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Restoration of Horizontal Motion Blurred Images Based on Wiener Filtering

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Abstract: This paper proposes to use Wiener filtering for restoring the horizontal motion blurred images and designs a method for estimating the required parameters. In this paper, the spectrum data of motion blurred images are processed, and an effective method for measuring the motion blurred length is designed. After the wavelet decomposition of the blurred image, suitable processing of low frequency coefficients and high frequency coefficients may get the noise to signal power ratio which is needed by Wiener filtering. MATLAB simulation results show that the motion blur length obtained by this method is more accurate, the restoration method proposed by this paper is suitable for horizontal motion blurred images, the effect is better and the ringing artifact is not obvious.

Key Words: Image restoration, motion blur, PSF, Wiener filtering, noise to signal power ratio

1 Introduction

The relative motion between camera and picture will produce the motion blurred image which is usually described by point spread function (PSF). PSF includes two factors: length and angle. A high speed imaging digital camera can reduce the influence of motion blurring on images very well only but the very high price. Another way is to use image deblurring technology, to which whether the blur angle and blur length in PSF can be correctly gotten is the key. If the motion blur angle is zero, for example the camera takes photos of the workpiece on the conveyor belt, or takes external shots in a moving vehicle, and so on, only the motion blur length is needed for restoration. This paper will focus on image deblurring in this case.

Many scholars have made meaningful research on restoration of motion blurred images: Reference[2] improved inverse filtering algorithm. Reference[3] improved Wiener filtering algorithm. Reference[4] used Lucy-Richardson algorithm. Several methods have their own advantages. In addition to the motion blur length,image restoration also needs the noise to signal power ratio (NSPR) of the blurred image. If there are only blurred images to be relied on, these parameters can only be gotten through estimating. Reference [5] used cepstrum and Radon changes to determine PSF parameters. Reference [6] used the radial basis function neural network to determine the blur length. Reference [7]got the motion blur length by measuring the positions of two zero frequency spike on the differential autocorrelation curve. Reference [8] estimated the NSPR with the wavelet high frequency coefficients. Reference [9] used the difference between the Wiener filtered image and the blurred image to estimate NSPR.

Considering all the above literature and because of its small amount of computation and the strong anti noise ability, Wiener filtering is adopted for restoring the horizontal motion blurred image in this paper. This paper also proposes that the blur motion length is determined by measuring the spectrum main lobe width of the blurred image, and NSPR is estimated by way of wavelet decomposing and inverse filtering. MATLAB simulation shows that the restoration is effective and the estimation of the blur length and NSPR are accurate.

2 Technology of Image Restoration

2.1 Blurred Image Model

Blurred images caused by motion blurring and random noise can be described with the following model:

$$g(x,y) = f(x,y) * h(x,y) + n(x,y) \tag{1}$$

Where $f(x,y)$ is an original clear image, the degradation model $h(x,y)$ is related to the reason of blurring, also known as point spread function (PSF), $n(x,y)$ is additive random noise in which Gaussian noise is the most common, $g(x,y)$ is a blurred image. Image $f(x,y)$ and $g(x,y)$ are shown in Fig. 1, the standard deviation of Gaussian noise mixed in $g(x,y)$ is $\sigma = 0.02$.

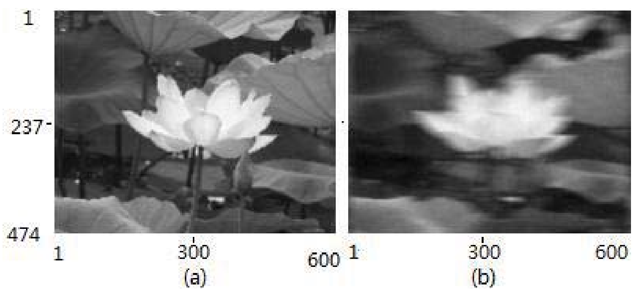

Fig. 1: (a) Original image $f(x,y)$;
(b) Blurred image $g(x,y)$ when $L = 40, \theta = 0^0$ and $\sigma = 0.02$

Fourier transform of formula (1) is shown as formula (2). If the pixel of an image is $M \times N$, then $0 \le u \le M-1$, $0 \le v \le N-1$.

$$G(u,v) = F(u,v)H(u,v) + N(u,v) \tag{2}$$

2.2 Wiener Filtering

The mean square error between the original image and the restored image is the smallest, which is the basic idea of

Wiener filtering. The mean square error between $f(x,y)$ and $f'(x,y)$ is

$$e^2 = E\{(f-f')^2\} \tag{3}$$

Where $E\{\cdot\}$ is the expected value, supposing $f(x,y)$ is not related to $n(x,y)$, and the mean value of $n(x,y)$ is 0. When e^2 is the smallest. Considering formula (1) and formula (2), the restored image $f'(x,y)$ in its frequency domain can be described as the following

$$F'(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \dfrac{P_n(u,v)}{P_f(u,v)}} \right] G(u,v) \tag{4}$$

Where $P_n(u,v)$ and $P_f(u,v)$ is the power spectrum of $n(x,y)$ and $f(x,y)$ respectively. $P_n(u,v)/P_f(u,v)$ is called noise signal power ratio (NSPR). $H(u,v)$ is the spectrum of $h(x,y)$ [6,10,11].

2.3 Degradation Model and Motion Blur Length

A motion blurred image can be described by PSF including two parameters: blur length L and blur angle θ. The degradation model is $h(x,y)$ as the following

$$h(x,y) = \begin{cases} 1/L & 0 \le |x| \le L\cos\theta, y = \sin\theta \\ 0 & \text{else} \end{cases} \tag{5}$$

If original image $f(x,y)$ is motion blurred in the horizontal direction, the blur angle is $\theta = 0$, so

$$h(x,y) = \begin{cases} 1/L & 0 \le |x| \le L, y = 0 \\ 0 & \text{else} \end{cases} \tag{6}$$

$H(u,v)$ is the discrete Fourier transform of $h(x,y)$, and

$$|H(u,v)| = |Sa(\pi u L)| \tag{7}$$

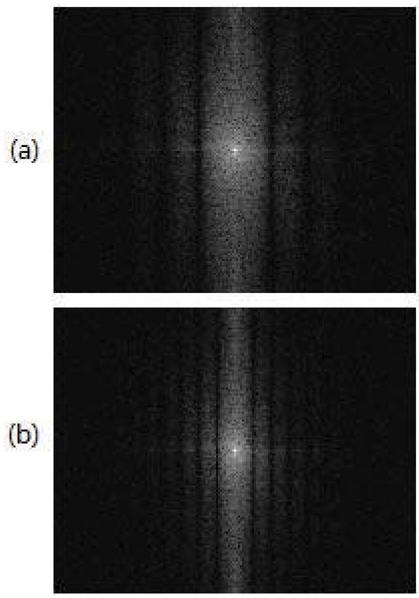

Fig. 2: The spectrum $G(u,v)$ of blurred image $g(x,y)$ with different motion blur length. (a) $L=10$; (b) $L=20$

There are zeros on the transverse axis of the spectrum $H(u,v)$ at $u = \pm 1/L, \pm 2/L, \pm 3/L, \cdots$. By formula (2) and formula (7), it can be seen that $|H(u,v)|$ and $|G(u,v)|$ get the maximum value at $u=0$; $|H(u,v)|=0$, $|G(u,v)|=0$ at $u = \pm 1/L, \pm 2/L, \pm 3/L, \cdots$, which means there are dark stripes at these points on the spectrogram of $G(u,v)$; $|H(u,v)| \ne 0$ at $u \ne \pm 1/L, \pm 2/L, \pm 3/L, \cdots$, which means there are bright stripes at these points on the spectrogram of $G(u,v)$. To calculate the number of dark stripes may obtain the motion blur length.

As shown in Fig. 2, it is difficult to determine the number of dark stripes in a spectrogram with naked eye observation. Furthermore, when the blurred image is mixed with random noise, the clarity and identification of the dark stripes will be affected, which will increase the difficulty of the accurate judgement of the motion blur length[7,8,11,12].

2.4 Noise to Signal Power Ratio

$P_n(u,v)/P_f(u,v)$ is NSPR. $|F_T(u,v)|$ and $|N_T(u,v)|$ are the modules of the Fourier transform of $f(x,y)$ and $n(x,y)$. The calculating of $P_n(u,v)$ and $P_f(u,v)$ is shown as the following

$$\begin{cases} P_f = \lim\limits_{T\to\infty} \dfrac{|F_T(u,v)|^2}{2\pi T} \\ P_n = \lim\limits_{T\to\infty} \dfrac{|N_T(u,v)|^2}{2\pi T} \end{cases} \tag{8}$$

3 MATLAB Simulations on Image Restoration

3.1 Ideas of Algorithm

In this paper, Wiener filtering is used to restore motion blurred images. The motion blur length is obtained through $G(u,v)$, and NSPR is obtained by wavelet decomposition. The steps of the algorithm are shown as the following:

- The motion blur length is obtained by the related formula in which the number of dark strips on $G(u,v)$ is estimated through the main lobe width of $G(u,v)$.
- The blurred image $g(x,y)$ is wavelet decomposed. The reconstruction of low frequency coefficients is restored by inverse filtering, the result is used to approximate the original image $f(x,y)$. The reconstruction of high frequency coefficients is used to approximate random noise, their power spectrum are calculated to get NSPR.
- The template size of Wiener filtering is selected as 5×5.

This paper will do a series of MATLAB simulation experiments to verified the restoration effect of the proposed method.

3.2 Estimation of Parameters

As shown in Fig. 1, 'Lotus' is the original image $f(x,y)$, whose size is $M\times N = 474\times 600$. $g(x,y)$ is a motion blurred image of $f(x,y)$ with a blur angle $\theta = 0^0$. So the motion blur length and NSPR (noise to signal power ratio) will be estimated on $g(x,y)$.

3.3 Estimation of Motion Blur Length

The discrete Fourier transform of the degradation function $h(x,y)$ is $H(u,v)$ whose module is $|H(u,v)|$. There are zero points on the transverse axis of the spectrum

$H(u,v)$: $u = \pm 1/L, \pm 2/L, \pm 3/L, \cdots$ where $G(u,v)$, the Fourier transform of blurred image $g(x,y)$, appears dark strips. The number of dark strips is motion blur length in pixels.

As shown in Fig. 2, it is unrealistic to count the number of dark fringes in the spectrum $G(u,v)$, but if the main lobe width of the spectrum is known, the following formula can be used to calculate the number of dark fringes in the spectrum.

$$L = \frac{N}{D/2} = 2N/D \quad (9)$$

The size of the spectrum $G(u,v)$ is $M \times N = 474 \times 600$. Every line of the spectrum $G(u,v)$ is a set of data of $1 \times N = 1 \times 600$ which is characterized by the light and dark fringes on the spectrum of Fig 2(b). When $M = 237$, the set of data $A(n)$ is a curve, as shown in Fig. 3 (a). Due to the complexity of the pixel value of the blurred image $g(x,y)$ and the influence of the noise, the position of the first dark stripe on both sides of the spectrum main lobe is not clear enough on curve $A(n)$.

In this paper, adding all the elements of each column in the spectrum matrix in Fig 2(b), it can get a set of data $B(n)$, whose size is $1 \times N = 1 \times 600$, as shown in Fig. 3(b). The fluctuation of the central region of the curve corresponds to the spectrum main lobe and the dark stripes on both sides of it apparently.

A piece of data from curve $B(n)$ on both sides of the central point $N/2 = 300$ is taken, and the valley data of the curve is needed, which corresponds to the first dark stripes of the spectrum. Using the MATLAB program to select the two minimum values of this piece of data and determine their positions x_1 and x_2, $D = x_2 - x_1$ is the width of the main lobe of the spectrum. The estimation of motion blur length is obtained by formula (9).

Table 1and Table 2 show the comparison of the true motion blur length values with the estimates obtained by the above method. Relevant literature [5, 6] have pointed out that when the motion blur length is small, the estimation is not very accurate. However, the data in this paper shows that the estimation of motion blur length is more accurate and the intensity of the mixed Gaussian noise has little effect on the estimation.

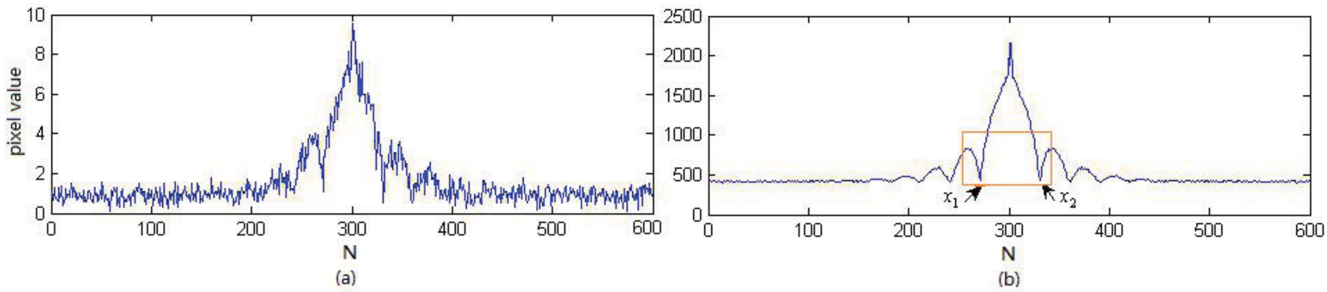

Fig. 3: (a) Curve $A(n)$; (b) Curve $B(n)$ where $D = x_2 - x_1$ is the width of the main lobe of the spectrum .
The motion blur length is $L = 20$ and the standard deviation of Gaussian noise $\sigma = 0.02$.

Table 1: Estimates of Very Small Motion Blurred Length (in pixel)

True value of L		2	3	4	5	6	7	8	9
Estimate	$\sigma = 0.01$	2	3	4	5	6	7	8	9
	$\sigma = 0.02$	2	3	4	5	6	7	8	9
	$\sigma = 0.04$	3	4	4	6	6	7	8	9

Table 2: Estimates of Motion Blurred Length of Selected Values (in pixel)

True value of L		10	25	45	50	60	75	85	100
Estimate	$\sigma = 0.01$	10	25	46	50	60	75	86	100
	$\sigma = 0.02$	10	25	46	50	60	75	86	100
	$\sigma = 0.04$	10	25	46	50	60	75	86	100

3.4 Estimation of Noise to Signal Power Ratio

How to estimate the original image information and noise information form $g(x,y)$ accurately, decides the value of NSPR, and has great influence on the restoration effect of Wiener filtering. This paper will propose the following four methods to get the estimates of the original image and mixed random noise, then to calculate their power spectrum by formula (8). The ratio $P_n(u,v)/P_f(u,v)$ is NSPR[13,14].

Method A: To do one-layer wavelet decomposition on the motion blurred image $g(x,y)$. After being restored by inverse filtering with the estimated motion blur length, the reconstruction of low frequency coefficients is used to approximate the original image. The reconstruction of high frequency coefficients is used to approximate random noise.

Method B. To do one-layer wavelet decomposition on image $g(x,y)$. The reconstruction of low frequency coefficients is used to approximate the original image, the reconstruction of high frequency coefficients is used to approximate random noise.

Method C: To do two-layer wavelet decomposition on image $g(x,y)$. The reconstruction of low frequency coefficients is used to approximate the original image, the reconstruction of all the high frequency coefficients is used to approximate random noise.

Method D: To do one-layer wavelet decomposition on image $g(x,y)$. The reconstruction of low frequency coefficients is used to approximate the original image. All the high frequency coefficients are denoised with soft threshold before they are reconstructed with low frequency coefficients together. The reconstruction is used to approximate the original image. The difference between $g(x,y)$ and the reconstruction is used to approximate random noise.

The true NSPR of $g(x,y)$ and its four kinds of estimates are shown in Table 3. It can be seen that the estimates of method A are closer to the true NSPR, therefore, method A will be used in this paper in the following image restoration.

3.5 Image Restoration

After L and P_n/P_f have been gotten, Wiener filtering is used to restore blurred images. The qualities of restored images will be measured with peak signal to noise ratio (PSNR). The restored image $f'(x,y)$ is good and relatively stable comparing with original image $f(x,y)$. The ringing artifact which often occurs on a restored image is not obvious and can be done without further processing [15,16,17].

Observing Fig. 4 and Table 4, for the blurred image $g(x,y)$ with various motion blurring lengths, the quality of blur length. Ratio P_n/P_f is estimated by method A proposed in this paper. Restored image $f'(x,y)$ is shown in Fig. 4, PSNR is shown in Table 4.

3.6 Application of Restoration Technology

The horizontal motion blurred image 'Car' whose size is $M \times N = 800 \times 601$ will be restored. Blurred image $g(x,y)$ and its restored image $f'(x,y)$, as well as their license plates are shown in Fig. 5 when $\sigma = 0.005$. It can be seen that the restored images of the vehicle interior as well as the letters and numbers on the license plates are very clear, even if the motion blur length is very large such as $L = 80$.

Image $f(x,y)$ of $M \times N = 512 \times 512$ is the original image 'Lena'. When the motion blur length is $L = 40,60$ respectively, as well as the noise standard deviation is $\sigma = 0.01$, the horizontal motion blurred image $g(x,y)$, its restored image $f'(x,y)$ and $f(x,y)$ are shown in Fig. 6

Table 3: True NSPR and Its Four Kinds of Estimates

Motion blur length	Standard deviation	True NSPR	estimate of PSNR			
			A	B	C	D
$L=10$	$\sigma=0.01$	0.0012	0.0002	0.00027	0.0031	0.9871
	$\sigma=0.02$	0.0004	0.00036	0.00035	0.0011	0.9876
$L=20$	$\sigma=0.01$	0.00012	0.00014	0.00017	0.0006	0.9883
	$\sigma=0.02$	0.00048	0.00029	0.00027	0.00075	0.9837
$L=30$	$\sigma=0.01$	0.00012	0.00014	0.00017	0.00049	0.9843
	$\sigma=0.02$	0.00048	0.00029	0.00027	0.0009	0.9852
$L=40$	$\sigma=0.01$	0.00012	0.00012	0.00015	0.0004	0.9822
	$\sigma=0.02$	0.00048	0.00027	0.00020	0.0009	0.9828
$L=60$	$\sigma=0.01$	0.00012	0.0001	0.00009	0.00051	0.9837
	$\sigma=0.02$	0.00048	0.00029	0.00021	0.00081	0.9871

Table 4: PSNR of Restored Images with Different Motion Blur Length

Standard deviation	PSNR(dB) with different motion blur length							
	$g(x,y)$	$f'(x,y)$	$g(x,y)$	$f'(x,y)$	$g(x,y)$	$f'(x,y)$	$g(x,y)$	$f'(x,y)$
	$L=10$		$L=20$		$L=30$		$L=40$	
$\sigma=0.01$	30.0848	33.5742	26.6310	32.1393	25.0297	31.1592	24.0021	30.2520
$\sigma=0.02$	29.9581	32.6373	26.5713	31.0483	24.9823	30.0084	23.9713	29.2026
	$L=50$		$L=60$		$L=70$		$L=80$	
$\sigma=0.01$	23.1879	29.5824	22.5177	28.8517	21.9652	29.5685	21.4989	29.1173
$\sigma=0.02$	23.1605	28.5521	22.4959	27.9363	21.9453	28.3985	21.4795	28.0946

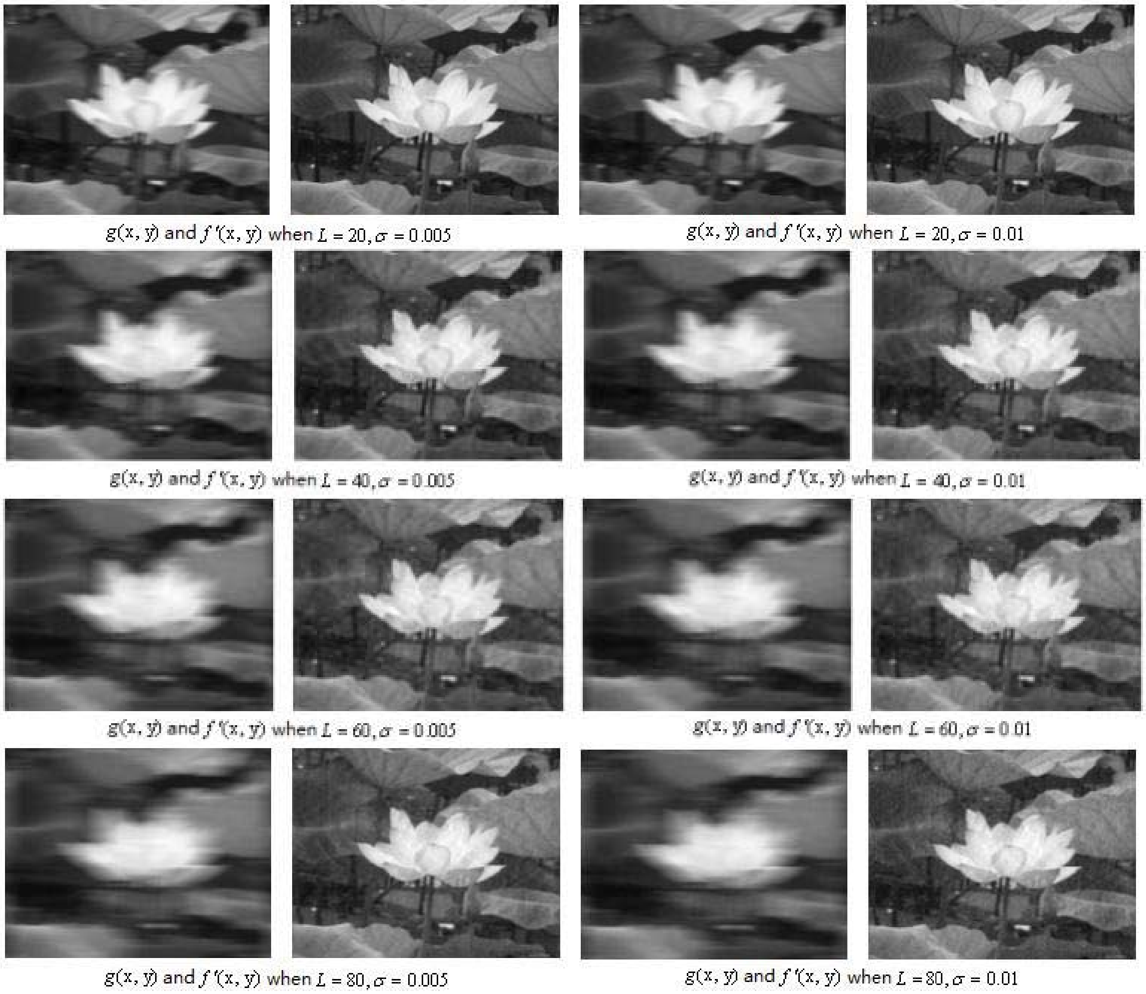

Fig. 4: Blurred image $g(x, y)$ and its restoration $f'(x, y)$ with different L and σ

Fig. 5: Blurred image $g(x, y)$, its restored image $f'(x, y)$ and their license plates


(a) (b) (c) (d) (e)

Fig. 6: (a) Original image $f(x,y)$; (b) and (c) blurred image $g(x,y)$ and its restored image $f'(x,y)$ when $L = 40$; (d) and (e) blurred image $g(x,y)$ and its restored image $f'(x,y)$ when $L = 60$.

From the above images and data of MATLAB simulation experiments it can be seen

- As show in Fig. 5 , Table 1 and Table 2, the estimates of the spectrum main lobe and the motion blur length are accurate.
- As shown in Table 3, the NSPR obtained through method A is closer to the true value than the other three methods.
- As shown in Fig. 4, Fig. 5, Fig. 6 and Table 4, under the condition of various motion blur length and not very large noise standard deviation, the quality of restored image $f'(x,y)$ is throughout stable, and PSNR is high. Especially in Fig. 5, the restored license plate can be identified easily, which indicates good restoring effect.
- During the series of MATLAB simulation for image restoration, it is found that, for blurred images with small pixels, when the true blur length is $L \geq 80$, the estimation error of very few blur length is 6 pixels. Although it will affect the restoration quality in this case, the way of estimating the motion blur length in this paper is feasible.

4 Conclusion

Image restoration is one of the important applications of Wiener filtering which needs some information about blurred images in advance, such as PSF, NSPR and so on. This paper propose to use the frequency spectrum of a blurred image to get the motion blur length. The NSPR of blurred image is gotten by wavelet decomposition, that is, the restoration of the reconstruction of low frequency coefficient is used to approximate the original image, the reconstruction of high frequency coefficients is used to approximate the noise. MATLAB simulations prove that, the method of estimating the motion blur length is simple, effective and adaptable, and when using the parameters estimated by this paper, the restoration quality of blurred images is good and stable, ringing artifacts is not obvious. Image restoration with Wiener filtering may be applied practically.

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