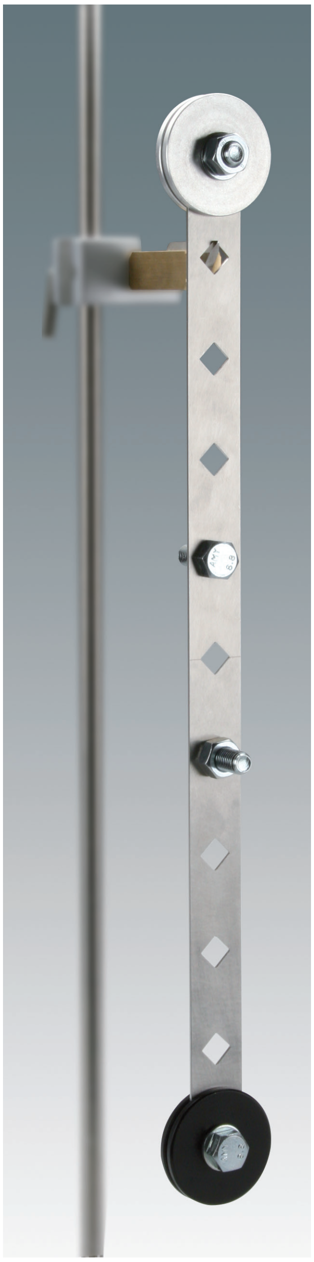
Manual for Physical pendulum / Bessel pendulum

18.04.13 2181.00 AE

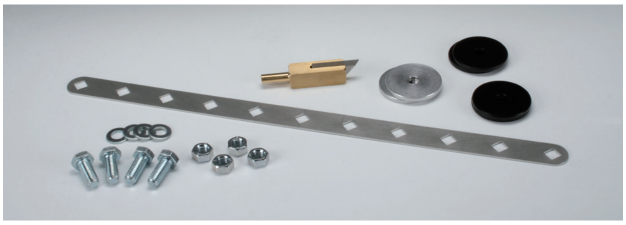


DESCRIPTION

The device consists of a steel rod with a row of holes, which are used as pivots of the pendulum and for attaching weights. The fixed part of the pivot consists of a robust knife edge, which is fastened to ordinary stand material or preferably a table edge.

The device is supplied with 4 discs of steel and 2 discs of aluminium. The discs are used in pairs - positioned on either side of the steel rod with a bolt. Two discs, a bolt and a nut is hereafter called a *weight.*

Two extra sets of nuts and bolts are included to be used as ”trimming weights” in connection with the Bessel pendulum. For the Bessel pendulum 4 washers are used with the big weights.



USE

1 – The physical pendulum is used in the study of moments of inertia and motion of rigid bodies.

Centre of gravity, pivot and moment of inertia can be varied in countless ways. For the calculations you will need Steiner's theorem and a number of formulae for the moments of inertia of the various components of the pendulum. These relationships can be found in the *Moments of Inertia* section. The practical calculations are most conveniently done in a spreadsheet.

2 – The reversion pendulum is used to determine the acceleration due to gravity. The name reflects that the pendulum can be turned upside down - there is a pivot at each end. The reversion pendulum is built with the pivots having different distances to the centre of mass, and is adjusted to have the same period for the two suspension points.

The reversion pendulum was developed by Henry Kater. The principle was refined by F.W. Bessel, and his version can be demonstrated with this device (although the accuracy is not as high as with the classical precision pendulums). **The Bessel pendulum** is a reversion pendulum, which is symmetric in terms of volume (i.e. geometrically symmetric), although its mass distribution is asymmetric. It turns out that two sources of error due to the presence of air thereby disappear. (These are, respectively, buoyancy and added inertia due to dragged air.) For details, please see the bibliography.

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A BIT OF THEORY

We will consider a rigid body as composed of a large number of small components.

The **moment of inertia** of the body about a given axis is then given as a sum of contributions from the individual components, each of the form

$$I_j = m_j \cdot r_j{}^2$$

where m_j is the mass of component no. j and r_j is the distance between this and the axis of rotation.
The total moment of inertia can thus be written in the form

$$I = \sum_j I_j$$

In practice, moments of inertia are determined by integration. If the body shape is sufficiently simple, this can be done analytically. There are a number of relevant examples of this in a later section.

If the moment of inertia about an axis through the body's centre of gravity is called I_G, the moment of inertia I about any other axis which is parallel to the first can be determined by using **Steiner's theorem:**

$$I = I_G + Ma^2$$

Here, M is the mass of the body and a is the distance between the two axes.

This theorem is extremely useful for calculating the moments of inertia, except for the simplest cases.

The term "physical pendulum" is used when a rigid body is suspended from an axis which does not pass through the centre of gravity.

The period of the physical pendulum is given by

$$T = 2\pi\sqrt{\frac{I}{Mag}}$$

where I is the moment of inertia about the axis of rotation, and M is the mass, a is the distance between the pivot and the centre of mass, g is the acceleration due to gravity.

MOMENTS OF INERTIA

It follows from the definition of moment of inertia that for a rigid body, the total moment of inertia about an axis can be found as a sum of the moments of inertia of the individual parts about this axis.

In this case, we will divide the pendulum rod into a rectangular piece and two semicircular ends. (The material from the 11 square holes should be subtracted from this, if you want to be exact.)

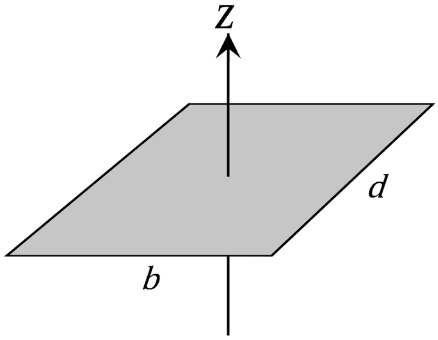
The mass of the rod is distributed among the three parts proportional to their areas.

The large discs have the form of a cylinder with a cylindrical hole in the centre. The weights are attached by a bolt which we can consider as a point mass. (Meaning that we ignore the bolt's moment of inertia about its own centre – but not its mass.)

Below are some formulae for moments of inertia.

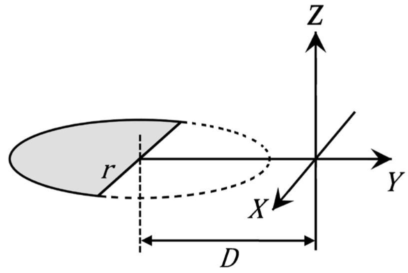
Rectangle with dimensions *b x d* and mass *m:*

$$I_Z = \frac{m}{12} \cdot (b^2 + d^2)$$



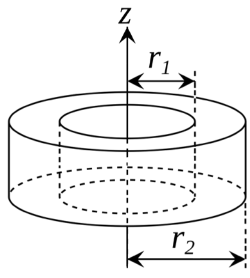
Semicircle of radius *r* and mass *m,* translated by *D:*

$$I_Z = m \cdot \left(D^2 + \frac{r^2}{2} + \frac{8Dr}{3\pi}\right)$$



Cylinder with outer radius r_2, inner radius r_1 and mass *m:*

$$I_Z = \frac{m}{2} \cdot (r_1^2 + r_2^2)$$



The work with keeping track of all the separate parts of the pendulum is best done in a spreadsheet.

MEASURING THE PERIOD

With a stopwatch

You measure the time for a number of complete oscillations and divide by the number. The accuracy is increased if you start and stop the watch when the pendulum passes its lowest point where its speed is highest. Fix your sight on something vertical behind the pendulum, and do not move your head between start and stop.

In practice, you can hardly achieve lower uncertainty than 0.2 s.

For an accuracy of e.g. 0.5%, the total measurement time must be at least 40 s.

With the Bessel pendulum you should not aim for less accuracy than 0.1%, corresponding to 200 seconds of measurement time.

With a photogate and a counter

Suspend the pendulum vertically, completely at rest. Place the photogate so the light is just ”touching” a vertical edge near the bottom of the pendulum (the edge of the rod or eventually the edge of a weight). Photocell type 1975.50 has a green LED that turns off when light beam is interrupted.

When the pendulum swings (small oscillations!) the light beam is to be blocked throughout one half oscillation period and to pass during the other. Thus the period is the time from the start of one interruption to the next one.

With counter **2002.50**, the procedure is the following:

1. Connect the photocell to DIN connector A
2. Pull the pendulum a little away from the photogate during the following steps
3. Press *Select* repeatedly until *Period* is lit
4. Wait until the lamp *Continuous* is lit, then press *Memory/Continuous*
5. Finally press *Start/Stop*
6. Now the pendulum can be released

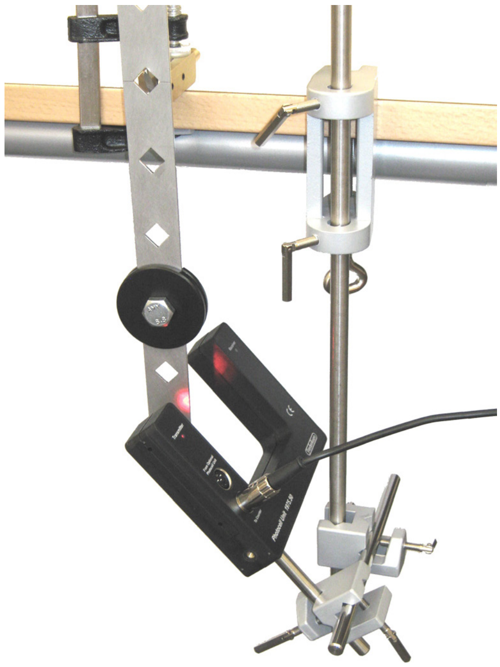
The results are shown as an average of two periods. Read out many values and use the average.

Using data logging

Place a motion sensor about 20 cm from the pendulum pointing at one of the weights. (It takes luck to measure the motion of the rod alone – but it can be done.) Set the data acquisition program to measure position with a sampling frequency of e.g. 100 Hz. Ensure that the data collected roughly shows a sinusoidal curve - there must not be odd spikes indicating that the sensor misses the target.

Continue measuring for “sufficiently long time”. With the Bessel pendulum, approx. one minute is needed. If the precision requirements are less strict, the time can be reduced.

Fit the data points with a damped sinus oscillation. Be sure to set the program to display sufficient number of digits of the parameters!



Some programs deliver the oscillation time T directly, others use $\omega = 2\pi/T$.

Physical pendulum – experiment procedure

When working with the general physical pendulum, neither washers nor the extra set of bolts are needed. There are enough variations even so!

Experiments 1, 2 and 3 below may be carried out independently of each other.

Fix the metal discs with the bolts - it is enough to tighten them with your fingers.

The knife edge pivot can for simple demonstration experiments be used with a retort stand rod and an A foot or a table clamp. More accurate results are achieved by clamping the knife edge pivot to the edge of a steady table with a cramp. Preferably just above a table leg. If possible, use a table that is secured to a wall.

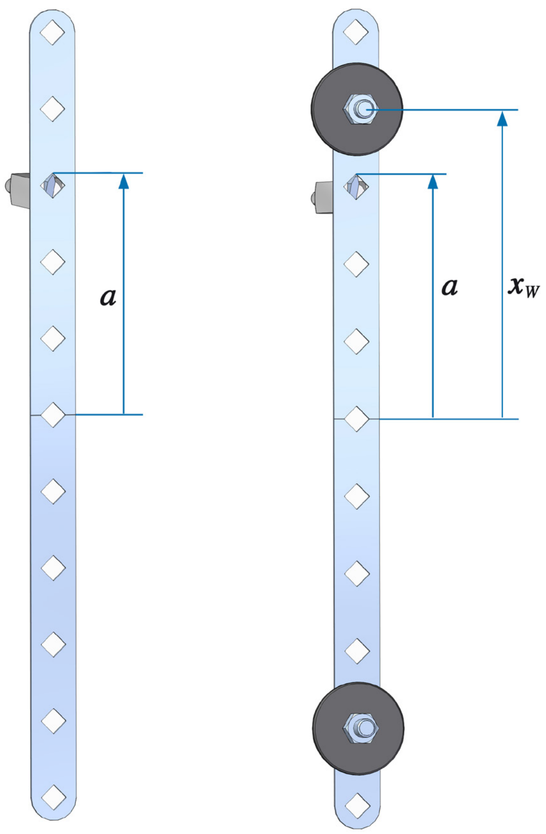
Note: There is a position difference when a hole is used for a pivot (upper edge) or as the location for a weight (centre). Be careful to use the right values.

The amplitude of the oscillations should be small. About a centimetre is fine.

1 – Moment of inertia of the rod

Measure the period as an average over at least 20 cycles for each of the 6 possible pivots (centre hole included).

Measure the distance from the centre of the rod to each of the five pivots precisely. The centre is marked with a thin line – it may be an idea to temporarily extend the line across the centre hole using the edge of a strip of adhesive tape. (These distances are also used in the following experiments.)



Calculations – Moment of inertia of the rod

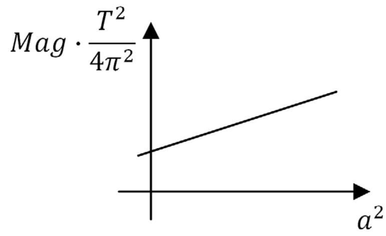
The formula for the period can be rewritten as follows

$$Mag \cdot \frac{T^2}{4\pi^2} = I$$

Since the centre of gravity is located in the centre of the rod, everything on the left-hand side is known. The total mass M is equal to the mass m_R of the rod. For the rod without weights the right-hand side can be rewritten using Steiner's theorem:

$$I = I_R + m_R \cdot a^2$$

It follows that if the values of $Mag \cdot \frac{T^2}{4\pi^2}$ are plotted as a function of a^2, the results should lie on a straight line with the rod mass m_R as slope and moment of inertia of the rod about its centre I_R as the intersection with the y-axis.



For comparison, calculate the moment of inertia of the rod about its centre by dividing it into two semicircular ends and a rectangular piece in the centre – see also "Inertia Moments". Ignoring the square holes only results in a relatively small error.

To determine the mass of the three parts, distribute the total mass of the rod proportional to the areas of the three parts.

2 – Symmetrical mass distribution

Select a position of the weights – e.g. the next outermost hole. The locations must be symmetrical, so the pendulum still has the centre of mass in the centre of the rod.

In order to avoid that the pendulum "capsizes", when using the centre hole as pivot, the bolts can be pointing in opposite directions.

Measure the period as an average over at least 20 cycles for each of the 5 possible pivots (centre hole included).

If you have not already determined these: Measure the distance from the centre of the rod to each of the five pivots precisely. The centre is marked with a thin line – it may be an idea to temporarily extend the line across the centre hole using the edge of a strip of adhesive tape. (These distances are also used in the following experiment.)

Repeat eventually the measurement series with the weights in a new – but still symmetrical – position.

Calculations – Symmetrical mass distribution

The formula for the period can be rewritten as follows

$$Mag \cdot \frac{T^2}{4\pi^2} = I$$

For symmetry reasons, the centre of gravity is located in the centre of the rod, so everything on the left-hand side is known. The right-hand side can be rewritten using Steiner's theorem:

$$I = I_G + M \cdot a^2$$

It follows that if the values $Mag \cdot \frac{T^2}{4\pi^2}$ are plotted as a function of a^2, the results should lie on a straight line with pendulum mass M as slope and the moment of inertia of the pendulum about its centre of mass I_G as the intersection with the y-axis.

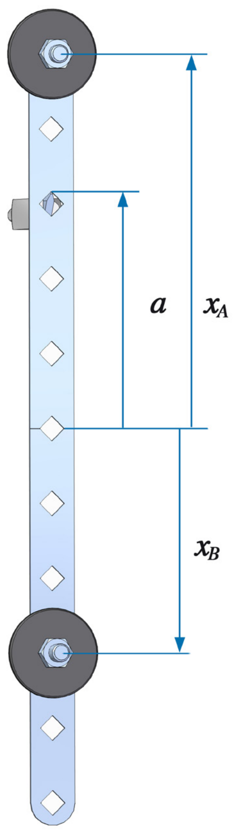
We can compare the value found for I_G with one calculated as

$$I_G = I_R + 2(I_W + m_W x_W^2)$$

where I_R is the moment of inertia of the rod about its centre, I_W is the moment of inertia of the weight about its centre axis, m_W is the mass of the weight and x_W is the distance between centres of the rod and the weight.

I_R and I_W are calculated using the formulae in the section "Moments of inertia". The weights are considered as two cylinders, to be treated as extended bodies, plus a bolt and a nut, which to a good approximation can be considered as point-shaped.

If more than one series of measurements are performed, the graphs will be parallel.



3 - arbitrary mass distribution

We are now dealing with the situation where there is no longer symmetry about the centre of the rod. Thus, we must not only determine the moment of inertia of the pendulum, but also the centre of mass.

Measure the distance from the centre of the rod to each of the possible pivots accurately. The centre is marked with a thin line – it may be an idea to temporarily extend the line across the centre hole using the edge of a strip of tape.

In order to specify positions unequivocally, we define a coordinate axis along the rod, with zero in the rod centre and the positive direction upward. Positions below centre of the rod are negative.

Call the pivot position x_P. The positions of the weights are designated x_A, resp. x_B, and their masses m_A, resp. m_B.

Note the experimental conditions carefully and measure the period averaged over at least 20 times.

Calculations - arbitrary mass distribution

Since the centre of mass of the rod has the coordinate 0, we get the centre of mass position x_G

$$x_G = \frac{m_A \cdot x_A + m_B \cdot x_B}{M}$$

The distance from the axis of rotation to the centre of mass is given by

$$a = x_P - x_G$$

The pendulum's moment of inertia about the pivot is calculated by summing the contributions from the rod and the two weights:

$$\begin{aligned} I = \; & I_R + m_R \cdot x_P^2 \\ & + I_A + m_A \cdot (x_P - x_A)^2 \\ & + I_B + m_B \cdot (x_P - x_B)^2 \end{aligned}$$

where I_A and I_B denotes the moment of inertia of weight A and B about the weight's own centre.

Now, a theoretical value for the oscillation time can be calculated and compared with the experimental one.

BESSEL PENDULUM – EXPERIMENT PROCEDURE

The mass of the weights must be increased slightly: Mount two steel discs with a bolt *and two washers* in the outermost hole in the rod at one end, and the two aluminium discs at the opposite end – also *with washers.*

The pendulum is suspended in the last but one hole at each end – i.e. the outermost free hole. The knife pivot is clamped to a table edge – normal stand material is not stable enough.

The pivot in the end with the steel weight is called P_1, the pivot near the aluminium weight is called P_2. The two corresponding periods are called T_1 and T_2.

It is necessary to measure the oscillation period very accurately. With a stopwatch at least 150 periods must be measured. Proper use of photogates or data acquisition equipment is better than using a stopwatch.

As before, the amplitude must be small.

The trimming weights, comprising a bolt with a nut, should always be located symmetrically around the centre of the rod. For a start, place them in the two holes 50 mm from the centre.

Use the pivot P_1 and determine T_1. Reverse the pendulum using P_2 and determine T_2.

Move the trimming weights symmetrically to the next hole and repeat. Ditto for the last vacant hole.

The period varies when the trimming weights are moved. When they are as close to each other as possible, T_1 will be less than T_2. Conversely, when the trimming weights are as far apart as possible, the period T_2 is less than T_1. For some distance, the two periods must be equal. We are looking for the exact value of this common period T:

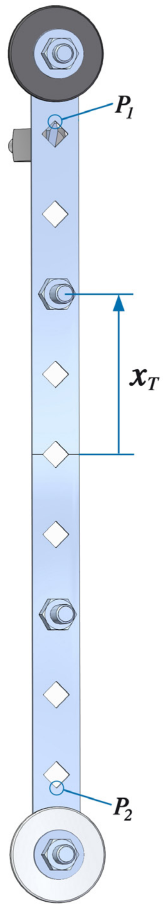
Use a spreadsheet to graph T_1 and T_2 as a function of the distance x_T from the centre to the trimming weights. Add parabolas (quadratic function, polynomial of degree 2) as "trend lines". Add sufficient horizontal subdivisions to the y axis, and read the common period where the graphs are crossing.

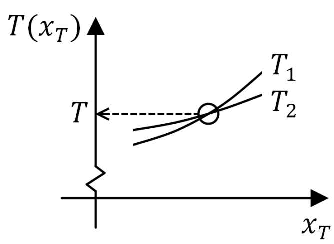
The last measurement needed is the distance Δx between P_1 and P_2. The measure is taken from the upper rim of the upper hole to the lower rim of the lower hole.

Now the local acceleration due to gravity can be found:

$$g = \Delta x \cdot \left(\frac{2\pi}{T}\right)^2$$

The period of oscillation must be determined so accurately that the overall accuracy is determined by the measurement of Δx. On the classical precision pendulums this distance could for instance be found by interferometry.





The acceleration due to gravity – comparison

The experimentally determined value of g can be compared with the theoretical value for the smooth earth ellipsoid, compensated for height:

$$\begin{aligned} g = &\quad 0{,}0002269\,\tfrac{\mathrm{m}}{\mathrm{s}^2}\cdot\sin^4(\varphi) \\ &+ \quad 0{,}0516323\,\tfrac{\mathrm{m}}{\mathrm{s}^2}\cdot\sin^2(\varphi) \\ &+ 9{,}780327\,\tfrac{\mathrm{m}}{\mathrm{s}^2} + C_{FA} + C_B \end{aligned}$$

Here, φ is the latitude. The height correction is split into two parts C_{FA} and C_B. (Free Air correction and Bouguer correction). They represent, resp. a reduction of g due to larger distance to the earth's centre and an increase in g due to the increased gravity from the extra layers of soil:

$$C_{FA} = -3{,}086\cdot 10^{-6}\ \mathrm{s}^{-2}\ \cdot h$$

$$C_B = 4{,}193\cdot 10^{-10}\ \mathrm{m}^3\mathrm{s}^{-2}\ \mathrm{kg}^{-1}\ \cdot \rho\cdot h$$

Here, h is the height above sea level.

In C_B we need the average density ρ of the soil layers between sea level and the position. Using a typical density of 2670 kg/m^3, these two corrections to be joined:

$$C(h) = C_{FA} + C_B = -1{,}966\cdot 10^{-6}\ \mathrm{s}^{-2}\ \cdot h$$

In practice, the easiest way to find the latitude is by using a service like Google Maps. The height above sea level can possibly be found via the municipal website – search for a section with geographical data ("GIS"). Or use a good map.

Reversion pendulum – theory

The pendulum has mass M and the moment of inertia about the centre of mass G is called I_G.

The moment of inertia about P_1 resp. P_2 is called I_1 resp. I_2.

The distance between G and P_1 is called x_1 and the distance between G and P_2 called x_2.

According to Steiner's theorem we now have that

$$I_1 = I_G + M\cdot x_1^2 \qquad I_2 = I_G + M\cdot x_2^2$$

The two periods are given by

$$T_1 = 2\pi\cdot\sqrt{\frac{I_G + M\cdot x_1^2}{M\cdot x_1\cdot g}} \qquad T_2 = 2\pi\cdot\sqrt{\frac{I_G + M\cdot x_2^2}{M\cdot x_2\cdot g}}$$

If $T_1 = T_2$, this means that

$$\frac{I_G + M\cdot x_1^2}{x_1} = \frac{I_G + M\cdot x_2^2}{x_2}$$

Assuming that $x_1 \neq x_2$, this equation can be solved in terms of I_G

$$I_G = M\cdot x_1\cdot x_2$$

From this we obtain the period as

$$T = 2\pi\cdot\sqrt{\frac{x_1 + x_2}{g}} = 2\pi\cdot\sqrt{\frac{\Delta x}{g}}$$

Where Δx denotes the distance between the pivots.

TEACHER NOTES

Science concepts

Center of mass - assumed known.

Moment of inertia, Steiner's theorem and period for physical pendulum - formulae are summarized.

Reversion pendulum - the formula for the oscillation period is derived.

Mathematical prerequisites

Equation Solving, trigonometric functions, use of spreadsheets, graphs.

Derivation of the formulae for moments of inertia requires integral calculus.

Didactic considerations

Calculation of the moment of inertia of the rod can be simplified by ignoring the holes. This causes an error in the rod moment of inertia about the centre of just over 1%.

Calculated oscillation periods of the rod alone will increase by up to 0.5%. (The greatest deviation for pivot in the centre hole.) With weights mounted on the rod, period deviations will be less.

An even more obvious simplification can be done by considering bolts, nuts and washers as point masses. The error in calculations of moment of inertia will be less than 0.08%, the period error is half of that.

These errors will probably be negligible compared to measurement uncertainties and other sources of error when working with the general physical pendulum.

Note that these errors are irrelevant when the device is used as a reversion pendulum !!!

It seems tempting to expand the calculations of experiment 2 – *Symmetrical mass distribution* by finding the moment of inertia of the weight about its own centre. This requires subtraction of two nearly equal values, and will therefore be subject to very high uncertainty. It is only recommended if you want to dig into uncertainty calculations – otherwise, the discrepancy between theory and measurement will only create frustration.

The best procedure is to repeat experiment 2 for all the possible values of x_L, then I_G can be plotted as a function of x_L^2. The interception with the y-axis is then $I_R + 2\, I_W$.

On www.frederiksen.eu you will find a complete spreadsheet for calculating moments of inertia etc...

Search for part number 2181.00

Literature

D. Candela, K. M. Martini, R. V. Krotkov, and K. H. Langley:

Bessel's improved Kater pendulum in the teaching lab

American Journal of Physics - June 2001 - Volume 69, Issue 6, pp. 714

Equipment list

The following is suggested:

2181.00 Physical pendulum / Bessel pendulum

0015.10 Clamp

2002.50 Electronic Counter

1975.50 Fotocell unit

0016.00 Table clamp

0023.10 Universal bosshead (2 are used)

0008.50 Retort Stand Rod 25 cm

0008.20 Retort Stand Rod 75 cm

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