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Mathematical Modeling and Dynamic Simulation of DC Motors using MATLAB/Simulink Environment

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Abstract— This paper focuses on the modeling and dynamic simulation of different types of DC motors using MATLAB and its toolbox Simulink. The current, Voltage, back emf, speed and torque equations which are in the form of algebraic and differential equations of Permanent Magnet DC motor, DC shunt motor and DC series motor are used to build the mathematical model of the respective motor and implemented in MATLAB/Simulink and mathematical equations of DC compound motor are also derived. With the aid of the developed model, the Steady and transient-state characteristics of speed and torque in addition to voltages and currents of different types of DC motor can be effectively examined and analyzed.

Keywords – Modeling, Dynamic simulation, PMDC motor, DC shunt Motor, and DC series Motor.

I. INTRODUCTION

The aim of mathematical modeling is an essential step in the analysis and design of physical systems. In this paper, the mathematical models of DC motors are obtained by applying the fundamental physical laws governing the nature of the components making these models. A electric motor is being an electromechanical system, Newton's laws are used in the mathematical modeling of mechanical systems. Similarly, Kirchhoff's laws are used in the modeling and analysis of electrical systems. Our mathematical treatment is limited to linear, time-invariant ordinary differential equations whose coefficients do not change in time. In real life many systems are nonlinear, but they can be linearized around certain operating ranges about their equilibrium conditions. In general, the mathematical model of a system is one which encompasses of algebraic and differential equations describing the steady state and dynamic behavior of the system.

DC motors are increasingly being used. Since there is a linear relationship between the input voltage and rotation in DC motors, it is possible to control precisely the speed by computer, microprocessors or even analog circuits. The DC motor is one of the first machines invented to convert electrical power into mechanical power motor [2 – 4]. Its origin, according

to [3], can be traced to disc-type machines conceived and tested by Michael Faraday, who formulated the fundamental concepts of electromagnetism. The DC motor uses electricity and a magnetic field to produce torque, which causes it to turn. It requires two magnets of opposite polarity and an electric coil, which acts as an electromagnet. The repellent and attractive electromagnetic forces of the magnets provide the torque that causes the motor to turn. It also consists of one set of coils, called armature winding, inside a set of permanent magnets, called the stator. Applying a voltage to the coils produces a torque in the armature, resulting in motion. Basically, there are three types of DC motors. The two commonly criteria usually use in classifying them are their characteristics and the connection of their exciting windings or circuits. Based on these criteria, the three common types are: shunt, series and compound motors. The models developed in this paper will be effective for monitoring and analyzing the performance of PMDC motor, DC shunt motor and series motor.

II. NOMENCLATURE

V_{dc}	Supply Voltage(V)
R_a	Armature winding Resistance(Ohm)
L_{AA}	Armature winding inductance(Henry)
L_{AF}	Armature field winding inductance(Henry)
J	Moment of Inertia (Kg-m^2)
R_f	Field winding Resistance(Ohm)
W_r	Angular velocity(rad/sec)
I_a	Armature current(A)
T_L	Load Torque(N-m)
T_e	Torque developed by armature(N-m)
N	Speed(rpm)
L_{FF}	Field inductance(Henry)
B	Viscous coefficient(N-m(rad/sec))
E_b	Back Emf(V)
T_e	Torque(N-m)
I_f	Field current(A)
R_{se}	Series field winding Resistance(Ohm)
L_{se}	Series field winding Inductance(Henry)
I_{se}	Series filed winding Current(A)
I_{sh}	Shunt field winding Current(A)
R_{sh}	Shunt field winding Resistance(Ohm)
L_{sh}	Shunt field winding Inductance(Henry)
V_a	Armature Voltage(V)
R_L	Load Resistance(Ohm)

III. Mathematical Model of Permanent Magnet Direct current Motor (PMDC)

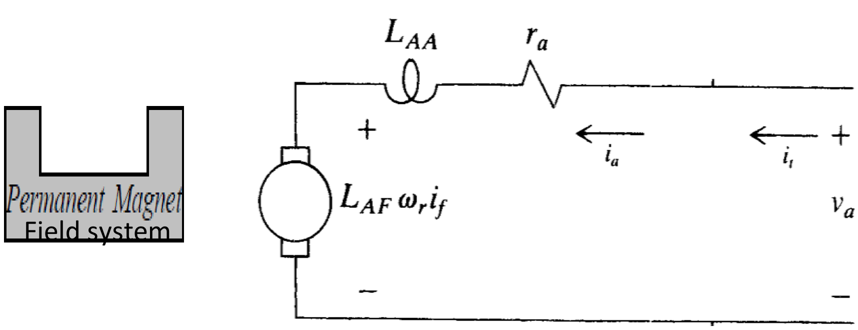

Figure 1. Equivalent circuit of PMDC Motor

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For a PMDC Motor, Field voltage equation is eliminated and $L_{AF} i_f$ is replaced by constant k_v which is given by the manufacturer.

This motor consists of two first order differential equation and two algebraic equations

$$v_a = i_a r_a + L_{AA} \frac{di_a}{dt} + k_v \omega_r \text{ ------------------------------ } (1)$$

$$L_{AA} \frac{di_a}{dt} = v_a - i_a r_a - k_v \omega_r$$

Armature current equation

$$, i_a = \int \frac{v_a - i_a r_a - k_v \omega_r}{L_{AA}} \, dt \text{ ---- } \quad (2)$$

Electromagnetic torque, $T_e = T_l + j \frac{d\omega_r}{dt} + B\omega_r$

$$j \frac{d\omega_r}{dt} = T_e - T_l - B\omega_r$$

Speed equation, $\omega_r = \int \frac{T_e - T_l - B\omega_r}{j} \, dt$ ---------------- (3)

$$T_e = k_v i_a \text{ -- } (4)$$

Back emf , $e = k_v \omega_r$ -------------------------------------- (5)

Motor Parameters

$r_a = 7\Omega$; $L_{AA} = 120mH$; $J = 1.06 \times 10^{-6} kg - m^2$; $B_m = 6.04 \times 10^{-6}\ N - m/(rad/sec)$
$k_v = 1.41 \times 10^{-2}\ v/(rad/sec)$; $No\ load\ armature\ current\ i_{a(no\ load)} = 0.15\ A$
$No\ load\ speed\ , \omega_{r(no\ load)} = 351.6(rad/sec)\ V_{DC} = 6\ V$
$Load\ torque, T_L = 0.00353\ N - m$

The mathematical equations (2), (3), (4) and (5) can be implemented in Simulink library , we can analysis the performance of the motor under various conditions .

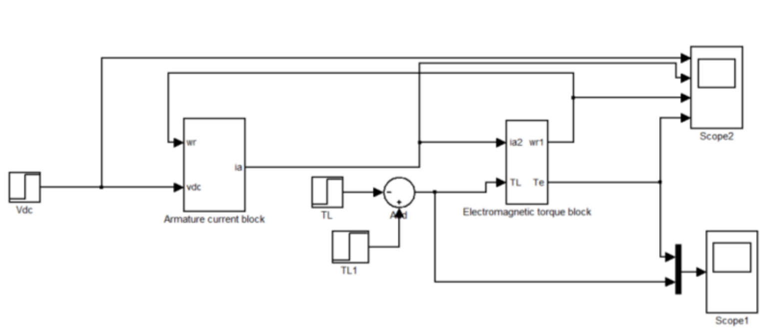

Figure 2. Simulink model of PMDC motor

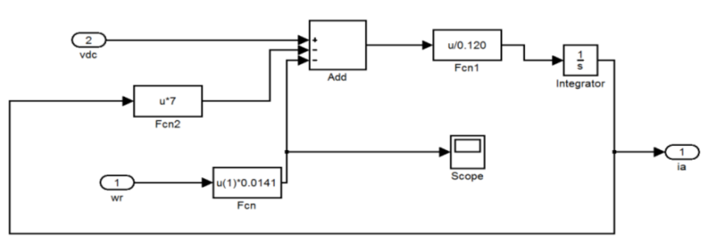

Figure 3. Solving armature current equation by Simulink blocks

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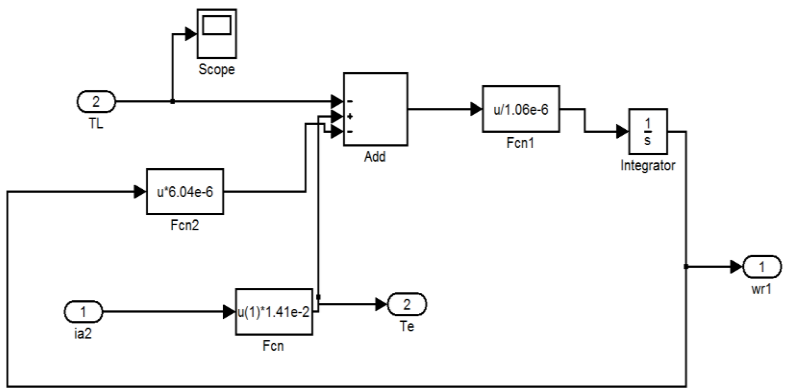

Figure 4. Solving speed equation by Simulink blocks

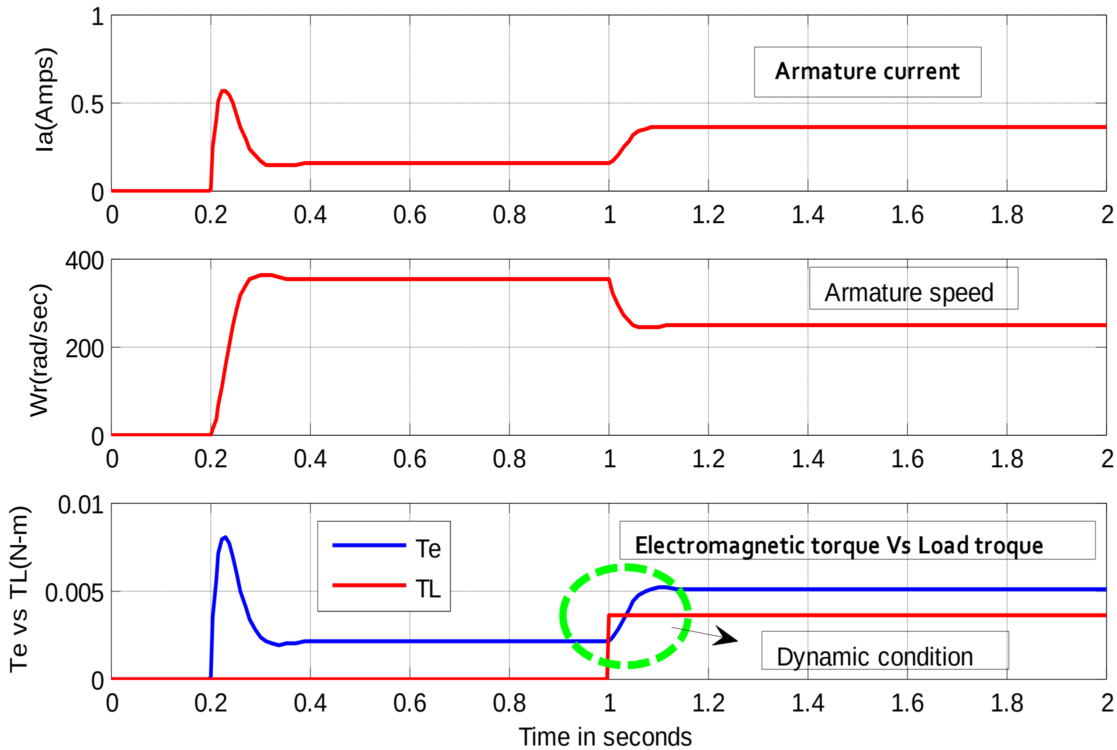

Figure 5. Dynamic performance during starting and sudden changes in load torque

(Time Varying Load Response T_L=0 (0→1sec) T_L= 0.00353 N-m (1→2secs))

From 0 to 0.2 seconds , there is no input voltage is applied to the motor. But , 0.2 to 1 second , 6 volt is applied to the armature terminals and load torque is zero. The peak transient armature current is limited to approximately 0.55 A by the inductance and resistance of the armature and the fact that the rotor is accelerating from stall, there by developing the back emf, $k_v \omega_r$ which opposes the applied voltage . After about 0.25 seconds, steady state operation is achieved with the no-load current of 0.15 A. It is noted that the rotor speed is slightly oscillatory i.e. ., under damped as illustrated by the small overshoot of the final value due to frictional coefficient of the motor.

At one second, the load torque is suddenly applied with the motor , the current is increased from 0.15 A to 0.3 A due to reduction in back emf i.e 30% decrease in speed for this increase in load torque.

IV. Mathematical Model of Direct current Shunt Motor

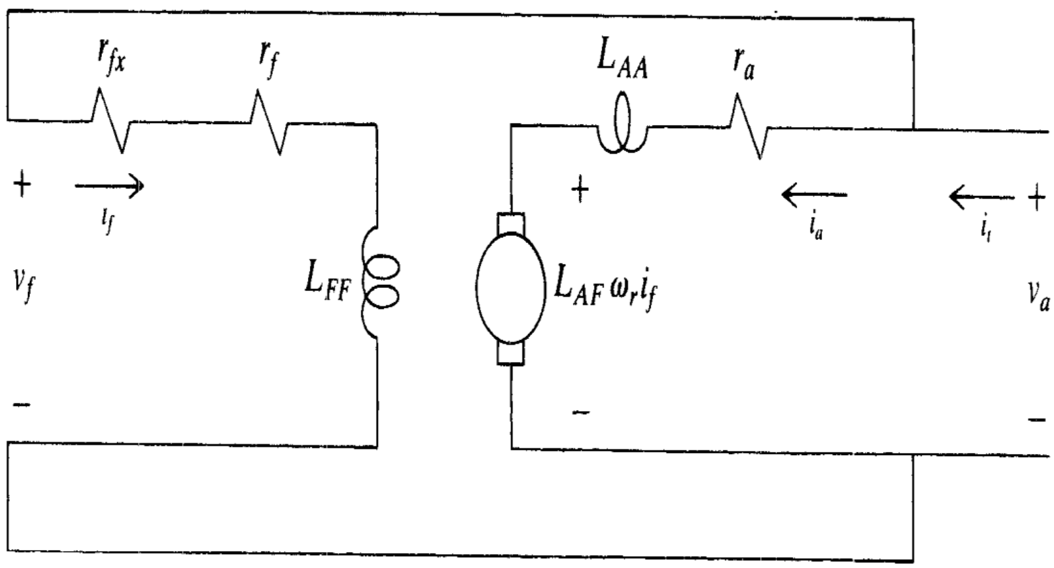

Figure 6. Equivalent circuit of DC shunt Motor

A figure shows the equivalent circuit of a DC shunt motor which consists of three first order differential equations and two algebraic equations.

$$v_a = i_a r_a + L_{AA}\frac{di_a}{dt} + L_{AF} i_f \omega_r$$

$$v_f = i_f r_f + L_{FF}\frac{di_f}{dt}$$

$$L_{AA}\frac{di_a}{dt} = v_a - i_a r_a - L_{AF} i_f \omega_r$$

Armature current equation

$$i_a = \int \frac{v_a - i_a r_a - L_{AF}\, i_f \omega_r}{L_{AA}}\, dt \qquad (6)$$

Electromagnetic torque, $T_e = T_l + j\,\frac{d\omega_r}{dt} + B\omega_r$

$$j\,\frac{d\omega_r}{dt} = T_e - T_l - B\omega_r$$

Speed equation, $\omega_r = \int \frac{T_e - T_l - B\omega_r}{j}\, dt$ --------------- (7)

$T_e = k_v i_a$ --- (8)

Back emf , $e = L_{AF}\, i_f \omega_r$ ------------------------------ (9)

Field current equation ,$i_f = \int \frac{v_f - i_f r_f}{L_{ff}}\, dt$----------- (10)

The equations from (6) to (10) are used to build the Simulink model of DC shunt motor

Motor Parameters

$r_a = 0.6\Omega$; $L_{AA} = 0.012H$; $J = 1kg - m^2$; $B_m = 1 \times 10^{-6}\ N - m/(rad/sec)$
$k_v = 1.41 \times 10^{-2}\ v/(rad/sec)$; Rated $armature\ current\ i_a = 16.2\ A$
$\omega_r = 127.7(rad/sec)$; $L_{FF} = 120H$; $L_{AF} = 1.8H$; $r_f = 240\Omega$
$Load\ torque, T_L = 29.2\ N - m$; $V_{DC} = 240\ V$

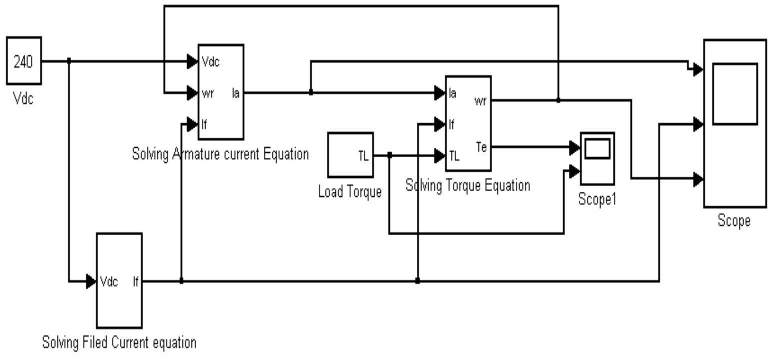

Figure 7. Simulink model of DC Shunt motor

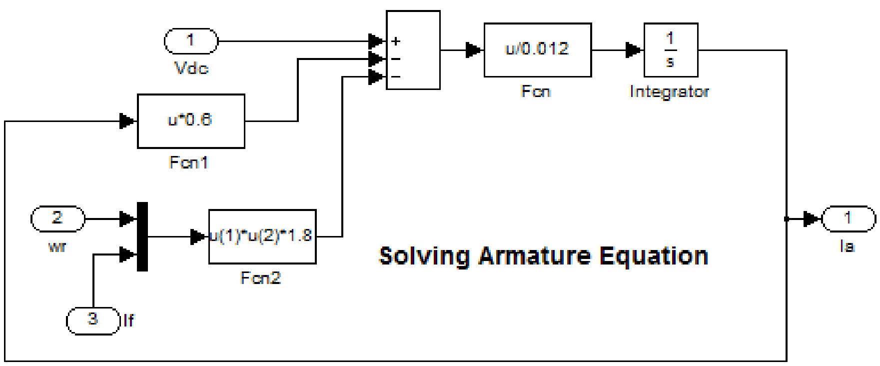

Figure 8. Solving armature current equation by Simulink blocks

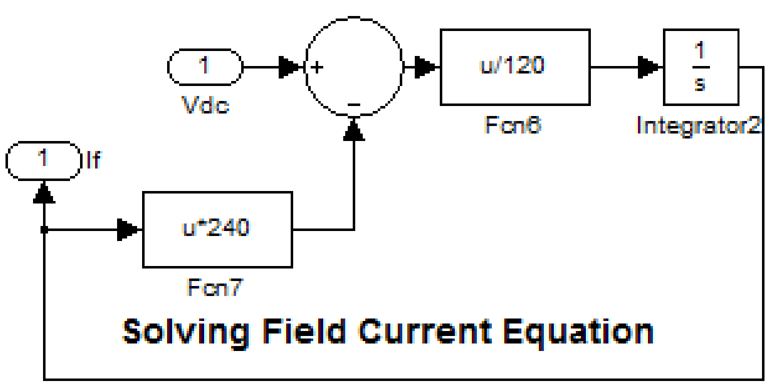

Figure 9. Solving field current equation by Simulink blocks

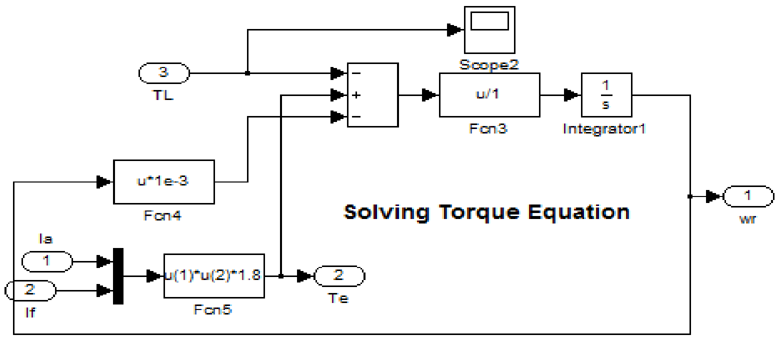

Figure 10. Solving torque equation by Simulink blocks

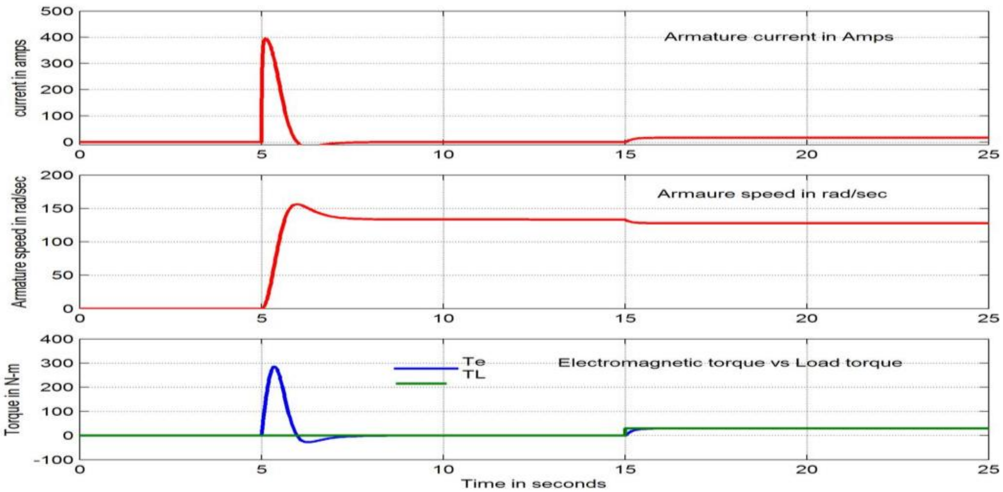

Figure 11. Dynamic performance during starting and sudden changes in load torque

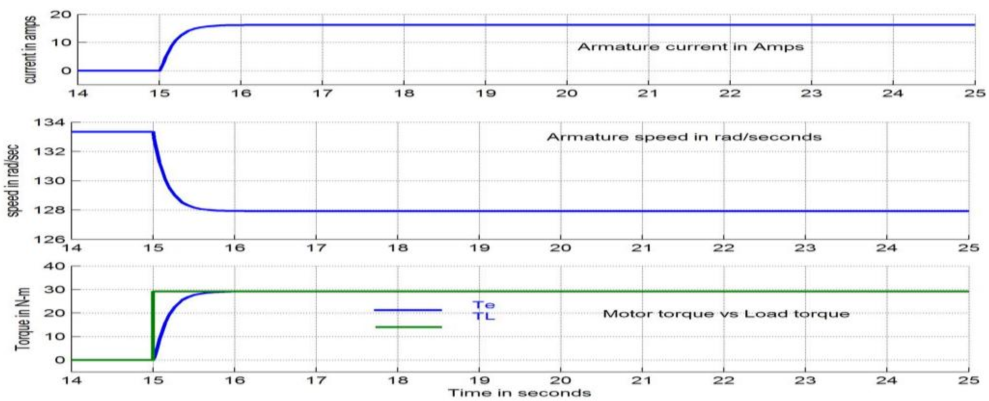

Figure 12. Dynamic performance during sudden changes in load torque (Expanded graph)

(Time Varying Load Response T_L=0 (5→15sec); T_L= 29.2 N-m (15→25secs)

From 0 to 5 seconds, there is no input voltage is applied to the motor. From 5 to 15 second, 2406 volt is applied to the armature terminals and load torque is zero. The peak transient armature current is limited to approximately 400 A by the inductance and resistance of the armature and the fact that the rotor is accelerating from stall, there by developing the back emf, $L_{AF} i_f \omega_r$ which opposes the applied voltage. After about 7 seconds, steady state operation is achieved with the no-load current of 3 A. It is noted that the rotor speed is slightly oscillatory i.e. ., under damped as illustrated by the small overshoot of the final value due to frictional coefficient of the motor.

At 15^{th} second, the load torque is suddenly applied with the motor , the current is increased from 3A to 16.2 A due to reduction in back emf i.e 30% decrease in speed for this increase in load torque.

V. Mathematical Model of Direct current Series Motor

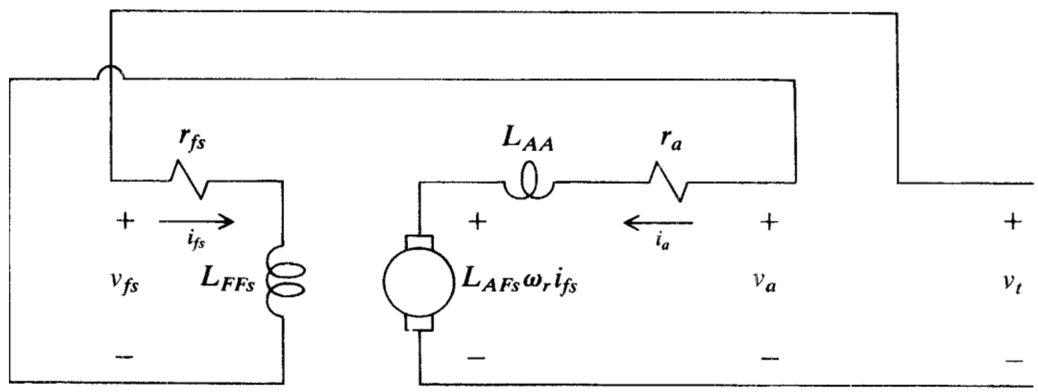

Figure 13. Equivalent circuit of DC series Motor

A figure shows the equivalent circuit of a DC series motor which consists of three first order differential equations and two algebraic equations.

$i_a = i_f$ − − − − − (11)

$v_t = i_a(r_a + r_f) + L_{AA}\frac{di_a}{dt} + L_{FF}\frac{di_a}{dt} + L_{AF} i_f \omega_r$

$i_a = \int \frac{v_t - i_a(r_a + r_f) - L_{AF} i_f \omega_r}{(L_{AA} + L_{FF})}\, dt$ -------- (12)

Electromagnetic torque, $T_e = T_l + j\frac{d\omega_r}{dt} + B\omega_r$

Speed equation, $\omega_r = \int \frac{T_e - T_l - B\omega_r}{j}\, dt$ -------------------- (13)

$T_e = L_{AF} I_a^2$ − − − − − − (14)

Motor Parameters

$r_a = 1.5\,\Omega\,; r_f = 0.7\,\Omega\, J = 0.02365\ kg - m^2\,; B_m = 0.0025\ N - m/(rad/sec)$
$k_v = 1.41 \times 10^{-2}\ v/(rad/sec)$; Rated $armature\ current\ i_a = 12.88\ A$
$\omega_r = 209.52(rad/sec); L_{FF} = 0.03H\,;\ L_{AF} = 0.0675H; r_f = 240\Omega\,; L_{AA} = 0.12H$
$Load\ torque, T_L = 10.675\ N - m\,; V_{DC} = 230\ V$

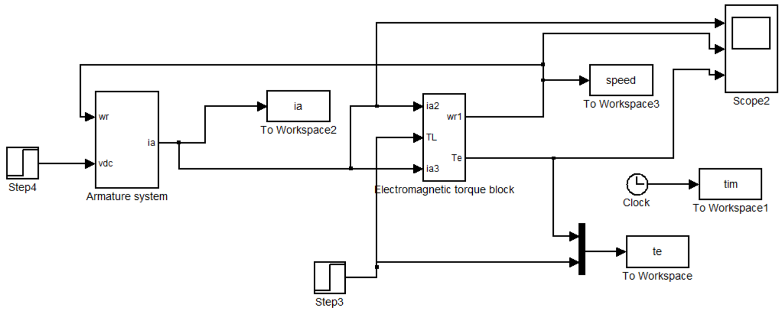

Figure 14. Simulink model of DC Shunt motor

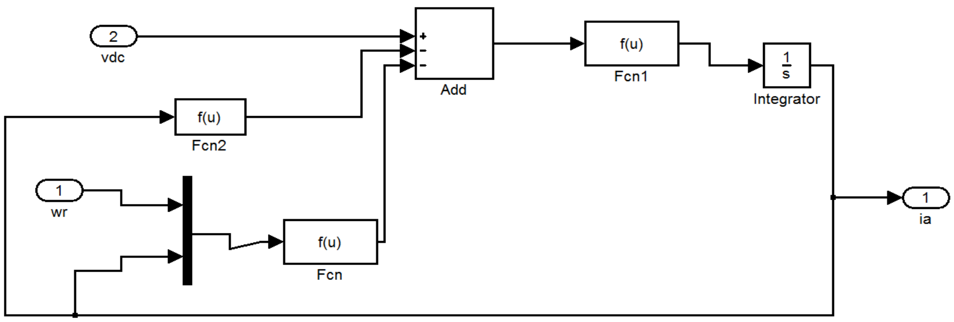

Figure 15. Solving armature current equation

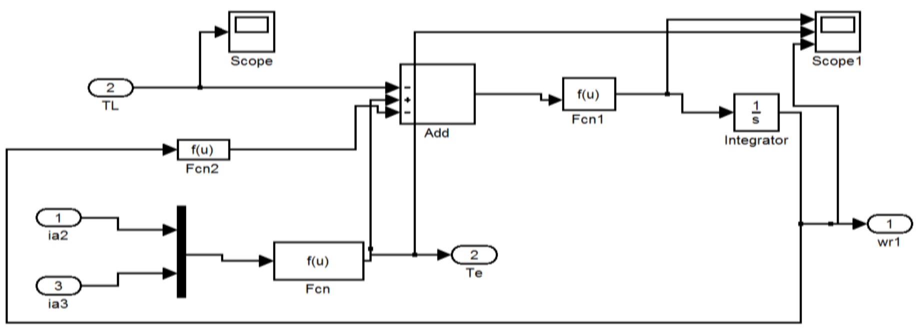

Figure 16. Solving speed equation

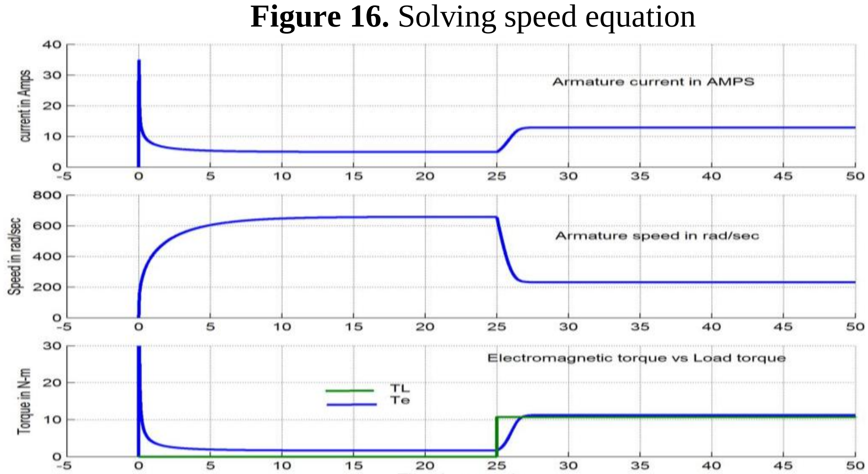

Figure 17. Dynamic performance during starting and sudden changes in load torque

In figure 17, from 0 seconds to 25 seconds motor operates under no load condition. As per theoretical concept, the motor tries to rotate at high speed which is gradually increased from 0 to 600 rad/sec. This speed is greater than nominal speed of the motor. Under this condition , the starting current and torque developed by the motor is very high. At 25 seconds , the full load is applied to the motor the current and torque are increased. The speed is reduced to rated value.

VI. Mathematical Model of Direct current Compound Motor

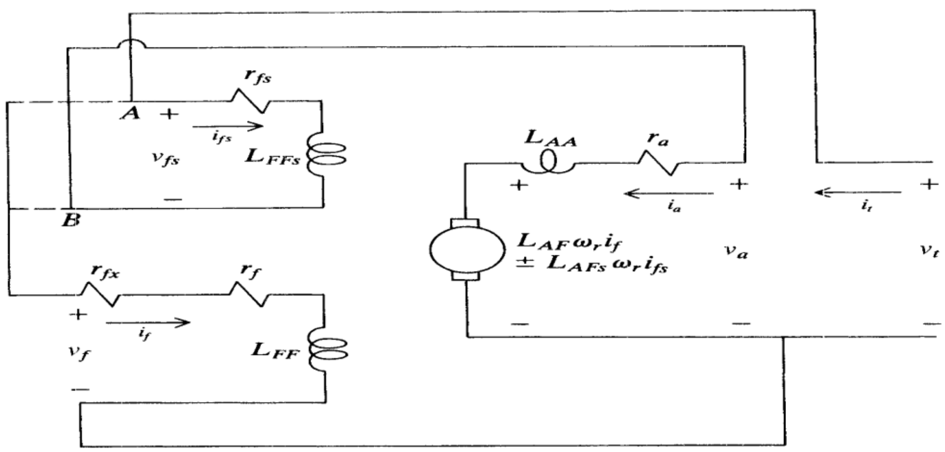

Figure 18. . Equivalent circuit of DC Compound Motor

$v_t = L_{Fs}\frac{di_f}{dt} + L_{AF}i_f\omega_r \pm L_{AFs}i_{fs}\omega_r + i_{fs}r_{fs} + L_{FFs}\frac{di_{fs}}{dt} + i_a r_a + L_{AA}\frac{di_a}{dt} ------ (15)$

$v_f = i_f r_f + L_{FF}\frac{di_f}{dt} \pm L_{Fs}\frac{di_{fs}}{dt} ------ (16)$ (+ for cumulative and – for differential connection)

The Long shunt connection is commonly used

$v_t = v_f = v_{fs} + v_a ------ (17)$

$i_{fs} = i_a -------- (18)$

$i_t = i_{fs} + i_f ------ (19)$

Electromagnetic torque, $T_e = T_l + j\frac{d\omega_r}{dt} + B\omega_r ------ (20)$

Speed equation, $\omega_r = \int \frac{T_e - T_l - B\omega_r}{j}\,dt$ --------------- (21)

$T_e = L_{AF}i_f i_a \pm L_{AF}i_{fs}i_a -------- (22)$

$I_f = \frac{V_t}{R_f} -------(23)$

The above equations from (15) to 23 can be used to build the Simulink model of compound motor. The equations pertain to long shunt connection with cumulative and differential compound motor.

VII. CONCLUSION

In this paper, modeling and dynamic simulation of PMDC motor , DC shunt Motor , and DC Series motor have been developed and the performance analysis of each machine has been verified and mathematical modeling of DC compound motor has been derived . The dynamic and steady state characteristics are effectively and more accurately analyzed. This analysis helps to design the starters

and controllers for DC closed loop drive systems. and it can be very well applied with a slight modification to other ac machine applications which incorporates the permanent magnet ac motor, the synchronous reluctance motor , induction motor and the like.

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