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Lagrangian Formulation of Dual Resonance Model

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In previous papers[1),2)] we have proposed the proper-time formulation of dual resonance model.[3),4)] In reference 2) we have introduced the new freedom (c^+, c) for the complete factorization. Further we have discussed the invariance properties of proper-time formulation of the dual resonance model under the transformations of three generators corresponding to Koba and Nielsen's Möbius transformation.[5),6)] In this paper, using these properties, we shall discuss the invariance properties of our Lagrangian formulation and derive the conservation of the currents, which corresponds to the Ward-like identities.[7)]

In II we obtain the proper-time equation in the interaction representation:

$$\left(\lambda V(z)-z\frac{\partial}{\partial z}\right)\Omega(P,z,n_\mu)=0\,, \qquad (1)$$

where

$$V(z)=\int\frac{d^3k}{\sqrt{2k_0(2\pi)^3}}A_k z^{(P^2-(P-k)^2)}$$

$$\times\exp\left\{-ik\left(\sum\sqrt{\frac{2}{n}}(a_n{}^+z^n+a_nz^{-n})\right)\right\}$$

$$\times\Gamma+\text{comp conj}$$

and

$$\Gamma=\sum_m f_m C^{+(m)}|0\rangle\langle 0|C^{(n)}f_n z^{(n-m)}\,,$$

$$f_m=\sqrt{(-1)^m\binom{\alpha_0-1}{m}}\,.$$

We have shown the invariance of (1) under the following three transformations $S_i(\delta)=\exp(-\delta L_i)$, since

$$S_i^{-1}(\delta)V(z)S_i(\delta)=\left(\frac{z}{z'}\right)\frac{dz'}{dz}V(z'),$$

where $i=\pm,0$. (2)*)

These three transformations S_+, S_-, S_0 correspond to

$$z'=z+\delta,\qquad z'=z(1-\delta z)^{-1},$$

$$z'=e^{\delta}z, \text{ respectively.} \tag{3}$$

We also derive the Ward-like identity[7] for

$$|P\rangle=V(1)\int_0^1 dzz^{-1}V(z)\int_0^z dz_1z_1^{-1}V(z_1)\cdots$$

$$\cdots\int_0^{z_{n-1}}dz_nz_n^{-1}V(z_n)|0\rangle, \tag{4}$$

namely

$$U|P\rangle=|P\rangle,$$

where $$U=\exp\delta(L_--L_0). \tag{4'}$$

This can be shown from (2) by $UZ^{-1}\times V(z)U^{-1}=z'^{-1}(dz'/dz)V(z')$, where $z'=(1-\delta')z(1-\delta'z)^{-1}$, since $U|P\rangle=(4)$ with z replaced by z', where $\delta'=(1-\exp\delta)$.

We consider the following unitary gauge transformations, which preserve $|z|=1$:

$$S_1=\exp(-i\delta(L_++L_-)),$$

$$S_2=\exp(-\delta(L_+-L_-)),$$

$$S_3=\exp(-i\delta L_0). \tag{5}$$

Using (3) and II, and noting Eqs. (5) which can be expressed as

$$S_1=\exp(-i\tanh\delta L_+)(\cos \mathrm{h}^2\delta)^{-L_0}$$

$$\times\exp(-i\tanh\delta L_-),$$

$$S_2=\exp(-\tanh\delta L_+)(\cosh^2\delta)^{-L_0}$$

$$\times\exp(+\tanh\delta L_-),$$

we obtain the following transformations of variable z in (1):

for S_1, $$z\to z_1'=(z+i\tanh\delta)\times(1-iz\tanh\delta)^{-1}, \tag{6·1}$$

S_2, $$z\to z_2'=(z+\tanh\delta)\times(1+z\tanh\delta)^{-1}, \tag{6·2}$$

S_3, $$z\to z_3'=e^{+i\delta}z \quad\text{or}\quad \tau'=\tau-\delta. \tag{6·3}$$

It is easily shown that we can define new τ' from $z'=\exp(-i\tau')$, since, from (6), $|z_i'|=1$ and $d\tau'/d\tau>0$, where $i=1,2,3$.

We go back to the Schrödinger representation, and consider the Lagrangian formulation. We assume the following Lagrangian density:

$$\mathcal{L}=\Big[-\partial_\mu\varphi^*\partial_\mu\varphi-\varphi^*(N+C^+C+m^2-\lambda V)$$

$$\times\varphi-\frac{i}{2}\left(\varphi^*\frac{\partial}{\partial\tau}\varphi-\frac{\partial\varphi^*}{\partial\tau}\varphi\right)\Big]. \tag{7}$$

By variational principle we have

$$i\frac{\partial\varphi}{\partial\tau}=(-L_0+\lambda V)\varphi,$$

$$-i\frac{\partial\varphi^*}{\partial\tau}=\varphi^*(-L_0+\lambda V). \tag{8}$$

We discuss the invariance of our Lagrangian under the following gauge transformations corresponding to (5) and (6):

$$S_1'\varphi=z_1'^{-L_0}\exp i\delta(L_++L_-)z^{L_0}\varphi, \tag{9·1}$$

$$S_2'\varphi=z_2'^{-L_0}\exp\delta(L_+-L_-)z^{L_0}\varphi, \tag{9·2}$$

$$S_3'\varphi=z_3'^{-L_0}\exp i\delta L_0z^{L_0}\varphi=\varphi. \tag{9·3}$$

The corresponding infinitesimal transformations are obtained from (6) and (9), by

$$\delta_1\varphi=i\varepsilon Q_1\varphi$$

$$=i\varepsilon\{(L_+z^{-1}+L_-z)-L_0(z+z^{-1})\}\varphi,$$

$$\delta\tau'=-\varepsilon(z+z^{-1}),$$

$$\delta_2\varphi=i\varepsilon Q_2\varphi$$

$$=\varepsilon\{(L_+z^{-1}-L_-z)-L_0(z^{-1}-z)\}\varphi,$$

$$\delta\tau^2=i\varepsilon(z^{-1}-z),$$

$$\delta_3\varphi=i\varepsilon Q_3\varphi=0,\qquad \delta\tau^3=-\varepsilon, \tag{10}$$

*) $L_+=\sum_{n=1}^{\infty}\sqrt{n(n+1)}a_{n+1}^+a_n-i\sqrt{2}Pa_1^+$
$-f_{n+1}/f_nC^{+(n+1)}|0\rangle\langle 0|C^{(n)}(n+1)$,
$L_-=(L_+)^+$, $\quad L_0=P^2+N+C^+C+m^2$.

where ε is infinitesimal δ in (6) and (9).

By the conventional variational principle, we obtain the current $J_\mu{}^i$ and $\rho_\tau{}^i$,

$$\delta\int d^4x\int d\tau L=\int d^4x\int d\tau \times\left(\frac{\partial \mathring{J}_\mu{}^i}{\partial x_\mu}+\frac{\partial \rho_\tau{}^i}{\partial \tau}\right)=0\,. \qquad (11)$$

However, the situation is somewhat different from the conventional one, since infinitesimal operator of (10) includes derivative with respect to x_μ. Thus, (11) is invariant only by adding suitable four divergence $\partial_\mu A_\mu$. A_μ is determined by real calculation. Thus we get the conserved current from (10).

$$J_\mu{}^i=\mathring{J}_\mu{}^i+A_\mu{}^i=-i(\partial_\mu\varphi^*\vec{Q}^i\varphi-\varphi^*\overleftarrow{Q}^i\partial_\mu\varphi) + (\partial_\mu\varphi^*\partial_\tau\varphi+\partial_\tau\varphi^*\partial_\mu\varphi)\left(\frac{\delta\tau^i}{\varepsilon}\right) +i\frac{1}{2}\partial_\mu(\varphi^*(\vec{Q^i}-\overleftarrow{Q^i})\varphi) +\frac{1}{2}\partial_\mu(\varphi^*\varphi)\frac{d}{d\tau}\left(\frac{\delta\tau^i}{\varepsilon}\right),$$

$$\rho_\tau{}^i=\varphi^*\frac{\vec{Q}^i+\overleftarrow{Q}^i}{2}\varphi +\left(\frac{i}{2}\varphi^*(\vec{\partial}_\tau-\overleftarrow{\partial}_\tau)\varphi+L\right)\left(\frac{\delta\tau^i}{\varepsilon}\right). \qquad (12)$$

They satisfy the conservation of currents

$$\frac{\partial}{\partial x_\mu}J_\mu{}^i+\frac{\partial}{\partial\tau}\rho_\tau{}^i=0\,. \qquad (13)$$

We can also derive Ward-like identity[7] from the following equation,

$$\int_{-\infty}^{+\infty}d^4x\int_{-\infty}^{0}d\tau\left(\frac{\partial J_\mu{}^i}{\partial x_\mu}+\frac{\partial\rho_\tau{}^i}{\partial\tau}\right) =\int d^4x\rho_\tau{}^i(\tau=0)=0\,. \qquad (14)$$

If we notice the z-independence of φ for multiparticle amplitude, which is easily verified by perturbation theory, then we get

$$\varphi^*((L_+-L_0)\pm(L_--L_0))\varphi=0\,, \qquad (15)$$

for Q_1, Q_2, which correspond to Ward-like identity (4′).

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