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Max-Sum: Graph Coloring Problem

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1 BASICS OF MAX-SUM

Essential Points

1. The message sent from a node v on an edge e is the sum of the local function at v (or 0 if v is a variable node) with all messages received at v on edges other than e , summarized for the variable associated with e .

2. From variable to function message:

$$Q_{i\to j}(x_i) = \sum_{k\epsilon M_i - j} R_{k\to i}(x_i)$$

3. From function to variable message:

$$R_{j\to i}(x_i) = max_{x_j - i}[U_j(x_j) + \sum_{k\epsilon N_j - i} Q_{k\to j}(x_k)]$$

4. Once the variable node has received a message from each of its connected function nodes, the optimal variable assignment is found by locally calculating the function, $z_i(x_i)$,

$$Z_i(x_i) = \sum_{j\epsilon M_i} R_{j\to i}(x_i)$$

where M_i is a set of function indexes, indicating which function nodes are connected to variable node x_i.

1.1 Example

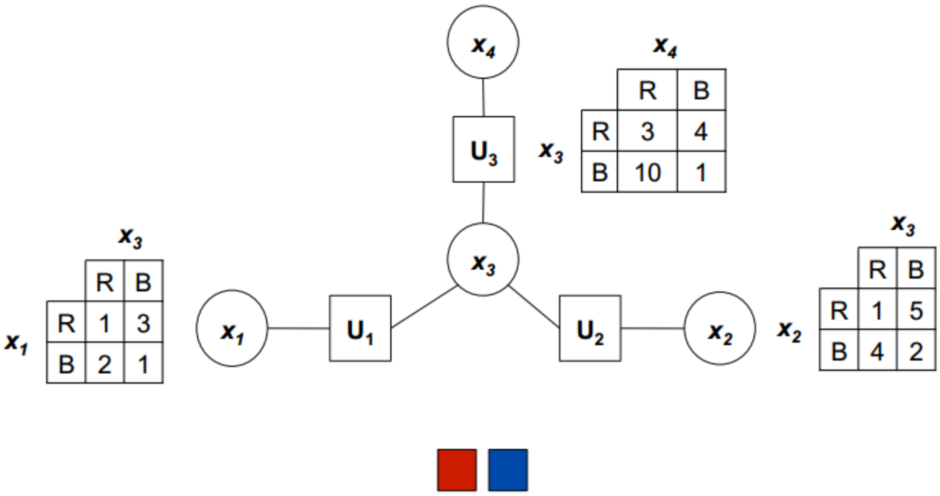

Figure 1.1: A Sample Factor Graph with 4 variables $(x_1, ..., x_4)$ and three utility functions $(U_1, ..., U_3)$. Each variable can takes value from the available domain: Red or Blue.

Complete Message Passing				
Steps	Messages			Message Type
1	$Q_{1\to1}(x_1) = \{0,0\}$	$Q_{2\to2}(x_2) = \{0,0\}$	$Q_{4\to3}(x_4) = \{0,0\}$	Variable to Function
2	$R_{1\to3}(x_3) = \{2,3\}$	$R_{2\to3}(x_3) = \{4,5\}$	$R_{3\to3}(x_3) = \{4,10\}$	Function to Variable
3	$Q_{3\to1}(x_3) = \{8,15\}$	$Q_{3\to2}(x_3) = \{6,13\}$	$Q_{3\to3}(x_3) = \{6,8\}$	Variable to Function
4	$R_{1\to1}(x_1) = \{18,16\}$	$R_{2\to2}(x_2) = \{18,15\}$	$R_{3\to4}(x_4) = \{18,10\}$	Function to Variable
5	$Z_1(x_1) = \{18,16\}$; $Z_2(x_2) = \{18,15\}$; $Z_3(x_3) = \{10,18\}$; $Z_4(x_4) = \{18,10\}$			At Completion (When each Variable has received messages from all of its neighbours)
6	$x_1 = R$; $x_2 = R$; $x_3 = B$; $x_4 = R$;			Final Color Assignment to Variables after finding $argmax_{x_i} z_i(x_i)$

Sample Calculations from Step 2:

$$R_{1\to3}(x_3) = max_{\{x_1,x_3\}-\{x_3\}}[U_1(x_1,x_3) + Q_{1\to1}(x_1)]$$
$$= max_{\{x_1\}}[U_1(x_1,x_3) + Q_{1\to1}(x_1)]$$

$$R_{1\to3}(x_3) = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$$
$$R_{1\to3}(x_3) = \{2,3\}$$

Sample Calculations from Step 3:

$$Q_{3\to3}(x_3) = R_{1\to3}(x_3) + R_{2\to3}(x_3)$$
$$Q_{3\to3}(x_3) = \{2,3\} + \{4,5\} = \{6,8\}$$

Sample Calculations from Step 4:

$$R_{3\to4}(x_4) = max_{\{x_3,x_4\}-\{x_4\}}[U_3(x_3,x_4) + Q_{3\to3}(x_3)]$$
$$= max_{\{x_3\}}[U_3(x_3,x_4) + Q_{3\to3}(x_3)]$$

$$R_{3\to4}(x_4) = \begin{bmatrix} 3 & 4 \\ 10 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 6 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} 9 & 10 \\ 18 & 9 \end{bmatrix}$$
$$R_{3\to4}(x_4) = \{18,10\}$$

Sample Calculations from Step 5:

$$Z_3(x_3) = R_{1\to3}(x_3) + R_{2\to3}(x_3) + R_{3\to3}(x_3)$$
$$Z_3(x_3) = \{2,3\} + \{4,5\} + \{4,10\} = \{10,18\}$$