

1.

Let  $f(n)$  and  $g(n)$  be asymptotically nonnegative functions. Using the basic definition of  $\Theta$ -notation, prove that  $\max(f(n), g(n)) = \Theta(f(n) + g(n))$ .

2.

3.1-2

Show that for any real constants  $a$  and  $b$ , where  $b > 0$ ,

$$(n + a)^b = \Theta(n^b).$$

3.

Explain why the statement, “The running time of algorithm  $A$  is at least  $O(n^2)$ ,” is meaningless.

4.

$$\text{Is } 2^{n+1} = O(2^n)? \text{ Is } 2^{2n} = O(2^n)?$$

5.

Prove that the running time of an algorithm is  $\Theta(g(n))$  if and only if its worst-case running time is  $O(g(n))$  and its best-case running time is  $\Omega(g(n))$ .

6.

Prove that  $o(g(n)) \cap \omega(g(n))$  is the empty set.