

Macaulay2, version 1.24.11-1695-gf35df1017f (vanilla)

with packages: ConwayPolynomials, Elimination, IntegralClosure, InverseSystems, Isomorphism, LLBases, MinimalPrimes, OnlineLookup, PackageCitations, Polyhedr

i1 : load "hhl (4) (5).m2"

i2 : needsPackage "NormalToricVarieties"

o2 = NormalToricVarieties

o2 : Package

i3 : M = matrix{{1, 1, -1, 0, 1}, {1, 0, 0, 0, 1}, {1, 0, 0, -1, 0}, {1, -1, 0, 0, 0}, {1, 0, -1, 1, 1}, {1, 0, -1, 0, 0}, {1, 0, 1, 0, 0}, {1, 0, -1, 0, 1},

$$o3 = \begin{pmatrix} 1 & 1 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

o3 : Matrix  $Z^9 \leftarrow Z^5$

i4 : MNew = submatrix(M, , {1,2,3,4})

$$o4 = \begin{pmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

o4 : Matrix  $Z^9 \leftarrow Z^4$

i5 : A = -1\*MNew

$$o5 = \begin{pmatrix} -1 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

o5 : Matrix  $Z^9 \leftarrow Z^4$

i6 : v = M\_0

$$o6 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

o6 :  $Z^9$

i7 : P = polyhedronFromHData(A, matrix v)

o7 = P

o7 : Polyhedron

i8 : X = normalToricVariety(P)

o8 = X

o8 : NormalToricVariety

i9 : dim X

o9 = 4

i10 : isWellDefined X

o10 = true

**i11** : isSmooth X

**o11** = true

**i12** : isFano X

**o12** = true

**i13** : isProjective X

**o13** = true

**i14** : classGroup(X)

**o14** =  $\mathbb{Z}^5$

**o14** :  $\mathbb{Z}$ -module, free

**i15** : rays X

**o15** = {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}, {0, 0, 0, 1}, {0, -1, 0, 1}, {1, -1, 0, 1}, {0, -1, 1, 1}}

**o15** : List

**i16** : max X

**o16** = {{0, 1, 3, 4}, {0, 1, 3, 6}, {0, 1, 4, 8}, {0, 1, 6, 8}, {0, 2, 3, 4}, {0, 2, 3, 5}, {0, 2, 4, 8}, {0, 2, 5, 8}, {0, 3, 5, 6}, {0, 5, 6, 8}, {1, 3, 4, 7}, {1, 3, 6, 7}, {1, 4, 7, 8},

**o16** : List

**i17** : Y = X\*\*X

**o17** = Y

**o17** : NormalToricVariety

**i18** : phi = diagonalToricMap(X)

**o18** = ToricMap { cache  $\Rightarrow$  (CacheTable{}), matrix  $\Rightarrow$   $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ , source  $\Rightarrow$  X, target  $\Rightarrow$  Y }

**o18** : ToricMap

**i19** : makeHHLResolution(Y, matrix phi)

$\left( \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}, (0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1) \right)$

Cells Complete, 8 cells found

Labels Complete

**o19** =  $(Q[x_0 \dots x_{17}])^1 \xleftarrow{(x_0x_5x_6x_8x_{13} - x_4x_9x_{14}x_{15}x_{17} \dots x_7x_9 - x_0x_{16})} (Q[x_0 \dots x_{17}])^{11} \xleftarrow{\begin{pmatrix} 0 & \dots & x_1x_{11} \\ \vdots & \ddots & \vdots \\ -x_2x_5x_{10}x_{13} + x_1x_4x_{11}x_{14} & \dots & 0 \end{pmatrix}} (Q[x_0 \dots x_{17}])^{27} \xleftarrow{\begin{pmatrix} 0 & \dots & -x_6x_8 \\ \vdots & \ddots & \vdots \\ x_{12} & \dots & 0 \end{pmatrix}}$

**o19** : ChainComplex

**i20** : L = for i from 0 to 4 list(-1\*degrees o19#i)

**o20** = {{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, -1, -1, -1, 0, -1, -1, -1}, {-1, 0, 0, 0, 0, -1, 0, 0, 0}, {0, -1, -1, -1, -1, 0, -1, -1, -1}, {-1, -1, 0, 0, 0, -1, -1, 0, 0}}

**o20** : List

**i21** : L = flatten L

**o21** = {{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, -1, -1, -1, 0, -1, -1, -1}, {-1, 0, 0, 0, 0, -1, 0, 0, 0}, {0, -1, -1, -1, -1, 0, -1, -1, -1}, {-1, -1, 0, 0, 0, -1, -1, 0, 0}}

**o21** : List

**i22** : LBs = for i from 0 to length L-1 list( for j from 0 to rank classGroup(X) - 1 list(L#i#j))

**o22** = {{0, 0, 0, 0, 0}, {0, -1, -1, -1, -1}, {-1, 0, 0, 0, 0}, {0, -1, -1, -1, -1}, {-1, -1, 0, 0, 0}, {0, 0, 0, 0, -1}, {-1, 0, 0, 0, 0}, {0, -1, -1, -1, -1}, {-1, 0, 0, 0, 0}, {0

**o22** : List

**i23** : LBs = unique LBs

**o23** = {{0, 0, 0, 0, 0}, {0, -1, -1, -1, -1}, {-1, 0, 0, 0, 0}, {-1, -1, 0, 0, 0}, {0, 0, 0, 0, -1}, {0, 0, 0, -1, 0}, {-1, -1, 0, -1, 0}, {-1, -1, 0, 0, -1}, {0, 0, 0, -1, -1}, {0,

o23 : List

i24 : length LBS

o24 = 20

i25 : classGroup(X)

o25 =  $Z^5$

o25 :  $Z$ -module, free

i26 : quiver = for i from 0 to length LBS-1 list( for j from 0 to length LBS-1 list( for k from -5 to 5 list(HH^k(X, OO\_X(LBS#j#0-LBS#i#0, LBS#j#1-LBS#i#1, LB

o26 = {{{0, 0, 0, 0, 0, 0,  $Q^1$ , 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0,

o26 : List

i27 : strong = for i from 0 to length LBS-1 list( for j from 0 to length LBS-1 list( for k from -5 to 5 do(if quiver#i#j#k!=0 then print(k,i,j))))

(5, 0, 0)

(5, 1, 0)

(5, 1, 1)

(5, 1, 4)

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(5, 2, 0)

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(5, 19, 19)
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o27 = {{null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null}, {null,null,null,null,null,null,null,null,null,null,
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o27 : List
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i28 : ----- It now remains to find an ordering of these line bundles for which the collection is exceptional
ReOrder = for i from 0 to length LBS-1 list( for j from 0 to length LBS-1 list( for k from -5 to 5 do(if (quiver#i#j#k!=0 and j>i) then print(k,i,j))))
```

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(5, 1, 4)
(5, 1, 5)
(5, 1, 8)
(5, 2, 4)
(5, 2, 5)
(5, 2, 8)
(5, 3, 4)
(5, 3, 5)
(5, 3, 8)
(5, 6, 8)
(5, 6, 10)
(5, 6, 11)
(5, 7, 8)
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(5, 15, 16)
(5, 15, 17)
(5, 15, 18)
(5, 16, 17)
(5, 16, 18)
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o28 = {{null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null}, {null,null,null,null,null,null,null,null,null,null,
```

```
o28 : List
```

```
i29 : -----so we define
ReOrderList = {0, 4, 5, 8, 1, 2, 3, 10, 11, 6, 9, 13, 7, 17, 18, 16, 12, 14, 15, 19}
```

```
o29 = {0, 4, 5, 8, 1, 2, 3, 10, 11, 6, 9, 13, 7, 17, 18, 16, 12, 14, 15, 19}
```

```
o29 : List
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```
i30 : LBSNew = for i from 0 to length LBS-1 list LBS#(ReOrderList#i)
```

```
o30 = {{0, 0, 0, 0, 0}, {0, 0, 0, 0, -1}, {0, 0, 0, -1, 0}, {0, 0, 0, -1, -1}, {0, -1, -1, -1, -1}, {-1, 0, 0, 0, 0}, {-1, -1, 0, 0, 0}, {-1, 0, 0, -1, 0}, {0, -1, -1, -2, -1}, {-1
```

```
o30 : List
```

i31 : length LBSNew

o31 = 20

i32 : -----now re-run quiver and strong with LBSNew:

quiver = for i from 0 to length LBSNew-1 list( for j from 0 to length LBSNew-1 list( for k from -5 to 5 list(HH^k(X, OO\_X(LBSNew#j-LBSNew#i, LBSNew

o32 = {{{0, 0, 0, 0, 0, Q<sup>1</sup>, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0,

o32 : List

i33 : strong = for i from 0 to length LBSNew-1 list( for j from 0 to length LBSNew-1 list( for k from -5 to 5 do(if quiver#i#j#k!=0 then print(k,i,j)))

(5, 0, 0)  
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o33 = {{null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null}, {null,null,null,null,null,null,null,null,null,null,

o33 : List

i34 : -----We now have that this corresponds to an exceptional ordering, since all Hom's go in one direction  
 ExceptionalityCheck = for i from 0 to length LBSNew-1 list( for j from 0 to length LBSNew-1 list( for k from -5 to 5 do(if quiver#i#j#k!=0 and j>i then

o34 = {{null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null,null}, {null,null,null,null,null,null,null,null,null,null,

o34 : List

i35 : -----Note that this list is empty!  
 -----Since we currently have all Hom's to the left, we'd then reverse the ordering of LBSNew to get an exceptional collection so that all Hom's go to th