Macaulay2, version 1.24.11-1695-gf35df1017f (vanilla) with packages: ConwayPolynomials, Elimination, IntegralClosure, InverseSystems, Isomorphism, LLLBases, MinimalPrimes, OnlineLookup, PackageCit

$$\underline{02} = \begin{pmatrix}
1 & 0 & -1 & 1 & 0 \\
1 & 0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 \\
1 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 \\
1 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & -3 & 1
\end{pmatrix}$$

$$\underline{\text{o2}}$$
 : Matrix  $Z^7 \leftarrow Z^5$ 

$$\underline{03} = \begin{pmatrix}
0 & -1 & 1 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 \\
-1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 \\
1 & 1 & -3 & 1
\end{pmatrix}$$

$$o3$$
: Matrix  $Z^7$  ←  $Z^4$ 

$$\underline{04} = \begin{pmatrix}
0 & 1 & -1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -1 & 0 \\
-1 & -1 & 3 & -1
\end{pmatrix}$$

$$\underline{\text{o4}}\,:\, \text{Matrix}\, Z^7\, \longleftarrow\, Z^4$$

$$\underline{o5}$$
 :  $Z^7$ 

i6 : needsPackage "NormalToricVarieties"

 $\underline{o6}$  = NormalToricVarieties

o6 : Package

 $\underline{i7}$  : P = polyhedronFromHData(A, matrix v)

<u>o7</u> = P

o7 : Polyhedron

i8 : X = normalToricVariety(P)

<u>08</u> = X

o8 : NormalToricVariety

<u>i9</u> : X

o9 = X

o9 : NormalToricVariety

```
4/29/25, 2:21 PM
    <u>i10</u> : dim X
    010 = 4
    i11 : isWellDefined X
    o11 = true
    i12 : isSmooth X
    012 = true
    <u>i13</u> : isFano X
    \underline{o13} = true
    i14 : classGroup(X)
    \underline{\mathsf{o14}} = Z^3
    <u>i15</u> : rays X
    <u>o15</u> : List
    <u>i16</u> : max X
    <u>o16</u> : List
```

o14 : Z-module, free

 $\underline{\texttt{o15}} = \{ \{-1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 1, 0\}, \{0, -1, 1, 0\}, \{0, 0, 0, -1\}, \{1, 1, -3, 1\} \}$ 

 $\underline{o16} = \{\{0, 1, 2, 5\}, \{0, 1, 2, 6\}, \{0, 1, 4, 5\}, \{0, 1, 4, 6\}, \{0, 2, 5, 6\}, \{0, 3, 4, 5\}, \{0, 3, 4, 6\}, \{0, 3, 5, 6\}, \{1, 2, 5, 6\}, \{1, 4, 5, 6\}, \{3, 4, 5, 6\}\}$ 

i17 : fromWDivToCl(X)

$$\underline{017} = \begin{pmatrix}
0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 1 & 0 & 0 \\
1 & 1 & -3 & 0 & 0 & 1 & 1
\end{pmatrix}$$

 $\underline{\text{o17}}$  : Matrix  $Z^3 \leftarrow Z^7$ 

 $\underline{i18}$  : Y = X\*\*X

018 = Y

o18 : NormalToricVariety

i19 : phi = diagonalToricMap(X)

$$\underbrace{\text{o19}}_{\text{cache}} = \text{ToricMap} \begin{cases} cache \Rightarrow (\text{CacheTable}\{\}), \ matrix \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \ source \Rightarrow X, \ target \Rightarrow Y \end{cases}$$

<u>o19</u> : ToricMap

$$\frac{\textbf{i20}}{\textbf{(0)}} : \mathsf{makeHHLResolution(Y, matrix phi)} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ \end{pmatrix}, (0 & 0 & 0 & 1 & 1 & 1 & 1) \\ \mathsf{Cells Complete, 10 cells found}$$

Labels Complete

$$\underbrace{ \begin{pmatrix} 0 & \cdots & 0 \\ 0 & \cdots & -x_{13} \\ x_1 & \cdots & x_{13} \\ -x_3x_8x_9x_{11} & \cdots & x_{1}x_2x_6x_{10} \end{pmatrix} }_{ Q[x_0 ..x_{13}])^{25} \leftarrow \underbrace{ \begin{pmatrix} 0 & \cdots & -x_7 & 0 & \cdots & 0 \\ 0 & \cdots & -x_7 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 &$$

```
o20 : ChainComplex
\underline{i21}: L = for i from 0 to 4 list(-1*degrees o20#i)
\underline{021} = \{ \{ \{-1, -1, 2, -1, -1, 1\}, \{-1, 0, 1, -1, 0, 1\}, \{-1, -1, 1, -1, -1, 2\}, \{0, 0, 0, 0, 0, 0, 0\} \}, \{ \{-1, 0, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, 2\}, \{-1, -1, 0, -1, 2\}, \{-1, -1, 0, -1, 2\}, \{-1, 0, 0, -1, -1, 2\}, \{-1, 0, 0, -1, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{-1, 0, 0, -1, 2\}, \{
<u>o21</u> : List
\underline{i22} : L = flatten L
\underline{022} = \{\{-1, -1, 2, -1, -1, 1\}, \{-1, 0, 1, -1, 0, 1\}, \{-1, -1, 1, -1, -1, 2\}, \{0, 0, 0, 0, 0, 0\}, \{-1, 0, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, -1, 2\}, \{-1, -1, 0, -1, 2\}, \{-1, -1, 0, -1, 2\}, \{-1, -1, 0, -1, 2\}, \{-1, -1, 0, 1, -1, 2\}, \{-1, -1, 0, 1, 2\}, \{-1, -1, 0, 1, 2\}, \{-1, -1, 0, 1, 2\}, \{-1, -1, 0, 1, 2\}, \{-1, -1, 0, 1, 2\}, \{-1, -1, 0, 1, 2\}, \{-1, -1, 0, 1, 2\}, \{-1, -1, 0, 1, 2\}, \{-1, -1, 0, 1, 2\}, \{-1, -1, 0, 1, 2\}, \{-1, -1, 0, 1, 2\}, \{-1, -1, 0, 1, 2\}, \{-1, -1, 0, 2\}, \{-1, -1, 0, 2\}, \{-1, -1, 0, 2\}, \{-1, -1, 0, 2\}, \{-1, -1, 0, 2\}, \{-1, -1, 0, 2\}, \{-
<u>o22</u> : List
i23 : LBs = for i from 0 to length L-1 list( for j from 0 to rank classGroup(X) - 1 list(L#i#j))
\underline{023} = \{\{-1, -1, 2\}, \{-1, 0, 1\}, \{-1, -1, 1\}, \{0, 0, 0\}, \{-1, 0, 0\}, \{-1, -1, 0\}, \{-1, -1, 0\}, \{-1, -1, 1\}, \{-1, 0, 0\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0, 0, -1\}, \{0
<u>o23</u> : List
<u>i24</u>: LBs = unique LBs
<u>o24</u> : List
i25 : length LBs
025 = 14
i26 : quiver = for i from 0 to length LBs-1 list( for j from 0 to length LBs-1 list( for k from -5 to 5 list(HH^k(X, O0_X(LBs#j#0-LBs#i#0, LBs
<u>o26</u> : List
<u>i27</u>:
                              strong = for i from 0 to length LBs-1 list( for j from 0 to length LBs-1 list( for k from -5 to 5 do(if quiver#i#j#k!=0 then print(k,i,j
(5, 0, 0)
(5, 0, 3)
(5, 0, 6)
(-3, 0, 9)
(-3, 0, 10)
(-3, 0, 11)
(-3, 0, 12)
(-3, 0, 13)
(5, 1, 0)
(5, 1, 1)
(5, 1, 3)
(5, 1, 6)
(-3, 1, 11)
(-3, 1, 12)
(-3, 1, 13)
(5, 2, 0)
(5, 2, 1)
(5, 2, 2)
(5, 2, 3)
(5, 2, 6)
(5, 2, 9)
(-3, 2, 12)
(-3, 2, 13)
(5, 3, 3)
(5, 4, 0)
(5, 4, 1)
(5, 4, 2)
(5, 4, 3)
(5, 4, 4)
(5, 4, 6)
(5, 4, 9)
(5, 5, 0)
(5, 5, 1)
(5, 5, 2)
(5, 5, 3)
(5, 5, 4)
(5, 5, 5)
(5, 5, 6)
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- (5, 5, 9)
- (5, 5, 12)
- (5, 6, 3)
- (5, 6, 6)
- (5, 7, 0)
- (5, 7, 1)
- (5, 7, 2)
- (5, 7, 3)
- (5, 7, 4)
- (5, 7, 5)
- (5, 7, 6)
- (5, 7, 7)
- (5, 7, 9)
- (5, 7, 12)
- (5, 8, 0)
- (5, 8, 1)
- (5, 8, 2)
- (5, 8, 3)
- (5, 8, 4)
- (5, 8, 5)
- (5, 8, 6)
- (5, 8, 7)
- (5, 8, 8)
- (5, 8, 9)
- (5, 8, 12)
- (5, 9, 3)
- (5, 9, 6)
- (5, 9, 9)
- (5, 10, 0)
- (5, 10, 1)
- (5, 10, 2)
- (5, 10, 3)
- (5, 10, 4)
- (5, 10, 5)
- (5, 10, 6)
- (5, 10, 7)
- (5, 10, 8)(5, 10, 9)
- (5, 10, 10)
- (5, 10, 12)
- (5, 11, 0)
- (5, 11, 1)
- (5, 11, 2)
- (5, 11, 3)
- (5, 11, 4)
- (5, 11, 5)
- (5, 11, 6)
- (5, 11, 7)
- (5, 11, 8)
- (5, 11, 9)
- (5, 11, 10)
- (5, 11, 11)
- (5, 11, 12)
- (5, 12, 3)
- (5, 12, 6)
- (5, 12, 9)
- (5, 12, 12)
- (5, 13, 0)
- (5, 13, 1)(5, 13, 2)
- (5, 13, 3)
- (5, 13, 4)
- (5, 13, 5)
- (5, 13, 6)
- (5, 13, 7)
- (5, 13, 8)
- (5, 13, 9)
- (5, 13, 10) (5, 13, 11)
- (5, 13, 12)
- (5, 13, 13)

o27 = {{null, null, null