

# CSCI 132:

# Basic Data Structures and Algorithms

Searching (Binary Search)

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Spring 2025

# Searching

We store values in data structures, but we also need to retrieve/search for values!

Today, we will discuss techniques for how to search for a value in a data structure

(We will be using arrays, but these techniques could also be used on Linked Lists, queues, stacks, etc)



# Searching

## Option 1: Linear Search

Check every spot until one by one until we find what we are looking for

```
public int linear_search(int[] array, int s) {  
    for(int i = 0; i < array.length; i++) {  
        if(array[i] == s) {  
            return i;  
        }  
    }  
    return -1;  
}
```

# Searching

## Option 1: Linear Search

Check every spot until one by one until we find what we are looking for

Not efficient for large data structures.  **$O(n)$**  running time

```
public int linear_search(int[] array, int s) {  
    for(int i = 0; i < array.length; i++) {  
        if(array[i] == s) {  
            return i;  
        }  
    }  
    return -1;  
}
```

# Searching

Option 1: Linear Search

Check every spot until we find it

Not efficient for large datasets

**Can we do better?**

```
public int  
for(int i = 0; i < arr.length; i++)
```

```
}
```

```
}
```

0												12
1	2	9	10	11	15	18	21	27	31	41	43	50

What if our array is sorted?

Target Value: 27

0												12
1	2	9	10	11	15	18	21	27	31	41	43	50

We can leverage the fact that this array is sorted to make searching more efficient

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- 1. Start at the middle of the array



Target Value: 27

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1	2	9	10	11	15	18	21	27	31	41	43	50

1. Start at the middle of the array
2. Compare to target value:
  - If the value is the target value, return
  - If the target value is greater than the middle, discard the “left section” of the array
  - If the target value is less than the middle, discard the “right section” of the array

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1	2	9	10	11	15	18	21	27	31	41	43	50
↑												↑
low												high

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We will define two pointers, `low` and `high` that point to the possible bounds of the target value



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  - If the target value is less than the middle, discard the “right section” of the array (move the high pointer)

Because we know the array is sorted, and the target value is greater than our mid point, then we know the target value must be located somewhere to the right.



We can eliminate half of the array!!!

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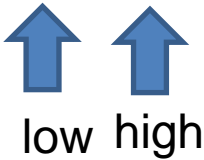
0							7	8				12
1	2	9	10	11	15	18	21	27	31	41	43	50

low high

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This algorithm is known as **Binary Search**

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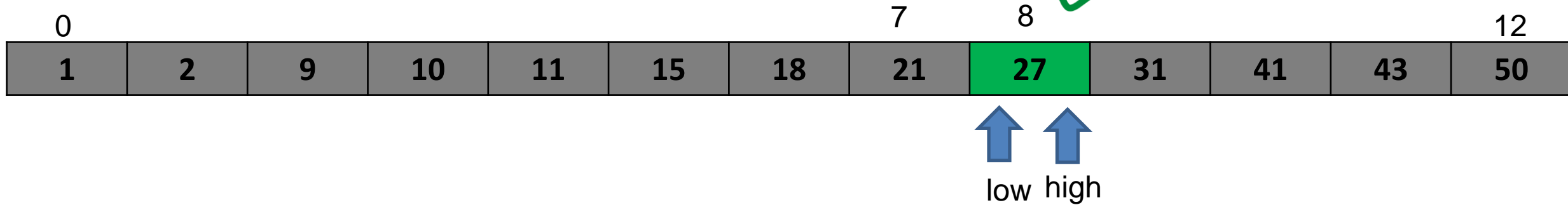
How to calculate the mid point?



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How to calculate the mid point?  $(low + high) / 2$

Target Value: 27

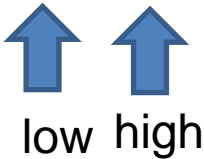
0							7	8				12
1	2	9	10	11	15	18	21	27	31	41	43	50

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How do we know when to stop looping?

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How do we know when to stop looping? If we find the target value, or if `low` and `high` cross each other (`low > high`)

Target Value: 27

0		7	8		12
1				43	50

LET'S CODE THIS

1. S

2. C

→ If

→ If

section

→ If

section

3. Rec

target value is found

How do we know when to stop looping? If we find the target value, or if `low` and `high` cross each other (`low > high`)

```
private static int binary_search(int[] array, int n) {  
    int low = 0;  
    int high = array.length - 1;  
    while(low <= high) {  
        int mid = (low + high) / 2;  
        if(n == array[mid]) {  
            return mid;  
        }  
        else if(n > array[mid]) {  
            low = mid + 1;  
        }  
        else {  
            high = mid - 1;  
        }  
    }  
    return -1;  
}
```

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```

## Running time?

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            low = mid + 1;  
        }  
        else {  
            high = mid - 1;  
        }  
    }  
    return -1;  
}
```

**Running time?** Each time we loop, we eliminate **half** the array

# Running time?

Initial length of array =  $n$

Iteration 1 - Length of array =  $n/2$



# Running time?

Initial length of array =  $n$

Iteration 1 - Length of array =  $n/2$

Iteration 2 - Length of array =  $(n/2)/2 = n/2^2$

# Running time?

Initial length of array =  $n$

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Iteration  $k$  - Length of array =  $n/2^k$

# Running time?

Initial length of array =  $n$

Iteration 1 - Length of array =  $n/2$

Iteration 2 - Length of array =  $(n/2)/2 = n/2^2$

Iteration  $k$  - Length of array =  $n/2^k$

After  $k$  iterations, eventually our array has been reduced to one element

Length of array =  $n/2^k = 1$

$$n = 2^k$$

*“Two to what power makes  $n$ ??”*

# Running time?

After  $k$  iterations, eventually our array has been reduced to one element

$$\text{Length of array} = n/2^k = 1$$

$$n = 2^k$$

*“Two to what power makes  $n$ ??”*

$$\log_2(n) = \log_2(2^k)$$

# Running time?

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$$\text{Length of array} = n/2^k = 1$$

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*“Two to what power makes  $n$ ??”*

$$\log_2(n) = \log_2(2^k)$$

$$\log_2(n) = k * \log_2 2 :$$

# Running time?

After k iterations, eventually our array has been reduced to one element

$$\text{Length of array} = n/2^k = 1$$

$$n = 2^k$$

*“Two to what power makes n??”*

$$\log_2(n) = \log_2(2^k)$$

$$\log_2(n) = k * \cancel{\log_2 2}$$

$$\log_2(n) = k$$

After K iterations, we will have done  $\log(n)$  divisions

```
private static int binary_search(int[] array, int n) {  
    int low = 0;  
    int high = array.length - 1;  
    while(low <= high) {  
        int mid = (low + high) / 2;  
        if(n == array[mid]) {  
            return mid;  
        }  
        else if(n > array[mid]) {  
            low = mid + 1;  
        }  
        else {  
            high = mid - 1;  
        }  
    }  
    return -1;  
}
```

## Running time?

Generally speaking, whenever we eliminate half of the problem each iteration, that will give us  **$O(\log n)$**  running time

```

private static int binary_search(int[] array, int n) {
    int low = 0; O(1)
    int high = array.length - 1; O(1)
    while(low <= high) { O(log n)
        int mid = (low + high) / 2; O(1)
        if(n == array[mid]) { O(1)
            return mid; O(1)
        }
        else if(n > array[mid]) { O(1)
            low = mid + 1; O(1)
        }
        else {
            high = mid - 1; O(1)
        }
    }
    return -1; O(1)
}

```

## Running time?

Generally speaking, whenever we eliminate half of the problem each iteration, that will give us **O(logn)** running time



```
private static int binary_search(int[] array, int n) {  
    int low = 0; O(1)  
    int high = array.length - 1; O(1)  
    while(low <= high) { O(log n)  
        int mid = (low + high) / 2; O(1)  
        if(n == array[mid]) { O(1)  
            return mid; O(1)  
        }  
        else if(n > array[mid]) { O(1)  
            low = mid + 1; O(1)  
        }  
        else {  
            high = mid - 1; O(1)  
        }  
    }  
    return -1; O(1)  
}
```

**Running time?**     $O(\log n)$

```

private static int binary_search(int[] array, int n) {
    int low = 0;
    int high = array.length - 1;
    while(low <= high) {
        int mid = (low + high) / 2;

        int result = x.compareTo(array[mid])

        if(result == 0) {
            return mid;
        }
        else if(result > 0){
            low = mid + 1;
        }
        else {
            high = mid - 1;
        }
    }
    return -1;
}

```

We can do binary search  
on an array of Strings  
using the `compareTo()`  
method

```

private static int binary_search(???????????) {

    if(low <= high) {
        int mid = (low + high) / 2;
        if(n == array[mid]) {
            return mid;
        }
        else if(n > array[mid]) {
            return binary_search(??????????);
        }
        else {
            return binary_search(??????????);
        }
    }
    else {
        return -1;
    }
}

```

Binary Search can also be implemented using recursion