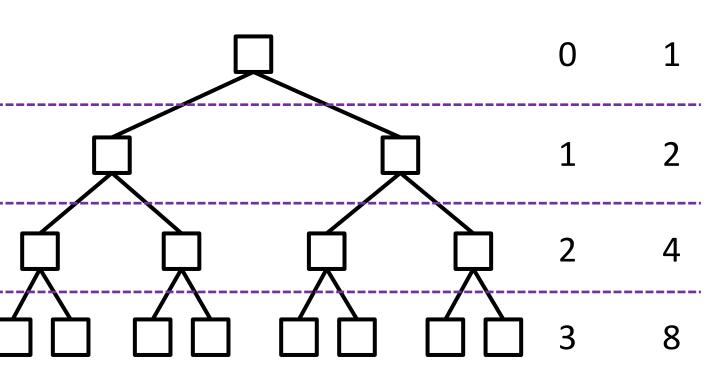


Num

Nodes

What is the point? Why use a BST?

In general, at depth d, there are at most 2^d nodes.



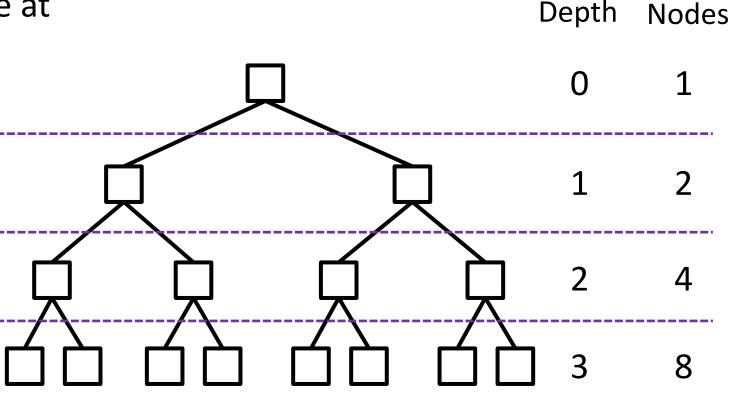
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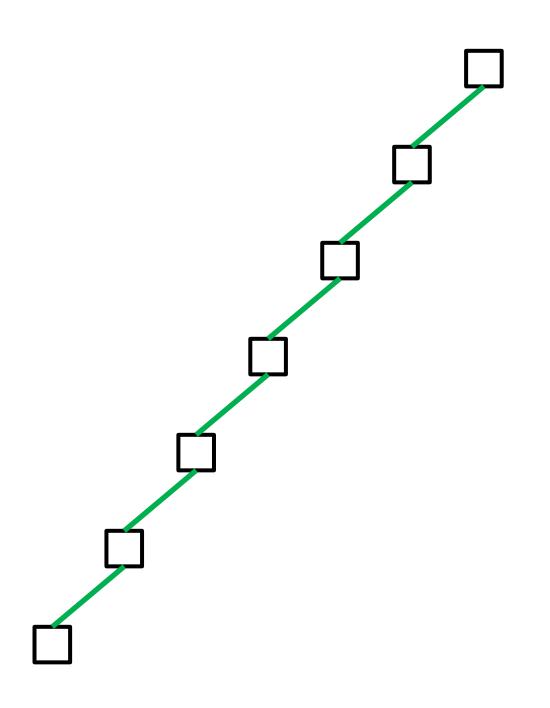
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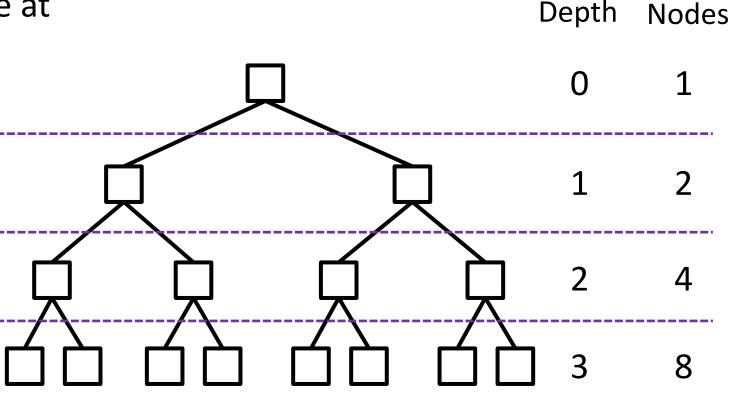
Given a BST with n nodes, what is the greatest number of edges we would have to traverse to go from the root to a leaf? n-1



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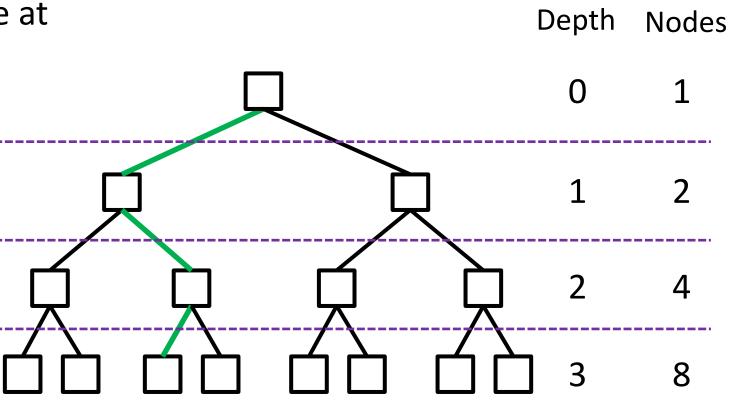
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In general, at depth d, there are at most 2^d nodes.

Given a BST with *n* nodes, what is the greatest number of edges we would have to traverse to go from the root to a leaf? *height of tree*.

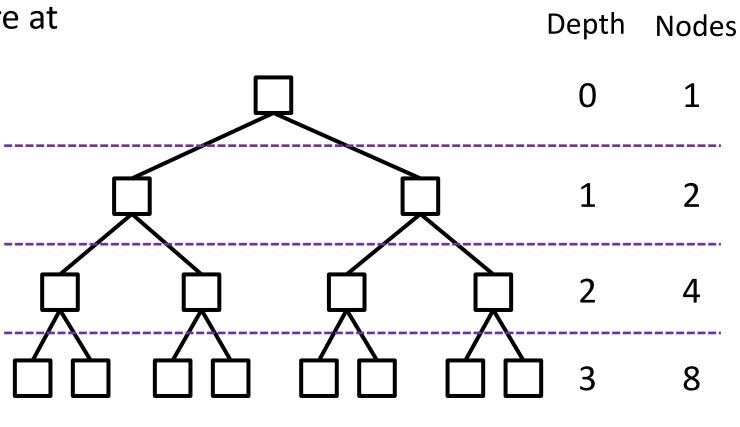


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Depth **Nodes** 3

$$n = 2^0 + 2^1 + 2^2 + \dots + 2^h$$

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Num
$$n = 2^{0} + 2^{1} + 2^{2} + \dots + 2^{h}$$

$$\Rightarrow n - 1 = 2^{1} + 2^{2} + \dots + 2^{h}$$

$$= 2(2^{0} + 2^{1} + \dots + 2^{h-1})$$

$$= 2(n - 2^{h}) = 2n - 2^{h+1}$$

$$\Rightarrow n - 1 = 2n - 2^{h+1}$$

$$\Rightarrow n = 2^{h+1} - 1$$

$$3$$

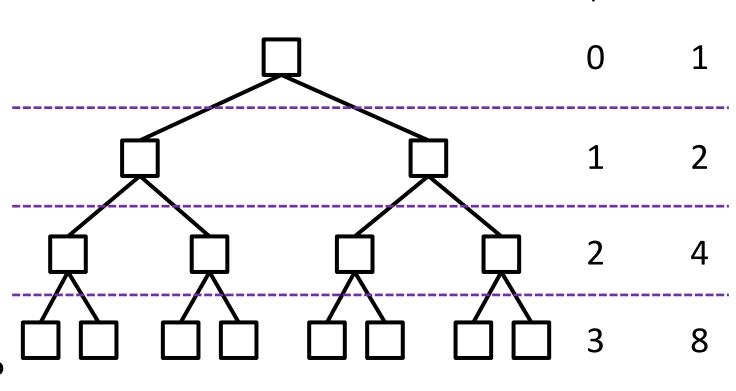
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Nodes

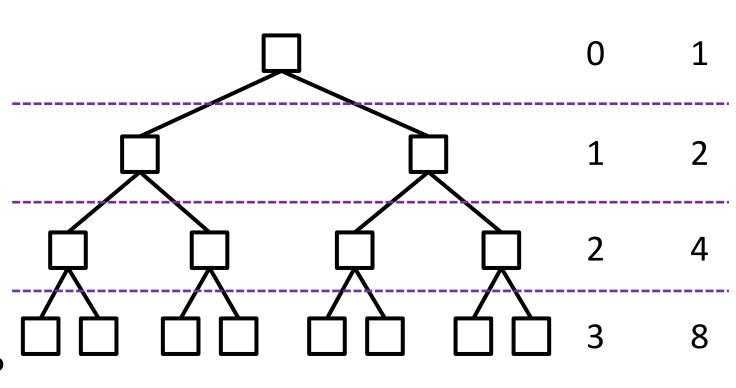
$$n = 2^{0} + 2^{1} + 2^{2} + \dots + 2^{h} = 2^{h+1} - 1 \Rightarrow n+1 = 2^{h+1}$$

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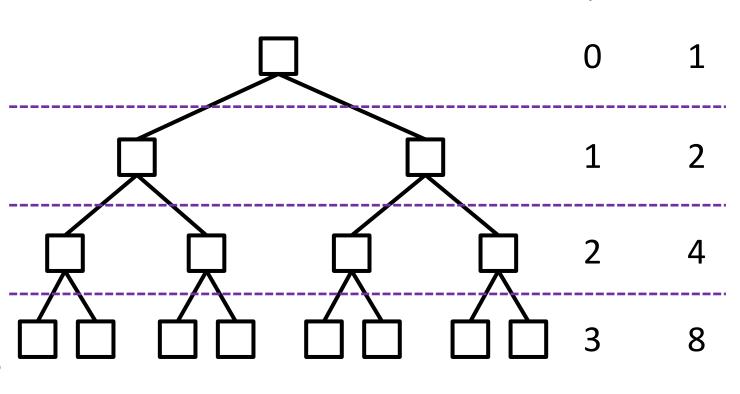
$$n = 2^{0} + 2^{1} + 2^{2} + \dots + 2^{h} = 2^{h+1} - 1 \Rightarrow \frac{n+1}{\log_{2}(n+1)} = n + 1$$

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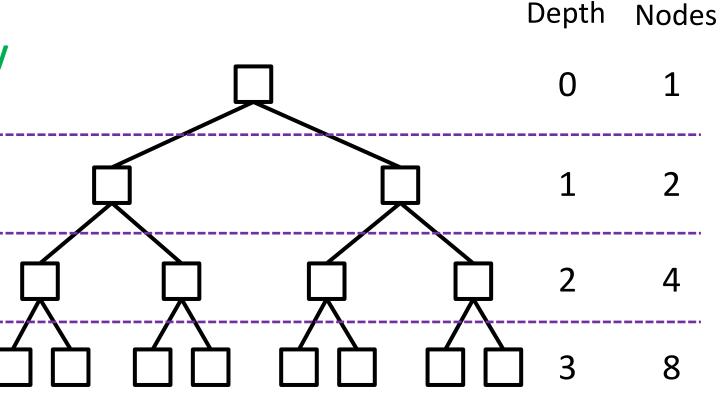
Num

Nodes

$$n = 2^{0} + 2^{1} + 2^{2} + \dots + 2^{h} = 2^{h+1} - 1 \Rightarrow \frac{n+1}{\log_{2}(n+1)} = h+1 \Rightarrow h \in O(\log n)$$

What is the point? Why use a BST?

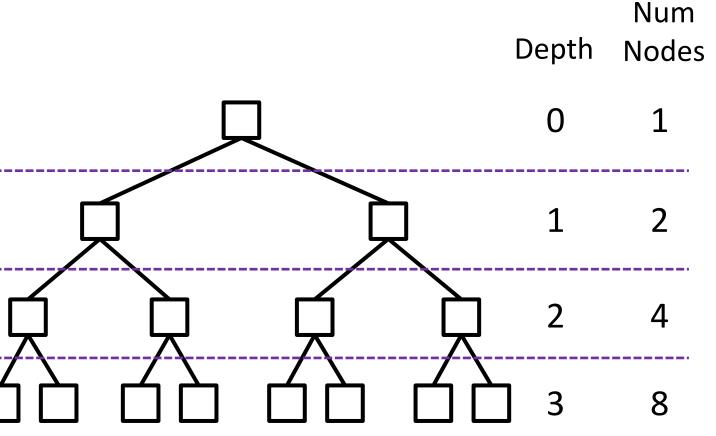
This means we can access any node in a specific type of binary tree in $\log n$ time.



What is the point? Why use a BST?

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Of note, we can test if a specific value is in a collection in $\log n$ time.

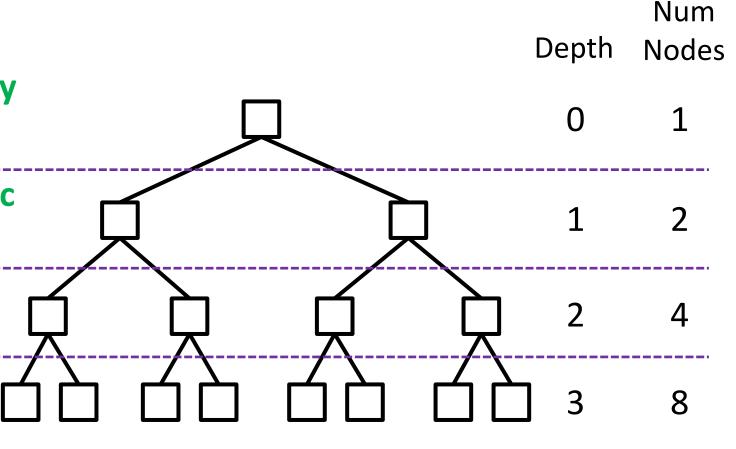


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But we can already do that with a sorted array and Binary Search!



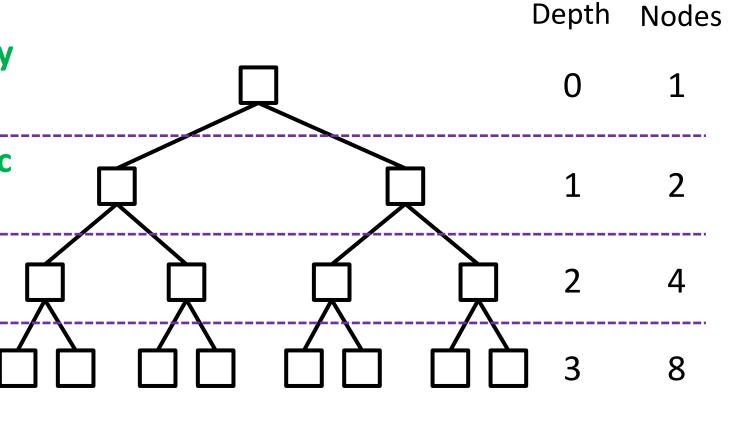
Binary Search Tree

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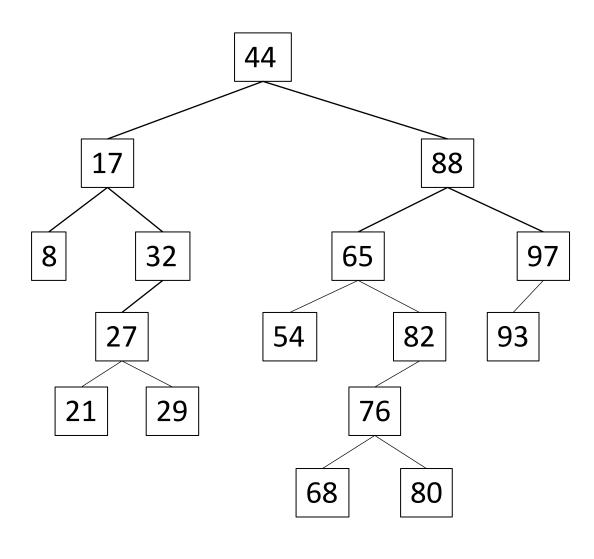
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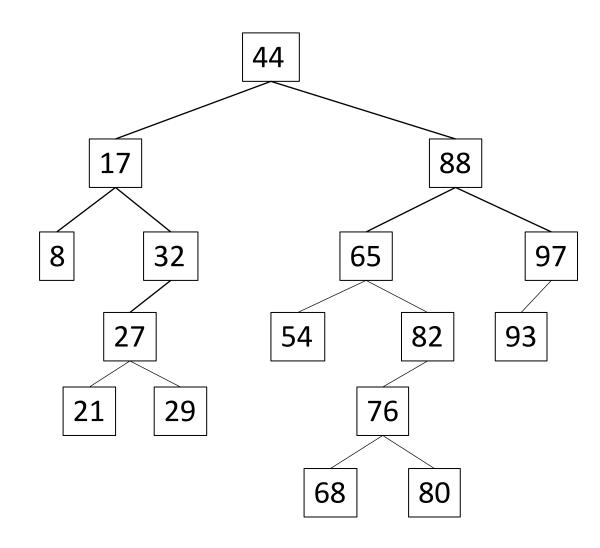
Num

Perhaps managing a BST is more efficient than managing an array.

```
public class Node {
    private int value;
    private Node left;
    private Node right;
    private Node parent;
    public Node(int value) {
        this.value = value;
   // getValue()
    // getLeft(), getRight()
    // getParent()
   // setLeft(), setRight()
    // setParent()
```



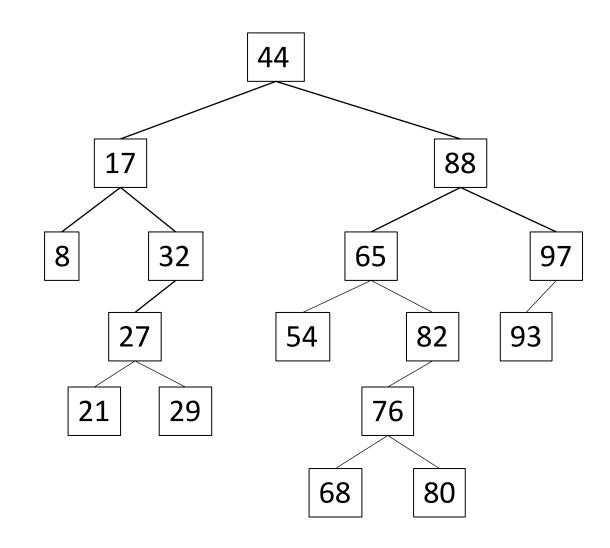
insert(31);



insert(31);

Step 1: Find where it should go.

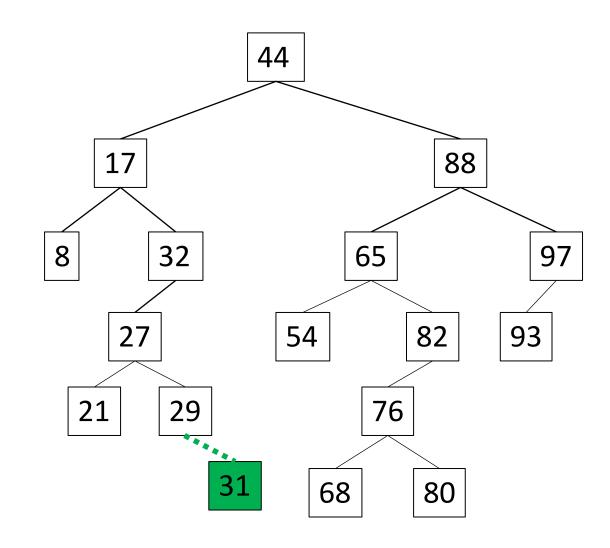
Step 2: Modify pointers.



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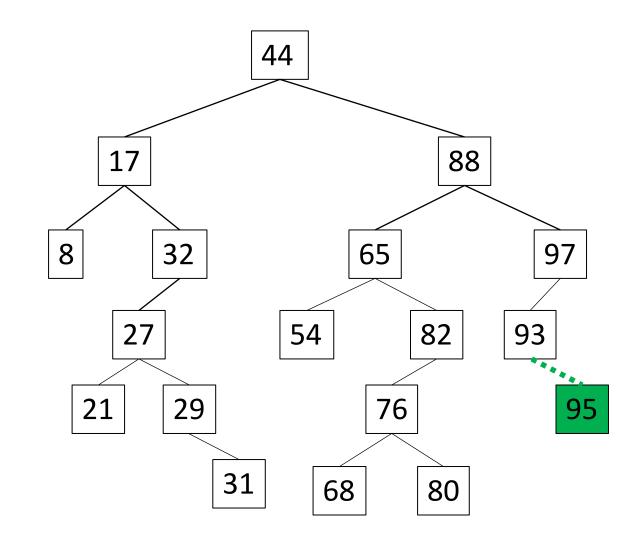
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insert(95);

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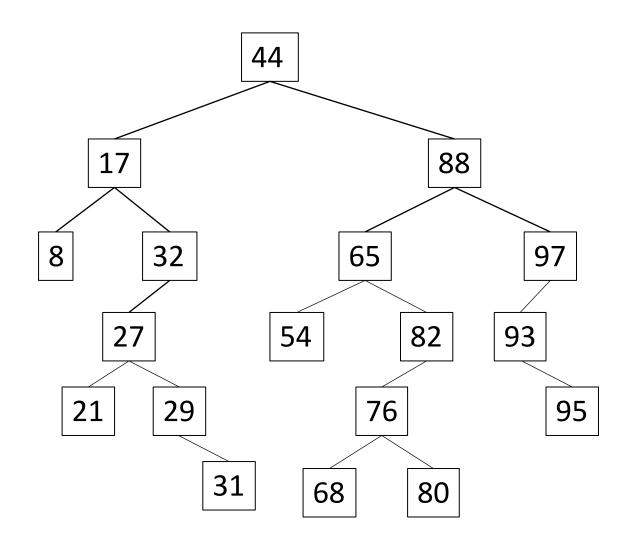


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Any trends??



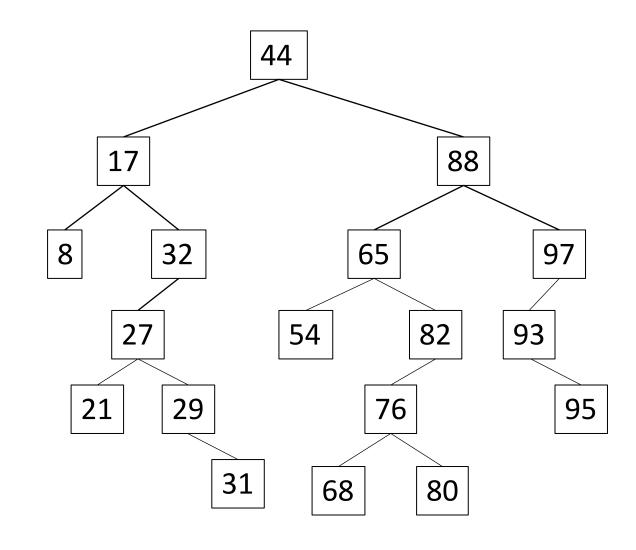
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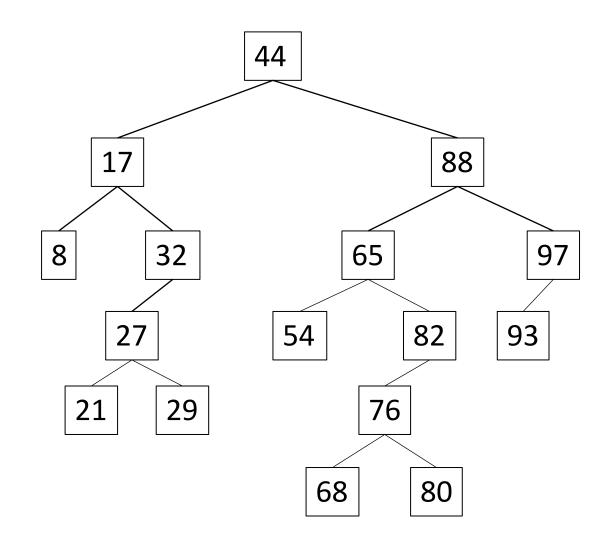
Any trends??

Always insert a new leaf!



insert(28);

public void insert(int newValue) {

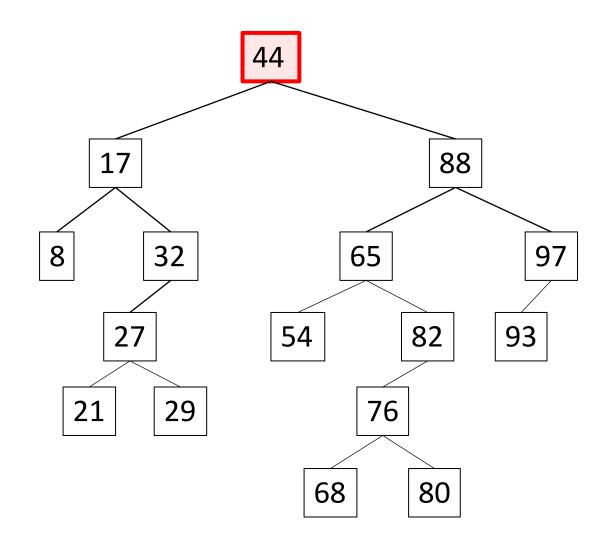


```
insert(28);
```

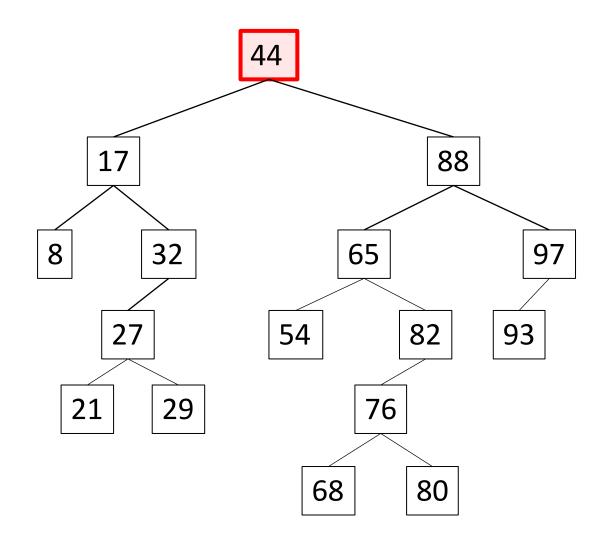
```
public void insert(int newValue) {
  if (root == null) {
    root
    } else {
```

```
public void insert(int newValue) {
  if (root == null) {
    root = new Node(newValue);
  } else {
```

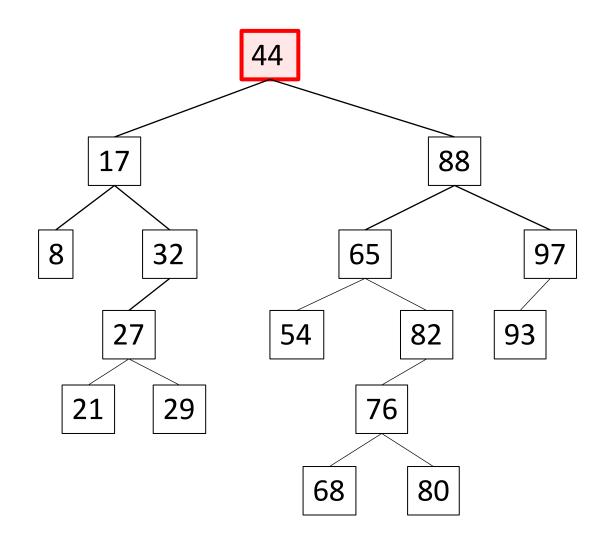
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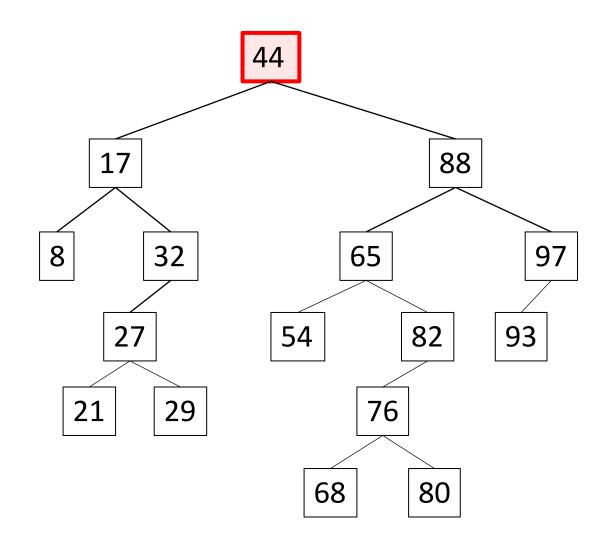
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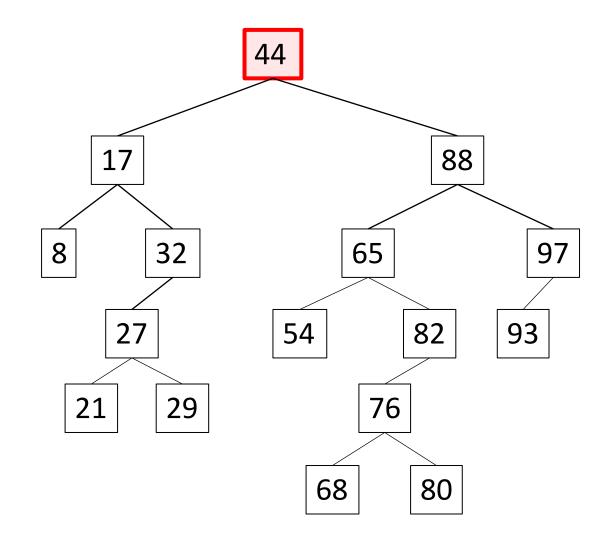
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```



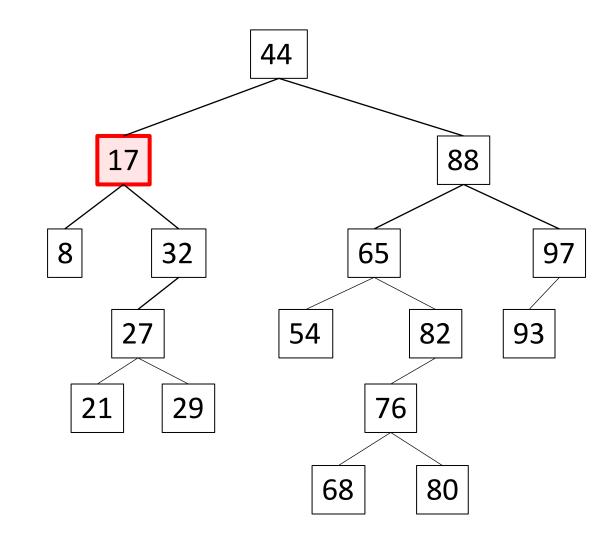
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```



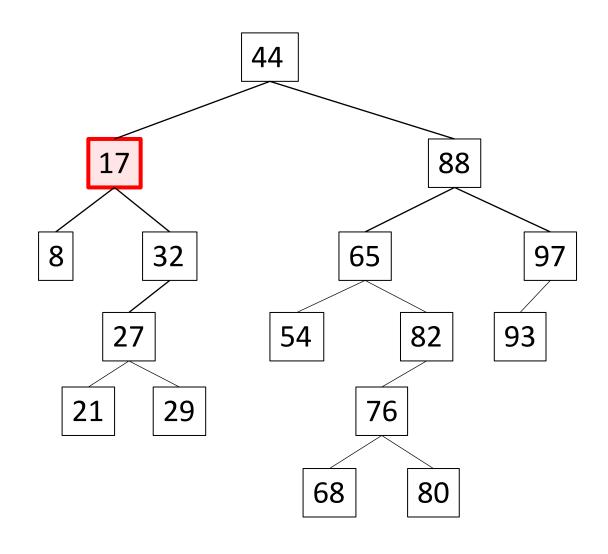
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        if (currentNode.getLeft() != null) {
        } else {
      } else {
```



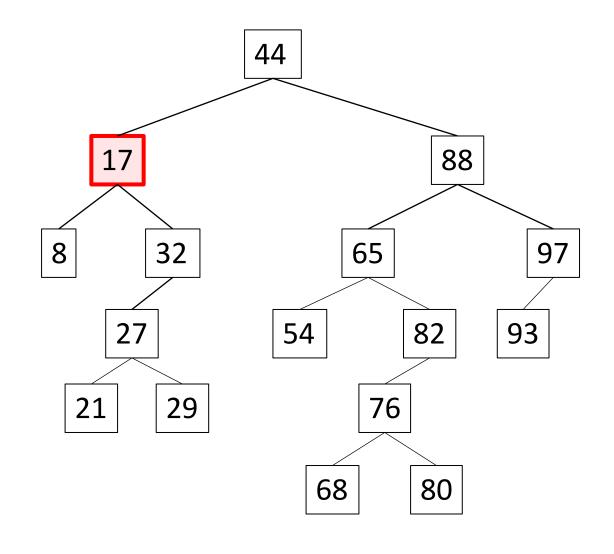
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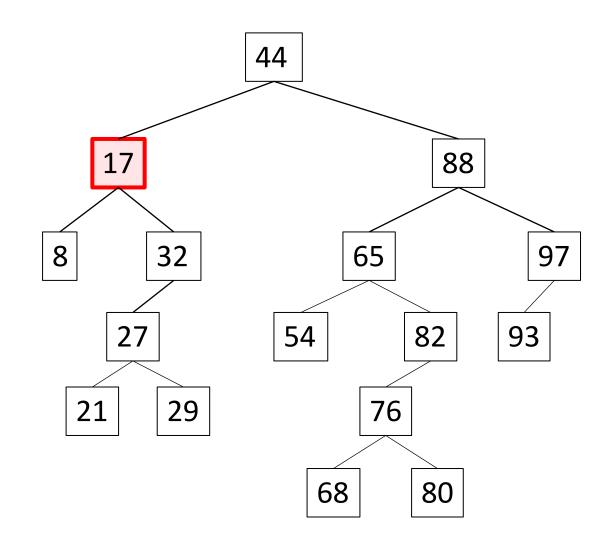
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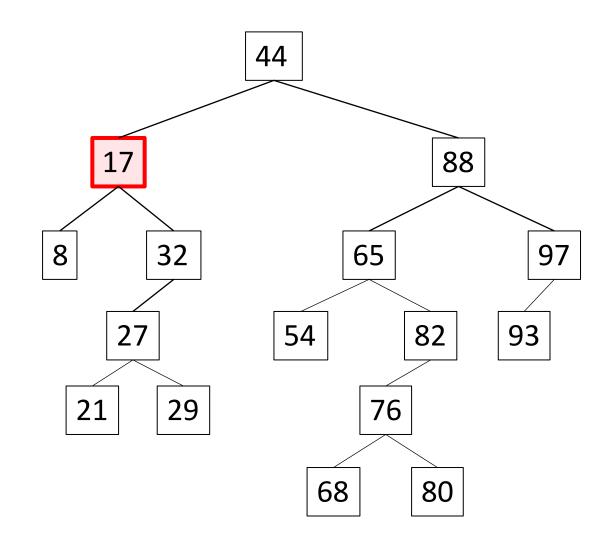
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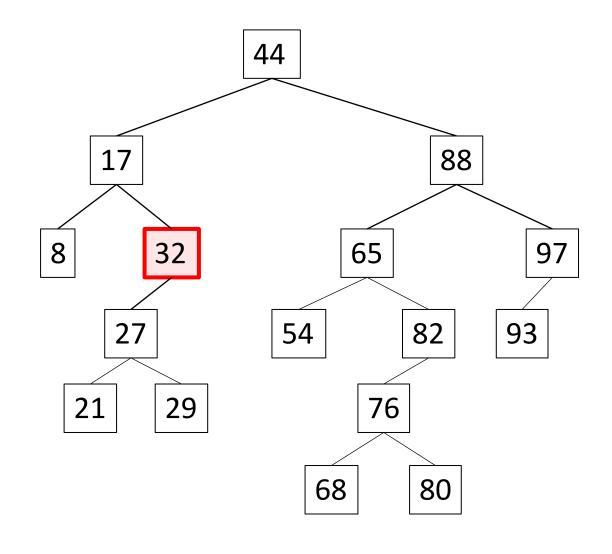
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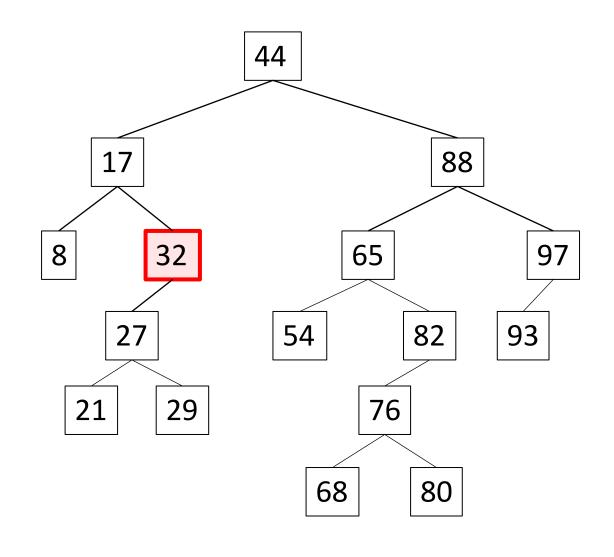
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      } else {
        if (currentNode.getRight() != null) {
        } else {
```



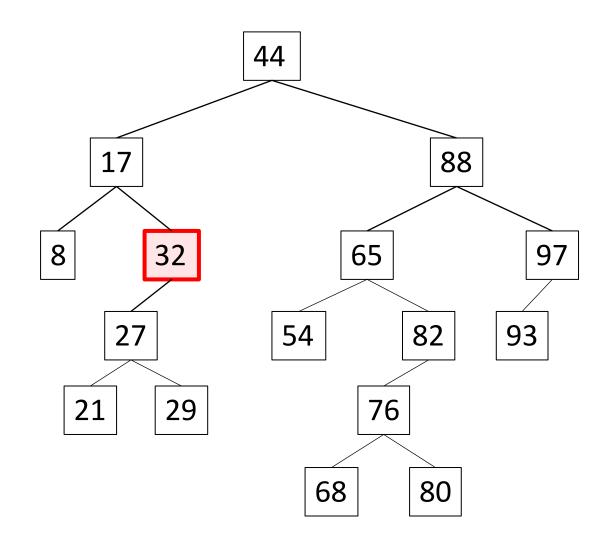
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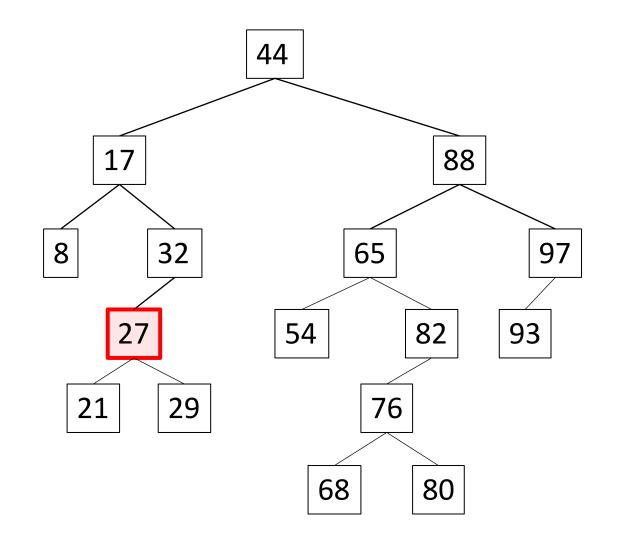
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        if (currentNode.getRight() != null) {
          currentNode = currentNode.getRight();
        } else {
```



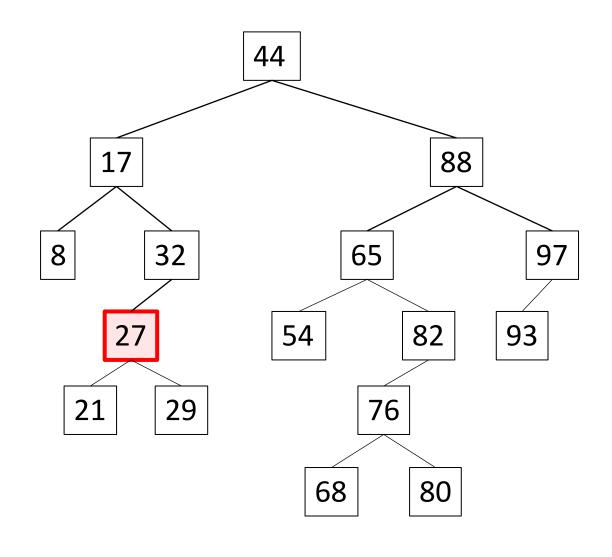
```
public void insert(int newValue) {
  if (root == null) {
    root = new Node(newValue);
 } else {
    Node currentNode = root;
    boolean placed = false;
    while (!placed) {
      if (newValue < currentNode.getValue()) {</pre>
        if (currentNode.getLeft() != null) {
          currentNode = currentNode.getLeft();
        } else {
      } else {
        if (currentNode.getRight() != null) {
          currentNode = currentNode.getRight();
        } else {
```



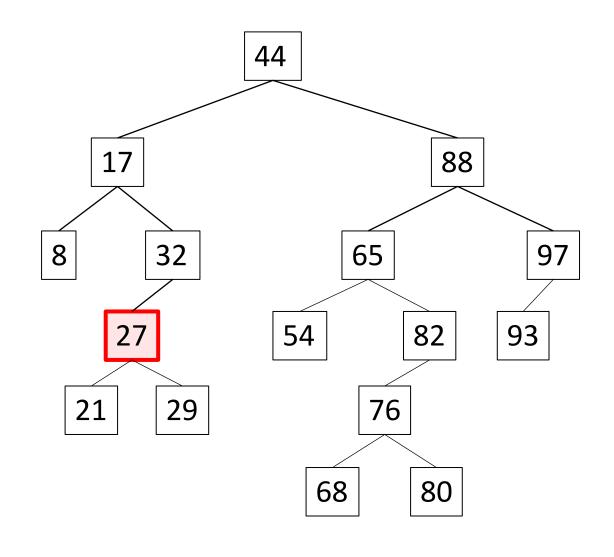
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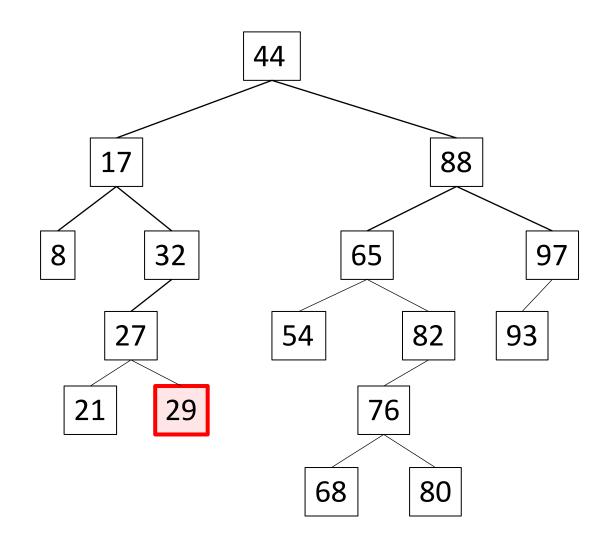
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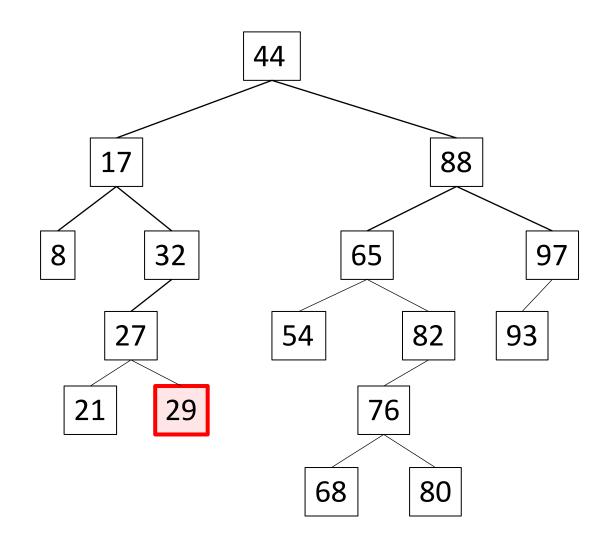
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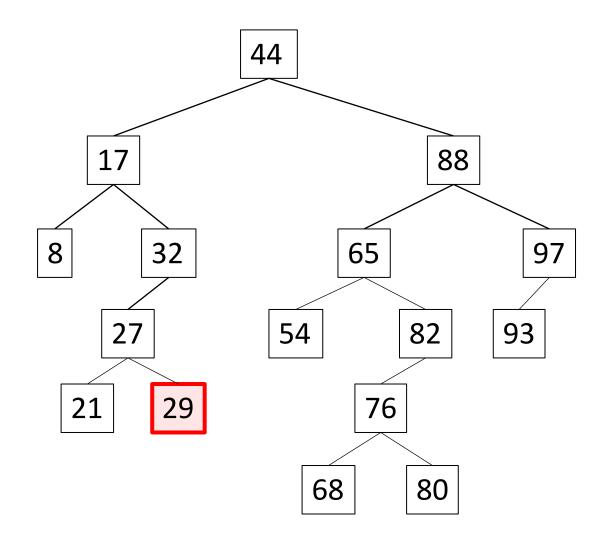
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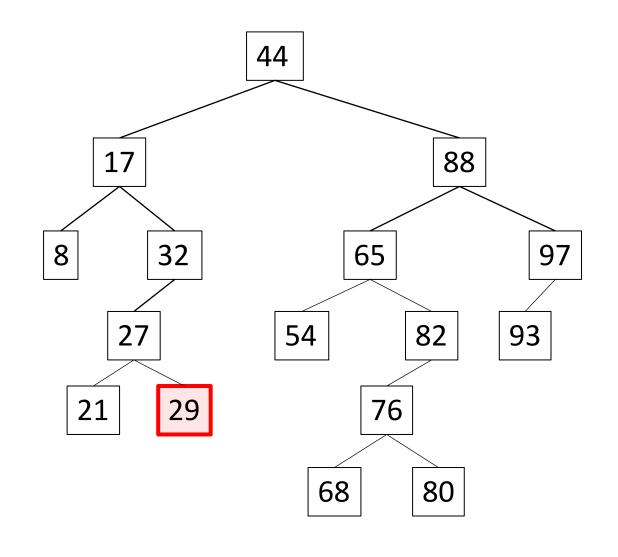
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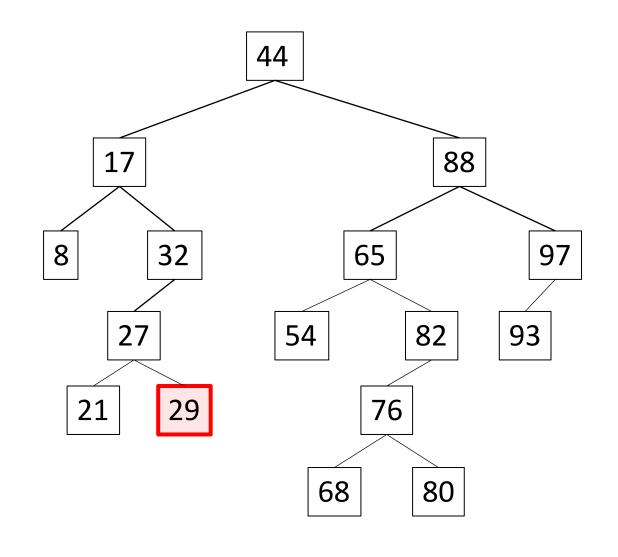
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```



```
public void insert(int newValue) {
                                                                      44
  if (root == null) {
    root = new Node(newValue);
 } else {
    Node currentNode = root;
                                                                                     88
                                                          17
    boolean placed = false;
    while (!placed) {
      if (newValue < currentNode.getValue()) {</pre>
        if (currentNode.getLeft() != null) {
                                                                              65
                                                                                             97
                                                              32
          currentNode = currentNode.getLeft();
        } else {
          currentNode.setLeft(new Node(newValue));
                                                           27
                                                                                   82
                                                                                          93
                                                                        54
                                                        21
                                                               29
                                                                               76
      } else {
        if (currentNode.getRight() != null) {
          currentNode = currentNode.getRight();
                                                            28
                                                                           68
                                                                                   80
        } else {
```

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                                                                                   82
                                                                                          93
                                                                        54
          currentNode.getLeft().setParent(currentNode);
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                                                       21
                                                               29
                                                                               76
      } else {
        if (currentNode.getRight() != null) {
          currentNode = currentNode.getRight();
                                                           28
                                                                           68
                                                                                   80
        } else {
          currentNode.setRight(new Node(newValue));
```

```
public void insert(int newValue) {
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