# Quiz 1 — Spring 2024

- Due Jan 30 at 3:30pm
- Points 8
- Questions 8
- Available Jan 26 at 11:59pm - Jan 30 at 3:30pm
- Time Limit 120 Minutes
- Allowed Attempts 2

# Instructions

# Instructions

To complete this quiz, you'll have **120 minutes** (2 hours) and a maximum of **two attempts** once you begin. (The two-hour limit is *per attempt*.) The quiz is due by **Tuesday, January 30, at 3:30 pm** (when the next class meets).

The quiz is open-note and open-book. You may use Google. However, you must not seek direct help or collaborate with others when taking the quiz. "Direct help" includes posting questions on sites like Stackoverflow, Chegg, ChatGPT, etc. Doing so is considered a violation of the Georgia Tech Honor Code that will result in a zero score on this quiz and an official report to the Dean of Students.

When questions involve 1-D dynamical systems, you may assume they take the form, $Dx = f(x)$, where:

- $x$ is the unknown state variable, which varies with time, i.e., $x = x(t)$;
- $D$ is the first-derivative operator with respect to $t$;
- and $f(x)$ is a continuous differentiable function.

Good luck!

This quiz was locked Jan 30 at 3:30pm.

## Attempt History

| | Attempt | Time | Score |
|---|---|---|---|
| **KEPT** | Attempt 2 | 1 minute | 8 out of 8 |
| **LATEST** | Attempt 2 | 1 minute | 8 out of 8 |

| Attempt 1 | 16 minutes | 7 out of 8 |
|---|---|---|

Score for this attempt: 8 out of 8

Submitted Jan 27 at 6:36pm

This attempt took 1 minute.

Question 1

1 / 1 pts

Which of the following is a valid motivation for simulating a system on a computer, rather than doing an experiment with the system itself? **Check all that apply**. (To get full credit, you must choose all of the correct options and no incorrect ones.)

Correct!

- [x] You might not have access to the system

Correct!

- [x] The real system may be too dangerous to observe or manipulate

Correct!

- [x] It may be immoral or unethical to observe or manipulate the system

Correct!

- [x] The system does not actually exist

See [SB09] Chapter 1

Question 2

1 / 1 pts

Consider the system, $Dx = -1 + 2\sin x$ , on the interval $0 < x < 2\pi$. Which of the following statements is true?

Correct!

- (•) There is an **unstable** fixed point at $\frac{\pi}{6}$ and a **stable** one at $\frac{5\pi}{6}$.
- ( ) There is a **stable** fixed point at $\frac{7\pi}{6}$ and an **unstable** one at $\frac{11\pi}{6}$ .
- ( ) There is an **unstable** fixed point at $\frac{7\pi}{6}$ and an **stable** one at $\frac{11\pi}{6}$ .
- ( ) There is a **stable** fixed point at $\frac{\pi}{6}$ and an **unstable** one at $\frac{5\pi}{6}$.

Question 3

1 / 1 pts

The Gompertz Law models the growth of a cancerous tumor over time as $Dx = -ax\ln(bx)$, where $x = x(t)$ is the unknown time-dependent state variable related to the size of the tumor, and $a, b > 0$ are constant parameters. The function $\ln(\cdot)$ denotes the natural logarithm.

Which of these claims about the Gompertz Law is true? Check all that apply.

Correct!

- [x] There is a **stable** fixed point at $\frac{1}{b}$.
- [ ] There is an **unstable** fixed point at $\frac{1}{b}$.
- [ ] There is a **stable** fixed point at 0.

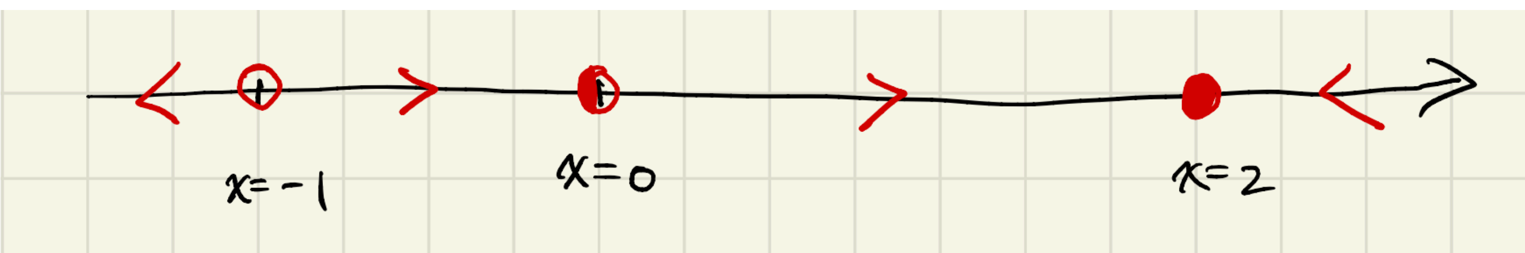
Correct!

- [x] There is an **unstable** fixed point at 0.

Question 4

1 / 1 pts

Suppose you are told that the vector-flow field of a 1-D system looks like the following:




Evidently, there is an unstable fixed point at x=-1, a left-semistable fixed point at x=0, and a stable fixed point at x=2.

Which of the following equations is a possible model of this system?

Correct!

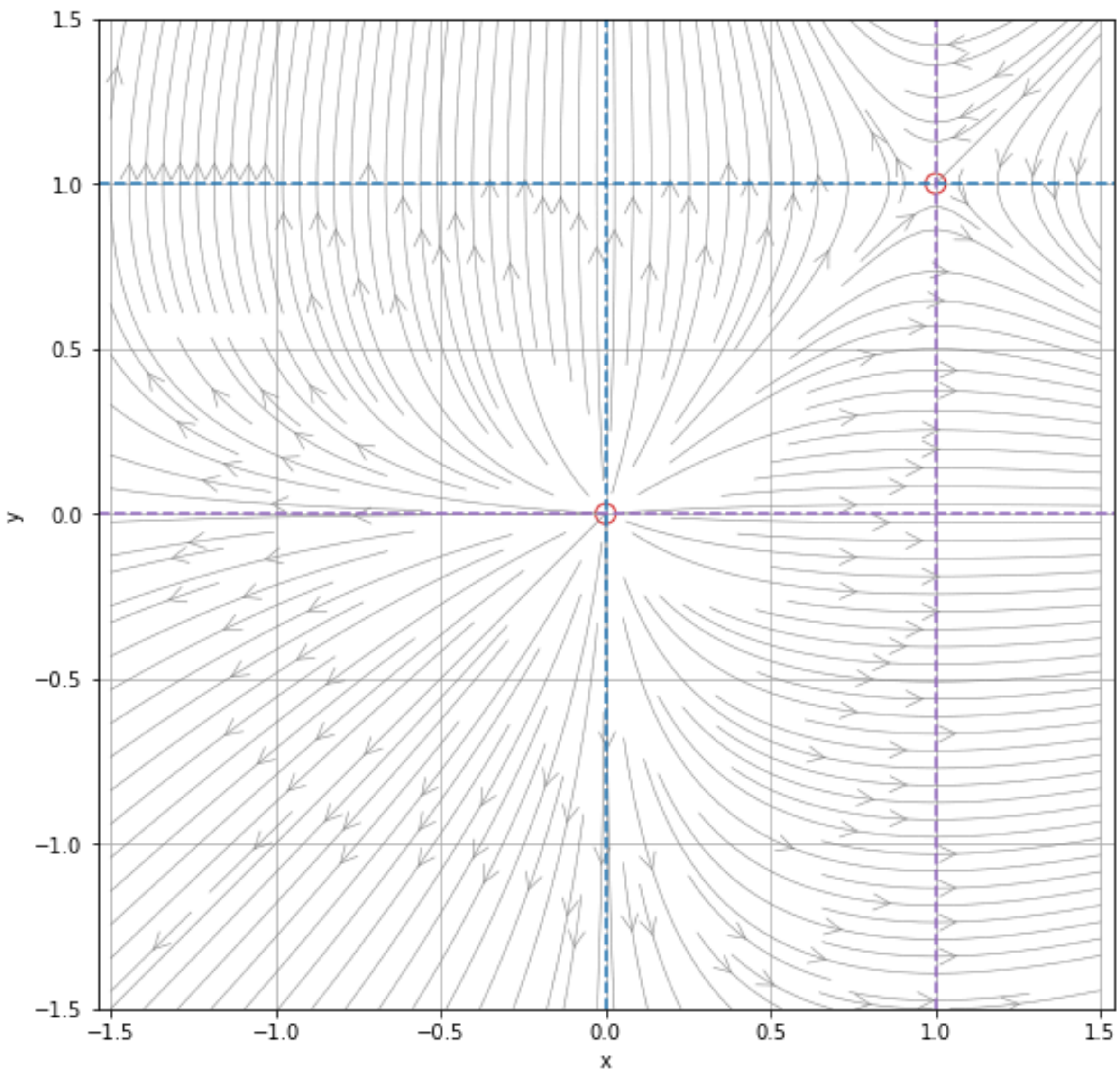
- [x] $Dx = (x+1)x^2(2-x)$
- [ ] $Dx = (x+1)^2(x)(x-2)$
- [ ] $Dx = (x+1)^2(x)(2-x)$

- [ ] $Dx = -(x+1)x^2(2-x)$

Question 5

1 / 1 pts

Consider the following phase diagram (or "vector flow field") for a 2-D continuous-time dynamical system. The gray lines with arrows show the flow of trajectories at each point in space; the dashed lines show nullclines; and circles indicate fixed points.




Which of the following systems is the most likely to have generated this flow field?

Correct!

- $D\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x(1-y) \\ y(1-x) \end{bmatrix}$
- $D\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ x-y \end{bmatrix}$
- $D\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x^2-y \\ x-y \end{bmatrix}$
- $D\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x^2+y \\ x-y \end{bmatrix}$

Question 6

1 / 1 pts

Some political scientists are studying the dynamics of shifts in political opinions among a population of "leftists," "rightists," and "centrists." Let $L = L(t)$, $R = R(t)$, and $C = C(t)$ represent the fraction of the total population in each of these groups, respectively, as a function of time.

Note that $L + R + C = 1$, and consider that only nonnegative values are meaningful.

These scientists discover that the population changes may be modeled approximately by the dynamical system,

$$D\begin{bmatrix} L \\ R \\ C \end{bmatrix} = \begin{bmatrix} \alpha LC \\ \alpha RC \\ -\alpha(L+R)C \end{bmatrix},$$

where $\alpha \neq 0$ is a real-valued model parameter that measures the "strength" of opinion shifts and may be positive or negative.

Which of the following statements is a valid conclusion of this model? Check all that apply.

- [ ] $(L_*, R_*, C_*) = (0, 0, 1)$ is a stable fixed point provided $\alpha > 0$.

Correct!

- [x] $(L_*, R_*, C_*) = (0, 0, 1)$ is a stable fixed point provided $\alpha < 0$ .

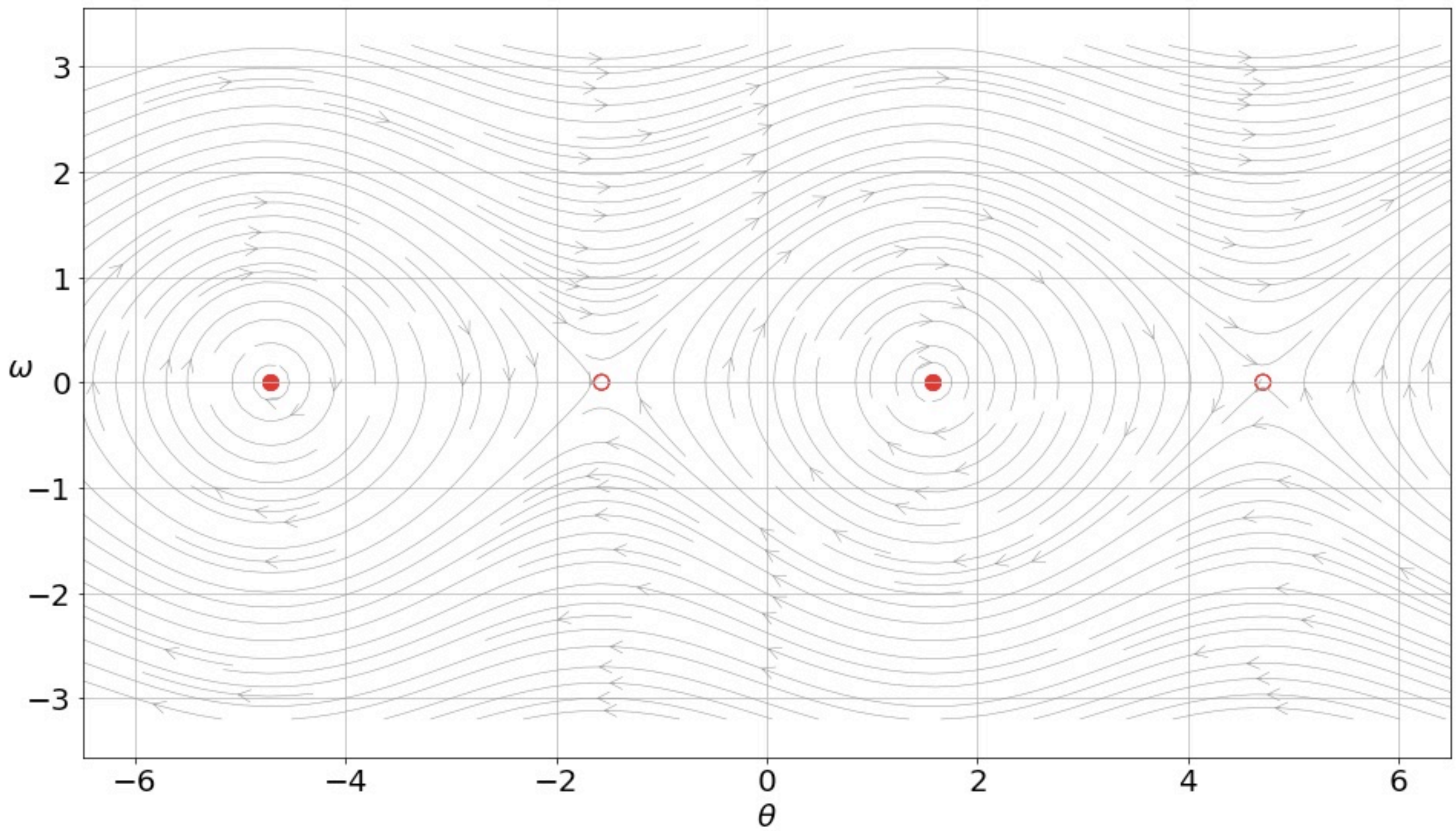
Correct!

- [x] All points such that $L_* + R_* = 1$ are stable fixed points provided $\alpha > 0$.
- [ ] All points such that$L_* + R_* = 1$ are stable fixed points provided $\alpha < 0$ .

Question 7

1 / 1 pts

Consider a two-dimensional dynamical system whose state variables are $x_1 = x_1(t)$ and $x_2 = x_2(t)$. Suppose this system's phase portrait looks like the following:




Solid red dots indicate stable circular fixed points and hollow red circles indicate unstable fixed points. Which of the systems below is the most likely to have produced this phase portrait?

- $D \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\sin x_1 \end{bmatrix}$
- $D \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \sin x_1 \end{bmatrix}$
- $D \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\cos x_1 \end{bmatrix}$

Correct!

- $D \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \cos x_1 \end{bmatrix}$

Question 8

1 / 1 pts

Consider the nonlinear dynamical system, $Dx = 1 - x^2$. Which of these statements best characterizes the fixed points of the system? Check all that apply. For full credit, you must select all of the correct options and none of the incorrect ones.

Correct!

- [x] The fixed point $x_* = 1$ is **stable**.
- [ ] The fixed point $x_* = 1$ is **unstable**.
- [ ] The fixed point $x_* = -1$ is **stable**.

Correct!

- [x] The fixed point $x_* = -1$ is **unstable**.

Quiz Score: 8 out of 8