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Building the Stemma Codicum from Geometric Diagrams

A Treatise on Optics by Ibn al-Haytham as a Test Case

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Abstract In view of the progress made in recent decades in the fields of stemmatology and the analysis of geometric diagrams, the present article explores the possibility of establishing the stemma codicum of a handwritten tradition from geometric diagrams alone. This exploratory method is tested on Ibn al-Haytham's *Epistle on the Shape of the Eclipse*, because this work has not yet been issued in a critical edition. Separate stemmata were constructed on the basis of the diagrams and the text, and a comparison showed no major differences. The greater reliability of a stemma codicum constructed on the basis of the diagrams rather than the text of a mathematical work is discussed and preliminary conclusions are drawn.

Keywords geometrical sciences · manuscripts · diagrams · stemma codicum

1 Introduction

In recent decades the history of science has witnessed two important, parallel movements. First, a growing interest has been taken in visual data, notably the geometric diagrams that illustrate mathematical texts. The second trend is the recent transfer of the techniques of cladistics, used in the field of biological taxonomy, to stemmatology with the aim of establishing the genealogical tree of a work by comparing its manuscripts (stemma codicum).

In view of the advances made in these two domains, this article poses the question as to whether it would be possible to establish the stemma codicum of a work from its geometric diagrams alone. Several mathematical examples are considered in the body of this article but Ibn al-Haytham's *Epistle on the Shape of the Eclipse* has been chosen as a test case because of the sophistication of its diagrams. A blind comparison between the diagram stemma and the text stemma reveals no major differences, thus suggesting the equivalence of the two procedures. Next, a comparison shows that there is a higher concentration of errors in a diagram than in the same area of text (Section 7, note 20). Therefore, diagrams make it easier to discriminate between and sort the manuscripts of a given work.

Building the diagram stemma appears to offer a highly promising way to study and uncover the phylogeny of complex sets of manuscripts in geometry and geometric sciences such as astronomy, optics, mechanics, architecture and perspective.

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1.1 Advances in Diagram Studies

From the pioneering work of Murdoch (1984), Edgerton (1985) and Mahoney (1985), increasing attention is now being paid to visual scientific data of any kind: photographs, diagrams, sketches and figures. The different functions of visual data have been clarified, as well as the way they connect to the text. One line of investigation specifically concerns the use of geometric diagrams.[1]

Progress has been made on gathering data from diagrams, by developing a software for directly capturing diagrams in manuscripts (Saito 2005). Then a team consisting of Gregg De Young, Takanori Suzuki, Ken'ichi Takahashi and Ken Saito was formed to study diagram variation in Euclid's treatises (*Elements, Optica, Phaenomena*) (Saito et al. 2011). The keen attention being paid to geometric diagrams is beginning to bear fruit in the form of critical editions of mathematical texts (Sidoli 2007; Sidoli and Berggren 2007).

Some general findings have already been reached. After preliminary discussions, scholars have studied more carefully the position, mode of composition and lettering of diagrams as well as the mutual dependence of text and diagrams (Decorps-Foulquier 1999; Netz 1999). Moreover, the tacit conventions applied to drawing geometric diagrams are now better known:

1. A diagram is schematized whenever possible—that is, provided it is not contrary to the hypotheses. For example in Apollonius' *Conics*, parabolas and hyperbolas are rendered by circular arcs (Decorps-Foulquier 1999, pp. 73–4). In Euclid's *Elements* III.10, the ellipse $EB\Delta\Theta M$ is drawn by simply joining two circular arcs (Netz 1999, p. 55). In most manuscripts of Euclid's *Elements* IV.16, the sides of the fifteen-angled figure are replaced by concave arcs (Saito et al. 2011, p. 171). These properties apply to many ancient and medieval manuscripts.
2. Geometric figures are usually overspecified. The rectangular triangle appearing in *Elements* I.47 is drawn as rectangular and isosceles (Saito 2005, p. 19). Similarly, in his commentary on the *Book of Lemmas*, al-Sijzī bisects the diameter of the semi-circle instead of considering an arbitrary division (Crozet 2005, p. 35).
3. Linear values as represented in the diagrams are not necessarily faithful to the mathematical content. Consider Euclid's *Phaenomena*, prop. 1. In the Berlin MS, radius DA is much smaller than radius DB, although it is explicitly stated that D is the center of circle $AB\check{G}E$ (Suzuki 2011, p. 17). Similar deviations are to be found in al-Sijzī's works (Crozet 2005, p. 35).

1.2 Advances in Stemmatology

Another major breakthrough took place when interest in transferring the methods of phylogenetics to philology was spurred by the development of computer-based analysis.

Conventionally, three different approaches have been used for the purpose of text editing. Lachmann's method is based on identifying "shared errors." This is a rigorous method, which nevertheless raises the question of how to reliably identify errors (Lachmann 1850). Bédier and Dom Quentin both criticized Lachmann's method and expanded "errors" into "variant locii" to take into account the smallest details of the text (Bédier 1929; Dom Quentin 1922).[2] The three methods by Lachmann, Bédier and Dom Quentin are not equally reliable, however. A critical test, comparing their outcomes against a benchmark text, showed the indisputable superiority of Lachmann's method, which was designed to identify errors (Huygens 2001). Errors can

[1] See, in particular, Cambiano (1992), Decorps-Foulquier (1999), Netz (1999), De Young (2005), Mascellani et al. (2005), Saito (2005), Saito (2006), Sidoli (2007), Manders (2008), Sidoli and Saito (2009), Jardine and Jardine (2010), Saito et al. (2011), Saito and Sidoli (2012), Mumma et al. (2013).

[2] For example, the simple rephrasing of statements or the transposition of the words *odientis oscula* to make the verse *Quam fraudulenta oscula odientis* rhyme with *Meliora sunt vulnera diligentis* (Huygens 2001, p. 55).

be regarded as a subset of text variants, and include: reading errors, storing errors, transcribing errors, clerical errors, and errors in returning to the text, that is, omission, homoioarkton, homoioteleuton, additions and dittographies (Bourgrain and Vielliard 2002, pp. 34–7).

Recently, philologists have come to the realization that their approach bears many similarities to the methods of evolutionary biology, where characteristics are compared in a pairwise manner to determine the common ancestor of a group of species. It has been pointed out that:

> "The stemmaticist who is analyzing the agreements and disagreements between manuscripts for evidence of relationship by descent, is doing the same thing as the evolutionary biologist who is analyzing the agreements and disagreements between species for evidence of relationship by descent [. . .] Indeed, several experiments in this area have proved that, for some manuscript traditions at least, the methods developed in evolutionary biology give remarkably accurate results" (Robinson 1998, p. 130).

Phylogenetics is divided into two main approaches.[3] Phenetics, which treats homoplasies and homologies on an equal footing, highlights current similarities between the taxa with no attention paid to the real phylogeny. Cladistics, which was developed from the work of Hennig (1950), is targeted at discovering the common ancestor of a group of species.

This focus brings computer-aided cladistics very close to traditional stemmatology. The connection can be summed up in one sentence: the researcher can build a stemma codicum/cladogram in order to reconstruct the common ancestor and the textual history/phylogeny from a group of manuscripts/species. This equation favours the transfer of methods from cladistics to stemmatology. As a result, cladistic analysis is now being used in the critical editing of both literary and scientific texts.[4]

1.3 Intersection of the Methods

Despite the parallel development of these two research areas, the intersection of diagram studies with stemmatology is still largely unexplored.

The fragments of knowledge that we possess regarding this intersection are limited to the following:

1. The actual size of the diagram does not constitute significant information. In general, it is adapted to the space left for the drawing, and even sometimes at the risk of altering the diagram, when the space, particularly in the margin, is small (Saito 2006, p. 10). The geometric diagram is then either cropped—an example is the diagram of *Elements* III.3 in Codex p (Saito et al. 2011, p. 53) or distorted—an example is prop. III.20 in Codex p (Saito et al. 2011, p. 66).
2. If a diagram is placed in the margin and covers a case not treated in the text, this strongly suggests a scribe's addition, e.g. Apollonius' *Conics* I.43 and III,1 (Decorps-Foulquier 1999, p. 79). The same applies to Euclid's *Elements* III.25 where, in all likelihood, the marginal diagrams were supplied by the copyist (Saito 2005, p. 95).

[3] This comparison builds on the difference between homology, i.e. a similarity inherited from a common ancestor, and homoplasy, i.e. a similarity shared by two individuals, whereas it is not found in the ancestor.

[4] On the application of cladistic analysis in different domains, the reader is directed to the following key studies: on the approach in general (Glenisson 1979; Reenen et al. 1996; Robinson 1996; Dees 1998; Huygens 2001; Woerther and Khonsari 2001; Macé et al. 2001; Reenen et al. 2004; Macé and Baret 2006; Cipolla et al. 2012); in literary texts (Robinson et al. 1996; Salemans 1996; Barbrook et al. 1998; Salemans 2000; Mooney et al. 2001; Windraw et al. 2008; Maas 2010); and in scientific texts (Brey 2009; Pietquin 2010; Cardelle de Hartmann et al. 2013).

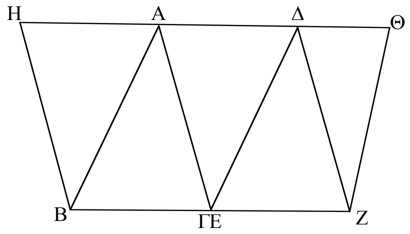

Fig. 1 I.38 (codex B)

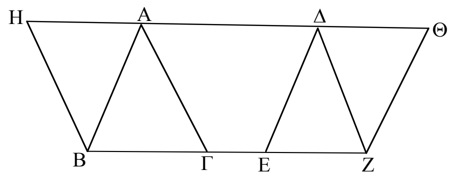

Fig. 2 I.38 (codex P)

3. More crucially, diagram errors can allow us to retrace a handwritten tradition, i.e. mistakes transmitted from copyist to copyist. The diagrams of *Elements* I.27, I.38, VI.19 point to different families of manuscripts. For example, the diagram appended to prop. I.38 differs in codex P ($E \neq \Gamma$, Fig. 1) and codex B ($E = \Gamma$, Fig. 2) (De Young 2004, p. 358; De Young 2005, p. 158; Saito 2005, p. 134). Consider a manuscript such as Mantova, ebreo MS 1, fol. 8, which includes a diagram of I.38 where points ΓE are separate. If this copy were to show several such features, it would suggest a connection to the family of codex P rather than to the family of codex B. Similar conclusions can be drawn from al-Sijzī's *Demonstrations of the Book of Euclid on the Elements* (Crozet 2005, p. 39).[5]

Such observations recently gave rise to the idea that "The diagrams should be presented as they are found in the MSS, accompanied by a critical apparatus [...] Where this is possible, we should seek to establish the text history of the diagrams and present this in a stemma" (Sidoli 2007, p. 546). As far as I know, establishing the critical apparatus with a view to building a diagram stemma has not yet been done. This is ironic, because if one takes seriously the suggestion that the study of the history of mathematics has been misguided because less attention has been paid to the diagrams than to the text, then the key issue becomes the reconstruction of the diagram's own phylogeny.

The aim of this paper is to explore the phylogeny of manuscript diagrams by constructing their diagram stemma. I will first clarify the data to be used, by comparing several types of diagrams (Section 2). Then I will introduce Ibn al-Haytham's *Epistle on the Shape of the Eclipse* as a test case (Section 3). Since the text has not yet been published in a critical edition, this provides us with a unique opportunity to conduct a test under ideal conditions: the unknown text stemma cannot influence the construction of the diagram stemma. Results will be reached by a blind, parallel construction of the diagram stemma (Section 4) and text stemma (Section 5). From their comparison, general conclusions will be drawn in the final section of this article (Section 6).

2 Overview

As is now well known to philologists, Lachmann's method must be preferred to other methods of critical editing (Huygens 2001). Could this approach serve as a model for searching through the

[5] After constructing the stemma on the basis of a review of text accidents, Crozet says: "Cette filiation, dont nous venons d'étayer l'affirmation par le menu en considérant le texte seul, peut également se lire à l'aide des figures." Al-Sijzī's *Barāhīn kitāb Uqlīdis fī al-'uṣūl (Demonstrations of the Book of Euclid on the Elements)* is known in three copies: B Dublin, Chester Beatty MS 3652, fols. 17r–28v, R Istanbul, Reshit MS 1191, fols. 84v–105v, and L London, India Office MS 1270, fols. 87r–100r. MSS R and B are linked to one another. In the third demonstration of prop. I.2, both carry the correct lettering of point Ǧ. In the fourth demonstration, both refer to point H, which does not appear in MS L. In prop. II.9 both manuscripts contain superfluous lines. Furthermore, MS L predates MS R for the latter is the only text where point Ǧ is placed on AH, in keeping with the text stemma.

diagrams and arranging them in a stemma? Before applying stemmatology to diagrams we must first define what "characters" are. Then the robustness of the method will be tested by applying these definitions to a wide range of situations.

2.1 Definitions

The establishment of the diagram stemma and the study of diagrams are two separate tasks. In terms of the use of diagrams, it is well known that a distorted diagram can serve as a basis for sound reasoning, which leads us to distinguish the simple carelessness of the copyist from major errors. From a stemmatological point of view, there is no reason to make such a distinction; we call an error any alteration that diminishes the chances of replicating the diagram. What are these errors? Geometric diagrams are not just simple images. Being built up step by step, they record a series of operations or properties, each one of which can potentially introduce an error. It follows that any witness (i.e. a surviving manuscript that testifies to the content of the original work) can lead to a true or false reading vis-à-vis each operation or property. Therefore, diagram errors can be counted in the same way as—or perhaps even more easily than—text errors. In this respect, geometrical works have some advantages over literary texts.

Definition 1. Let us call "stated characters" all the geometric properties of a diagram that are based on explicit statements in the text, otherwise they will be called "unstated characters."

Either subjects ("square $ABCD$") or predicates ("$ABCD$ is a square") can serve to isolate a stated character in a geometrical treatise. Consider the following problem: "To construct an isosceles triangle having each of the angles at the base double of the remaining one" (*Elements* IV.10; Fig. 3) (Heath 1956, p. 96). The text states that $EB\varDelta$ is a circle. Thus a scribe faced with the oval $EB\varDelta$ in Codex B can restore it to a circle. Likewise, though missing in Codex F, line $B\varDelta$ must be drawn. Since it is stated that $A\varGamma = B\varDelta$, the scribe can restore line $B\varDelta$, even if his source is Codex F. These are (true) stated characters. However, it is not stated whether triangle $AB\varDelta$ is oriented downwards or upwards. Thus, a scribe copying Codex V cannot know that triangle $AB\varDelta$ is turned downwards in all the other manuscripts. This is an unstated character.

Definition 2. Let us call "true characters" those characters that conform to the mathematical content of the text, otherwise they will be called "false characters."

In Euclid's *Elements* IV.10 (Fig. 3), AB is the semi-diameter of circle $EB\varDelta$, which is not the case in Codex F. Similarly, no manuscript illustrates the point $A\varGamma = B\varDelta$, which however is clearly stated in the text. These are false (stated) characters. Furthermore, a correct construction would require that circles $A\varGamma\varDelta$ and $EB\varDelta$ meet at two points.[6] This is a false (unstated) character in all manuscripts, except in Codex F where it is true.

Definition 3. Let us call "separative characters" those characters found in only one manuscript, "conjunctive characters" those in which several copies agree, and "common" or "shared characters" those which are common to all manuscripts.

Let us consider again Euclid's *Elements* IV.10 (Fig. 3). It is stated that $A\varGamma\varDelta$ is a circle. This is a (true) conjunctive character appearing in Codices p, F, b and V—not in Codices P and B, which have a separative (false) character here. Furthermore, $A\varGamma\varDelta$ appears to be right-angled at $\varGamma$ in all the codices, despite the fact that this is inconsistent with proposition IV.10, which instead requires that $\widehat{\varGamma} = \pi - \frac{2\pi}{5} = 108°$. This is a shared (false) character.

Types of errors. True and false characters refer to the concept of "being mathematically true" (i.e., true to the mathematical content of the text), whereas shared and separative characters

[6] Heath assumes that $\varDelta$ and E are the two points of intersection of $A\varGamma\varDelta$ and $EB\varDelta$, though it is not explicit in Euclid: "With centre A and distance AB let the circle $B\varDelta E$ be described..." (Heath 1956, p. 96).

Table 1 Euclid's *Elements* IV.10

Character	MSS	P	B	p	F	b	V
1-01_s* Circle $A\Gamma\Delta$ drawn		0	0	0	0	0	0
1-02_s Line $B\Delta$ drawn (f: false)		0	0	0	f	0	0
1-03_u Triangle $AB\Delta$ down (u: up)		0	0	0	0	0	u
1-04_s AB radius of circle $EB\Delta$ (g: greater)		0	0	0	g	0	0
1-05_u Circle $A\Gamma\Delta$ cuts $EB\Delta$ (f: false)		f	f	f	0	f	f
1-06_u* Angle $A\Gamma\Delta = 108°$ (s: smaller)		s	s	s	s	s	s
1-07_u ΔB clockwise (c: counter-clockwise)		0	0	0	c	c	0

refer to the concept of "being codicologically true" (i.e. true to the content of the manuscripts). Mathematical and codicological properties do not coincide in all cases because a mathematical error is detected by comprehension, while a codicological error is detected by comparing the manuscripts. All scholars involved in critical editing have come across these two types of truth. When the manuscripts are corrupted by the same errors, the codicological study is unable to detect them all and the definitions provided above could help to identify such overlappings. The cross analysis of mathematical truth/error and codicological truth/error yields four possible cases:

1. When the mathematical truth and the codicological truth coincide, no comment is needed.
2. Codicologically true characters, i.e. shared characters, may all repeat the same mathematical error: e.g. $\widehat{\Gamma}$ is right-angled in all codices instead of being equal to 108° as we can deduce from the text (Fig. 3).
3. The mathematical truth cannot be reconstructed codicologically false characters in a diagram—this does not, however, prevent us from inferring the truth from false diagrams.
4. When mathematical and codicological error coincide, no comment is necessary.

Therefore only one case must be specified: when diagram characters that are codicologically true reflect a mathematical error, they are "shared false characters." In conclusion, the ambiguous concept of codicological truth gives rise to two cases: "true characters" when they correspond to mathematical truth, "shared false characters" when they differ from it.[7]

Notations. For clarity's sake, in the remainder of the text all characters have been labelled using the format "Diagram-Number$_{\text{Index}}$." The index is assigned a value among s, u, t or f, where s denotes a "character stated in the text," u a "character unstated in the text," t a "mathematically true character," and f a "mathematically false character." As the true characters are of little use in establishing the stemma (they consist of a simple series of 0, such as circle $A\Gamma\Delta$ in Table 1), we can dispense with discussing indices t and f. A character common to all manuscripts is marked *. Thus a code such as 1-06_u* denotes the 6th unstated character appearing in Diagram 1, which is common to all manuscripts. A variance between manuscripts is taken into account by coding 0 for "the diagram's being mathematically true" and x otherwise.[8] In order to highlight only the most relevant graphical errors, those representing less than five percent of the linear ($\Delta x/x < 0.05$) or angular ($\Delta\alpha/\alpha < 0.05$) measurements are ignored. This notation is applied to a few characters of *Elements* IV.10 for illustration (Table 1).

[7] Shared true characters also include the so-called "co-exact" geometric properties. "Co-exact attributes are those conditions which are unaffected by some range of every continuous variation of a specified diagram" (Manders 2008, p. 92). Typically, these are topological relations, which are more stable than magnitudes or ratios of magnitudes.

[8] With a view towards creating a matrix of characters, I have used straightforward codes: f: false, s: smaller, g: greater, e: equal, p: positive, n: negative, u: up, d: down, r: right, l: left, and ? for a missing character.

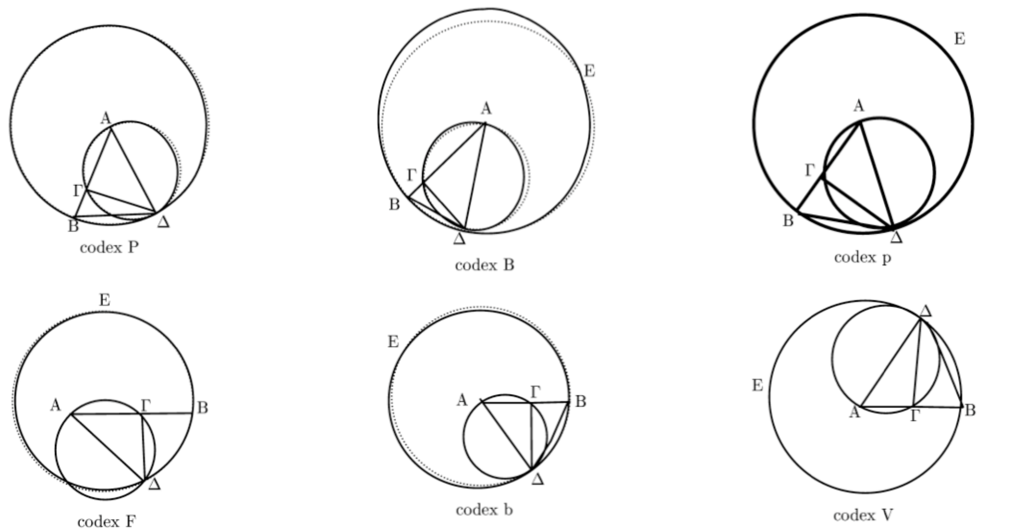

Fig. 3 Euclid's *Elements* IV.10 (Saito 2011, p. 168)

2.2 Preliminary Tests

Preliminary tests of the above definitions were conducted on a wide range of situations in order to establish their reliability. These tests focused on diagrams from Euclid's *Elements*, Books I–IV, VI, XI–XIII (Saito 2005; Saito et al. 2011; Saito 2013), Euclid's *Phaenomena* (Suzuki 2011), Theodosius' and Menelaus' *Spherics* (Sidoli and Saito 2009; Sidoli and Li 2011), and included a few comparisons of the stemmata built from stated and unstated errors.

Based on these preliminary tests it is possible to state the following:

1. The manuscript with the least errors should be taken as a basis for diagram editing. This holds true even if this copy differs from the autograph (indeed, the original autograph could have been a rough draft, with the entire set of copies being corrected much later).

2. A key property of unstated false characters is that, if a scribe reproduces a corrupted model, he cannot correct the error by referring to the text. Therefore unstated characters are of particular interest when reconstructing the history of a mathematical text.

3. The chance of finding unstated false characters depends directly on the complexity of the diagram, which can be roughly estimated by the number of lines needed to draw the diagram. For anyone wishing to study the phylogeny of a set of diagrams, it is advantageous to focus on the more complicated ones.

4. The chance of finding unstated false characters also depends on the kind of problem examined: arithmetics, plane geometry or solid geometry.

Compared to other situations, diagrams illustrating ratios and proportions provide only a limited number of usable characters. Consider the proposition: "Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides" (*Elements* VI.23; Fig. 4). The number of distinctive characters can be counted on one hand. Lines K, Λ, M are equal in Codex P (false) vs unequal in Codex V (true). The parallelogram $\Gamma\Delta\Theta H$ is a square in Codex P vs a rectangle in Codex V. Now if $\Gamma\Delta\Theta E$ is a square, then Codex P is false, because if $\Delta\Gamma = \Gamma H$, the relation $B\Gamma : \Gamma H :: \Delta\Gamma : \Gamma E$ implies $B\Gamma = \Gamma E$. Lines K, Λ, M are on the left side in Codex P vs on the right side in Codex V. Consequently, a large number of diagrams would be needed to build the stemma.

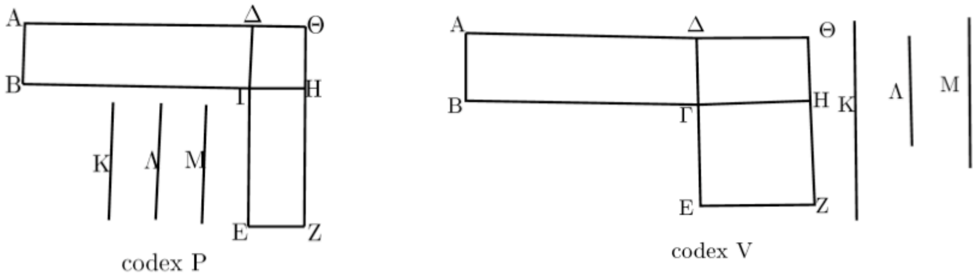

Fig. 4 Euclid's *Elements* VI.23 (Saito 2011, p. 190)

Diagrams of plane geometry illustrate the standard situation, with an acceptable number of usable false unstated characters.

Except in the event of very basic diagrams,[9] problems of solid geometry are particularly conducive to false unstated characters because a plane representation of a three-dimensional object is always subject to conventions (rabattement, orthogonal projection, axonometric view, perspective drawing, etc.). Since these conventions are generally tacit, they increase the number of unstated characters and thus the chance that the scribe will misinterpret the diagram.

3 A Test Case

Ibn al-Haytham's *Maqāla fī ṣūrat al-kusūf (Epistle on the Shape of the Eclipse)* has been chosen as a test case for several reasons. Firstly, there is no critical edition available, and therefore the unknown text stemma cannot influence the construction of the diagram stemma. Secondly, Diagrams 1 and 3 are complicated diagrams. Thus the chance of finding false unstated charaters is high. Thirdly, Diagrams 1, 3 and 4 are three-dimensional representations which are conducive to false unstated characters, and therefore of special interest in reconstructing a diagram's own phylogeny.

Versions of the *Epistle* are extant in five manuscripts:

F Istanbul, Fātiḥ, MS 3439, fols. 117r–123v. Size 190×130 mm. Incomplete. Ends at time 0.922.[10] This copy, written in poor *naskhī*, was completed in Mosul by Ibrāhīm ar-Rūjānī al-Bakrī on the night of 'Ashūrā' A.H. 587/7 February 1191 (Krause 1936, p. 458). It is extremely corrupted up to time 0.061. The diagrams are legible enough but often distorted. Diagram 4 is missing.

B Oxford, Bodleian, MS Arch. Seld. A32, fols. 81v–100v. Size 180×115 mm. B is written in a hurried *naskhī*. "[It] was transcribed before A.H. 633 (1235-6), being contained in a volume which came into the possession of Yaḥyā ibn Muḥammad ibn al-Labūdī in that year. In the colophon we are informed that the copyist transcribed the text from a copy claiming to have been transcribed from the prototype" (Sabra 1971, p. ix). The diagrams often impinge on the text. They are distorted by a lack of parallelism and squareness. The intersection points are rough.

[9] Compare in this respect Menelaus' *Spherics*, props. III, 1, III, 2a, III, 2b vs I, 51, I, 54, I, 57 (Sidoli and Li 2011). Other examples appear in Theodosius' *Spherics* (Sidoli and Saito 2009).

[10] In order to locate a passage parallel to a given passage in any extant manuscript, we define a new measurement called "time," which counts 0 as the beginning of the text and 1 as the end of the text in any complete version. Thereby one can precisely define a given passage from any text. As regards the transliteration of the Arabic, I have adopted DIN-31635 throughout, except for G 'ayn, which does not interfer with Ğ ğīm or Ġ ġayn.

P St. Petersburg, Institute of Oriental Manuscripts, MS B 1030, fols. 21r–47v. Incomplete. Ends at time 0.925. Size 170×92 mm (Khalidov 1986, No. 9749). MS P predates the mid-fourteenth century, for it was checked against Ibn al-Haytham's autograph in A.H. 750/1349. "This collection, written in mediocre *nasta'līq*, is of great scientific quality" (Rashed 2005, p. 15). All of its diagrams are geometrically clear and accurate.

O London, India Office, MS 1270 (Loth 734), fols. 79r–86v, ends at time 0.998 seven words before the end. Size 279×114 mm.[11] The manuscript is in good *naskhī*, evidently from the sixteenth century. The copy is well written in a small hand, with neat diagrams (Loth 1877, p. 214). MS O was initially part of the library of Richard Johnson, who came back to England in 1799. MS O was purchased by the India Office at the nabob's death in 1807.

L London, India Office, MS 461 (Loth 767), fols. 8v-34r. Size 229×140 mm. The manuscript is written in good *nasta'līq*. Its date can be deduced for it includes a copy of al-Ṭūsī's *Risāla al-asṭurlābiyya (Treatise on the Astrolabe)*, which was revised on 14 Shawwāl A.H. 1198/31 August 1784. It could have belonged to Governor-General Warren Hastings (1773–1785), before it passed into the collection of the London Library (Loth 1877, p. 223). MS L has finely drawn diagrams, which appear on a separate sheet.

Ibn al-Haytham's *Epistle* has been uncritically edited by Wiedemann (1914) and independently commented on by Naẓīf 1942/3). Neither of these meets the requirements of a critical edition, which is currently under preparation.

Ibn al-Haytham's treatise has four diagrams (Appendix B, Fig. 15). Diagram 1 shows the partially eclipsed sun $AB\check{G}D$ projected through the pinhole $\underset{.}{H}\underset{.}{T}H$ of a camera obscura on the wall opposite to the pinhole. The projected crescent-shaped images of the sun are $\check{S}Y\underline{H}F$ through point $\underset{.}{H}$, $KLMN$ through point $\underset{.}{T}$, and $\underline{T}\underset{.}{Z}GQ$ through point H. Diagram 2 is a lemma stated in view of the next proposition. Diagram 3 reproduces Diagram 1 with some additional features showing how the crescent-shaped figures overlap one another. Diagram 4 is a detailed view of the two lunes $\check{S}Y\underline{H}F$ and $KLMN$ appearing in Diagram 3.

Starting from the definitions given in Section 2.1, several tables of characters can be built. Let us set aside the list of all "true characters" (a series of 0) and "shared characters" (a series of the same letter, denoting the same accident), which can contribute little to the establishment of the stemma. We are able to construct two tables of characters:

Table 5 (Appendix A) presents forty-eight "stated characters" to be found in at least one manuscript of the *Epistle*. As said before, these stated characters are explicitly referred to in the text.

Table 6 (Appendix A) similarly provides a table of "unstated characters," which are not referred to in the text. The search for those properties of the diagrams that are unstated in the text yields forty-eight characters.

4 Building the Diagram Stemma

4.1 Diagram Errors

The most notable alterations affecting the diagrams of Ibn al-Haytham's *Epistle on the Shape of the Eclipse* are the following:

1. Diagram 4 is missing in MS F, because the text breaks off at time 0.922, switching to a copy of Ibn al-Haytham's *Maqāla fī kayfiyyat al-aẓlāl (Epistle on the Formation of Shadows).*

[11] O for Oblongus.

Table 2 The Main Shared and Conjunctive Errors

Character	MSS	F	B	P	O	L
Shared Errors (*)						
1-07$_s$ $A\check{G}$ collinear to KM *		f	f	f	f	f
3-14$_s$ $A\check{G}$ collinear to KM *		f	f	f	f	f
1-11$_s$ FT radius of arc $\check{S}F\underline{H}$ *		s	s	s	s	s
1-12$_s$ FT radius of $\check{S}Y\underline{H}$ *		g	g	g	g	g
1-02$_u$ Arc $\check{S}F\underline{H}$ = arc $\check{S}Y\underline{H}$ *		g	g	g	g	g
1-03$_u$ Arc KNM = arc KLM *		g	g	g	g	g
1-04$_u$ Arc $\underline{T}QG$ = arc $\underline{T}\d{Z}G$ *		g	g	g	g	g
Conjuctive Errors						
3-08$_s$ $\check{S}K$ radius of circle $\check{S}\underline{T}$		g	s	g	?	0
3-10$_s$ $\underline{H}M$ radius of circle $\underline{H}G$		?	s	g	?	0
3-16$_s$ Point F on circle $\check{S}\underline{T}$		f	f	f	?	0
3-17$_s$ Point F on circle $\underline{H}G$		?	f	f	?	0

2. In most manuscripts, Diagrams 1 and 3 show the outer rays of the visible part of the sun as lines $D\d{H}F$ and $D\d{T}N$. We find a constant error in MS F, which has lines $B\d{H}F$ and $B\d{T}N$ instead. These errors appear in Table 6 as 3-04$_u$, 3-05$_u$.

3. Diagram 3 of MS B offers a textbook case of the interplay between text and diagram. This diagram is very corrupted: $\d{S}\d{T}F$ and $D\d{H}F$ appear as broken lines, while $D\d{T}$ cuts the vertical axis $T\d{Z}$ between N and L. However, these accidents cannot be counted as proper errors, because a marginal addition appended to MS B expressly states that "line $\d{S}\d{T}F$ must be straight, as well as the line $D\d{H}F$" and "line $D\d{T}N$ is tangent to arc KNM at point N" (MS B, fol. 96r, time 0.757). Thus a scribe copying MS B can restore a correct diagram. These features are missing in Table 6 because they were not included in the construction of the stemma.

4. Even though MS L is the latest extant copy (1784), it contains several true characters which are missing in all the other manuscripts (1-03$_s$, 3-08$_s$, 3-10$_s$, 3-15$_s$, 3-16$_s$, 3-17$_s$) or are almost never seen (2-05$_s$, 4-02$_s$, 4-06$_s$, 1-14$_u$, 3-05$_u$, 3-12$_u$, 3-15$_u$). The overcorrectness of MS L suggests that the scribe redrew the lines of the diagrams by following the textual instructions found or by having at his disposal another set of diagrams.

When all the diagrams agree across manuscripts, they either agree or do not agree with the text. In the second case there are common or shared errors, i.e. conjunctive errors found in all the manuscripts (Maas 1958). In general, such errors belong to the ancestor: if all MSS agree on an erroneous line, there is a low likelihood that they represent independent departures from the ancestor. Bearing in mind L's overcorrectness, let us focus on the errors common to all the other manuscripts (F, B, P and O). As the errors reported in Table 2 are common to all manuscripts, they should be seen as a list of errors/choices made by the hand that copied out the common ancestor which, due to the limited number of manuscripts, was perhaps written by Ibn al-Haytham himself. Since past scholars have not mentioned these characters (Wiedemann 1914, p. 158; Naẓīf 1942/3, pp. 190–6; Sabra 1972, p. 196), they must be addressed in detail.

Errors 3-08$_s$, 3-10$_s$, 3-16$_s$ and 3-17$_s$ (absence or deformation of the flanking circles $\check{S}\underline{T}$, $\underline{H}G$) seem to result from the lack of space on the manuscript. Since Diagram 3 touches the edge of the folio in MSS F, P and O, it would have extended beyond the edge of the folio if it be had been drawn in accordance with what was written in the text. However these errors have little effect on the demonstration.

Errors 1-07$_s$ and 3-14$_s$ (lack of collinearity of $A\check{G}KM$) seem to go back a very long way. Even though these points are stated to be collinear in the text, the scribe—Ibn al-Haytham?—did not

Table 3 MSS Ranking from Diagram Errors

Type of Graphical Error	MSS	F	B	P	O	L
S Errors						
Diagram 1 (27 loci)		15	15	8	9	12
Diagram 2 (15 loci)		5	3	0	5	3
Diagram 3 (23 loci)		16	12	8	15	5
Diags. 1–3 (65 loci)		36	30	16	29	20
U Errors						
Diagram 1 (18 loci)		15	14	5	8	9
Diagram 2 (4 loci)		3	0	0	3	4
Diagram 3 (18 loci)		14	10	1	9	6
Diags. 1–3 (40 loci)		32	24	6	20	19
SU Errors						
Diags. 1–3 (105 loci)		68	54	22	49	39
Ordinal Ranking		5	4	1	3	2

feel impelled to explicitly draw the line AM. As a consequence, all copyists erred in locating $A\check{G}$ and KM in a haphazard fashion.

Errors 1-11$_s$, 1-12$_s$, 1-02$_u$, 1-03$_u$ and 1-04$_u$ (the inequality of the inner/outer arcs of the crescent-shaped figures) are more interesting because they have a rationale. According to the text, *FN* is the radius of both arcs *KNM* and *KLM.* It necessarily follows that arc *KLM* is horseshoe-shaped. But the scribe—Ibn al-Haytham?—had in mind to draw the two flanking lines *ŠKṮ* and *H̱MG* on which the lunes are chained. Therefore if the arcs are drawn based on what is stated in the text, the two flanking lines cut the arc *KLM* (*ŠYH̱* or *ṮẒG*) in the vicinity of points *K M* (*Š H̱* or *Ṯ G*). As a result, the diagram is unclear because the lines overlap each other. Visually, the result can be appreciated in the mathematically true diagram (Appendix B, Fig. 15). The agreement between all the extant copies means that the scribe of the common ancestor—Ibn al-Haytham?—took the decision to clean up the diagram so that *K* and *M* (*Š* and *H̱, Ṯ* and *G*) constitute both the end-points of the lune and its points of tangency with lines *ŠKṮ* and *H̱MG.* The simplest way to achieve this without changing the magnitude of the eclipse was to increase the radius of arcs *KNM, ŠFH̱* and *ṮQ̣G*, a decision fully implemented in MS P, Diagram 3 (Appendix B, Fig. 12).

4.2 MS P Contains the Fewest Errors

If one accepts as valid the idea that a stemma codicum can be constructed from diagrams, the manuscript with the fewest diagrammatic errors is to be regarded as the most faithful witness to the autograph. Let us therefore count all the errors, omitting those of Diagram 4 which are missing in MS F (Table 3). Errors are classified depending on whether they are stated (S) or unstated (U).

Whatever the criterion chosen (S or U Errors), the best witness is definitely MS Petersburg with only 16 S Errors out of 65 susceptible loci and 6 U Errors out of 40 susceptible loci found in Diagrams 1–3. Then follows MS India 461 with 20 S Errors and 19 U Errors. Extended to all manuscripts and all types of errors, this comparison provides the ordinal ranking of the five manuscripts of Ibn al-Haytham's *Epistle.* If "A ≺ B" denotes "A is preferred to B," this ranking may be written:

$$\text{Petersburg} \prec \text{India 461} \prec \text{India 1270} \prec \text{Bodleian} \prec \text{Fātiḥ}$$

Table 3 shows that the distributions of S, U and SU Errors provide the same ordinal ranking. Nevertheless, it is helpful to assess their effectiveness in sorting the manuscripts. At face value, S Errors are more severe than U Errors because they mean that the scribe passed over the instructions given in the text. However:

1. The average number of errors at the susceptible loci is 0.51 (U) $>$ 0.40 (S Errors).[12] The risk of error is thus higher in the case of unstated characters.

2. The maximum difference in the number of errors by susceptible loci is 0.65 (U) $>$ 0.31 (S Errors).[13] The manuscripts are therefore better discriminated by unstated characters than by stated ones.

These two differences are easy to understand. When a diagram is drawn, some lines meet the requirements stated in the demonstration and others do not. If a geometric property is explicitly stated in the text, it can be checked and the chance of error is smaller. If the line is drawn at random, then the scribe has no means of recovering the figure through the text and there is a higher chance of error. Consequently, in the event of a difference between S and U rankings, the best classification is given by U Errors alone, in consideration of statement 2, just above.

How should we classify the manuscripts? The greater the number of characters, the greater the accuracy of the stemmatological analysis. Assuming that stability is an indication of an exact result, let us try to establish the threshold at which the correct ranking of codices is provided by single diagrams. By ordering the manuscripts according to the number of U Errors, MS P is ranked first regardless of the diagram used. This is an indication that a set of 20 susceptible loci is needed to identify the best witness to a handwritten tradition. The final ranking of the manuscripts happens to be the same as the one produced by considering the errors of Diagram 3—while the ranking for Diagram 1 is different. As a merging of errors from Diagrams 1–3 is needed to arrive at the correct ranking, it is safe to assume that a set of 40 susceptible loci (at least) is required to provide a stable ranking of the manuscripts, whatever the text to be edited.

This outcome is of great interest from a methodological point of view because it means that, once the text has been read to understand its meaning and scope, there is no need to codicologically analyze the text errors in detail. We just need to pick errors at random in the diagrams,[14] provided the number of susceptible loci is great enough: say over fifty to rank all the manuscripts. This preliminary conclusion should be addressed more fully in further research.

4.3 The Diagram Stemma

Next, the diagram stemma should be built from the list of characters (Appendix A) using standard cladistics programs such as PAUP* (Swofford 2003) or PHYLIP (Felsenstein 2009). I chose PHYLIP, which has been adopted and tested by several philologists, one recent trial being that of Pietquin (2010) in his critical edition of Ibn al-Haytham's *Optics*, Book VII.

The matrix of characters is made of six taxa (the five extant manuscripts of Ibn al-Haytham's *Epistle* and the out-group, which simply consists of the list of common features) and of as many characters as we decide to examine. Fig. 5 shows the matrix corresponding to the SU Errors

[12] This is the result of $(36 + 30 + 16 + 29 + 20)/5/65 = 0.40$ and $(32 + 24 + 6 + 20 + 19)/5/40 = 0.51$.

[13] This is the result of $(36 - 16)/65 = 0.31$ and $(32 - 6)/40 = 0.65$.

[14] A random selection of errors in diagrams will yield a certain combination of stated and unstated errors. At one extreme, it is possible that the random draw will include stated errors only, in which case the result will be the same as if we based our analysis on the errors the text. Otherwise, the random draw will include some unstated errors as well, and in this case the result will be better than if we had simply selected errors in the text, because unstated errors are more discriminating than stated errors (Property 2 in Section 4.2). Therefore the resolution capability of a random selection of the errors contained in diagrams is greater than or equal to the the resolution capability of an analysis based on stated errors.

```
      6   96
outG       0000000000 SG00000000 0000000000 0000000F00 000000000G GG00000000 0000000000 0000000000 0000000000 000000
India461   0000SSFPS0 SGGNGGS00G G00N0000SS 000000FF00 0PG00GG0GG GGSS000000 G0GS00SSGF 00G00F0000 00FS0SG000 00000F
India1270  00G0SSF000 SG0PSS00GG GNS0000SSS F?F???FFG? ?PGSFSSFSG GG0SS00000 0US000SSG0 0US0FF0000 0FFGSS000E E000FF
Petersbg   00G000F0S0 SG0NSS0000 0000000000 0G0GFF0FGF F0000GS0SG GG00S00000 0000000000 00S0000000 0000000000 00SS00
Bodleian   FGGSSSF0G0 SGSP0GGG00 0NSN0SG0GG 0S0S00FFSF FFGS0SGFGG GGGGSG00LS SUGS000000 F0G00F0RR0 00FGSSG0GE EGS00F
Fatih      00GSS0FNGP SGSPSSGS0G GPGPSSSS0G 0GF?F?FFGF ?N??????SG GGGG00RRRS GUS0FFSSG0 00SFFFFRRR F0FGS0GF?? ??????
```

Fig. 5 Matrix of the Diagram Characters

found in Diagrams 1–4, but the matrix consisting of all unstated characters is sufficient. I have checked and verified that they provide a similar result.

Next a cladistic method must be chosen, which brings us to the important issue of what this choice should be based on. Thirteen major cladistics techniques have recently been compared by estimating the similarity between the stemmata they provided on three independent data sets (Roos and Heikkilä 2009).[15] This comparison showed the superiority of two methods (maximum parsimony and RHM). In view of its effectiveness and wide diffusion, I have adopted the maximum parsimony method.

Given a set of manuscripts, several stemmata can account for the same distribution of characters. Since the number of stemmata increases rapidly with the number of manuscripts, the parsimony rule—i.e. the assumption that nature chooses the simplest course—selects the simplest stemma among all possible stemmata that may be constructed. The simplest stemma is the one that requires the fewest steps to explain the observed distribution of characters. The parsimony rule does not necessarily reveal the true phylogeny, but it provides the arrangement best supported by the data. A main criticism levelled against the parsimony rule is that this method is reliable only if homoplasies are rare. When the ancestor is known, the number of homoplasies can be counted, otherwise it must be put up for discussion. In the present case, one can be confident that homoplasies are rare, because a manuscript exists that is very close to the out-group: MS Petersburg, with a total of 6 U Errors (Table 3). Thus PHYLIP's maximum parsimony algorithm can be used with confidence.

When this program is applied to the matrix of characters described above, one most parsimonious tree is found (Fig. 6). The manuscripts connect to several ancestors by means of branches whose (horizontal) length is proportional to the number of transformed characters between the ancestor and the manuscript.[16] Therefore, it is not surprising that the above stemma provides the same ranking of the manuscripts.

In the stemma above, MS Petersburg directly connects to the common ancestor [1]. MS India 461 stems from an intermediate node [2] between MSS Petersburg and India Office 1270. MSS Bodleian and Fātiḥ share a common intermediate ancestor [4].

This finding may be checked in multiple ways. I have redrawn the stemma codicum on the basis of S, U or SU Errors separately, as well as from Diagram 1 and 3 separately. It appears that the results based on the type of error are roughly the same, except as regards the length of the branches. When Diagram 1 or 3 is used alone, the result is less stable, because the branching of MS India 461 moves from one node to another. This confirms that the method is reliable only in analyses involving more than fifty characters.

[15] The techniques are RHM, PAUP Parsimony, Parsimony Bootstrap, Neighbour Joining, Neighbour Joining Bootstrap, Least Squares, Least Squares Bootstrap; n-Gram Clustering; SplitsTree4 NeighborNet, SplitDecomp, ParsimonySplits, CompLearn, Hierarchical Clustering, and seven manual methods. The similarity between the stemmata produced is estimated by the average sign distance ASD, which provides a value ranging from 0 (the two stemmata are different) to 1 (the two stemmata are identical).

[16] A branch of zero length means that there is no difference between the manuscript (terminal node) and the progenitor (intermediary node). The closest fit is MS Petersburg, which is very similar to the out-group.

```
+outG
|
1-----Petersbg
|
|              +----------India461
+--------------2
               |        +------India1270
               +---------3
                         |            +-----------------Bodleian
                         +------------4
                                      +------------------Fatih
```

Fig. 6 Diagram Stemma

In any case, the present example indicates that a stemma codicum can be constructed with confidence from geometric diagrams alone.

5 Building the Text Stemma

As the text and diagrams are physically linked to each other, a powerful test can be effected by comparing the diagram stemma (Section 4) with the stemma codicum independently reconstructed from the text alone by standard methods.

Either the stemma codicum constructed from diagrams will coincide with that constructed from the text, or it will not. If they do coincide, this could be an indication that the two methods are equivalent; otherwise, the differences should be addressed.

5.1 Text Accidents

As is well known to philologists, long omissions and additions are of special interest for building the text stemma (Viré 1986; Woerther and Khonsari 2001). While a scribe can compensate for the omission of one word, he cannot restore a passage of several words without referring to a source. Therefore all the descendants of a corrupted model will carry the same corruption. We shall focus here on long omissions and additions, because this is the most reliable way to obtain the stemma from the text alone.

I first collated all the known extant manuscripts of Ibn al-Haytham's *Epistle* from time 0.000 to 0.922 (which coincides with the end of MS Fātiḥ). A count of all the textual accidents—with the exception of diacritics, orthographical variants, and articles—yielded the following results (Table 4).

We can limit ourselves to the three categories *alteravit, addidit, omisit*, because other alterations are less significant. Marginal corrections and transpositions are not marks of imperfection, nor are repetitions. The latter are a convention used in Arabic manuscripts to return to the main text after restoring an omission; such repetitions are preceded by the oblivion mark . The total number of errors ($\text{Total}^{1\,2\,3}$) and the *omisit* $\geqslant 3w$ (which records the number of long omissions over three words) both yield the same ordinal ranking:

$$\text{Petersburg} \prec \text{India 1270} \prec \text{Bodleian} \prec \text{Fātiḥ} \prec \text{India 461}$$

The collated manuscripts thus provide an ordinal ranking which is very similar to the one reached by the analysis of diagrams alone, except for MS India Office 461—a case to which we shall return shortly.

Table 4 MSS Ranking from Text Accidents

Type of Textual Error	Codex F	B	P	O	L
Correxit/marginalia	13	158	126	3	0
Transposuit	10	0	0	0	6
Repetivit	8	2	1	8	9
Delevit	5	26	1	2	0
Alteravit [1]	43	50	7	57	33
Addidit [2]	7	30	1	3	7
Omisit [3]	300	247	30	147	363
Total [1,2,3]	350	327	38	207	403
Omisit $\geqslant 3w.$	270	230	21	139	319
Ordinal Ranking	4	3	1	2	5

```
+outG
|
1----Petersbg
|
|                    +--India1270
+--------------------2
                     |  +------------Bodleian
                     +--3
                        |        +------------------Fatih
                        +--------4
                                 +---------------------India461
```

Fig. 7 Text Stemma

5.2 The Text Stemma

Text accidents can be encoded in various ways. I have chosen the weighted omission system, which replaces any textual omission 1/0 by the number of words omitted. This is both a rapid and reliable method. A matrix of characters is built as previously. It consists of six taxa (the five MSS and the out-group, that is, the text without errors) and of as many characters as there are omissions in the text from time 0.000 to time 0.922. PHYLIP's maximum parsimony algorithm is then applied as in Section 4.3. Again, a single most parsimonious tree is found (Fig. 7). In this stemma, MS Petersburg is directly connected to the ancestor [1]. MSS India Office 1270 and Bodleian connect to intermediate nodes [2] and [3]. MSS Fātiḥ and India Office 461 have a common intermediate ancestor [4].

6 Discussion

In this stemmatological analysis of the manuscripts of Ibn al-Haytham's *Epistle on the Shape of the Eclipse*, the text stemma (Fig. 7) was found to be quite similar to the diagram stemma (Fig. 6), except for one manuscript—MS India Office 461. From the diagram stemma, in which it was ranked extremely high (just below MS Petersburg, the best witness), in the text stemma MS India Office 461 jumped over three manuscripts to arrive last in the ranking. Its text has now been determined to stem from the same archetype as MS Fātiḥ, which would explain its receding from third to fifth place in the ranking of the manuscripts. Succinctly put, it means that MS India Office 461 is a reliable witness to the diagrams but unreliable with regard to the text, while MS Bodleian is a reliable witness to the text but unreliable as to the diagrams. This difference is understandable:

1. The diagrams of MS B are very faulty. The scribe himself was aware of this situation. In the marginal annotation to Diagram 3, he criticizes his own work and describes the figure as it was in the archetype he used: "The line $\d{S}\d{T}F$ must be straight, as well as the line $D\d{H}F$... the extension of the line $D\d{T}$ meets N, so that the line $D\d{T}N$ is tangent to arc KNM at point N" (MS B, fol. 96r, time 0.757). This brief note is hardly sufficient. Many more accidents should have been corrected and these errors are largely responsible for the recession of MS B in the diagram ranking.
2. The diagrams of MS L have all been drawn together on a separate sheet (fol. 34r), which has a different format than the text (plate 215×280 mm; text 229×140 mm). By contrast, the diagrams are embedded in the text in all other extant manuscripts of the *Epistle*. This strongly suggests that the diagrams were corrupted or absent in the ancestor of MS L. Its diagrams could have been redrawn either from textual instructions, or from another exemplar (Section 4.1).[17]

Accordingly, MS L presents a textbook example of a distinct lineage for the text and diagrams because the switch of MSS B L in the stemma is confirmed by direct physical inspection of the manuscripts.

Further confirmation of external contamination of the diagrams is provided by another reconstruction of the stemmata after having first removed MS L (India Office 461) from both the diagram and the text matrices (Fig. 8–9).

The withdrawal of MS L provides two stemmata that are topologically identical, excepting a variation in length of the branches.[18] MS Petersburg connects to the lost ancestor [1]. MS India Office 1270 stems from node [2]. MSS Fātiḥ and Bodleian connect to node [3].[19]

These results are in line with other scholarly studies of Ibn al-Haytham's works. In the critical edition of Ibn al-Haytham's *Maqāla mustaqṣāt fī al-ashkāl al-hilāliyya (Comprehensive Treatise on Crescent-Shaped Figures)*, MSS P and O (there denoted MSS L and A) were found by Rashed to independently derive from the lost ancestor: MS P directly connects to the ancestor, while MS O stems from it through two intermediate archetypes (Rashed 1993, pp. 24-5). In the critical edition of Ibn al-Haytham's *Qawl fī birkār al-dawā'ir al-'iẓām (Discourse on the Compass of Great Circles)*, MSS P and O (there denoted MSS L and B) belong to two separate branches. MS P stems from the ancestor, while MS O has two intermediate archetypes (Rashed 2006, p. 856–7). These findings are in accordance with the present independent analysis.

Returning now to the scholarly literature on Ibn al-Haytham's *Epistle*, it appears that the diagrams are quite often faulty in the editions of Wiedemann (1914), Naẓīf (1942/3) and Sabra (1972). Wiedemann is wrong on characters 1-01_s, 1-07_s, 1-12_s, 1-16_s; 2-02_s, 2-03_s, 2-01_u, 2-02_u, 2-03_u; 3-07_s, 3-08_s, 3-09_s, 3-10_s, 3-11_s, 3-12_s, 3-13_s, 3-14_s, 3-15_s, 3-16_s, 3-17_s; 4-04_s, 4-05_s, 4-26_s,

[17] This conclusion does not hold for later periods. With the spread of copperplate engraving, diagrams were frequently grouped together on pages distinct from the text in printed editions (Rider 1993, Barrow-Green 2006). Prof. Gregg de Young informs me that later manuscripts could at times reproduce this arrangement, such as Cairo, Dār al-kutub, handasa turkiyya 42, ca. 1300 H./1834 or Ṭal'at Riyāda, handasa turkiyya 3, ca. 1250 H./1882. Quite possibly, these two manuscripts were copied from the *Usūl-i hendese*, a Turkish translation by Hüseyin Rıfkı Tamānī of Bonnycastle's *Elements of Geometry* that was printed by the Maṭba'at Būlāq in 1241 H./1825 (De Young 2012, p. 51).

[18] This variation in length means that there are more text differences than diagram differences between, say, node [2] and MS India 1270 (Fig. 8–9).

[19] Another way to proceed might have been to search for the consensus tree between the diagram and text stemmata. We did not proceed any further in this direction, because the consensus tree algorithm is based on bootstrapping, which has certain limitations (Wiesemiller et al. 2006, pp. 161–5; Kitching 1998, pp. 129–31).

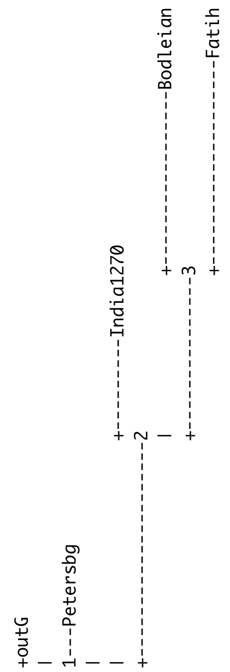

Fig. 8 Diagram Stemma–L

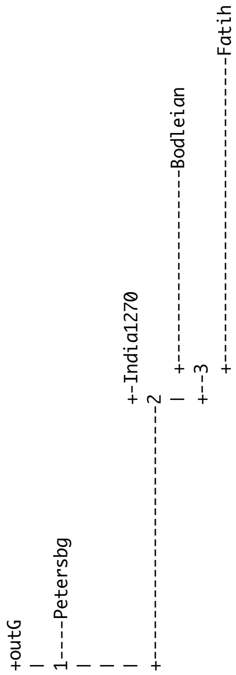

Fig. 9 Text Stemma–L

4-07$_u$, 4-08$_u$. Naẓīf was not aware of the diagrams of Ibn al-Haytham's *Epistle*, since he relied instead on the 1928 Hyderabad edition of Kamāl al-Dīn's *Tanqīḥ al-Manāẓir*. Sabra in his turn copied Naẓīf's diagrams.

Accordingly a key result of the present study is that we should in any case distinguish between the (mathematically true) diagrams drawn in order to explain the text and the (codicologically true) diagrams produced for a critical edition worthy of the name. Historians of science ought at least to be conscious of and state clearly what they are doing. The critical editing of Ibn al-Haytham's *Epistle* should be based on MS P (the *lectio praeferenda*) and MS O, these being the most reliable witnesses, with the aid on occasion of MSS B and L.

7 Conclusions

The purpose of this paper was to explore the phylogeny of diagrams contained in mathematical manuscripts by constructing the first stemma of diagrams ever to be published. After preliminary tests and clarification, Ibn al-Haytham's *Epistle on the Shape of the Eclipse* was selected as the test case.

The principal finding from the stemmatological analysis of this set of manuscripts was the strong similarity between the diagram and text stemmata, with the exception of one manuscript whose diagrams were drawn on a separate sheet. The results were reached by a blind, parallel construction of the diagram and text stemmata. From the analysis and discussion presented above, we can draw the following conclusions:

C1. The codicological study of diagrams is needed in all cases, because it can help us find features that might otherwise be overlooked. Specifically, the critical editing of diagrams should retain all the shared characters and all the unstated characters from the manuscript that constitutes the best witness. This method is seldom applied in the critical editing of geometrical texts, which usually adopts the text as its reference and replaces the original diagrams with mathematically true diagrams. There is a simple reason for this bias: to choose which diagram to reproduce, a comprehensive stemmatological study of diagrams is required.

C2. It is sufficient to analyze the unstated characters in a small number of diagrams, provided the number of susceptible loci is great enough (i.e. over 50) to calculate the manuscript ranking. Once the text has been read and understood, there is no need to study in detail the errors in the text, for unstated characters are more discriminating than stated characters. Even if the ranking would improve in reliability by including an ever larger number of susceptible loci, there is no need to undertake an overly long codicological analysis. Hence this suggested guideline.

C3. Contrary to the general study of mathematical diagrams, stemmatology is not concerned with the meaning of errors, but with the fact that they can provide us with information regarding a manuscript's phylogeny. The link between the text and diagram stemmata suggests that the phylogeny shows a low sensitivity with regard to the number of hands involved, i.e. whether the copy was made by one or more scribes. Copyists make errors of their own, but the extent of the possible errors depends on the existence of unstated characters in the source. Thus the diagram and text stemmata can vary, but they cannot be utterly different.

C4. The diagram stemma and text stemma are similar when the text and diagrams are created at the same time. When physical inspection of the manuscripts reveals diagrams properly embedded in the text, there is no reason to presume a shift between the text and diagrams and the stemma can be constructed from either the text or diagrams. If physical inspection reveals discrepancies such as marginal diagrams, emended diagrams, different drawing styles, diagrams appearing on separate sheets, etc., both stemmata are needed.

C5. Any geometric diagram has more susceptible loci of error than a text of the same size. In a text, the quantity of information roughly varies with its length (L), whereas in a geometric diagram the quantity of information depends on its area (L^2). The chance of error is thus higher in a geometrical diagram than in a text taking up the same amount of space on the page. In the case under discussion here, the density of errors is seven to eight times greater in the diagrams than in the text.[20] This strongly suggests that constructing the stemma codicum from diagrams could be faster than building the stemma from the text.

C6. Under standard conditions[21] *it is advantageous to draw the stemma codicum of a handwritten mathematical tradition from its diagrams.* Since in general the stemmatological study provides similar results for the text and diagrams (C4) and errors are more highly concentrated in the diagrams than in the text (C5), it is more efficient to construct the stemma from the diagrams. This furthermore helps to clarify the relationship between the text and diagrams. A mathematical understanding of the text is needed with a view to establishing the diagram stemma, but the text stemma is not needed. The two stemmatological procedures are independent.

What the latter conclusions suggest is an efficient and convenient procedure for building the stemma codicum from a set of manuscripts. At the present stage this outcome should be regarding

[20] The diagrams in MS O contain 48 errors within a total area of 38.26 cm^2 and the error density is therefore $\delta_D = 1.25$ per cm^2, while the text contains 138 errors within a written area of 847.20 cm^2: $\delta_T = 0.16$ per cm^2. The diagrams in MS B contain 57 errors within a total area of 134.29 cm^2: $\delta_D = 0.42$ per cm^2, while the text has 262 errors within an area of 4034.84 cm^2: $\delta_T = 0.06$ per cm^2. Therefore the number of errors is $7.0 \leqslant \delta_D/\delta_T \leqslant 7.8$ times greater in a diagram than in a text of the same area.

[21] That is, when the diagrams are embedded in the text and show no obvious signs of emendation or alteration.

as tentative pending further research. But in any event the guidelines proposed here could foster significant advances in the critical editing of diagrams not only in the area of geometry, but also in allied sciences such as astronomy, optics, mechanics, architecture and perspective, especially where the handwritten tradition is rich but divergent and hence confusing.

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Table 5 List of Stated Characters in Ibn al-Haytham's *Epistle on the Shape of the Eclipse*

Character	MSS	F	B	P	O	L
Diagram 1						
1-01$_s$ Line AǦ drawn		0	f	0	0	0
1-02$_s$ SB radius of circle ABǦ		0	g	0	0	0
1-03$_s$ ṢD radius of circle ADǦ		g	g	g	g	0
1-04$_s$ Line KW = WM		s	s	0	0	0
1-05$_s$ FN radius of arc KNM		s	s	0	s	s
1-06$_s$ YL radius of arc KLM		0	s	0	s	s
1-07$_s$ AǦ collinear to KM *		f	f	f	f	f
1-08$_s$ FT parallel to ḤH		n	0	0	0	p
1-09$_s$ Line FT = FN		g	g	s	0	s
1-10$_s$ FT parallel to ṢD		p	0	0	0	0
1-11$_s$ FT radius of arc ŠFH̱ *		s	s	s	s	s
1-12$_s$ FT radius of ŠYH̱ *		g	g	g	g	g
1-13$_s$ Chord ŠH̱ = KM		s	s	0	0	g
1-14$_s$ KM parallel to AǦ		p	p	n	p	n
1-15$_s$ Line ẒL = FN		s	0	s	s	g
1-16$_s$ Line QN = FN		s	g	s	s	g
1-17$_s$ ṮG = KM		g	g	0	0	s
1-18$_s$ ŠH̱ = ṮG deduced		s	g	0	0	0
Diagram 2						
2-01$_s$ Circle ABǦ = circle BHǦ		0	0	0	g	0
2-02$_s$ Line ḤṬ = radius AḤ		g	0	0	g	g
2-03$_s$ Line ḤṬ = radius HṬ		g	0	0	g	g
2-04$_s$ DM parallel to AḤ		p	n	0	n	0
2-05$_s$ Line DK = ML		g	s	0	s	0
2-06$_s$ Line ML ⊥ AḤ		p	n	0	0	n
Diagram 3						
3-01$_s$ Line ŠR = RH̱		s	0	0	0	0
3-02$_s$ Line KW = WM		s	s	0	0	0
3-03$_s$ Line ṮL = LG		s	g	0	0	0
3-04$_s$ Line KŠ = FN		s	0	0	s	0
3-05$_s$ Line KŠ = KṮ		0	g	0	s	s
3-06$_s$ Line H̱M = MG		g	g	0	s	s
3-07$_s$ Circle ŠṮ drawn		0	0	0	f	0
3-08$_s$ ŠK radius of circle ŠṮ		g	s	g	?	0
3-09$_s$ Circle H̱G drawn		f	0	0	f	0
3-10$_s$ H̱M radius of circle H̱G		?	s	g	?	0
3-11$_s$ Circle ŠṮ ⊥ circle ṮẒG at Ṯ		f	0	f	?	0
3-12$_s$ Circle H̱G ⊥ circle ṮẒG at G		?	0	f	?	0
3-13$_s$ Ḍ near K on arc LĠ		f	f	0	f	f
3-14$_s$ AǦ collinear to KM *		f	f	f	f	f
3-15$_s$ Line KF = KŠ		g	s	g	g	0
3-16$_s$ F on circle ŠṮ		f	f	f	?	0
3-17$_s$ F on circle H̱G		?	f	f	?	0
3-18$_s$ FT parallel to ḤH		n	f	0	p	p
Diagram 4						
4-01$_s$ TF, radius of arc ŠFH̱		?	g	0	g	g
4-02$_s$ Line KG = KŠ		?	s	0	s	0
4-03$_s$ Chord ŠH̱ < diam. ŠFH̱		?	0	0	f	0
4-04$_s$ Line QḌ = QR		?	s	g	s	g
4-05$_s$ Line QH = 2 GH		?	g	s	s	g
4-06$_s$ Point H on circle Q,QR		?	f	0	f	0
Total		42	35	18	35	23

Table 6 List of Unstated Characters in Ibn al-Haytham's *Epistle on the Shape of the Eclipse*

Character	MSS	F	B	P	O	L
Diagram 1						
1-01$_u$ Lune ABǦD = lune KLMN		s	g	s	s	g
1-02$_u$ Arc ŠFẖ = arc ŠYẖ *		g	g	g	g	g
1-03$_u$ Arc KNM = arc KLM *		g	g	g	g	g
1-04$_u$ Arc ṮQG = arc ṮẒG *		g	g	g	g	g
1-05$_u$ Chord Šẖ = chord KM		g	g	0	0	s
1-06$_u$ Chord ṮG = chord KM		g	g	0	s	s
1-07$_u$ Sagitta QẒ = sagitta NL		0	s	s	s	0
1-08$_u$ Sagitta FY = sagitta NL		0	g	0	0	0
1-09$_u$ Center of ŠYẖ on TẒ		r	0	0	0	0
1-10$_u$ Center of KLM at Y on TẒ		r	0	0	0	0
1-11$_u$ Center of ṮẒG at L on TẒ		r	l	0	0	0
1-12$_u$ Chord KM = 2 diam. ḤH		s	s	0	0	0
1-13$_u$ Distance 2 DḤ = 3 ḤF		g	s	0	0	g
1-14$_u$ Point Ṣ in circle ABǦ		u	u	0	u	0
1-15$_u$ Wall Eclipse Magnitude ≈ 0.779		s	g	0	s	g
1-16$_u$ Sky Eclipse Mag. as in 1-15$_u$		0	s	0	0	s
1-17$_u$ Arcs KLM, KNM meet in K		f	0	0	0	0
1-18$_u$ Arcs ŠYẖ, ŠFẖ meet in H̱		f	0	0	0	0
Diagram 2						
2-01$_u$ 3 KD > 2 radius AḤ		s	0	0	s	s
2-02$_u$ Line ZN = line AZ		s	0	0	s	s
2-03$_u$ Line NH = line ZN		g	0	0	g	g
2-04$_u$ Point Ǧ drawn		0	0	0	0	f
Diagram 3						
3-01$_u$ Chord AǦ drawn		0	f	0	0	0
3-02$_u$ Point Ṣ in circle ABǦ		0	0	0	u	0
3-03$_u$ Lune ABǦD = lune KLMN		s	g	s	s	g
3-04$_u$ Line DḤF drawn		f	0	0	0	0
3-05$_u$ Line DṬN drawn		f	0	0	f	0
3-06$_u$ Chord Šẖ parallel to KM		f	f	0	f	f
3-07$_u$ Chord TG̣ parallel to KM		f	0	0	0	0
3-08$_u$ Center of ŠYẖ on TẒ		r	r	0	0	0
3-09$_u$ Center of KLM at Y on TẒ		r	r	0	0	0
3-10$_u$ Center of ṮẒG at L on TẒ		r	0	0	0	0
3-11$_u$ Arcs ṮẒG, ṮQG meet in Ṯ		f	0	0	0	0
3-12$_u$ Line FK drawn		0	0	0	f	0
3-13$_u$ Arc ḌḎ drawn		f	f	0	f	f
3-14$_u$ Wall Eclipse Magnitude ≈ 0.779		g	g	0	g	s
3-15$_u$ Sky Eclipse Mag. as in 3-14$_u$		s	s	0	s	0
3-16$_u$ Chord KM = 2 diam. ḤH		0	s	0	s	s
3-17$_u$ Distance DḤ = ḤF		g	g	0	0	g
3-18$_u$ Point Ṭ center of HḤ		f	0	0	0	0
Diagram 4						
4-01$_u$ Arc ŠFG̱ = arc KNM		?	g	0	0	0
4-02$_u$ Line QN < radius KQ		?	e	0	e	0
4-03$_u$ Line RF < radius ŠR		?	e	0	e	0
4-04$_u$ Line QN = RF		?	g	0	0	0
4-05$_u$ Line ŠR = RḤ		?	s	s	0	0
4-06$_u$ Line KQ = QM		?	0	s	0	0
4-07$_u$ Radius TŠ drawn		?	0	0	f	0
4-08$_u$ Rectangle on TṢ = 2 KG		?	f	0	f	f
Total		40	32	8	24	20

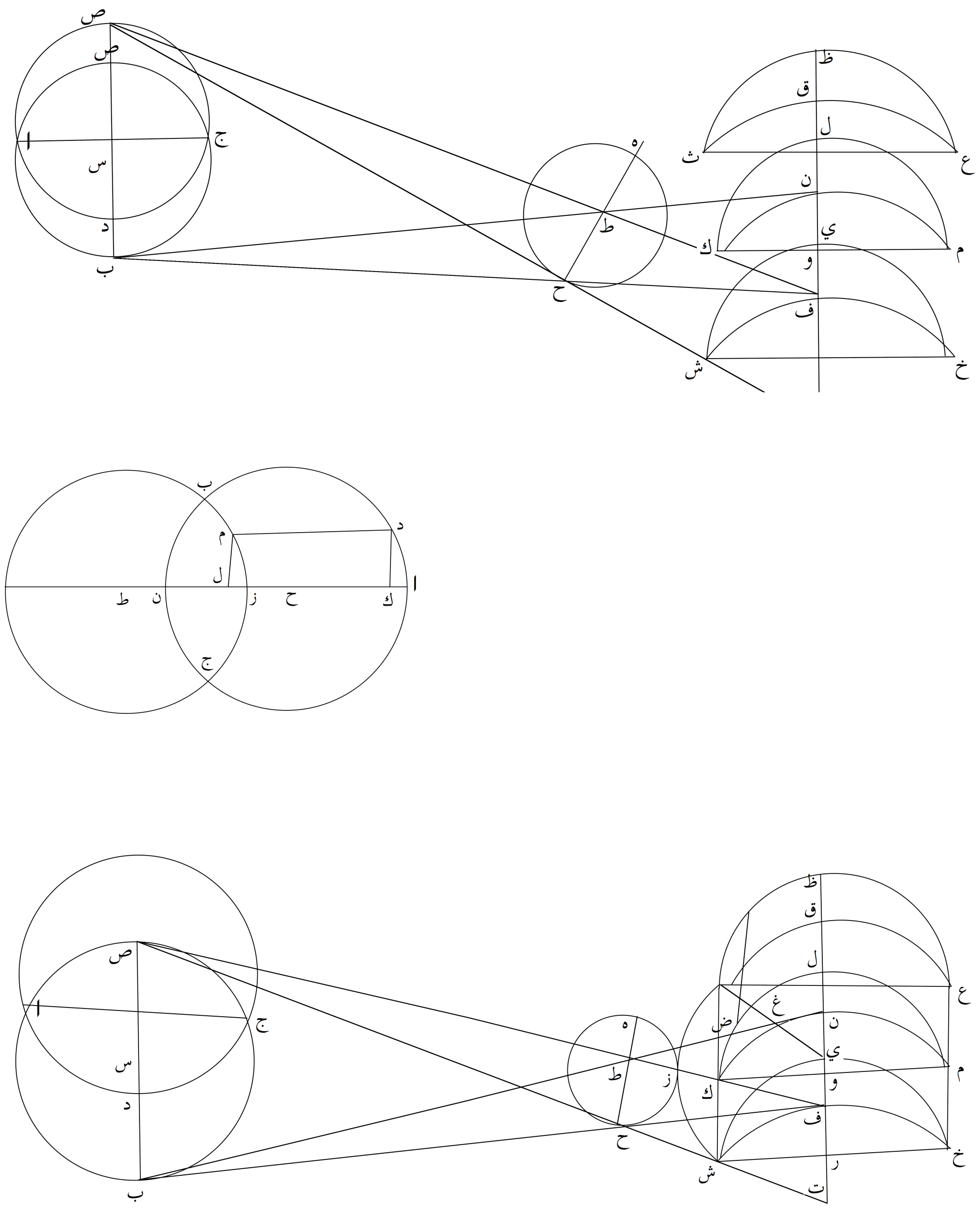

Fig. 10 MS F Fātiḥ 3439

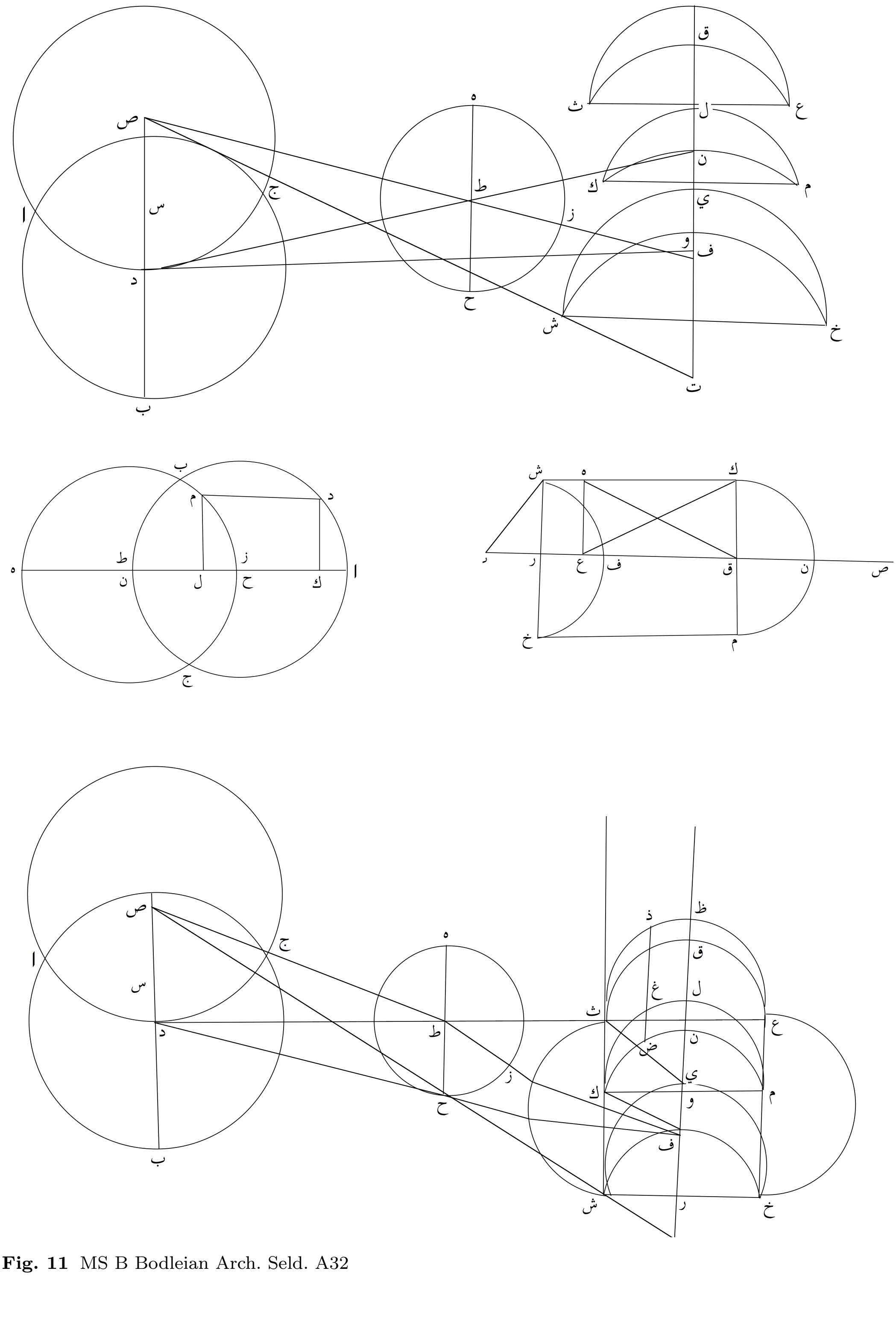

Fig. 11 MS B Bodleian Arch. Seld. A32

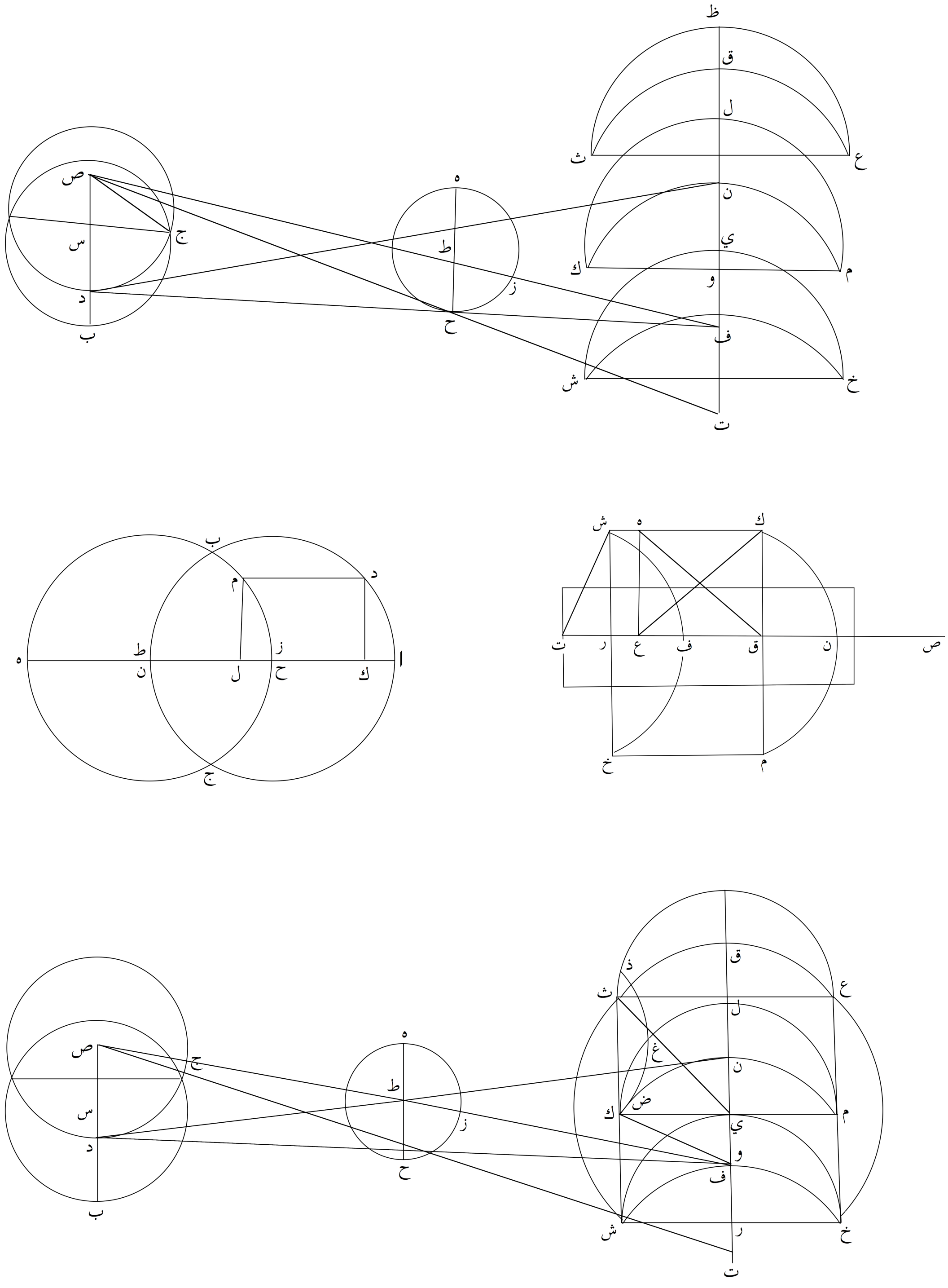

Fig. 12 MS P St. Petersburg B1030

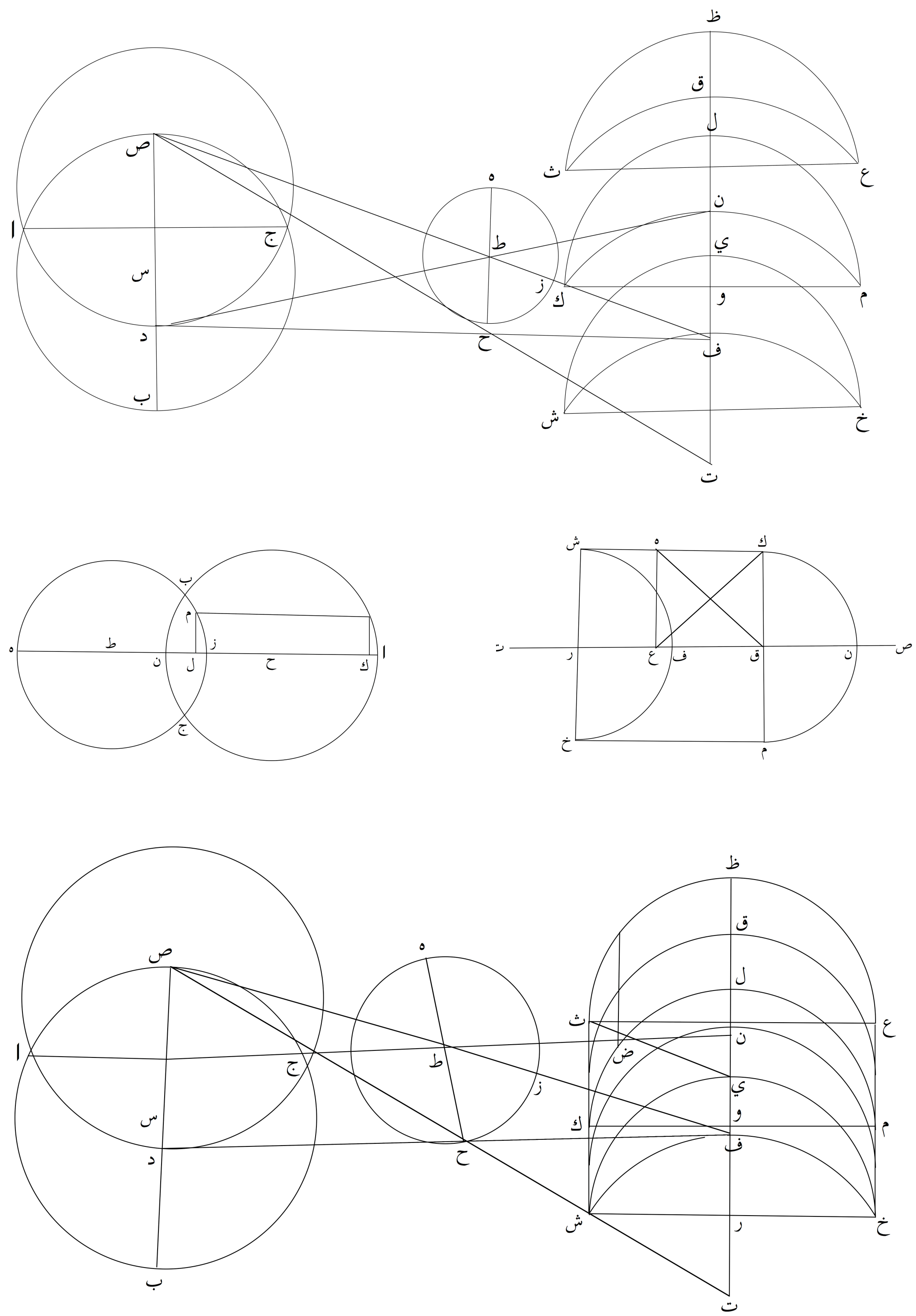

Fig. 13 MS O India Office 1270

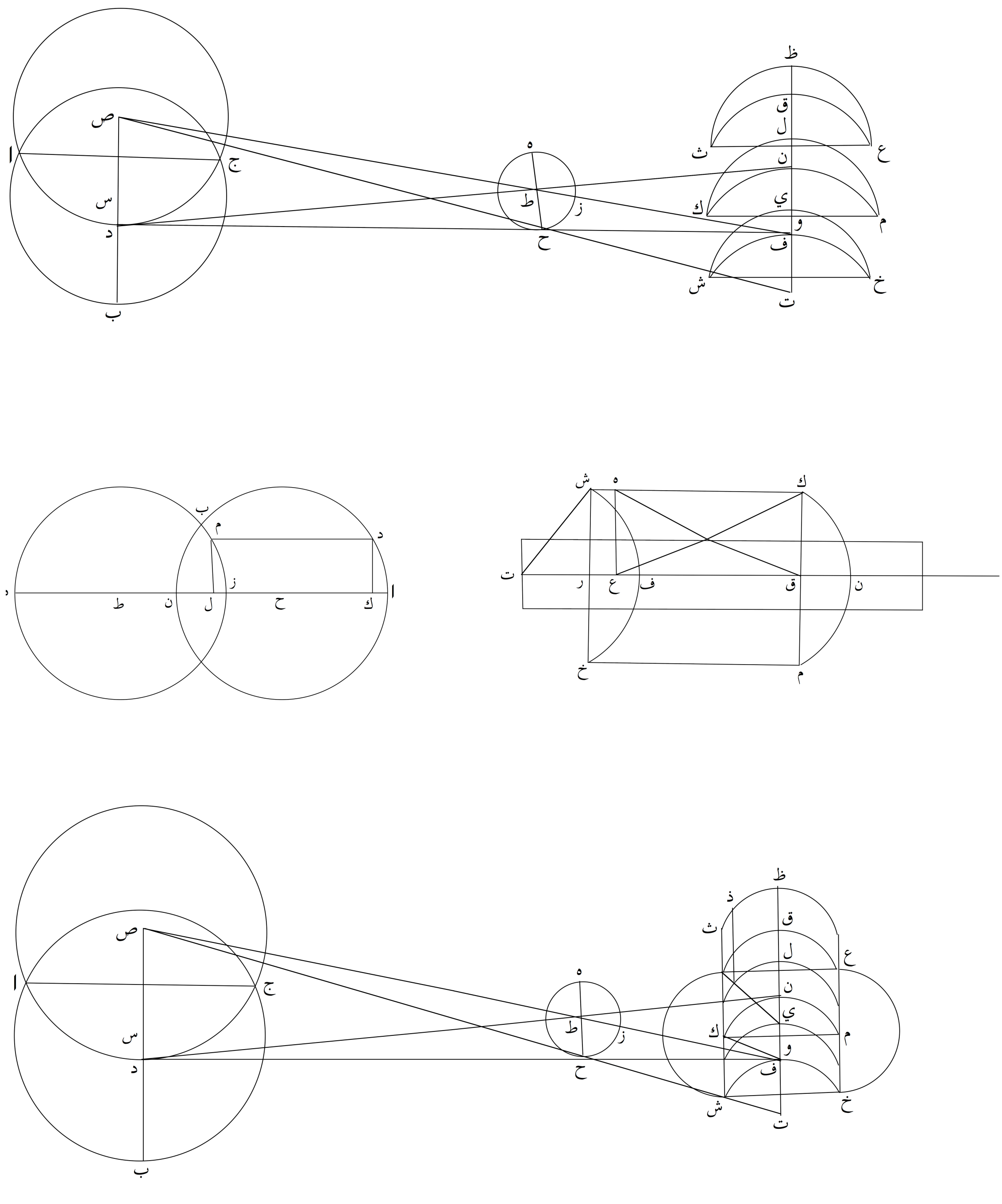

Fig. 14 MS L India Office 461

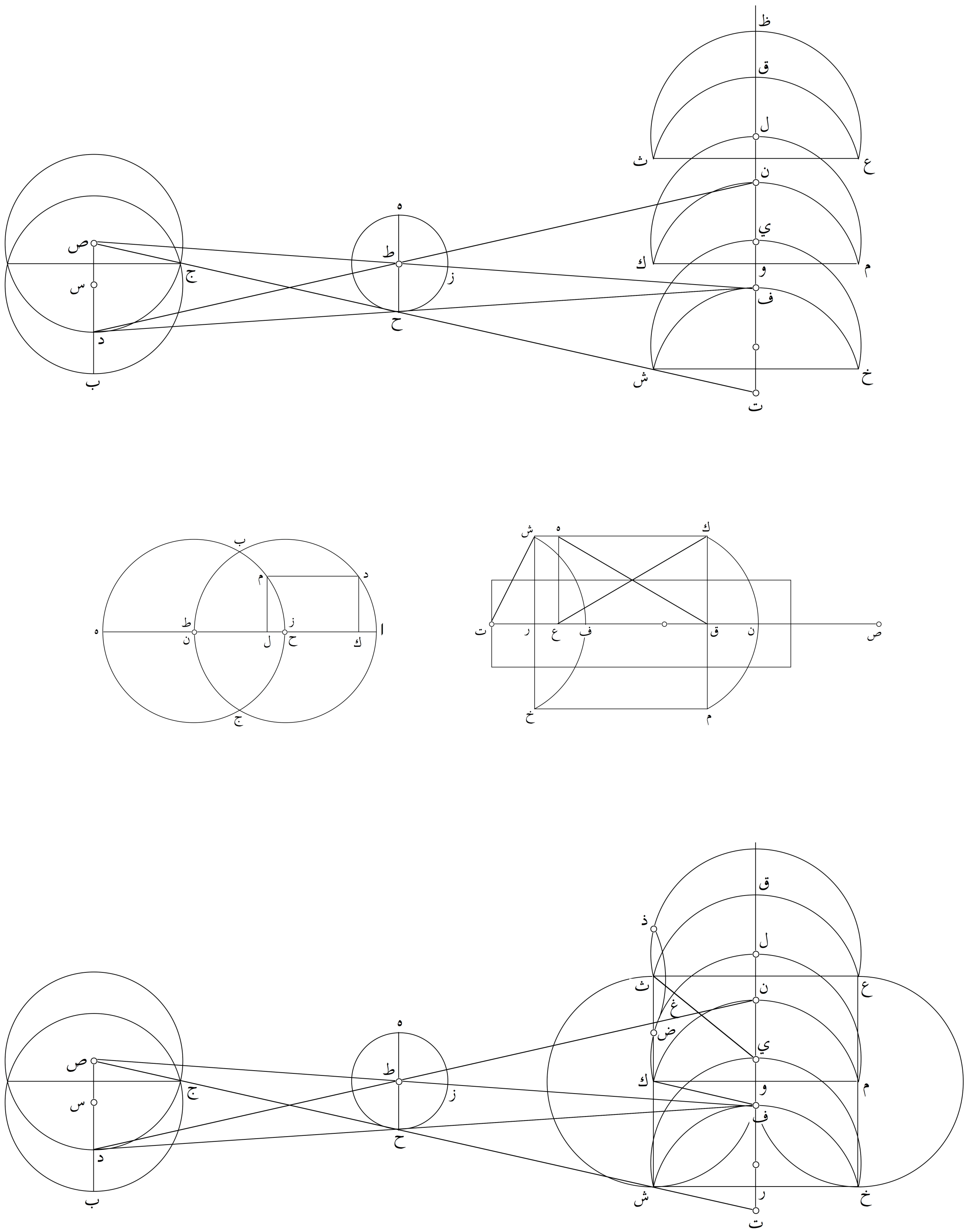

Fig. 15 Edited Diagrams