---This is code to iterate over the database of smooth projective toric Fano 5-folds in M2, and ask whether each yields a full strong exceptional collection of line bundles from those appearing on the left-hand side of the box products. The collection can be taken without repeats. The code conjures up a smooth projective toric Fano 5-fold from the M2 database, then applies the HHL resolution to the diagonal toric map for X -> X x X. It then constructs a collection of line bundles on one side, and asks whether the directed graph given by all graded homomorphisms between objects (line bundles) of this collection has any directed cycles of length greater than 1 (i.e., we remove all self-loops from each vertex of the directed graph). The command "isCyclic(-) for digraphs asks whether the corresponding digraph has **any** cycles, so a result of "true" from isCyclic(H) below means that the directed graph **does** have cycles, so that the collection is not exceptional. That is, an output of "false" from isCyclic(H) at the very end denotes that the Hanlon-Hicks-Lazarev resolution of the diagonal does yield a full strong exceptional collection of line bundles on the smooth projective toric Fano 5-fold X.   
  
The output is stored as a list of "true" and "false" in OutputForDim5Indicesxxtoxx.  
  
  
---First we load the "HHL Resolutions" package from the correct location:

needsPackage("HHLResolutions",FileName=>"HHL Resolutions/ HHLResolutions.m2");

needsPackage "Graphs";

cList = {};

--- To call a given smooth projective toric Fano 5-fold, we use the database internal to M2:

for q from xx to 865 do(

X = smoothFanoToricVariety(6,q);

dim X;

isSmooth X;

isWellDefined X;

classGroup(X);

-----We construct the product Y = X x X and apply the diagonal toric map:

Y = X\*\*X;

phi = diagonalToricMap(X);

output = makeHHLResolution(Y, matrix phi);

L = for i from 0 to 5 list(-1\*degrees output#i);

L = flatten L;

LBs = for i from 0 to length L-1 list( for j from 0 to rank classGroup(X) - 1 list(L#i#j));

LBs = unique LBs;

length LBs;

---quiver constructs the directed graph of all directed homomorphisms of objects in this collection

quiver = for i from 0 to length LBs-1 list( for j from 0 to length LBs-1 list( for k from 0 to 5 list(HH^k(X, OO\_X( toSequence( for u from 0 to rank classGroup(X) -1 list (LBs#j#u-LBs#i#u) ) ) ) ) ) );

strong = for i from 0 to length LBs-1 list( for j from 0 to length LBs-1 list( for k from 0 to 5 do(if quiver#i#j#k!=0 then print(k,i,j))));

m = diagonalMatrix (splice {length LBs:0});

n = mutableMatrix m;

----To avoid considering self-loops at each vertex, we'll omit the i=j case (i.e., the main diagonal of the adjacency matrix) when constructing the directed graph:

adjList = for i from 0 to length LBs -1 list( for j from 0 to length LBs -1 list( for k from 0 to 5 do( if quiver#i#j#k != 0 and i!= j then n\_(i,j) = 1 )));

H = digraph matrix n;

----We now ask whether this directed graph has **any** directed cycles:

print isCyclic(H);

cList = append( cList, isCyclic(H) );

print cList;

f = "OutputForDim5Indicesxxtoxx" << "";

f << cList << endl;

f << close;

)

----Remember: this will say whether the graph \*\*IS\*\* cyclic, which would make NO i.e., failure of HHL res’n to yield FSEC of LBs