Solutions to Cpt S 450 Homework #10

Please print your name!

1. Describe a proof that, for any three NP-problems A, B, C, we have
$A \leq_m B$ and $B \leq_m C$ implies $A \leq_m C$.

Given $A \leq_m B$ and $B \leq_m C$, according to the definition of $\leq_m$, we have two translations T_{AB} and T_{BC} such that, for any x,
$x \in A$ iff $T_{AB}(x) \in B$
and for any y,
$y \in B$ iff $T_{BC}(y) \in C$.
Now, consider a new translation $T_{AC}(x) = T_{BC}(T_{AB}(x))$. Clearly, since both T_{AB} and T_{BC} run in polynomial time, so is T_{AC}, Also, one can show, for any x,
$x \in A$ iff $T_{BC}(T_{AB}(x)) \in C$.
Hence, $A \leq_m C$.

2. Here is a nondeterministic algorithm (running poly-time):
Guess a path p not longer than n (the total number of nodes in G), and check p covers every node exactly once.

3. Here is a nondeterministic algorithm (running poly-time):
Guess a path p not longer than n^2 (n the total number of nodes in G), and check p covers every node.
Why it works? Explain!

4. Notice that C_1 and C_2 are NOT equivalent iff
C is satifiable, where C is the circuit $(C_1 \wedge \neg C_2) \vee (C_2 \wedge \neg C_1)$.
Result follows from the assumption that we have a deterministic polynomial time algorithm that decides whether a given circuit C is satisfiable.