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CS350
HW 2

1.p=high q=low
```
partition(A,p,q){
x=A[p]
y=q-1
for z=1;z<=p-1;z++{
	if(A[z]<=x){
		y++
		swap A[y] and A[z]
	}
}
swap A[y+1] and A[p]
return i+1
}
```

2.Tavg as the average case and Tw as the worst case when the input is monotonically decreasing which is θ(n^2). So:
Tavg(n) = 1/100 *Tw(n) + 99/100*Tavg(n)
We recall that Tw(n) is θ(n^2) and Tavg(n) is O(n^2)
θ(n^2)+0 <= Tavg(n) <= θ(n^2)+O(n^2) = θ(n^2)
So, Tavg(n) = θ(n^2).

3. Tqs(n) as quicksort time and Tis as insertsort time.

Worst Case:
Tw(n) = Twqs(r-1)+Twis(n-r)+ θ(n)
To find the worst case, we need to know both worst case of quicksort and insertsort. We know that the worst case of Tis is θ((n-r)^2) and the worst case of Tqs is θ((r-1)^2). From this we can get the worst time by checking if one of them reaches θ((n-1)^2). Thus, the worst case is going to be θ(n)+ θ((n-1)^2) = θ(n^2)

Best Case:
T(n) = Tqs(r-1)+Tis(n-r)+ θ(n)
We should find best case of qs and is. We know that Tqs best case is θ((r-1)log(r-1)) and Tis best case is θ(n-r). We can see that Tis best case is better than Tqs, so we can conclude that the best case for T(n) = θ(n)

Avg Case:
Tavg(n) =Σ(1<=r<=n)1/n( Tavgqs(r-1)+Tavgis(n-r)+ θ(n))
Tavg(n) <=Σ(1<=r<=n)1/n(a*(r-1)log(r-1)+b(n-r)^2+ a*n)
<=2/n(Σ(1<=r<=n)a(r-1)^2+a*n
<=2/n$\int_0^n ax^2dx + an = \frac{2a}{n} * \frac{1}{3}n^3 + a * n$=O(n^2)

4. $T_{ms}(n)$ as mixsort time and $T_{is}$ as insertsort time.

Best case:

$T(n) = T_{ms}(r-1)+T_{is}(n-r)+ \theta(n)$

Best case of insertsort is $\theta(n-r)$ and the best case of mixsort is at least linear. So we can imagine from the equation that the equation will work best when low part is empty and insertsort get the best case. So best case is, $T(n) = \theta(n)+\theta(n-1)=\theta(n)$.

Worst case:

$Tw(n) = \max(1<r<n)\ Twqs(r-1)+Twis(n-r)+ \theta(n)$

$Tw(n)<\max(1<r<n)(a(n-r)^2+c(r-1)^2+an)$ we can say this as $F(r)$.

We can find max by double derivative of $F(r)$ which is $2a+2c$. When double derivative is plus, the diagram will look like a bowl. From this we can say that the max is $F(1)$ and $F(n)$.

Derivative of $F(1)$ is $a(n-1)^2+an$ and $F(n)$ is $c(n-1)^2$. From this we can say that the worst case is $O(n^2)$

Avg Case:

$Tavg(n) =\Sigma(1<=r<=n)1/n( Tavgms(r-1)+Tavgis(n-r)+ \theta(n))$

$Tavg(n) <=\Sigma(1<=r<=n)1/n(Tavgms(r-1)+b(n-r)^2+ a*n)$

$<=an+b*1/n*\Sigma(1<=r<=n)(n-r)^2+(1/n* \Sigma(1<=r<=n)Tavgms(r-1)$

Then we guess that Tavgms is $O(n^2)$ then $Tavgms<cn^2$ for some c and we do integral on both $(n-r)^2$ and $(r-1)^2$.

$<=an+ ((b(n-1)^3)/3n) + c/n*(n-1)^3/3$

$<= an+bn^2/3+cn^2/3$

$c>>a$

$<=cn^2$

Thus mixsort average case is $O(n^2)$