Implementation of FDM in Solving EM Field Problem over Exact Solution and Accuracy Analysis

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***Abstract*— This paper is a discussion on the implementation of Finite Difference Method (FDM) in solving electromagnetic (EM) field problem. The iterative method and successive over relaxation (SOR) method is discussed and implemented as a technique of FDM. Here the exact solution, using separation of variables is also discussed and implemented. And finally, a comparison of accuracy is shown and discussed among these methods.**

I. INTRODUCTION

The finite difference method (FDM) is one of the most powerful numerical techniques for solving partial differential equations (PDE) of any kind. Because all electromagnetic (EM) field problems are represented by scalar or vector PDEs, the FDM can solve both the electric and magnetic fields in various media approximately. Of course, the most satisfactory solution of EM field problem is an exact mathematical one. But in many practical cases, such solution cannot be applied – thus numerical approximate solution is needed. Though it is named 'approximate', the data is almost accurate and in usable format. This paper has an intension to prove this level of accuracy.

II. BOUNDARY CONDITION

Most EM field problems deal with three boundary conditions:

1. Dirichlet boundary condition,
2. Neumann boundary condition
3. mixed boundary condition

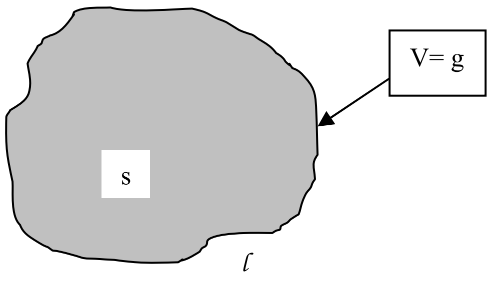

Fig. 1 Dirichlet boundary condition

In this paper, we use the Dirichlet boundary condition. Let us consider a region s bounded by a curve l as shown in fig. 1. To determine the potential distribution V in the region s such that the potential along l is V=g, where g is a continuous potential function. Thus the condition along the boundary l is known as the Dirichlet boundary condition.

III. EXACT SOLUTION (SEPARATION OF VARIABLES)

For exact solution in solving EM problems analytical methods are used. The most powerful analytical method is the separation of variables which is very convenient for solving a PDE.

Consider an infinitely long rectangular conducting trough whose cross section is shown in fig. 2. The length of the solution domain is b=15m and the width is a=10m.

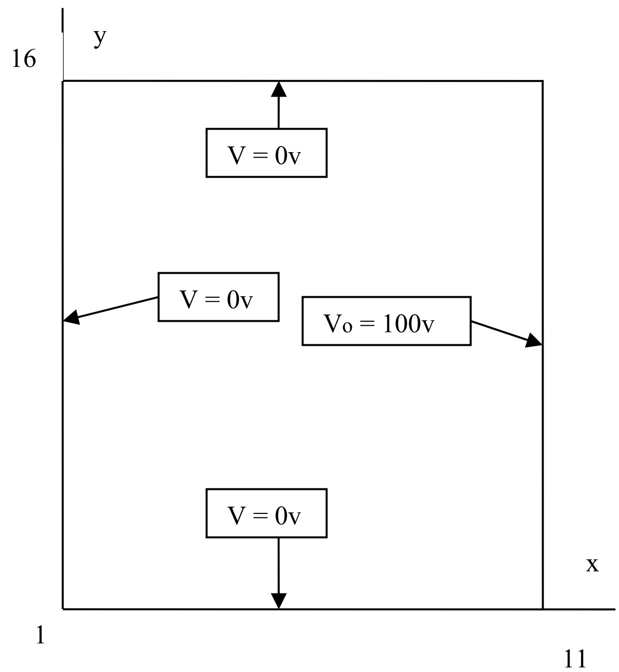

Fig. 2 Cross section of conducting trough with boundary value

Consider a 2-D Laplace equation,

$(del)^2V= \partial^2V(x,y)/\partial x^2 + \partial^2V(x,y)/\partial y^2 = 0$

;subject to Dirichlet boundary conditions,

$V(x,1)=0$, $V(x,16)=0$, $V(1,y)=0$, $V_0(16,y)=100v$

$$V(x,y) = 4\, V_0 /\pi \sum_{n=odd}^{\infty} \sin (n\pi x/a) \sinh (n\pi y/a) / n \sinh (n\pi b/a)$$

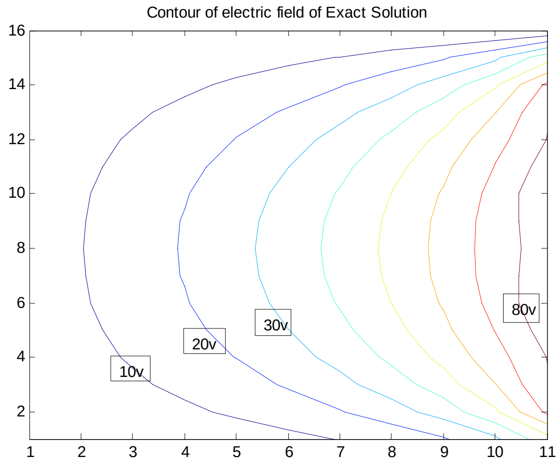

Fig. 3 Contour of electric field of Exact Solution

IV. Finite Difference Method (FDM)

FDM is a technique that divides the solution domain into finite discrete points, named as grid points, and replaces the PDE with a set of difference equations. Hence the solution is approximate; the error can be minimized to an acceptable level if the grid points are selected very close to one another.

In this paper, the length 15m is divided into 15 and the width 10m is divided into 10. So, both of them have same mesh size which is 0.1m.

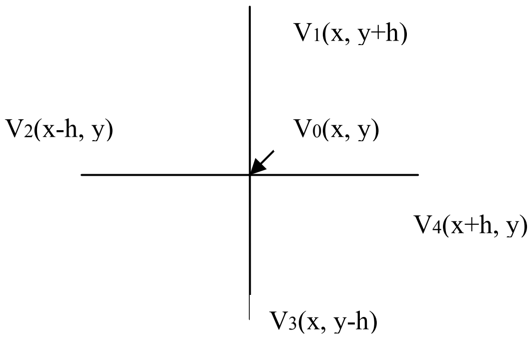

Fig. 4 The square mesh configuration (h = mesh size)

For a sqare mesh configuration, in fig. 4, we get,

$1/h^2\ (V_1 + V_2 + V_3 + V_4 - 4V_0) = 0$

So, $V_0 = \frac{1}{4}\ (V_1 + V_2 + V_3 + V_4)$

The mesh size is sufficiently small as per the solution domain. Here the FDM method is implemented after 50 times iteration for finding the accurate result of each grid points.

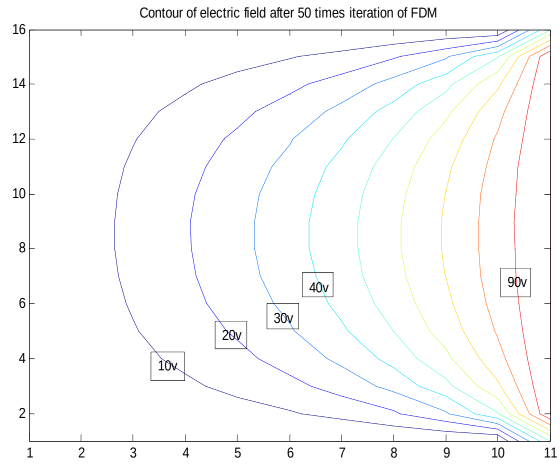

Fig. 5 Contour of electric field after 50 times iteration of FDM

V. Successive Over Relaxation (SOR)

The SOR method is basically an iterative algorithm that requires an initial guess for the potential at each node. Let V_n be the potential of a node after n-th iteration. Then the modified potential for the (n+1)-th iteration according to SOR method becomes,

$V_{n+1} = V_n + \alpha/4\ R_n$

Here α is the acceleration factor and for successful convergence $1 \leq \alpha < 2$. R is the residual. To achieve the accuracy R will be zero. For calculation time consumption, an error criterion $|\ V_{n+1} - V_n\ | << 1$ is set in the beginning of the iteration. When this criterion is satisfied, the iteration process is stopped.

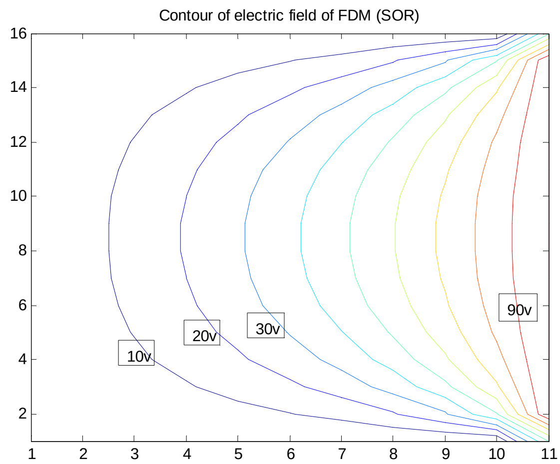

Fig. 6 Contour of electric field of FDM (SOR)

VI. Accuracy Analysis

The fig. 7 and fig. 8 shows the comparison of each node's potential of the solution domain of FDM after 50 times iteration and SOR respectfully to exact solution. The both comparisons assert that there is not significant error if we reject the exact solution. But the advantage is that we can avoid complex mathematical calculation for getting the exact solution by analytical method. Of course, SOR is more error free than FDM 50 times iteration.

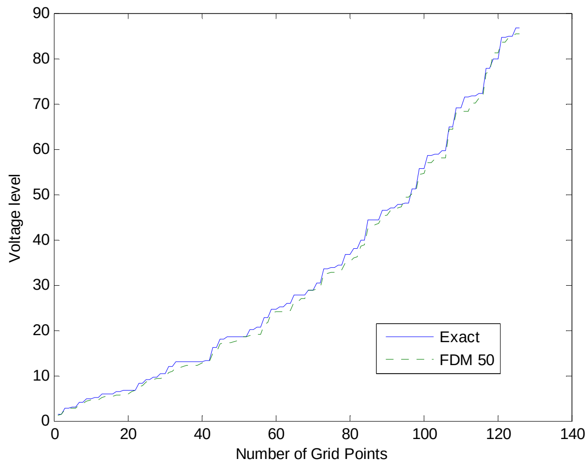

Fig. 7 Comparison between Exact and FDM after 50 times iteration

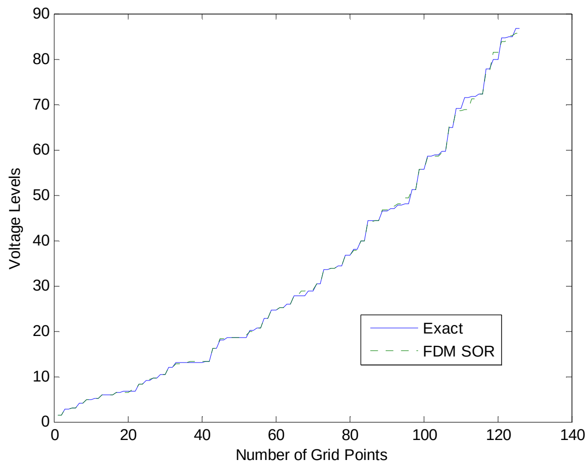

Fig. 8 Comparison between Exact and FDM SOR

VII. CONCLUSIONS

The FDM is a very powerful method, mentioned earlier of this paper. Analytical solution will be failed if PDE is non-linear, the solution region is a complex one, the boundary conditions are mixed types and the medium is inhomogeneous or anisotropic. Here the example is very simple but give us the assurance to rely on the approximate results achieved by the FDM.

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