Computational Photography

- Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.

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Image Processing and Filtering, via Convolution and Cross-Correlation

- Point-process and Neighboring Pixels Computations on an Image using Cross-Correlation and Convolution

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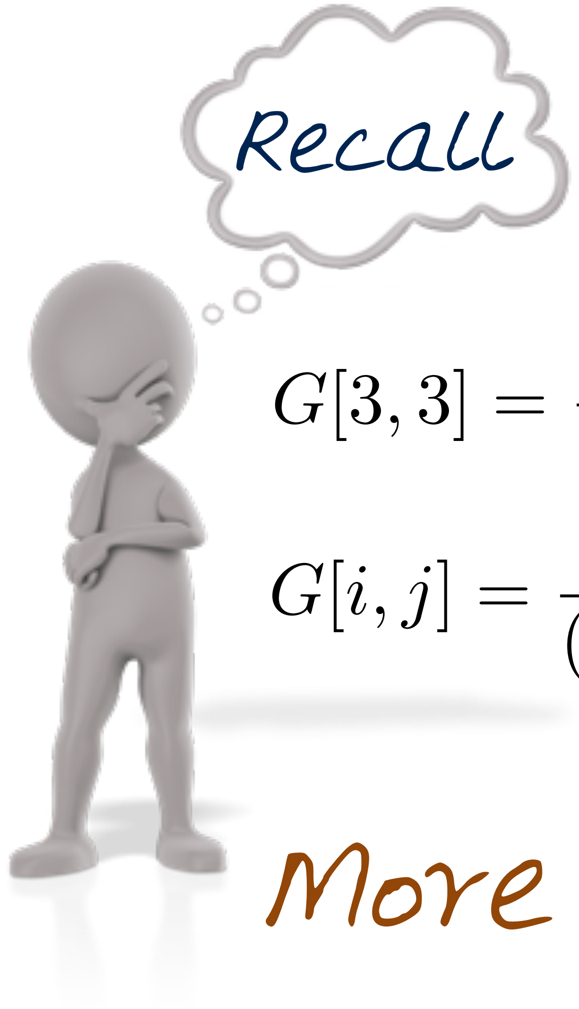
1. Cross-Correlation
2. Convolution
3. Difference between Cross-Correlation and Convolution
4. Properties of these methods!



Recall A Mathematical Representation for Smoothing

$$G[3,3] = \frac{1}{9}(A + B + C + D + E + F + G + H + I)$$

$$G[i,j] = \frac{1}{(2k+1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v]$$

a	b	c
d	e	f
g	h	i

20	20	10	20	10	20	10	10	13
30	0	0	0	0	0	0	0	30
20	0	A	B	C	90	90	0	20
20	0	D	E	F	90	90	0	20
10	0	G	H	I	90	90	0	10
10	0	90	90	90	90	90	0	10
10	0	90	90	90	90	90	0	10
20	0	0	0	0	0	0	0	20
20	20	10	20	10	20	10	10	13

More Generally,

$$G[3,3] = a * A + b * B + c * C + d * D + e * E + f * F + g * G + h * H + i * I$$

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v] F[i+u, j+v]$$

Referred to as Cross-correlation, which we will cover later

Cross-Correlation Method

- ★ In signal processing, cross-correlation is a measure of similarity of two waveforms as a function of a time-lag applied to one of them.
- ★ Also known as a sliding dot product or sliding inner-product.

Cross-Correlation Method

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v] F[i+u, j+v]$$

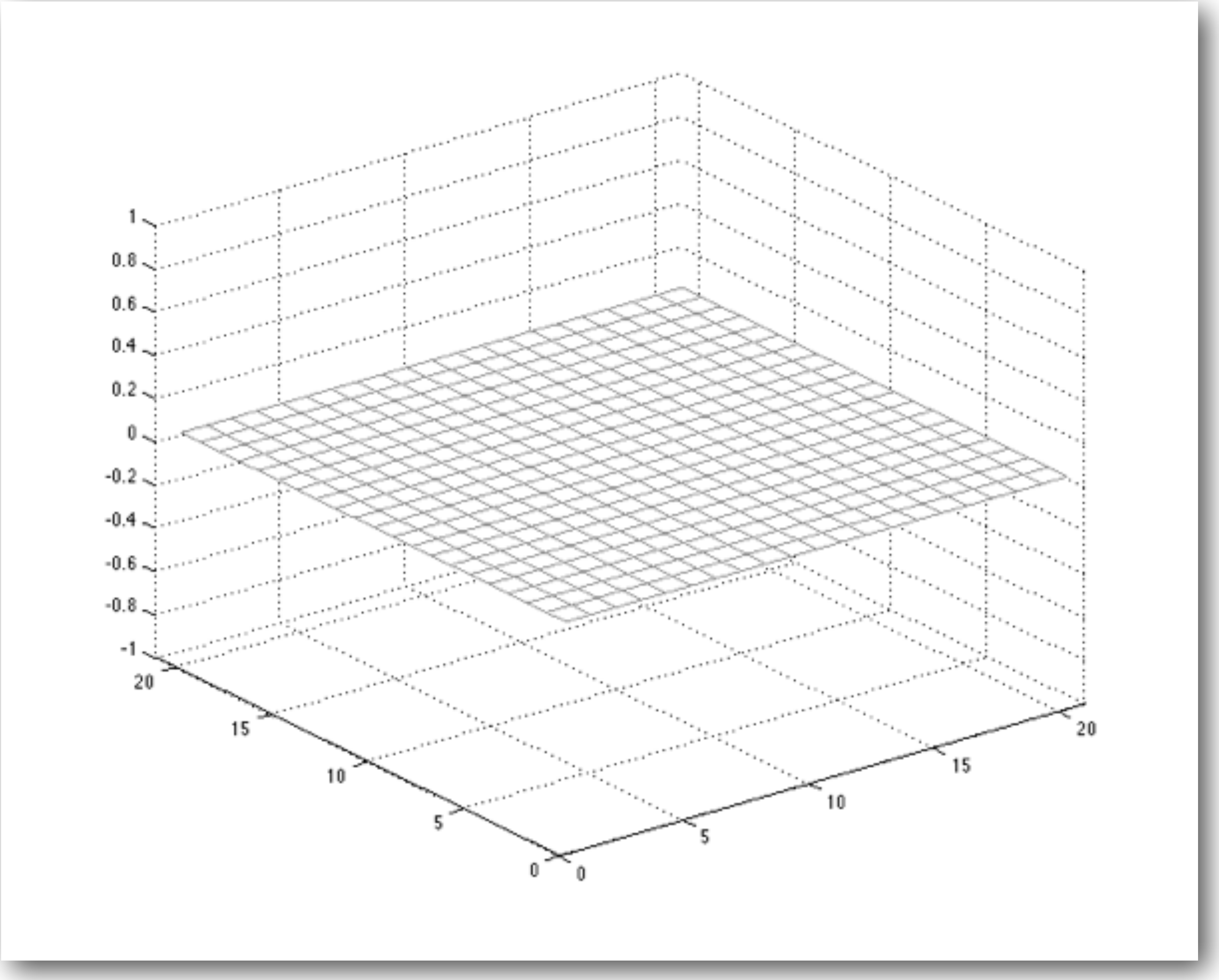
Denoted by $G = h \otimes F$

- Filtering an image:
  - Replace each pixel with a linear combination of its neighbors
- Filter "kernel" or "mask"
  - h[u,v] is the prescription for weights in the linear combination

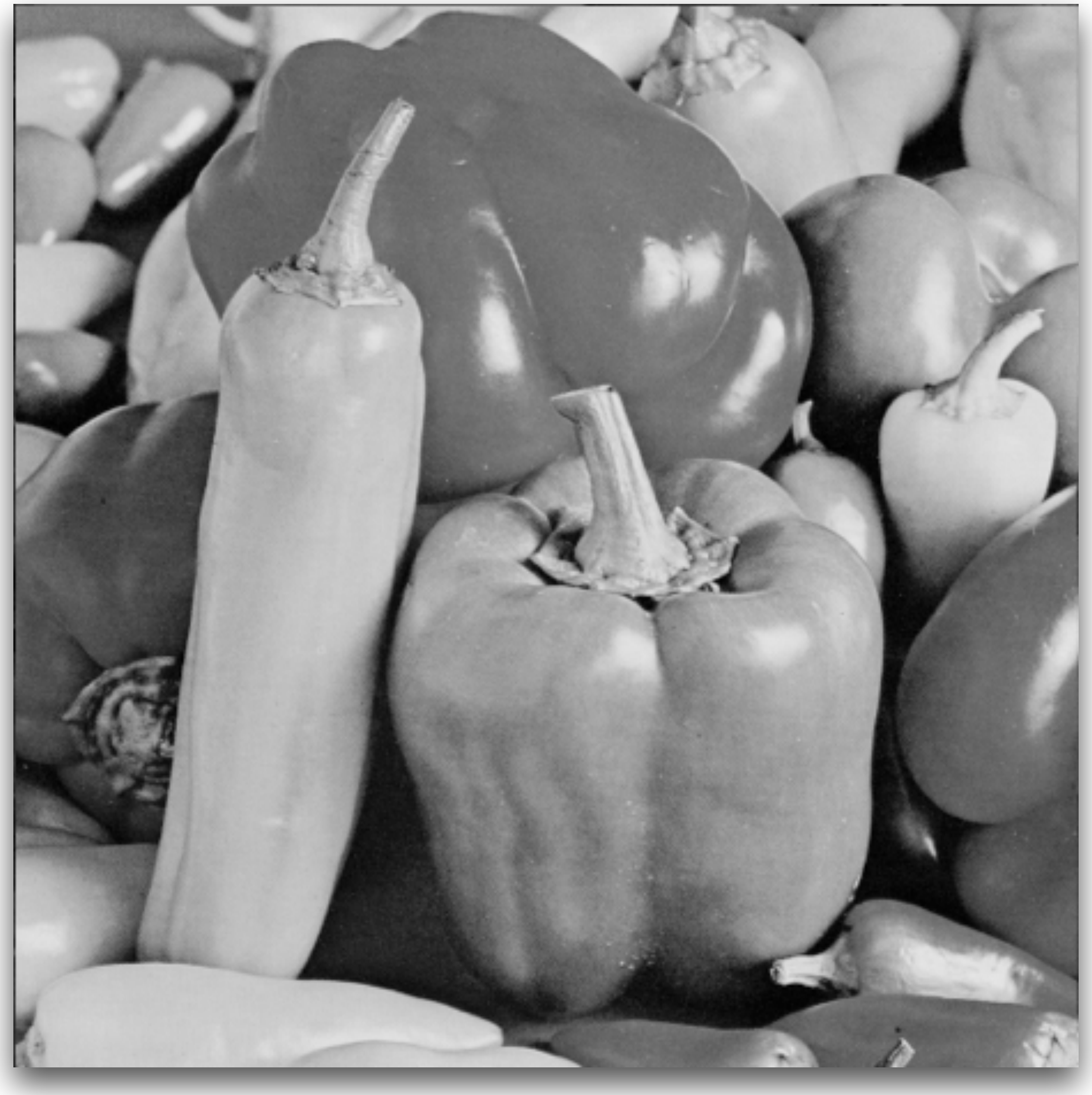
Example: Box Filter

Box/Average Filter

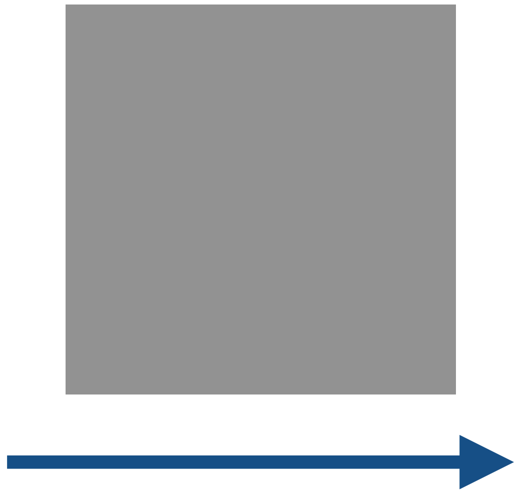
- Size: 21x21
- Values: Uniform



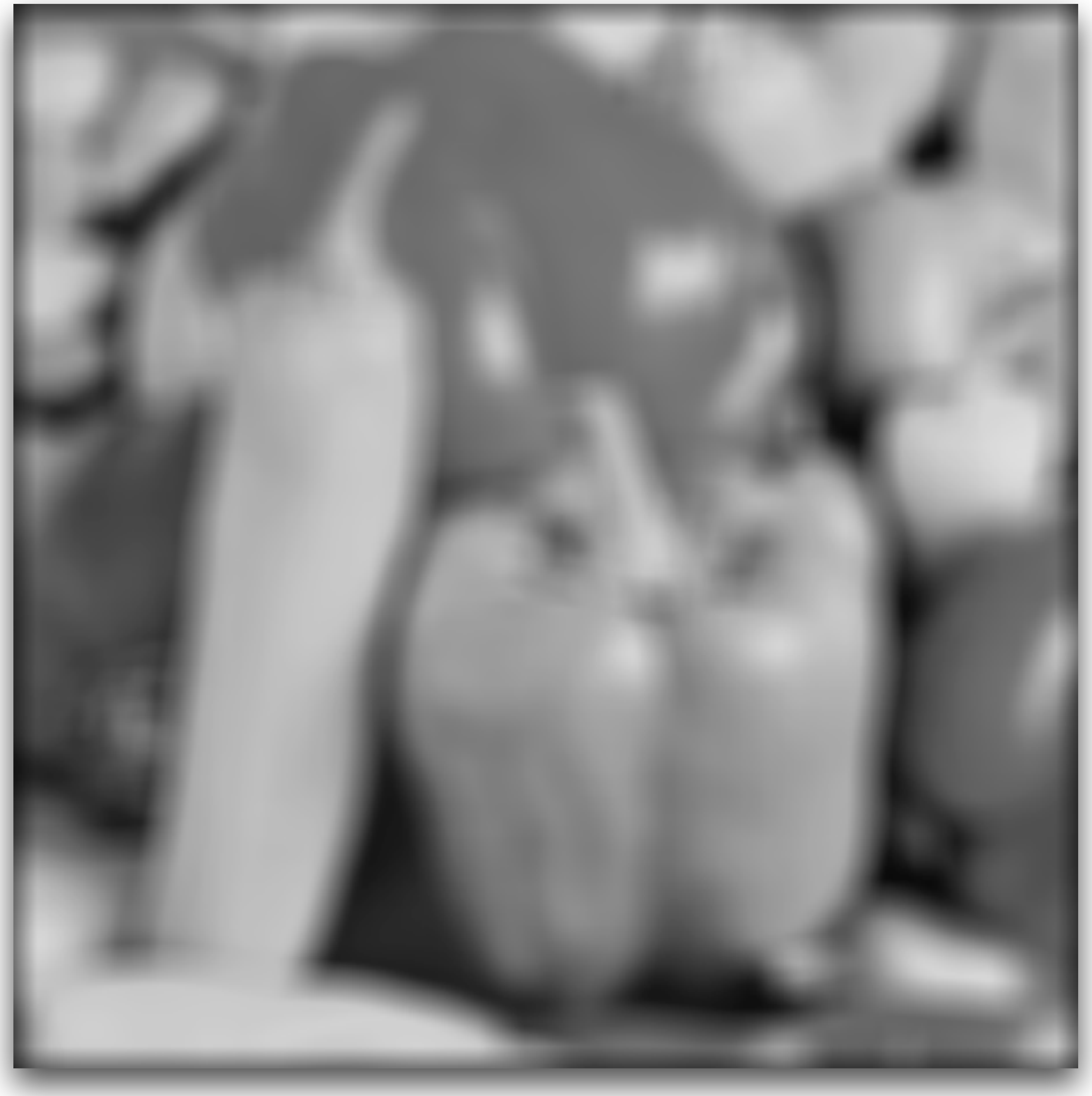
Example: Box Filter



Original



Filter

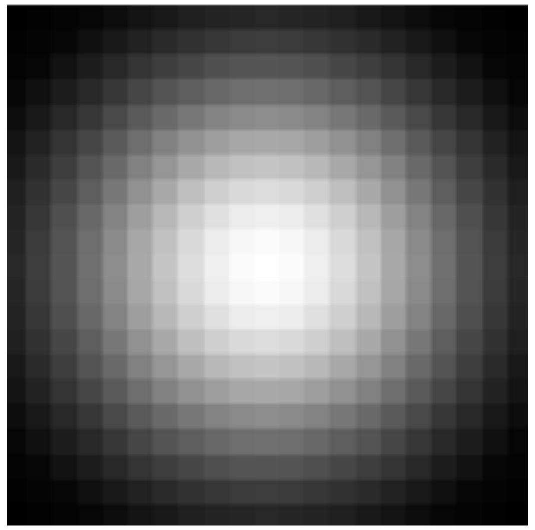


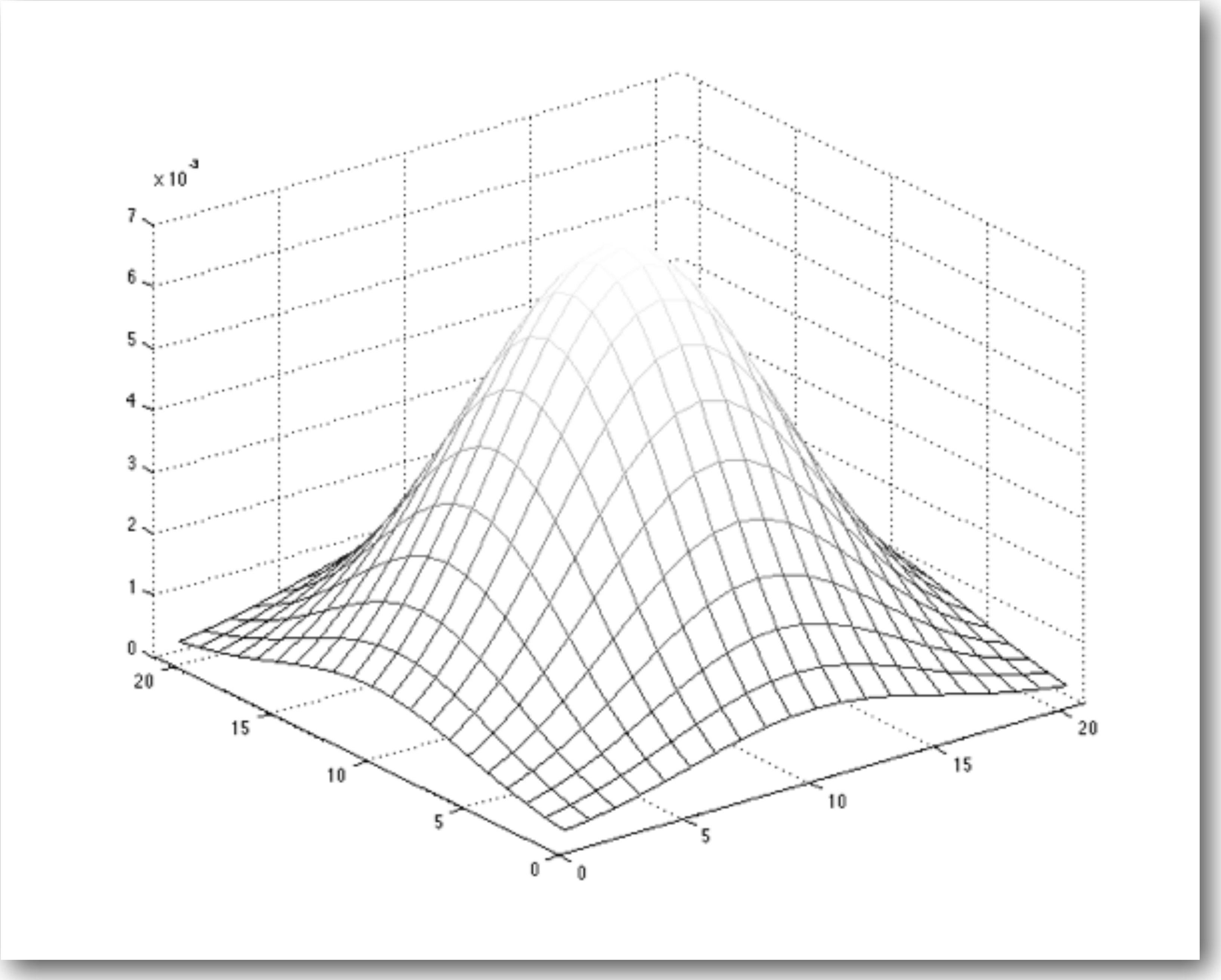
Result

Example: Gaussian Filter

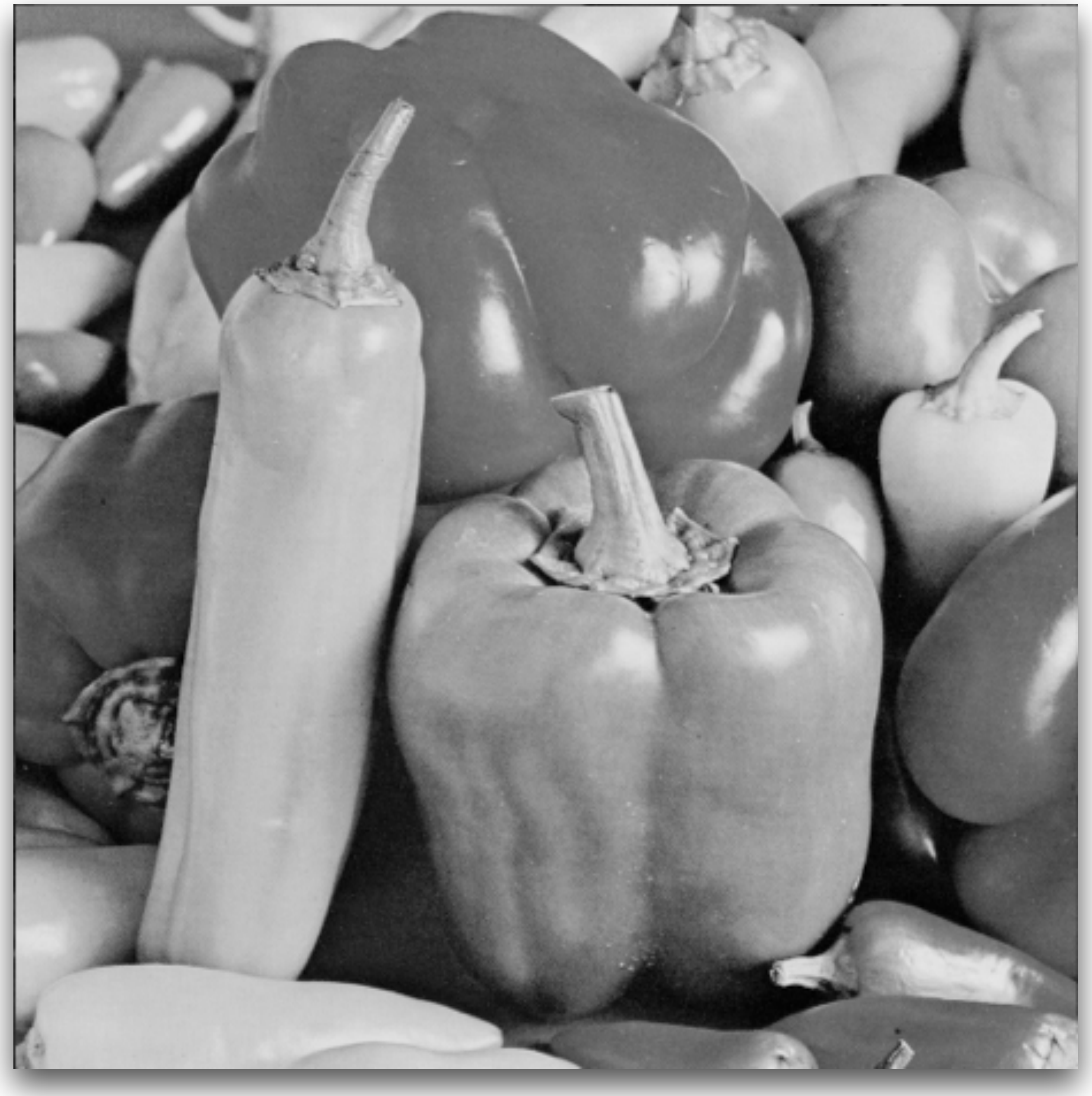
Gaussian Filter

- Size: 21x21
- Values: Gaussian or Normal distribution

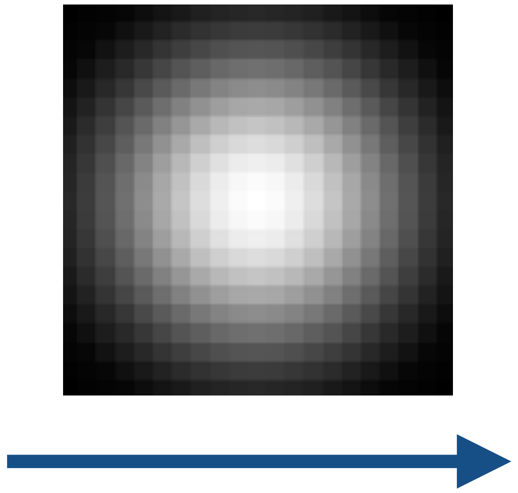




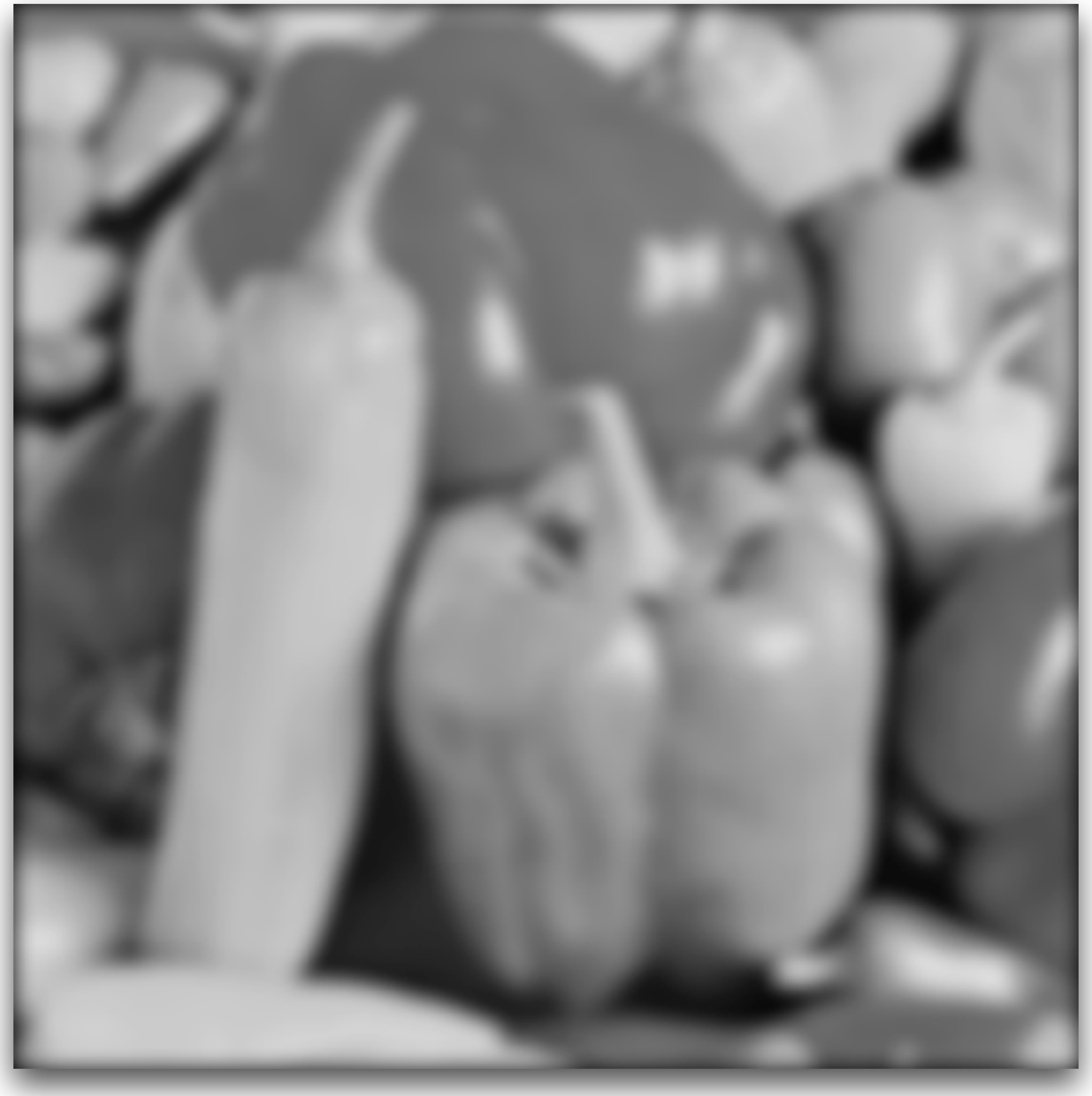
Example: Gaussian Filter



Original

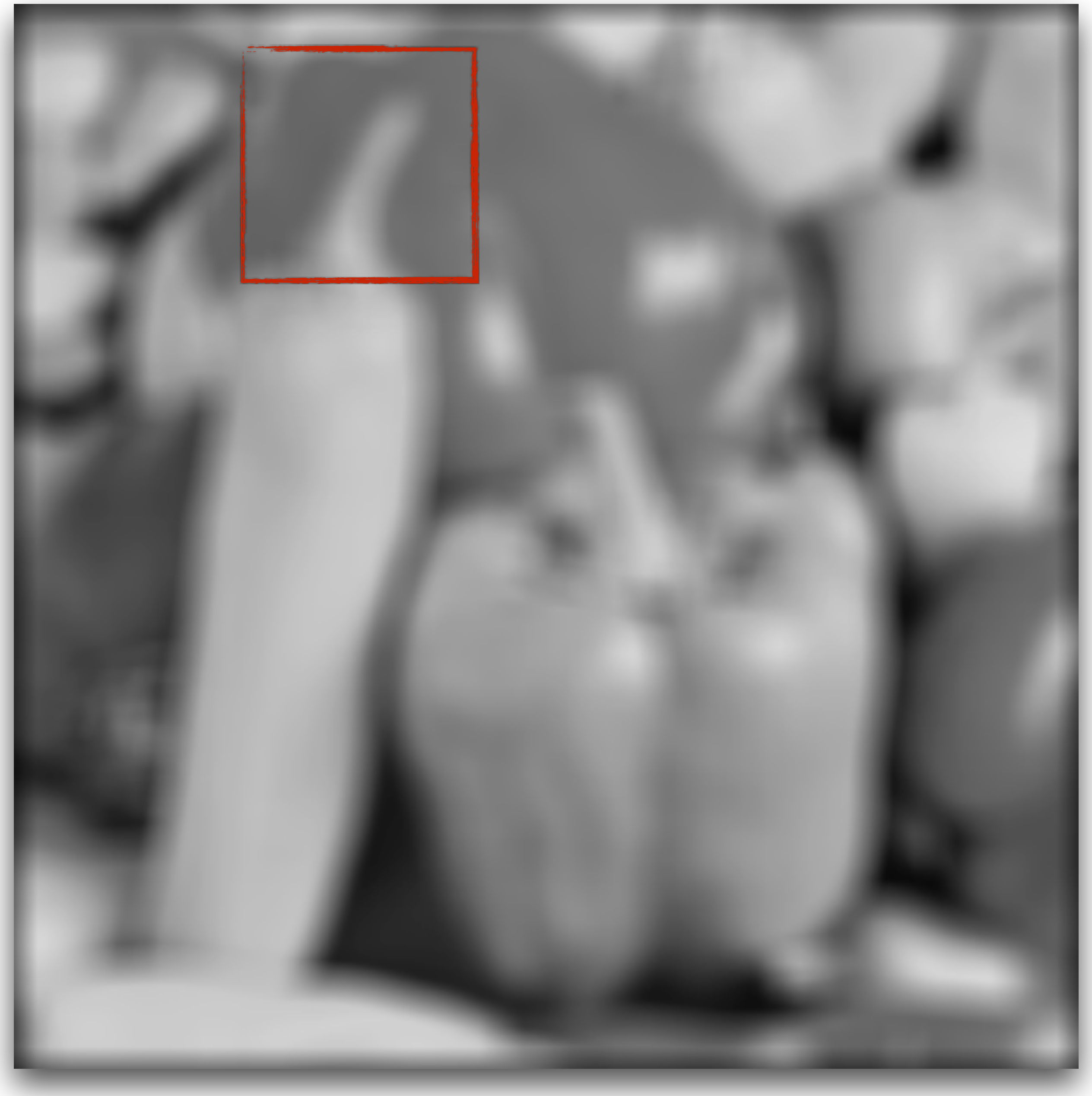


Filter

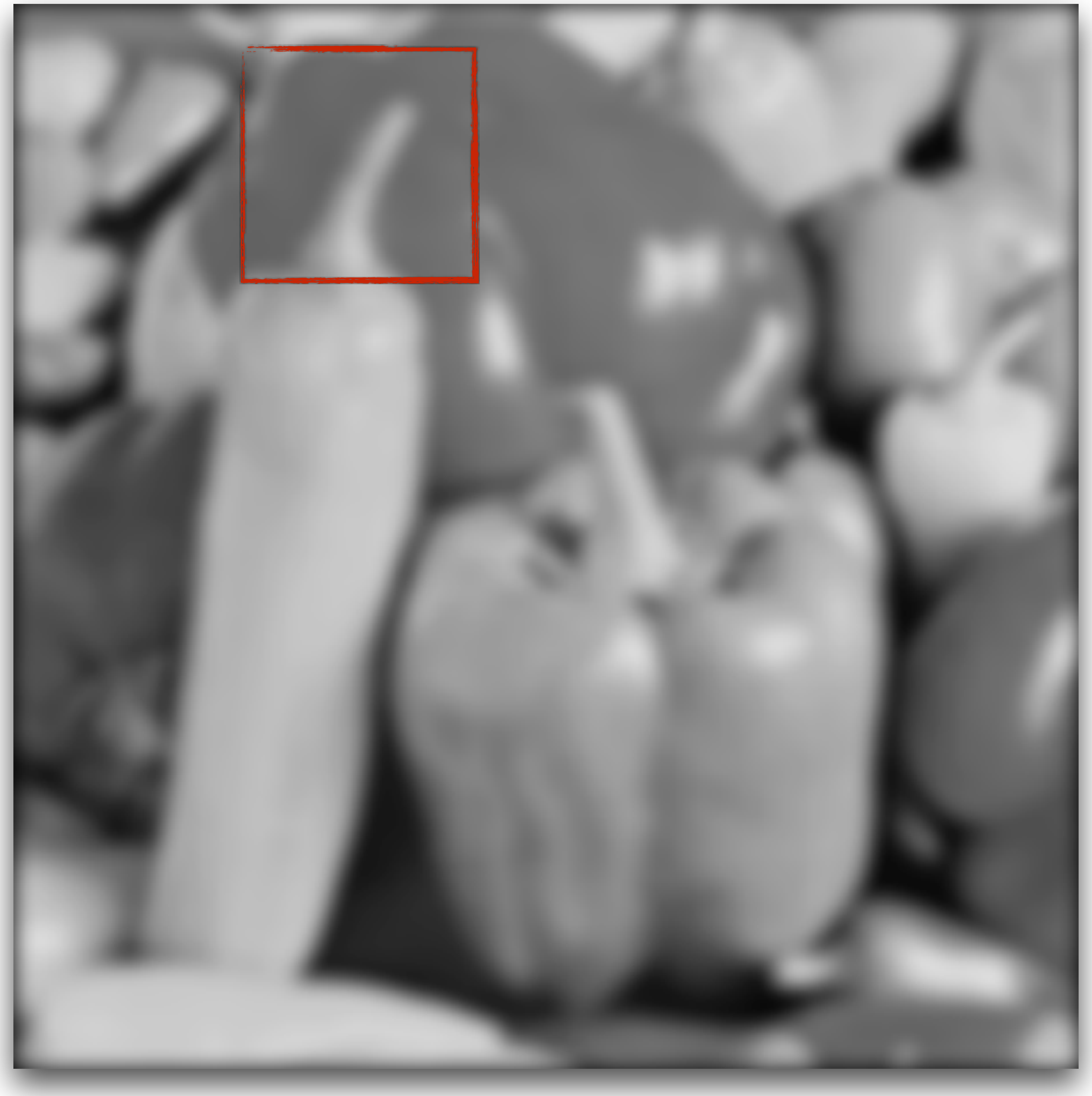


Result

Compare: Average vs. Gaussian

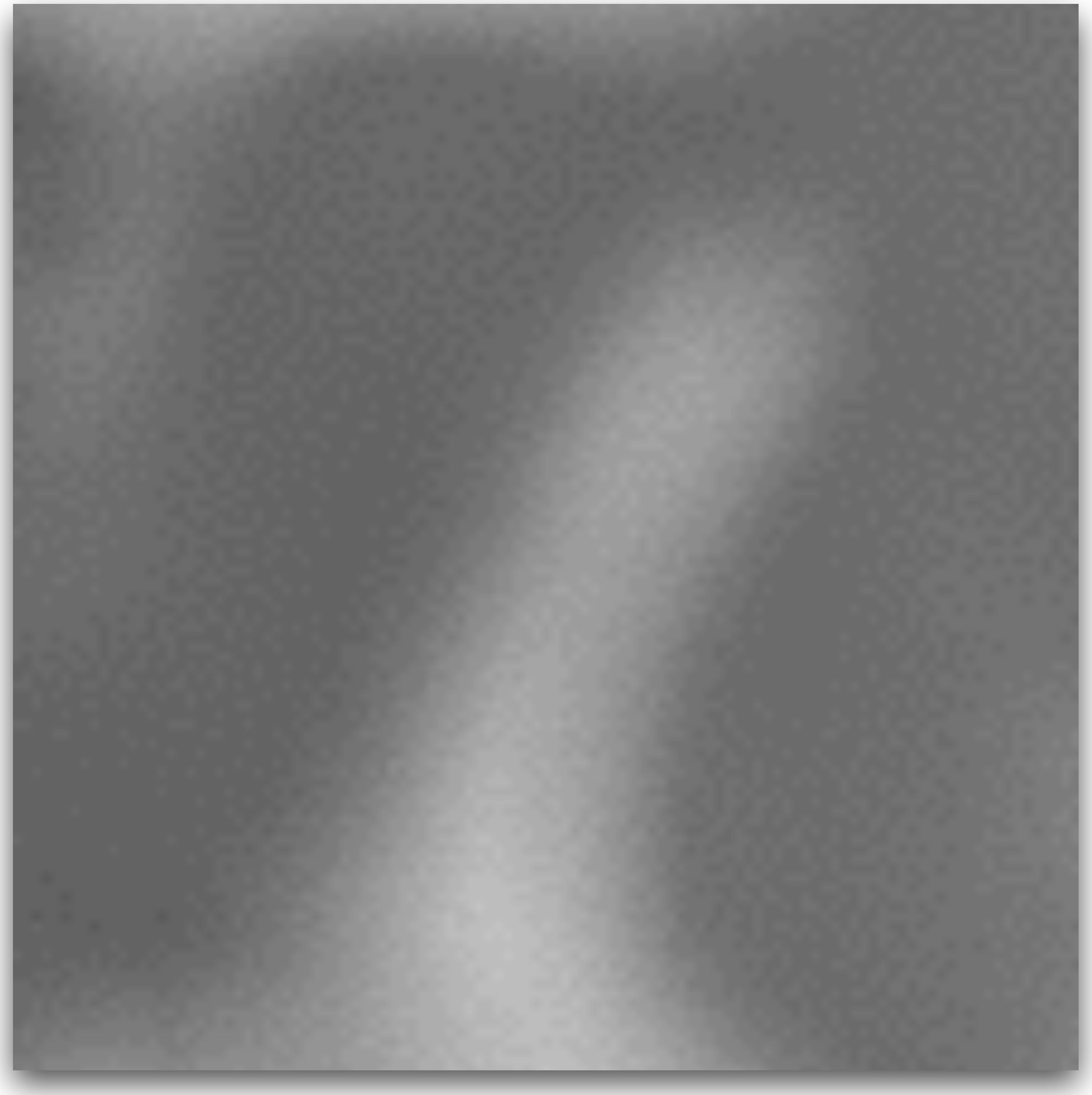


Average filter result

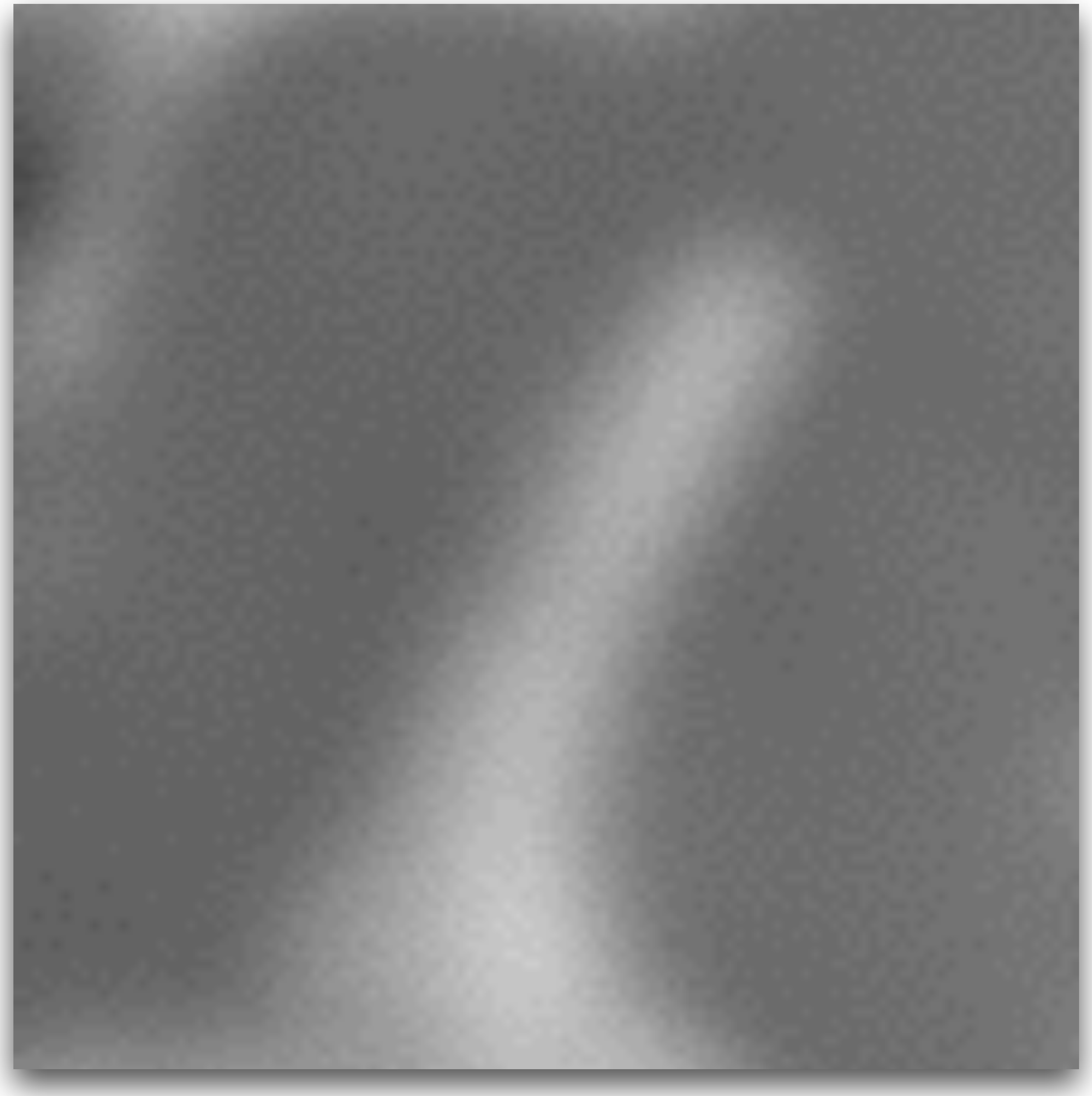


Gaussian filter result

Compare: Average vs. Gaussian



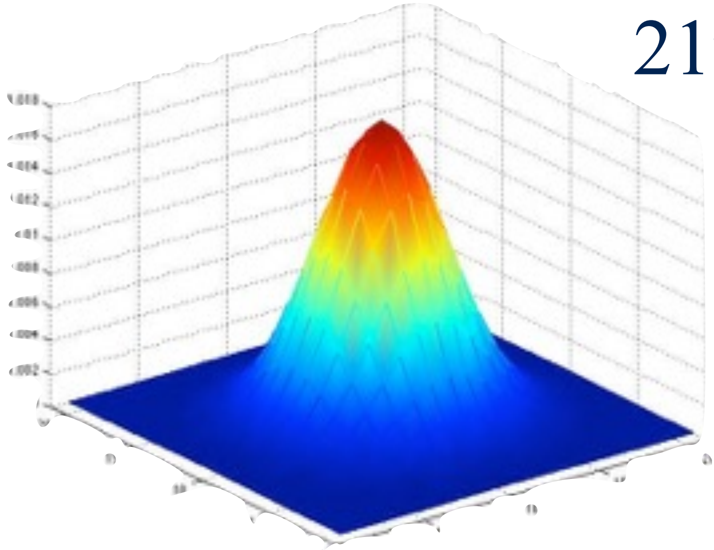
Average filter result



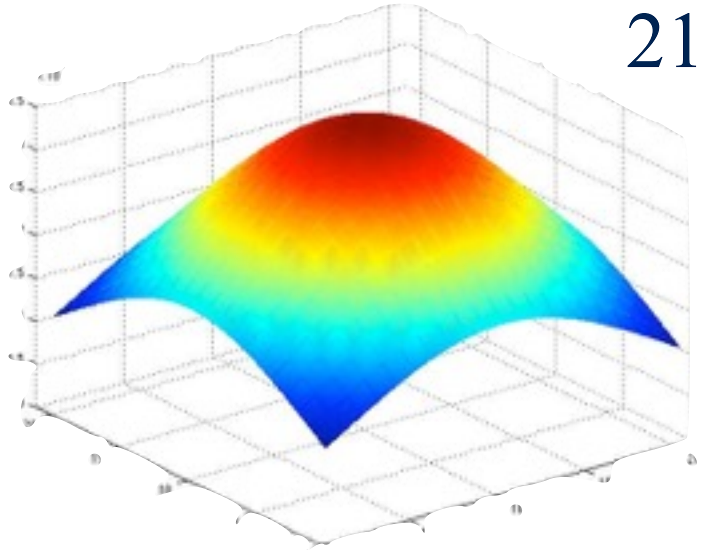
Gaussian filter result

Using Gaussian Filters?

- Square kernels are NOT smooth
- Average filter not eq. to a defocussed lens
  - A single point of light viewed in a defocussed lens looks like a fuzzy blob; the averaging process is square.
- Gaussian function in 2D, with σ as the Variance, centered at (0,0):



21x21, σ=3



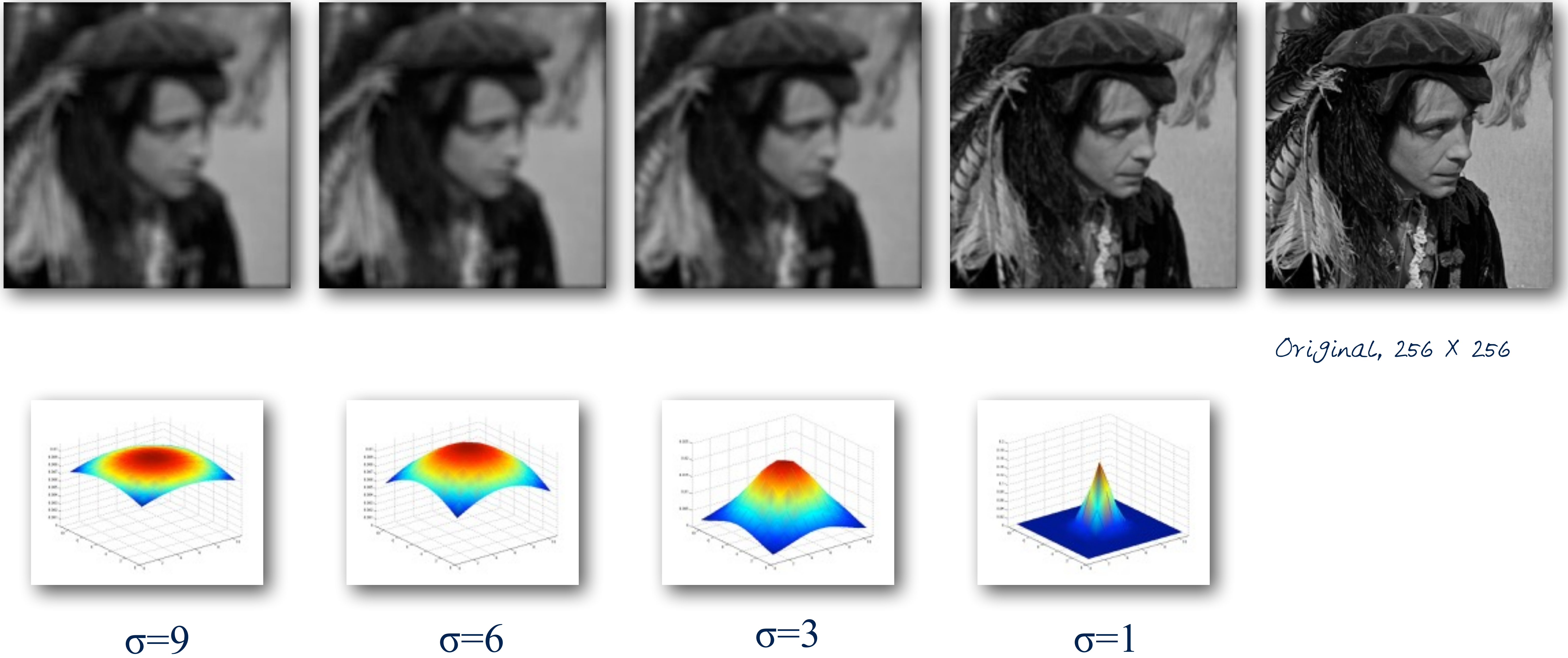
21X21, σ=9

$$h(u,v) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{(u^2+v^2)}{2\sigma^2}}$$

Slide Adapted from Aaron Bobick and David Forsyth

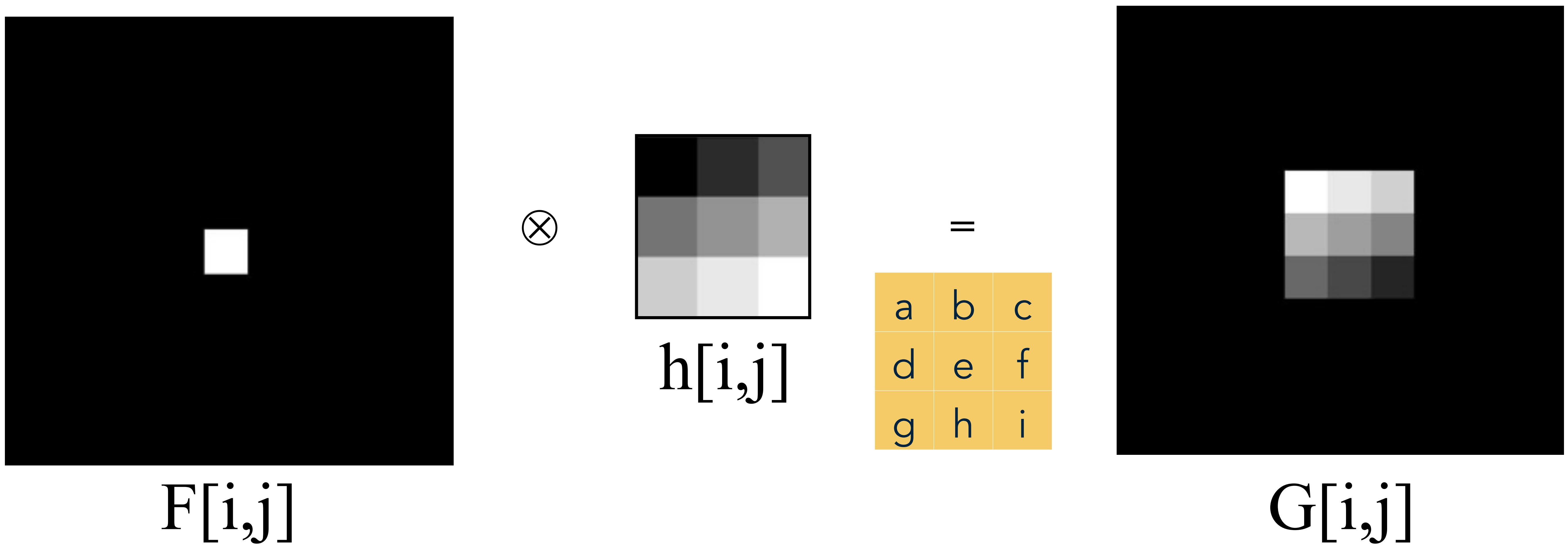
For more information on Gaussian see http://en.wikipedia.org/wiki/Gaussian_function

Using Gaussian Filters for Smoothing



σ determines extent of smoothing

Filtering by a Kernel (defining Convolution)



a	b	c
d	e	f
g	h	i

- ✳ 'Filter' means to slide the kernel over the image
- ✳ Results in a reversed response

Slide Adapted from Aaron Bobick

Convolution Method

- Convolution is a mathematical operation on two functions **F** and **h**,
- Produces a third function that is typically viewed as a modified version
- Gives the area of overlap between the two functions
- In a form of the amount that one of the original functions is translated.

Convolution Method

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v] F[i-u, j-v]$$

Denoted by $G = h * F$

- Flip filter in both dimensions:
  - Bottom to top
  - Right to left
- Then apply cross-correlation

a	b	c
d	e	f
g	h	i

g	h	i
d	e	f
a	b	c

i	h	g
f	e	d
c	b	a

Convolution vs. Cross-Correlation

Cross-Correlation:

$$G = h \otimes F$$

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v]F[i+u,j+v]$$

a	b	c
d	e	f
g	h	i

g	h	i
d	e	f
a	b	c

i	h	g
f	e	d
c	b	a

Convolution:

$$G = h * F$$

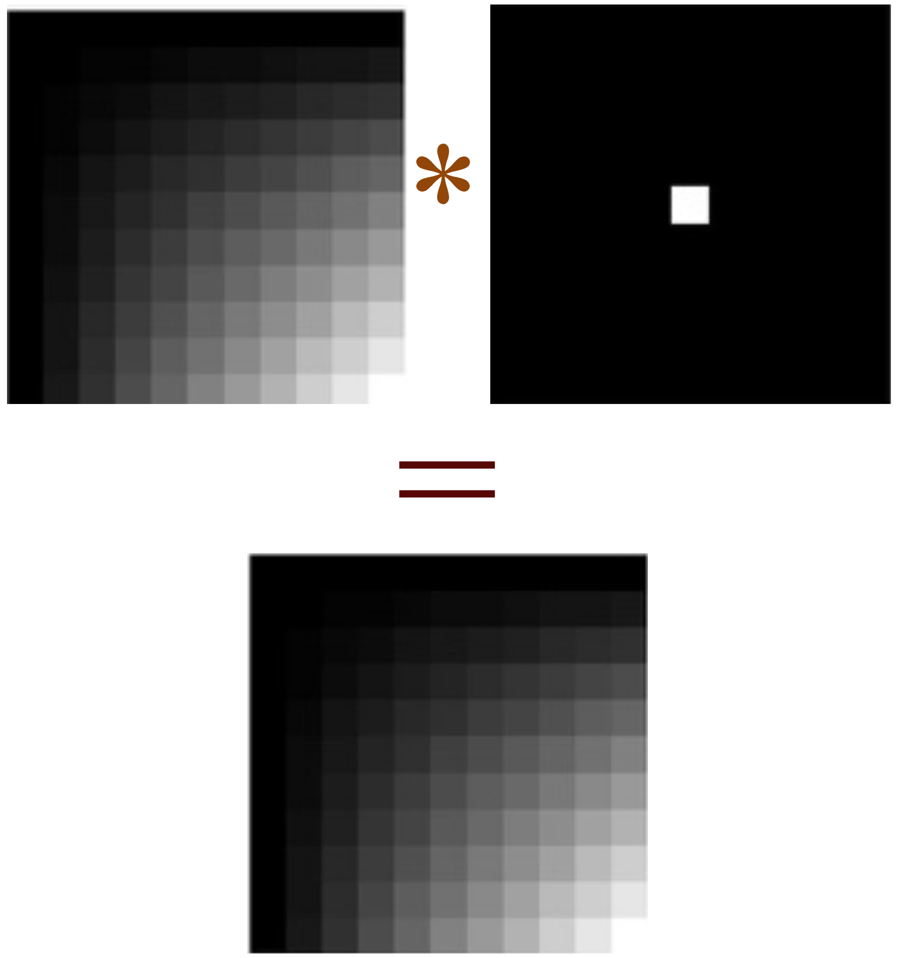
$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v]F[i-u,j-v]$$

Properties of Convolution

- Linear and Shift Invariants:
  - Behaves the same everywhere (i.e., the value of the output depends on the pattern in the image neighborhood, not the position of the neighborhood).
- Commutative: $F * G = G * F$
- Associative: $(F * G) * H = F * (G * H)$

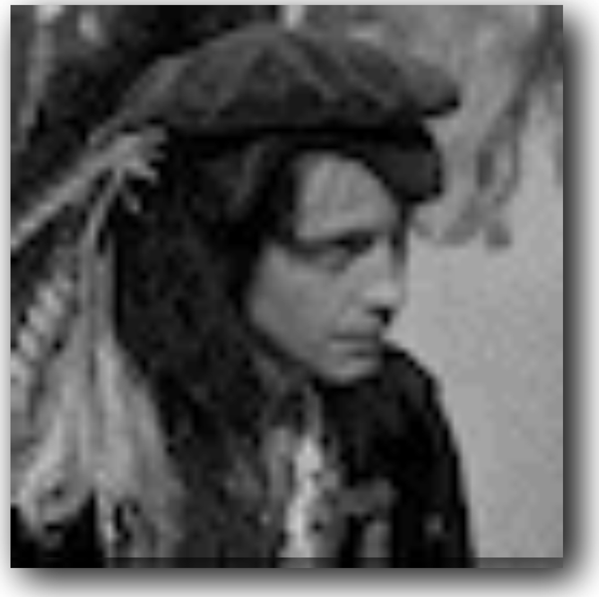
Properties of Convolution

- Identity: Unit Impulse
  - E = [....0,0,1,0,0....],
  - F * E = F
  - True of Cross-Correlation?
- Separable:
  - If the filter is separable, convolve all rows, then convolve all columns.

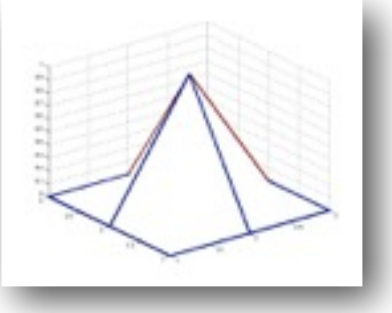


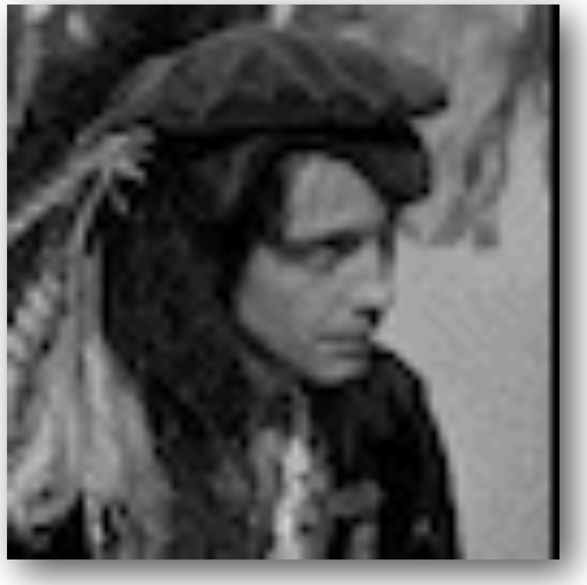
Linear Filters

original, 64 X 64

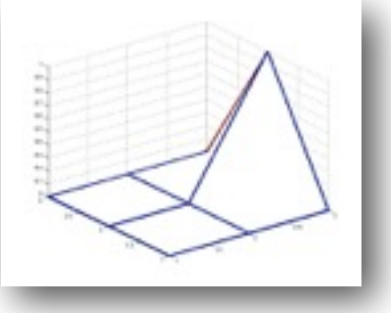


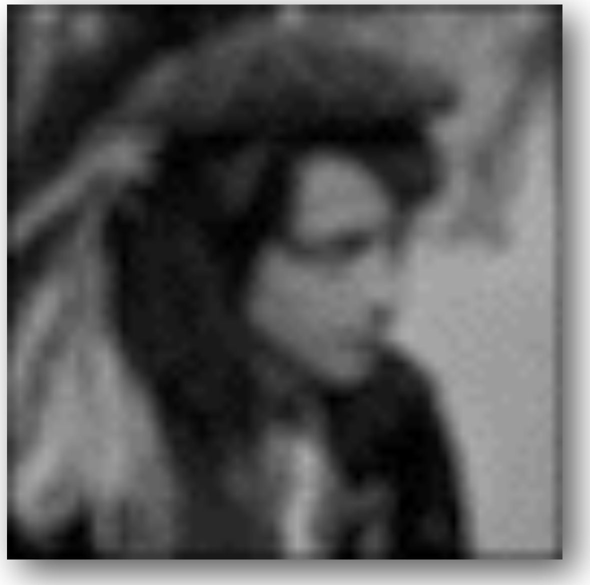
0	0	0
0	1	0
0	0	0



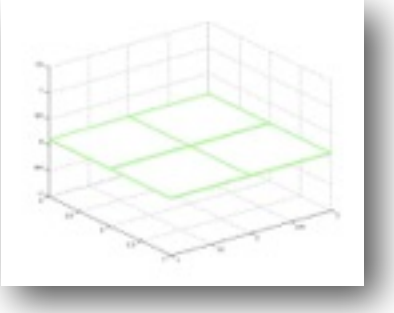


0	0	0
0	0	1
0	0	0



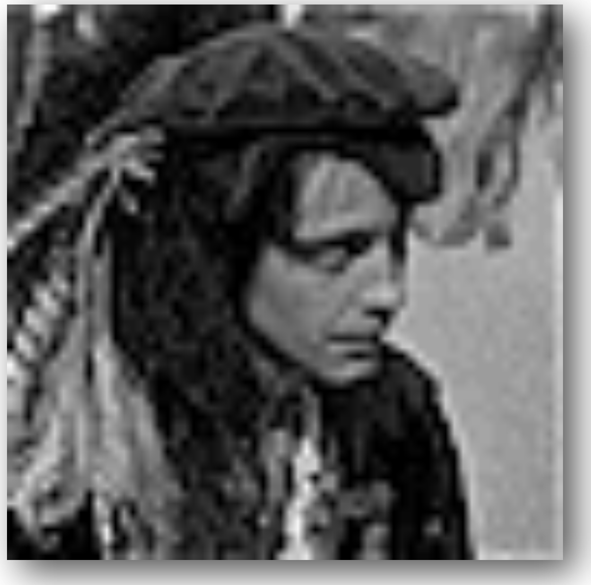


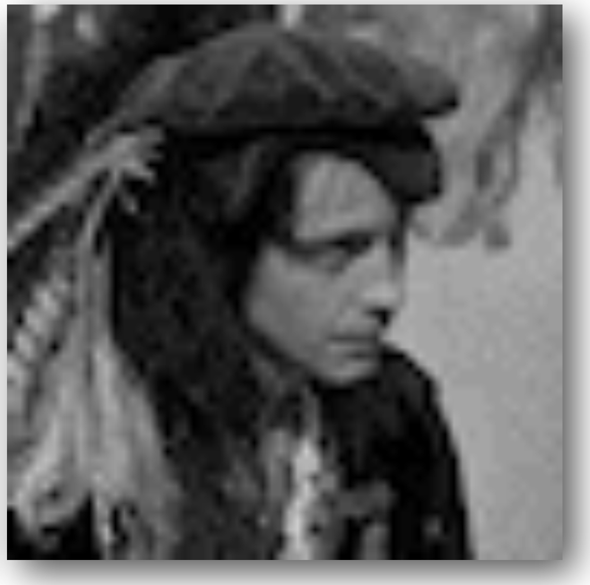
1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9



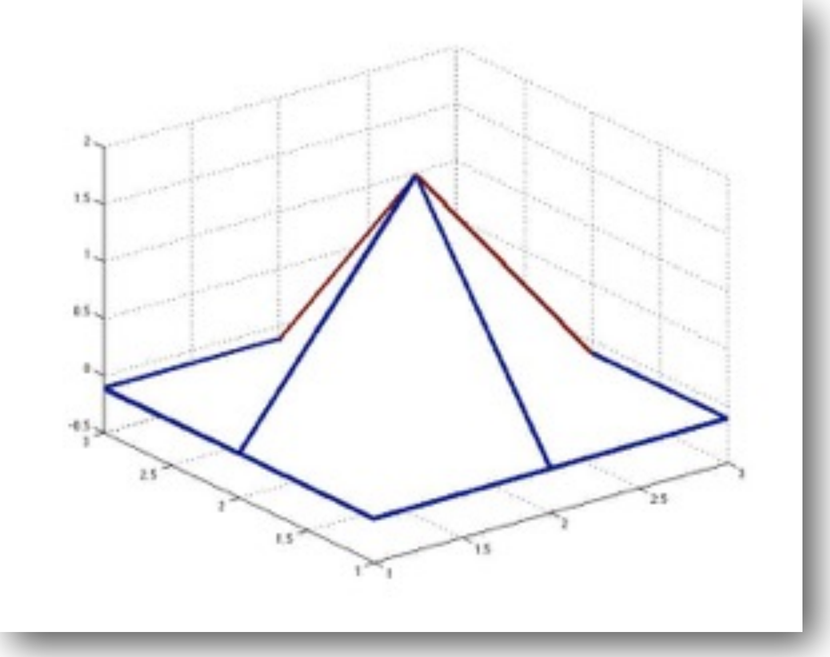
Linear Filters

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times 2 - \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix} =$$





-0.11	-0.11	-0.11
-0.11	1.9	-0.11
-0.11	-0.11	-0.11



original,
64x64

Summary

- Cross-Correlation
- Convolution
- Differences between the Cross-Correlation and Convolution methods for filtering images.
- Properties of the Convolution method for filtering images.

Next Class

Image Analysis:
Edge Detection

Credits

- Matlab software by Mathworks Inc.
- Some Slides adapted from Aaron Bobick, Steve Seitz, Steve Marschner & David Forsyth.
- Images used from USC's Signal and Image Processing Institute's Image Database

Computational Photography

- Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.

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