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Algorithm and Complexities Lab Report 03

on

**‘Brute-force, Greedy and Dynamic Programming implementation in
0/1 and Fractional KnapSack’**

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1. Purpose

Solving Knapsack problems using different algorithm design strategies.

2. Tasks

Solving the Knapsack problem using the following strategies:

Pseudocodes for each of these methods along with the source code of my program (and test cases).

2.1. Brute-force method (Both fractional and 0/1 Knapsack)

2.1.1. Brute-force Fractional Knapsack

2.1.1.1. Pseudocode:

```
FRACTIONAL-KNAPSACK-BRUTE-FORCE(profits,
weights, capacity)

1  n = profits.length // Number of items

2  max_profit = 0 // Initialize maximum
   profit

3  best_fractions = new Array(n).fill(0) //
   Array to store best fractions

4  for each possible combination of
   fractions:

5      current_profit = 0

6      current_weight = 0

7      for j = 0 to n - 1

8          current_profit = current_profit
+ profits[j] * fractions[j]
```

```

9             current_weight = current_weight
+ weights[j] * fractions[j]

10         if current_weight ≤ capacity AND
current_profit > max_profit

11             max_profit = current_profit

12             best_fractions =
fractions.copy()

13     return max_profit, best_fractions

```

2.1.1.2. Code:

```

from tabulate import tabulate

def bfmKSfract(p, w, m):

    # Ensure that the profit list and weight list are of the
same length

    assert len(p) == len(w), "p and w differ"

    # Get the number of items

    n = len(p)

    # Initialize the maximum profit to 0

    max_profit = 0

    # Set the total weight limit to m

    total_weight = m

    # Initialize the solution string to an empty string

    soln = ''

```

```

# Initialize table data list

table_data = []

# Iterate through all possible combinations of items

for i in range(2**n):

    # Convert the current combination number to a binary
string
    s = bin(i)[2:].rjust(n, '0')

    # Initialize profit and weight to 0

    profit = 0

    weight = 0

    # Calculate profit and weight for the current
combination
    for j in range(n):

        if s[j] == '1':

            profit += p[j]

            weight += w[j]

    # Calculate fractional weight for '0' elements in s

    if weight > total_weight:

        continue

    remaining_weight = total_weight - weight

    if remaining_weight > 0:

```

```

        total_zero_weight = sum(w[j] for j in range(n)
if s[j] == '0')

        if total_zero_weight > 0:

            fraction = remaining_weight /
total_zero_weight

            profit += fraction * sum(p[j] for j in
range(n) if s[j] == '0')

            weight += fraction * total_zero_weight

        # Append combination, profit, and weight to table
data

        table_data.append([s, round(profit, 2),
round(weight, 2)])

        # Update maximum profit and solution if the current
combination yields higher profit

        if profit > max_profit:

            max_profit = profit

            soln = s

        # Print the combinations, profits, and weights in a
tabular format

        print(tabulate(table_data, headers=['Combination',
'Profit', 'Weight'], tablefmt='grid'))

        # Print the best combination and the maximum profit

        print(f"\nBest combination: {soln}")

```

```
print(f"Maximum profit: {int(max_profit)}")

# Return the maximum profit found (converted to integer)

return int(max_profit)

# Function to run test cases automatically

def run_test_cases():

    # Test case 1: Basic example

    p1 = [60, 100, 120]

    w1 = [10, 20, 30]

    m1 = 50

    print("Test Case 1:")

    bfmKSfract(p1, w1, m1)

    # Test case 2: All items can be selected

    p2 = [10, 20, 30]

    w2 = [1, 2, 3]

    m2 = 6

    print("\nTest Case 2:")

    bfmKSfract(p2, w2, m2)

    # Test case 3: Fractional selection (similar behavior to
test edge case)

    p3 = [10, 20, 30]
```

```
w3 = [2, 3, 4]

m3 = 5

print("\nTest Case 3:")

bfmKSfract(p3, w3, m3)


# Test case 4: Edge case with zero capacity

p4 = [10, 20, 30]

w4 = [1, 2, 3]

m4 = 0

print("\nTest Case 4:")

bfmKSfract(p4, w4, m4)


# Uncomment the following test cases to use them in the
future


# Test case 5: High profit low weight items

# p5 = [100, 200, 300]

# w5 = [1, 2, 3]

# m5 = 4

# print("\nTest Case 5:")

# bfmKSfract(p5, w5, m5)


# Test case 6: Low profit high weight items

# p6 = [10, 20, 30]
```

```

# w6 = [10, 20, 30]

# m6 = 15

# print("\nTest Case 6:")

# bfmKSfract(p6, w6, m6)

# Run the test cases

if __name__ == "__main__":

    run_test_cases()

```

2.1.1.2.1. Inputs:

```

Maximum profit: 0
● (base) reewajkhanal.rk10@RK10 LAB03 % python bfmKSfracttestcases.py

```

2.1.1.2.2. Outputs:

```

Test Case 1:
+-----+-----+-----+
| Combination | Profit | Weight |
+-----+-----+-----+
| 000 | 233.33 | 50 |
+-----+-----+-----+
| 001 | 226.67 | 50 |
+-----+-----+-----+
| 010 | 235 | 50 |
+-----+-----+-----+
| 011 | 220 | 50 |
+-----+-----+-----+
| 100 | 236 | 50 |
+-----+-----+-----+
| 101 | 230 | 50 |
+-----+-----+-----+
| 110 | 240 | 50 |
+-----+-----+-----+

Best combination: 110
Maximum profit: 240

Test Case 2:
+-----+-----+-----+
| Combination | Profit | Weight |
+-----+-----+-----+
| 000 | 60 | 6 |
+-----+-----+-----+
| 001 | 60 | 6 |
+-----+-----+-----+
| 010 | 60 | 6 |
+-----+-----+-----+
| 011 | 60 | 6 |
+-----+-----+-----+
| 100 | 60 | 6 |
+-----+-----+-----+
| 101 | 60 | 6 |
+-----+-----+-----+
| 110 | 60 | 6 |
+-----+-----+-----+
| 111 | 60 | 6 |
+-----+-----+-----+

Best combination: 000
Maximum profit: 60

```

```

Test Case 3:
+-----+-----+-----+
| Combination | Profit | Weight |
+-----+-----+-----+
| 000 | 33.33 | 5 |
+-----+-----+-----+
| 001 | 36 | 5 |
+-----+-----+-----+
| 010 | 33.33 | 5 |
+-----+-----+-----+
| 100 | 31.43 | 5 |
+-----+-----+-----+
| 110 | 30 | 5 |
+-----+-----+-----+

Best combination: 001
Maximum profit: 36

Test Case 4:
+-----+-----+-----+
| Combination | Profit | Weight |
+-----+-----+-----+
| 000 | 0 | 0 |
+-----+-----+-----+

Best combination:
Maximum profit: 0

```


2.1.2. Brute-force 0/1 Knapsack

2.1.2.1. Pseudocode:

```
BRUTE-FORCE-KNAPSACK(profits, weights,
capacity)

1  n = profits.length // Number of items

2  max_profit = 0 // Initialize maximum
   profit

3  best_items = [] // List to track included
   items

4  for i = 0 to  $2^n - 1$  // Iterate through
   all combinations

5      current_profit = 0

6      current_weight = 0

7      items_included = []

8      for j = 0 to n - 1 // Check each
   item in the combination

9          if (i >> j) & 1 == 1 // If jth
   bit in i is 1, item is included

10             current_profit =
   current_profit + profits[j]

11             current_weight =
   current_weight + weights[j]

12             items_included.append(True)

13         else

14             items_included.append(False)
```

```

15         if current_weight ≤ capacity AND
current_profit > max_profit
16             max_profit = current_profit
17             best_items = items_included
18     return max_profit, best_items

```

2.1.2.2. Code:

```

from tabulate import tabulate

def bfmKS01(p, w, m):

    # Ensure that the profit list and weight list are of the
same length

    assert len(p) == len(w), "p and w differ"

    # Get the number of items

    n = len(p)

    # Initialize the maximum profit to 0

    max_profit = 0

    # Set the total weight limit to m

    total_weight = m

    # Initialize the solution string to an empty string

    soln = ''

    # Initialize table data list

    table_data = []

```

```

# Iterate through all possible combinations of items

for i in range(2**n):

    # Convert the current combination number to a binary
string
    s = bin(i)[2:].rjust(n, '0')

    # Calculate the total profit for the current
combination
    profit = sum((int(s[j])) * p[j] for j in range(n))

    # Calculate the total weight for the current
combination
    weight = sum((int(s[j])) * w[j] for j in range(n))

    # Append combination, profit, and weight to table
data
    table_data.append([s, profit, weight])

    # If the current combination's profit is greater
than the max profit

    # and its weight is within the allowed limit, update
the max profit and solution

    if profit > max_profit and weight <= total_weight:

        max_profit = profit

        soln = s

    # Print the combinations, profits, and weights in a
tabular format

```

```

    print(tabulate(table_data, headers=['Combination',
'Profit', 'Weight'], tablefmt='grid'))

# Print the best combination and the maximum profit

print(f"\nBest combination: {soln}")

print(f"Maximum profit: {max_profit}")

# Return the maximum profit found

return max_profit

# Function to run test cases automatically

def run_test_cases():

    # Test case 1: Basic example

    p1 = [60, 100, 120]

    w1 = [10, 20, 30]

    m1 = 50

    print("Test Case 1:")

    bfmKS01(p1, w1, m1)

    # Test case 2: All items can be selected

    p2 = [10, 20, 30]

    w2 = [1, 2, 3]

    m2 = 6

    print("\nTest Case 2:")

```

```
bfmKS01(p2, w2, m2)

# Test case 3: Fractional selection (similar behavior to
test edge case)

p3 = [10, 20, 30]

w3 = [2, 3, 4]

m3 = 5

print("\nTest Case 3:")

bfmKS01(p3, w3, m3)

# Test case 4: Edge case with zero capacity

p4 = [10, 20, 30]

w4 = [1, 2, 3]

m4 = 0

print("\nTest Case 4:")

bfmKS01(p4, w4, m4)

# Uncomment the following test cases to use them in the
future

# Test case 5: High profit low weight items

# p5 = [100, 200, 300]

# w5 = [1, 2, 3]

# m5 = 4

# print("\nTest Case 5:")
```

```

# bfmKS01(p5, w5, m5)

# Test case 6: Low profit high weight items

# p6 = [10, 20, 30]

# w6 = [10, 20, 30]

# m6 = 15

# print("\nTest Case 6:")

# bfmKS01(p6, w6, m6)

# Run the test cases

if __name__ == "__main__":

    run_test_cases()

```

2.1.2.2.1. Inputs:

```

● (base) reewajkhanal.rk10@RK10 LAB03 % python bfmKS01testcases.py

```

2.1.2.2.2. Outputs:

```

Test Case 1:
+-----+-----+-----+
| Combination | Profit | Weight |
+-----+-----+-----+
| 000 | 0 | 0 |
+-----+-----+-----+
| 001 | 120 | 30 |
+-----+-----+-----+
| 010 | 100 | 20 |
+-----+-----+-----+
| 011 | 220 | 50 |
+-----+-----+-----+
| 100 | 60 | 10 |
+-----+-----+-----+
| 101 | 180 | 40 |
+-----+-----+-----+
| 110 | 160 | 30 |
+-----+-----+-----+
| 111 | 280 | 60 |
+-----+-----+-----+

Best combination: 011
Maximum profit: 220

Test Case 2:
+-----+-----+-----+
| Combination | Profit | Weight |
+-----+-----+-----+
| 000 | 0 | 0 |
+-----+-----+-----+
| 001 | 30 | 3 |
+-----+-----+-----+
| 010 | 20 | 2 |
+-----+-----+-----+
| 011 | 50 | 5 |
+-----+-----+-----+
| 100 | 10 | 1 |
+-----+-----+-----+
| 101 | 40 | 4 |
+-----+-----+-----+
| 110 | 30 | 3 |
+-----+-----+-----+
| 111 | 60 | 6 |
+-----+-----+-----+

Best combination: 111
Maximum profit: 60

```

```

Test Case 3:
+-----+-----+-----+
| Combination | Profit | Weight |
+-----+-----+-----+
| 000 | 0 | 0 |
+-----+-----+-----+
| 001 | 30 | 4 |
+-----+-----+-----+
| 010 | 20 | 3 |
+-----+-----+-----+
| 011 | 50 | 7 |
+-----+-----+-----+
| 100 | 10 | 2 |
+-----+-----+-----+
| 101 | 40 | 6 |
+-----+-----+-----+
| 110 | 30 | 5 |
+-----+-----+-----+
| 111 | 60 | 9 |
+-----+-----+-----+

Best combination: 001
Maximum profit: 30

Test Case 4:
+-----+-----+-----+
| Combination | Profit | Weight |
+-----+-----+-----+
| 000 | 0 | 0 |
+-----+-----+-----+
| 001 | 30 | 3 |
+-----+-----+-----+
| 010 | 20 | 2 |
+-----+-----+-----+
| 011 | 50 | 5 |
+-----+-----+-----+
| 100 | 10 | 1 |
+-----+-----+-----+
| 101 | 40 | 4 |
+-----+-----+-----+
| 110 | 30 | 3 |
+-----+-----+-----+
| 111 | 60 | 6 |
+-----+-----+-----+

Best combination:
Maximum profit: 0

```

2.2. Greedy method (Fractional Knapsack)

2.2.1. Pseudocode:

```
FRACTIONAL-KNAPSACK-GREEDY(profits, weights,
capacity)

1  n = profits.length // Number of items

2  ratio = new Array(n) // Array to store
profit/weight ratios

3  for i = 0 to n - 1

4      ratio[i] = profits[i] / weights[i] //
Calculate profit/weight ratio

5  sorted_indices = argsort(ratio, reverse=True)
// Sort items by ratio in descending order

6  max_profit = 0

7  current_weight = 0

8  for i in sorted_indices

9      if current_weight + weights[i] ≤ capacity
// Can take the entire item

10         current_weight = current_weight +
weights[i]

11         max_profit = max_profit + profits[i]

12     else // Can only take a fraction

13         remaining = capacity - current_weight

14         fraction = remaining / weights[i]

15         max_profit = max_profit + profits[i] *
fraction
```

```
16         break // Knapsack is full
17     return max_profit
```

2.2.2. Code:

```
from tabulate import tabulate

def gmKSfract(p, w, m):

    # Ensure that the profit list and weight list are of the same
length

    assert len(p) == len(w), "p and w differ"

    # Get the number of items

    n = len(p)

    items = list(range(n))

    # Sort the items in descending order of their profit/weight
ratio

    items.sort(key=lambda i: p[i] / w[i], reverse=True)

    total_weight = 0

    max_profit = 0

    # Initialize table data list

    table_data = []

    for i in items:
```



```

# Check if adding the entire item exceeds the capacity

if total_weight + w[i] <= m:

    total_weight += w[i]

    max_profit += p[i]

    # Append selected item's data to table data with
fraction 1 indicating the entire item is selected

    table_data.append([i+1, p[i], w[i], 1.0])

else:

    # Calculate the remaining capacity

    remaining_weight = m - total_weight

    # Calculate the fraction of the item that can be added
to the knapsack

    fraction = remaining_weight / w[i]

    # Update the profit and weight considering the fraction

    max_profit += p[i] * fraction

    # Append selected item's data to table data with the
calculated fraction

    table_data.append([i+1, p[i], w[i], round(fraction,
2)])

    # Break the loop as the knapsack is now full

    break

# Print the items selected and their fractions (if any) in a
tabular format

print(tabulate(table_data, headers=['Item', 'Profit', 'Weight',
'Fraction'], tablefmt='grid'))

```

```
# Print the maximum profit

print(f"\nMaximum profit: {int(max_profit)}")

# Return the maximum profit found (converted to integer)

return int(max_profit)

# Function to run test cases automatically

def run_test_cases():

    # Test case 1: Basic example

    p1 = [60, 100, 120]

    w1 = [10, 20, 30]

    m1 = 50

    print("Test Case 1:")

    gmKSfract(p1, w1, m1)

    # Test case 2: All items can be selected

    p2 = [10, 20, 30]

    w2 = [1, 2, 3]

    m2 = 6

    print("\nTest Case 2:")

    gmKSfract(p2, w2, m2)

    # Test case 3: Fractional selection
```

```
p3 = [10, 20, 30]

w3 = [2, 3, 4]

m3 = 5

print("\nTest Case 3:")

gmKSfract(p3, w3, m3)


# Test case 4: Edge case with zero capacity

p4 = [10, 20, 30]

w4 = [1, 2, 3]

m4 = 0

print("\nTest Case 4:")

gmKSfract(p4, w4, m4)


# Uncomment the following test cases to use them in the future


# Test case 5: High profit low weight items

# p5 = [100, 200, 300]

# w5 = [1, 2, 3]

# m5 = 4

# print("\nTest Case 5:")

# gmKSfract(p5, w5, m5)


# Test case 6: Low profit high weight items

# p6 = [10, 20, 30]
```

```

# w6 = [10, 20, 30]

# m6 = 15

# print("\nTest Case 6:")

# gmKSfract(p6, w6, m6)

# Run the test cases

if __name__ == "__main__":

    run_test_cases()

```

2.2.2.1. Inputs:

```

(base) reewajkhanal.rk10@RK10 LAB03 % python gmKSfracttestcases.py

```

2.2.2.2. Outputs:

```

Test Case 1:


| Item | Profit | Weight | Fraction |
|------|--------|--------|----------|
| 1    | 60     | 10     | 1        |
| 2    | 100    | 20     | 1        |
| 3    | 120    | 30     | 0.67     |


Maximum profit: 240

Test Case 2:


| Item | Profit | Weight | Fraction |
|------|--------|--------|----------|
| 1    | 10     | 1      | 1        |
| 2    | 20     | 2      | 1        |
| 3    | 30     | 3      | 1        |


Maximum profit: 60

Test Case 3:


| Item | Profit | Weight | Fraction |
|------|--------|--------|----------|
| 3    | 30     | 4      | 1        |
| 2    | 20     | 3      | 0.33     |


Maximum profit: 36

Test Case 4:


| Item | Profit | Weight | Fraction |
|------|--------|--------|----------|
| 1    | 10     | 1      | 0        |


Maximum profit: 0

```

2.3. Dynamic programming (0/1 Knapsack)

2.3.1. 0/1 Knapsack using Tabulation - Bottom Up (Iterative) Method

2.3.1.1. Pseudocode:

```
KNAPSACK_01_TABULATION(profits, weights,
capacity)

1  n = profits.length // Number of items

2  dp = new table[n+1][capacity+1] // Create
a table to store solutions

3  for i = 0 to n

4      for w = 0 to capacity

5          if i == 0 OR w == 0 // Base case:
no items or no capacity

6              dp[i][w] = 0

7          else if weights[i-1] > w
// Current item's weight exceeds capacity

8              dp[i][w] = dp[i-1][w]
// Exclude the current item

9          else // Choose max value: include
or exclude current item

10             dp[i][w] = max(dp[i-1][w],
profits[i-1] + dp[i-1][w - weights[i-1]])

11 max_profit = dp[n][capacity]

12 taken = new array[n] filled with false
// Track which items are taken
```

```

13 w = capacity
14 for i = n downto 1
15     if dp[i][w] != dp[i-1][w] // Item was
        included
16         taken[i-1] = true
17         w = w - weights[i-1]
18 return max_profit, taken // Return max
        profit and items taken

```

2.3.1.2. Code:

```

import pandas as pd

# Function to solve the knapsack problem using Tabulation
# (Bottom-Up) approach

def knapsack_tabulation(wt, val, W, n):

    # Initialize a table to store results of subproblems

    t = [[0 for _ in range(W + 1)] for _ in range(n + 1)]

    # Build the table in bottom-up manner

    for i in range(n + 1):

        for w in range(W + 1):

            if i == 0 or w == 0:

                t[i][w] = 0

            elif wt[i - 1] <= w:

```

```

        t[i][w] = max(val[i - 1] + t[i - 1][w - wt[i - 1]], t[i - 1][w])

    else:

        t[i][w] = t[i - 1][w]

    return t[n][W], t

# Function to print the memoization table in tabular format
using pandas

def print_tabulation_table(table, n, W):

    df = pd.DataFrame(table)

    df.index = ["Item " + str(i) for i in range(n + 1)]

    df.columns = ["W" + str(i) for i in range(W + 1)]

    print("\nMemoization Table:")

    print(df)

# Function to run test cases

def run_test_cases():

    # Test case 1: Basic example

    profit1 = [60, 100, 120]

    weight1 = [10, 20, 30]

    W1 = 50

    n1 = len(profit1)

    max_profit1, memoization_table1 = knapsack_tabulation(weight1, profit1, W1, n1)

```

```

print(f"\nTest Case 1: Maximum profit = {max_profit1}")

print_tabulation_table(memoization_table1, n1, W1)

# # Test case 2: Edge case with zero capacity

# profit2 = [10, 20, 30]

# weight2 = [1, 2, 3]

# W2 = 0

# n2 = len(profit2)

# max_profit2, memoization_table2 =
knapsack_tabulation(weight2, profit2, W2, n2)

# print(f"\nTest Case 2: Maximum profit =
{max_profit2}")

# print_tabulation_table(memoization_table2, n2, W2)

# # Test case 3: All items have zero profit

# profit3 = [0, 0, 0]

# weight3 = [10, 20, 30]

# W3 = 50

# n3 = len(profit3)

# max_profit3, memoization_table3 =
knapsack_tabulation(weight3, profit3, W3, n3)

# print(f"\nTest Case 3: Maximum profit =
{max_profit3}")

# print_tabulation_table(memoization_table3, n3, W3)

# Run the test cases

```



```
if __name__ == "__main__":

    run_test_cases()
```

2.3.1.2.1. Inputs:

```
● (base) reewajkhanal.rk10@RK10 LAB03 % python dp01KStestcasesTabulationBottomUpIterative.py
```

2.3.1.2.2. Outputs:

```
Test Case 1: Maximum profit = 220
Memoization Table:
W50  W0 W1 W2 W3 W4 W5 W6 W7 W8 W9 W10 W11 W12 W13 W14 W15 W16 ... W34 W35 W36 W37 W38 W39 W40 W41 W42 W43 W44 W45 W46 W47 W48 W49
Item 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ... 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
Item 1 0 0 0 0 0 0 0 0 0 0 0 60 60 60 60 60 60 60 ... 60 60 60 60 60 60 60 60 60 60 60 60 60 60 60
Item 2 0 0 0 0 0 0 0 0 0 0 0 60 60 60 60 60 60 60 ... 160 160 160 160 160 160 160 160 160 160 160 160 160 160
Item 3 0 0 0 0 0 0 0 0 0 0 0 60 60 60 60 60 60 60 ... 160 160 160 160 160 160 180 180 180 180 180 180 180 180 180
220
[4 rows x 51 columns]
```

2.3.2. 0/1 Knapsack using Memoization - Top Down (Recursive) Method

2.3.2.1. Pseudocode:

```
KNAPSACK_01_MEMOIZATION(profits, weights,
    capacity, n, memo)
```

```
1  if memo[n][capacity] is not null // Check
    if the result is already calculated
```

```
2      return memo[n][capacity] // Return
    the cached value
```

```
3  if n == 0 OR capacity == 0 // Base case:
    no items or no capacity
```

```
4      result = 0
```

```

5  else if weights[n - 1] > capacity
    // Current item's weight exceeds capacity

6      result =
    KNAPSACK_01_MEMOIZATION(profits, weights,
    capacity, n - 1, memo) // Exclude

7  else                                     //
    Choose max value: include or exclude current
    item

8      include = profits[n - 1] +
    KNAPSACK_01_MEMOIZATION(profits, weights,
    capacity - weights[n - 1], n - 1, memo)

9      exclude =
    KNAPSACK_01_MEMOIZATION(profits, weights,
    capacity, n - 1, memo)

10     result = max(include, exclude)

11     memo[n][capacity] = result // Store the
    result in the memoization table

12     return result

```

2.3.2.2. Code:

```

#MemoizationTopDownRecursive

from tabulate import tabulate

# Class to represent a node in the decision tree

class TreeNode:

    def __init__(self, n, W, profit, include=None):

        self.n = n # The number of items being considered

```

```

        self.W = W # The current capacity of the knapsack

        self.profit = profit # The current profit at this
node

        self.include = include # Whether the current item
is included (True/False)

        self.left = None # Left child node (include the
current item)

        self.right = None # Right child node (exclude the
current item)

# Function to solve the knapsack problem using memoization
and build the decision tree

def knapsack(wt, val, W, n, root):

    # Base conditions: if there are no items or the capacity
of the knapsack is 0, return 0

    if n == 0 or W == 0:

        return 0

    # Check if the result is already in the memoization
table

    if t[n][W] != -1:

        return t[n][W]

    # If the weight of the nth item is less than or equal to
the knapsack capacity W,

    # we have two options: include the nth item or exclude
it

    if wt[n-1] <= W:

```

```

        # Create left child (include the item)

        root.left = TreeNode(n-1, W-wt[n-1], val[n-1], True)

        include_profit = val[n-1] + knapsack(wt, val,
W-wt[n-1], n-1, root.left)

        # Create right child (exclude the item)

        root.right = TreeNode(n-1, W, 0, False)

        exclude_profit = knapsack(wt, val, W, n-1,
root.right)

        # Store the maximum of including or excluding the
item

        t[n][W] = max(include_profit, exclude_profit)

    else:

        # If the weight of the nth item is greater than the
knapsack capacity W,

        # we cannot include the nth item in the knapsack

        root.right = TreeNode(n-1, W, 0, False)

        t[n][W] = knapsack(wt, val, W, n-1, root.right)

    # Return the value stored in the memoization table

    return t[n][W]

# Function to print the decision tree in a tree-like format

def print_tree(node, indent="", branch="Root"):

    if node is not None:

        print(f"{indent}└─ {branch}: Item {node.n},
Capacity {node.W}, Profit {node.profit}, Include
{node.include}")

```

```

        if node.left:

            print_tree(node.left, indent + "    ", "Left")

        if node.right:

            print_tree(node.right, indent + "    ", "Right")

# Function to run test cases

def run_test_cases():

    # Test case 1: Example from textbooks or online
    tutorials

    profit = [60, 100, 120]

    weight = [10, 20, 30]

    W = 50

    n = len(profit)

    global t

    t = [[-1 for i in range(W + 1)] for j in range(n + 1)]

    root = TreeNode(n, W, 0)

    max_profit = knapsack(weight, profit, W, n, root)

    print(f"\nTest Case 1: Maximum profit = {max_profit}\n")

    print("Decision Tree:")

    print_tree(root)

```

```

# # Test case 2: Custom scenario with different values

# profit = [70, 20, 39, 100, 50]

# weight = [31, 10, 20, 40, 50]

# W = 100

# n = len(profit)


# t = [[-1 for i in range(W + 1)] for j in range(n + 1)]


# root = TreeNode(n, W, 0)

# max_profit = knapsack(weight, profit, W, n, root)


# print(f"\nTest Case 2: Maximum profit =
{max_profit}\n")

# print("Decision Tree:")

# print_tree(root)


# # Test case 3: Edge case with very small capacity and
weights

# profit = [10, 20, 30]

# weight = [1, 2, 3]

# W = 2

# n = len(profit)


# t = [[-1 for i in range(W + 1)] for j in range(n + 1)]

```

```

# root = TreeNode(n, W, 0)

# max_profit = knapsack(weight, profit, W, n, root)

# print(f"\nTest Case 3: Maximum profit =
{max_profit}\n")

# print("Decision Tree:")

# print_tree(root)

if __name__ == '__main__':

    run_test_cases()

```

2.3.2.2.1. Inputs:

```

[1 rows x 51 columns]
● (base) reewajkhanal.rk10@RK10 LAB03 % python dp01KTestcasesMemoizationTopDownRecursive.py

```

2.3.2.2.2. Outputs:

```

Test Case 1: Maximum profit = 220

Decision Tree:
├─ Root: Item 3, Capacity 50, Profit 0, Include None
│   └─ Left: Item 2, Capacity 20, Profit 120, Include True
│       ├── Left: Item 1, Capacity 0, Profit 100, Include True
│       └─ Right: Item 1, Capacity 20, Profit 0, Include False
│           ├── Left: Item 0, Capacity 10, Profit 60, Include True
│           └─ Right: Item 0, Capacity 20, Profit 0, Include False
│   └─ Right: Item 2, Capacity 50, Profit 0, Include False
│       ├── Left: Item 1, Capacity 30, Profit 100, Include True
│       │   ├── Left: Item 0, Capacity 20, Profit 60, Include True
│       │   └─ Right: Item 0, Capacity 30, Profit 0, Include False
│       └─ Right: Item 1, Capacity 50, Profit 0, Include False
│           ├── Left: Item 0, Capacity 40, Profit 60, Include True
│           └─ Right: Item 0, Capacity 50, Profit 0, Include False

```