GITHUB FOURFOLDS REPOSITORY

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1. Data collection

1.1. Polytope F.4D.0000. Let P denote the polytope F.4D.0000 in polymake with half-space representation:

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & -3 & 1 \end{bmatrix}$$

with vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & -3 & 1 & -5 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & -6 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & 1 & 1 \\ 0 & -5 & -3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,1,0),(0,-1,1,0),(0,0,0,-1),(1,1,-3,1)\}$$

and presentation of the class group

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & -3 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{10} \to S^{36} \to S^{47} \to S^{25} \to S^4 \to 0.$$

The Hanlon-Hicks-Lazarev resolution yields the collection of line bundles $\{\mathcal{O}(a_1,a_2)\}$ for (a_1,a_2) appearing in

$$\begin{aligned} \mathcal{E} = &\{(-1,-1,1),(0,0,0),(-1,-1,2),(-1,0,1),(-1,0,0),(-1,-1,0),(0,0,-1),(-1,-1,-1),\dots \\ &\dots(-1,0,-1),(0,0,-2),(0,0,-3),(-1,-1,-2),(-1,0,-3)\} \end{aligned}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,0,-3)$ and $\mathcal{O}(0,-1,4)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,-3),\mathcal{O}(0,-1,4)) = \begin{cases} \mathbb{C} & \text{in degree 3} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,4),\mathcal{O}(-1,0,-3)) = \begin{cases} \mathbb{C}^{20} & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Here, we see that there does **not** exist an ordering of $\mathcal{E}$ for which we get an exceptional collection of line bundles.

Furthermore, we verify that Bondal's numerical criterion fails for this variety, since the torus-invariant curve corresponding to the 3-cone with primtitive ray generators $\{(-1,0,0,0),(0,-1,0,0),(0,-1,1,0)\}$ intersects the torus-invariant divisor corresponding to primitive ray generator $(0,-1,0,0)$ in -2.

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1.2. **Polytope F.4D.0001.** Let P denote the polytope F.4D.0001 in polymake with half-space representation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1/3 & 1/3 & 1/3 & 1/3 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

with vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 & -6 & 1 \\ 1 & 0 & 1 & 1 & 1 & -6 & 1 & 1 \\ 0 & 1 & 1 & 1 & -6 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(1,1,1,3)\}$$

and presentation of the class group

$$\mathbb{Z}^6 \xrightarrow{\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 3 & 0 & 1 \end{pmatrix}} \mathbb{Z}^2.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{23} \to S^{32} \to S^{18} \to S^3 \to 0.$$

The Hanlon-Hicks-Lazarev resolution yields the collection of line bundles $\{\mathcal{O}(a_1, a_2)\}$ for (a_1, a_2) appearing in

$$\mathcal{E} = \{(0,0),(-1,-2),(-1,-1),(0,-1),(-1,-3),(-1,-4),(0,-2),(-1,-5),(0,-3),(-1,-6)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-2)$ and $\mathcal{O}(-1,-6)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-2),\mathcal{O}(-1,-6)) = \begin{cases} \mathbb{C} & \text{in degree } 3 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-6),\mathcal{O}(-1,-2)) = \begin{cases} \mathbb{C}^{35} & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.3. **Polytope F.4D.0002.** Let P denote the polytope F.4D.0002 in the polymake database with the half-space representation:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & -1 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & -2 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} -3 & 1 & -3 & 1 & 1 & 1 & 0 & 1 & 1 & -2 & 1 & -2 & 1 & 0 & 1 & 0 & 1 \\ -2 & -2 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & -2 & -2 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3 & -3 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

The primitive ray generators of the fan Σ for X are

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(0,0,1,-1),(1,0,-2,1),(0,1,-1,1)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 \\ 1 & 0 & -2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{12} \to S^{40} \to S^{47} \to S^{22} \to S^3 \to 0$$

There does not exist an ordering of the 21 line bundles

$$\{(0,0,0,0),(-1,-1,1,0),(-1,-1,1,-1),(-1,-1,0,0),(-1,0,0,-1),(-1,0,0,0),(0,0,-1,-1),(0,0,-1,0),\ldots$$
$$\ldots(0,0,0,-1),(-1,0,-1,0),(-1,-1,-1,0),(0,0,-1,-2),(0,0,-2,-1),(-1,0,0,-2),(-1,0,-1,-1),\ldots$$
$$\ldots(-1,-1,0,-1),(-1,-1,-1,-1),(-1,0,-1,-2),(0,0,-2,-2),(-1,0,-2,-1),(-1,0,-2,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,1,0)$ and $\mathcal{O}(-1,0,-2,-1)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,-1,1,0),\mathcal{O}(-1,0,-2,-1)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,0,-2,-1),\mathcal{O}(-1,-1,1,0)) = \begin{cases} \mathbb{C}^3 & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.4. **Polytope F.4D.0003.** Let P denote the polytope F.4D.0003 in the polymake database with the half-space representation:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1/2 & 1/2 & 0 & 1/2 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & -4 & 1 & -4 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -3 & -3 & 1 & 1 \\ -2 & 0 & 0 & 1 & 1 & 1 & 1 & -3 & -3 & 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

The primitive ray generators of the fan Σ for X are

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(0,1,1,1),(1,0,1,2)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 2 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{10} \to S^{36} \to S^{45} \to S^{22} \to S^{3} \to 0.$$

The collection $\mathcal{E}$ of 17 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution are

$$\begin{aligned}\mathcal{E} = \{&(0,0,0),(-1,-1,-1),(-1,-1,-2),(-1,-2,-1),(-1,-2,-2),(0,-1,-1),(0,-1,0),(0,0,-1),(-1,-2,-3),\dots\\ &\dots(-1,-1,-3),(-1,-3,-2),(0,-2,-1),(0,-1,-2),(-1,-3,-3),(0,-2,-2),(-1,-2,-4),(-1,-3,-4)\}\end{aligned}$$

This collection is not exceptional since, for instance, there is a pair of objects with nonzero Hom's in both directions: $\mathcal{O}(-1,-1,-1)$ and $\mathcal{O}(-1,-2,-4)$ have

$$Hom_{D^b(X)}(\mathcal{O}(-1,-1,-1),\mathcal{O}(-1,-2,-4)) = \begin{cases}\mathbb{C} & \text{in degree } 2,\\ 0 & \text{else}\end{cases}$$

and

$$Hom_{D^b(X)}(\mathcal{O}(-1,-2,-4),\mathcal{O}(-1,-1,-1)) = \begin{cases}\mathbb{C}^{11} & \text{in degree } 0,\\ 0 & \text{else .}\end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.5. **Polytope F.4D.0004.** Let P denote the polytope F.4D.0004 in the polymake database with the half-space representation:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 1\\ 1 & 0 & 1 & 0 & 0\\ 1 & 0 & -1 & 0 & 0\\ 1 & 0 & 0 & -1 & 0\\ 1 & 0 & -1 & 0 & 1\\ 1 & 0 & 0 & 0 & -1\\ 1/2 & 1/2 & -1 & 1/2 & 1\end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 1 & -2 & 1 & 1 & 1 & -6 & 1 & -2 & 1\\ 1 & 1 & 1 & 0 & -1 & 0 & 0 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 1\\ 0 & 1 & 1 & 0 & -2 & 1 & 1 & 1 & 1 & -6 & -2 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(0,-1,0,1),(1,-2,1,2)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0\\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0\\ 1 & -2 & 0 & 1 & 2 & 0 & 0 & 1\end{pmatrix}} \mathbb{Z}^4.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{8} \to S^{28} \to S^{35} \to S^{18} \to S^{3} \to 0.$$

There does not exist an ordering of the collection of line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal, since each of the following pairs of line bundles have non-zero Hom's in both directions, where $(a_1,\dots,a_4)$ denotes $\mathcal{O}_X(a_1,a_2,a_3,a_4)$:

$$\begin{aligned}\{&\{(0,1,1,1),(0,1,1,4)\},\{(1,0,0,-1),(1,0,0,2)\},\{(1,1,0,-1),(1,1,0,2)\},\{(1,1,1,1),(1,1,1,4)\},\dots\\ &\dots\{(0,1,1,4),(0,1,1,1)\},\{(1,0,0,2),(1,0,0,-1)\},\{(1,1,0,2),(1,1,0,-1)\},\{(1,1,1,4),(1,1,1,1)\}\}.\end{aligned}$$

For instance, the pair $\{(0,1,1,1),(0,1,1,4)\}$ has

$$Hom_{D^b(X)}((0,1,1,1),(0,1,1,4)) = \begin{cases} \mathbb{C}^{10} & \text{in degree } 0, \\ 0 & \text{else} \end{cases}$$

and

$$Hom_{D^b(X)}((0,1,1,4),(0,1,1,1)) = \begin{cases} \mathbb{C} & \text{in degree } 2, \\ 0 & \text{else.} \end{cases}$$

Thus, there does not exist an ordering of the collection of line bundles appearing on one side of the Hanlon-Hicks-Lazarev resolution of the diagonal which yields a full strong exceptional collection of line bundles, since each pair above represents a directed cycle of length 2 in the directed quiver of all Hom's between line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal.

1.6. **Polytope F.4D.0005.** Let P denote the polytope F.4D.0005 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1/2 & 1/2 & -1 & 1/2 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & -4 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & -2 & 1 & 1 & 1 & -4 & 1 & -2 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 & 0 & -1 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & -4 & 1 & 1 & 1 & 1 & 0 & -2 & 1 & 1 & 1 & 1 & -4 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(0,0,0,1),(1,-2,1,2)\}$$

We use as presentation of the class group of X:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & -2 & 0 & 1 & 2 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{28} \to S^{35} \to S^{18} \to S^3 \to 0.$$

There does not exist an ordering of the 22 line bundles

$$\{(0,0,0,0,0),(-1,-1,0,0,1),(0,0,-1,-1,-1),(-1,-1,-1,0,1),(0,0,-1,-1,-2),$$
$$(-1,-1,0,0,0),(-1,0,-1,0,0),(0,0,0,0,-1),(-1,0,-1,-1,-1),(-1,0,-1,0,-1),$$
$$(-1,0,-1,-1,-2),(-1,-1,-1,0,0),(0,0,-1,-1,-3),(0,0,0,0,-2),(-1,-1,0,0,-1),$$
$$(-1,0,-1,-1,-3),(0,0,-1,-1,-4),(-1,-1,0,0,-2),(-1,0,-1,0,-2),(-1,-1,-1,0,-1),$$
$$(-1,-1,-1,0,-2),(-1,0,-1,-1,-4)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,0,0,1)$ and $\mathcal{O}(-1,-1,0,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0,0,1),\mathcal{O}(-1,-1,0,0,-2)) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0,0,-2),\mathcal{O}(-1,-1,0,0,1)) = \begin{cases} \mathbb{C}^{10} & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.7. **Polytope F.4D.0006.** Let P denote the polytope F.4D.0006 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1/2 & 1/2 & -1 & 1/2 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 1 & -4 & 1 & 1 & 1 & -4 & 1 & -2 & 1 \\ 1 & 1 & 1 & 0 & -2 & 0 & 0 & -2 & -2 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & -4 & 1 & 1 & 1 & 1 & -4 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(0,0,0,1),(1,-2,1,2)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & -2 & 1 & 2 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{28} \to S^{35} \to S^{18} \to S^3 \to 0.$$

There does not exist an ordering of the 18 line bundles

$$\{(0,0,0,0),(-1,0,0,1),(0,-1,-1,-1),(-1,-1,0,1),(0,-1,-1,-2),(-1,0,0,0),(0,-1,0,0),(0,0,0,-1),\dots$$
$$\dots(0,-1,0,-1),(-1,-1,0,0),(0,-1,-1,-3),(0,0,0,-2),(-1,0,0,-1),(0,-1,-1,-4),(-1,0,0,-2),\dots$$
$$\dots(0,-1,0,-2),(-1,-1,0,-1),(-1,-1,0,-2)\}$$

which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,0,0,1)$ and $\mathcal{O}(-1,0,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,0,1),\mathcal{O}(-1,0,0,-2)) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,0,-2),\mathcal{O}(-1,0,0,1)) = \begin{cases} \mathbb{C}^{10} & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.8. **Polytope F.4D.0007.** Let P denote the polytope F.4D.0007 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 1 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -4 & -4 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & -2 & 1 & 0 & 1 & -2 & 1 & -1 & 1 & -2 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & -3 & -3 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -3 & -3 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(1,0,0,-1),(-1,0,0,1),(-1,1,0,1),(-1,0,1,2)\}$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 2 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{12} \to S^{40} \to S^{47} \to S^{22} \to S^3 \to 0.$$

There does not exist an ordering of the 18 line bundles

$$\{(0,0,0,0),(0,-1,-1,-1),(0,0,0,-1),(0,0,-1,0),(-1,0,0,0),(0,-1,-2,-1),(0,-1,-1,-2),(0,-1,-1,-3),\ldots$$
$$\ldots(-1,0,-1,0),(-1,0,0,-1),(0,0,0,-2),(0,-1,-2,-2),(0,0,-1,-1),(-1,0,-1,-1),(-1,0,0,-2),(0,-1,-2,-3),$$
$$\ldots(0,0,-1,-2),(-1,0,-1,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,-1,-1,-1)$ and $\mathcal{O}(-1,0,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,-1,-1),\mathcal{O}(-1,0,0,-2)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,0,-2),\mathcal{O}(0,-1,-1,-1)) = \begin{cases} \mathbb{C}^6 & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.9. **Polytope F.4D.0008.** Let P denote the polytope F.4D.0008 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & -4 & 1 & -4 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & -2 & 1 & -2 & -2 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & -4 & -4 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(0,1,0,1),(1,0,1,2)\}$$

We use as presentation of the class group of X:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 2 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{28} \to S^{35} \to S^{18} \to S^3 \to 0.$$

There does not exist an ordering of the 14 line bundles

$$\{(0,0,0),(-1,-1,-1),(-1,-1,-2),(-1,-2,-1),(0,0,-1),(0,-1,0),(-1,-2,-2),(0,-1,-1),$$
$$(-1,-1,-3),(0,0,-2),(-1,-1,-4),(-1,-2,-3),(0,-1,-2),(-1,-2,-4)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,-1)$ and $\mathcal{O}(-1,-1,-4)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,-1),\mathcal{O}(-1,-1,-4)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,-4),\mathcal{O}(-1,-1,-1)) = \begin{cases} \mathbb{C}^{10} & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.10. **Polytope F.4D.0009.** Let P denote the polytope F.4D.0009 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & -2 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -2 & 1 & -6 & 1 & 0 & 1 & 0 & 1 \\ -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -2 & 0 & 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & -6 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,-1,1,0),(0,0,0,-1),(1,0,-2,1)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & -2 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{28} \to S^{35} \to S^{18} \to S^3 \to 0.$$

There does not exist an ordering of the 15 line bundles

$$\{(0,0,0),(0,-1,1),(-1,0,1),(0,-1,0),(-1,0,0),(0,0,-1),(-1,-1,1),$$
$$(-1,0,-1),(-1,-1,0),(0,-1,-1),(0,0,-2),(-1,-1,-1),(-1,0,-2),(0,-1,-2),$$
$$(-1,-1,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,-1,1)$ and $\mathcal{O}(0,-1,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,1),\mathcal{O}(0,-1,-2)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,-2),\mathcal{O}(0,-1,1)) = \begin{cases} \mathbb{C}^{10} & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.11. **Polytope F.4D.0010.** Let P denote the polytope F.4D.0010 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & -1 & \frac{1}{2} & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & -4 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & -4 & 1 & -2 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & -4 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & -4 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,-1,1,0),(0,0,0,-1),(1,0,-2,1)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & -2 & 0 & 1 & 2 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{28} \to S^{35} \to S^{18} \to S^3 \to 0.$$

There does not exist an ordering of the 18 line bundles

$$\{(0,0,0,0),(-1,-1,0,1),(0,0,-1,-1),(0,0,-1,-2),(-1,-1,0,0),(-1,0,0,0),(0,0,0,-1),$$
$$(-1,0,-1,-1),(-1,0,0,-1),(-1,0,-1,-2),(0,0,-1,-3),(0,0,0,-2),(-1,-1,0,-1),(-1,0,-1,-3),$$
$$(0,0,-1,-4),(-1,-1,0,-2),(-1,0,0,-2),(-1,0,-1,-4)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,0,1)$ and $\mathcal{O}(-1,-1,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0,1),\mathcal{O}(-1,-1,0,-2)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0,-2),\mathcal{O}(-1,-1,0,1)) = \begin{cases} \mathbb{C}^{10} & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.12. **Polytope F.4D.0011.** Let P denote the polytope F.4D.0011 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -4 & -4 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & -2 & 1 & -3 & 1 & -3 & -2 & 1 & 1 \\ 0 & 0 & 1 & 1 & -5 & 1 & -1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -3 & -3 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(-1,0,1,0),(0,0,0,-1),(1,0,0,-1),(-1,0,0,1),$$
$$(-1,1,0,2)\}$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 2 & 0 & 0 & 1 \end{pmatrix}.} \mathbb{Z}^4.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{12} \to S^{39} \to S^{44} \to S^{19} \to S^{2} \to 0.$$

There does not exist an ordering of the 18 line bundles

$$\{(0,0,0,0),(0,0,-1,-1),(0,0,-1,-2),(0,-1,0,0),(0,0,0,-1),(-1,0,0,0),(-1,0,-1,-1),$$
$$(-1,0,-1,-2),(-1,0,0,-1),(0,0,-1,-3),(-1,-1,0,0),(0,0,0,-2),(0,-1,0,-1),(-1,-1,0,-1),$$
$$(-1,0,-1,-3),(0,-1,0,-2),(-1,0,0,-2),(-1,-1,0,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,0,-1,-1)$ and $\mathcal{O}(-1,-1,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,0,-1,-1),\mathcal{O}(-1,-1,0,-2)) = \begin{cases} \mathbb{C} & \text{in degree } 3 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0,-2),\mathcal{O}(0,0,-1,-1)) = \begin{cases} \mathbb{C}^{22} & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.13. **Polytope F.4D.0012.** Let P denote the polytope F.4D.0012 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & -2 & -2 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & -3 & -3 & 1 \\ 0 & -3 & 1 & 1 & 0 & -1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & -4 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & -1 & -1 & -3 & -3 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(-1,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(0,0,-1,1),$$
$$(1,0,1,1)\}$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{12} \to S^{38} \to S^{42} \to S^{18} \to S^{2} \to 0.$$

There does not exist an ordering of the 18 line bundles

$$\{(0,0,0,0),(0,-1,0,-1),(-1,-1,0,-1),(0,-1,0,-2),(0,-1,-1,-1),(0,0,0,-1),(-1,0,0,0),$$
$$(-1,-1,-1,-1),(-1,0,0,-1),(-1,-1,0,-2),(0,-1,-1,-2),(0,-1,0,-3),(0,0,0,-2),(-1,-1,-1,-2),$$
$$(-1,-1,0,-3),(0,-1,-1,-3),(-1,0,0,-2),(-1,-1,-1,-3)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,-1,0,-1)$ and $\mathcal{O}(-1,-1,-1,-3)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(0,-1,0,-1),\mathcal{O}(-1,-1,-1,-3)) = \begin{cases} \mathbb{C} & \text{in degree } 3 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,-1,-1,-3),\mathcal{O}(0,-1,0,-1)) = \begin{cases} \mathbb{C}^{16} & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.14. **Polytope F.4D.0013.** Let P denote the polytope F.4D.0013 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & -4 & 1 & -4 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & -4 & 1 & 1 & -4 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 & 1 & 0 & 0 & -5 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(-1,0,1,0),(0,0,0,-1),(0,0,0,1),(1,1,0,2)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 2 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{27} \to S^{32} \to S^{15} \to S^2 \to 0.$$

There does not exist an ordering of the 14 line bundles

$$\{(0,0,0),(0,-1,-1),(0,-1,-2),(-1,0,0),(0,0,-1),(-1,-1,-1),(-1,-1,-2),\dots$$
$$\dots,(-1,0,-1),(0,-1,-3),(0,0,-2),(-1,-1,-3),(0,-1,-4),(-1,0,-2),(-1,-1,-4)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,-1,-1)$ and $\mathcal{O}(-1,-1,-4)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(0,-1,-1),\mathcal{O}(-1,-1,-4)) = \begin{cases} \mathbb{C} & \text{in degree } 3 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,-1,-4),\mathcal{O}(0,-1,-1)) = \begin{cases} \mathbb{C}^{30} & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.15. **Polytope F.4D.0014.** Let P denote the polytope F.4D.0014 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 1 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & -1 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 0 & 1 & 0 & 1 & -4 & 0 & 1 & 1 & 1 & 1 & -4 & 0 & 0 & 1 & 0 & 1 \\ -3 & -2 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & -3 & -2 & 1 & 1 \\ 1 & 1 & 1 & 1 & -3 & -1 & 0 & 0 & 0 & 0 & -3 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -4 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(1,0,-1,0),(-1,0,1,0),(-1,1,2,0),(0,0,0,-1),(0,0,-1,1)\}$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} -1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{12} \to S^{40} \to S^{47} \to S^{22} \to S^3 \to 0.$$

There does not exist an ordering of the 18 line bundles

$$\{(0,0,0,0),(0,-1,-1,0),(0,-1,-2,0),(0,-1,-1,-1),(1,-1,-2,1),(0,0,-1,0),(0,0,0,-1),$$
$$(1,-1,-2,0),(0,0,-1,-1),(0,-1,-3,0),(0,-1,-2,-1),(1,-1,-3,1),(0,0,-2,0),(1,-1,-3,0),$$
$$(0,-1,-3,-1),(0,0,-2,-1),(1,-1,-4,1),(1,-1,-4,0)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,-1,-1,0)$ and $\mathcal{O}(1,-1,-4,1)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,-1,0),\mathcal{O}(1,-1,-4,1)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(1,-1,-4,1),\mathcal{O}(0,-1,-1,0)) = \begin{cases} \mathbb{C}^{10} & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.16. **Polytope F.4D.0015.** Let P denote the polytope F.4D.0015 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 1 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -4 & -4 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & -3 & -3 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -3 & -3 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(1,0,0,-1),(-1,0,0,1),(-1,0,1,2)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & 2 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{21} \to S^{26} \to S^{13} \to S^2 \to 0.$$

There does not exist an ordering of the 18 line bundles

$$\{(0,0,0,0),(0,0,-1,-1),(0,0,-1,-2),(-1,0,0,0),(0,-1,0,0),(0,0,0,-1),(-1,0,-1,-1),$$
$$(-1,0,-1,-2),(-1,-1,0,0),(-1,0,0,-1),(0,0,-1,-3),(0,-1,0,-1),(0,0,0,-2),(-1,0,0,-2),$$
$$(-1,-1,0,-1),(-1,0,-1,-3),(0,-1,0,-2),(-1,-1,0,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,0,-1,-1)$ and $\mathcal{O}(0,-1,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,0,-1,-1),\mathcal{O}(0,-1,0,-2)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,0,-2),\mathcal{O}(0,0,-1,-1)) = \begin{cases} \mathbb{C}^6 & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.17. **Polytope F.4D.0016.** Let P denote the polytope F.4D.0016 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 1 \\ 1 & 1 & 1 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -4 & -4 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ -2 & 1 & -2 & 1 & 0 & 1 & -2 & 1 & 0 & 1 & -1 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & -3 & -3 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -3 & -3 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(1,0,0,-1),(1,1,0,-1),(-1,0,0,1),(-1,0,1,2)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 2 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{12} \to S^{40} \to S^{47} \to S^{22} \to S^3 \to 0.$$

There does not exist an ordering of the 18 line bundles

$$\{(0,0,0,0),(0,0,-1,-1),(0,0,-1,-2),(-1,-1,0,0),(0,-1,-1,-1),(0,-1,0,0),(0,0,0,-1),$$
$$(0,-1,0,-1),(0,-1,-1,-2),(-1,-1,0,-1),(0,0,-1,-3),(-1,-2,0,0),(0,0,0,-2),(-1,-1,0,-2),$$
$$(-1,-2,0,-1),(0,-1,-1,-3),(0,-1,0,-2),(-1,-2,0,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,0,-1,-1)$ and $\mathcal{O}(-1,-1,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,0,-1,-1),\mathcal{O}(-1,-1,0,-2)) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0,-2),\mathcal{O}(0,0,-1,-1)) = \begin{cases} \mathbb{C}^6 & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.18. **Polytope F.4D.0017.** Let P denote the polytope F.4D.0017 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 1 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & -1 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 1 & -6 & 0 & -1 & 1 & -1 & 1 \\ 0 & 1 & 1 & -2 & 1 & 0 & 1 & 1 & 1 & -4 & -2 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 & 0 & -4 & -1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(-1,0,1,0),(-1,1,2,0),(0,0,0,-1),(1,0,-1,1)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 2 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^3$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{12} \to S^{40} \to S^{47} \to S^{22} \to S^3 \to 0.$$

There does not exist an ordering of the 15 line bundles

$$\{(0,0,0),(-1,-1,0),(-1,-2,0),(-1,-1,-1),(0,0,-1),(0,-1,0),(0,-1,-1),$$
$$(-1,-3,0),(0,0,-2),(-1,-2,-1),(0,-2,0),(0,-1,-2),(0,-2,-1),(-1,-3,-1),$$
$$(0,-2,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,0)$ and $\mathcal{O}(0,-2,-1)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0),\mathcal{O}(0,-2,-1)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-2,-1),\mathcal{O}(-1,-1,0)) = \begin{cases} \mathbb{C}^6 & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.19. **Polytope F.4D.0018.** Let P denote the polytope F.4D.0018 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & -4 & 1 & -4 & 1 \\ -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & -4 & -4 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(1,0,1,2)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 2 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{15} \to S^{20} \to S^{11} \to S^2 \to 0$$

There does not exist an ordering of the 14 line bundles

$$\{(0,0,0),(0,-1,-1),(0,0,-1),(-1,0,0),(0,-1,-2),(-1,-1,-1),(-1,-1,-2),$$
$$(0,-1,-3),(0,0,-2),(-1,0,-1),(-1,0,-2),(-1,-1,-3),(0,-1,-4),(-1,-1,-4)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,-1,-1)$ and $\mathcal{O}(0,-1,-4)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,-1),\mathcal{O}(0,-1,-4)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,-4),\mathcal{O}(0,-1,-1)) = \begin{cases} \mathbb{C}^{10} & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.20. **Polytope F.4D.0019.** Let P denote the polytope F.4D.0019 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & -4 & 1 & -4 & 1 \\ -2 & 1 & -2 & -2 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & -4 & -4 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(0,0,0,1),(1,0,1,2)\}$$

We use as presentation of the class group of X:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 2 & 1 & 0 & 2 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{28} \to S^{35} \to S^{18} \to S^3 \to 0.$$

There does not exist an ordering of the 14 line bundles

$$\{(0,0,0),(0,-1,-1),(0,-1,-2),(1,-2,-3),(0,0,-1),(1,-1,-2),(1,-1,-3),(1,-2,-4),$$
$$(0,-1,-3),(0,0,-2),(1,-2,-5),(0,-1,-4),(1,-1,-4),(1,-2,-6)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,-1,-1)$ and $\mathcal{O}(0,-1,-4)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,-1),\mathcal{O}(0,-1,-4)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,-4),\mathcal{O}(0,-1,-1)) = \begin{cases} \mathbb{C}^{10} & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.21. **Polytope F.4D.0020.** Let P denote the polytope F.4D.0020 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & -6 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & -2 & -2 & -2 & 1 & 1 \\ 0 & 1 & 1 & 0 & -6 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(1,-2,1,0),(0,0,0,-1),(0,1,0,1)\}$$

We use as presentation of the class group of X:

$$\mathbb{Z}^6 \xrightarrow{\begin{pmatrix} -1 & 2 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^2$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{28} \to S^{35} \to S^{18} \to S^3 \to 0.$$

There does not exist an ordering of the 11 line bundles

$$\{(0,0),(-1,-1),(0,-1),(-1,-2),(1,0),(1,-1),(0,-2),(2,0),(1,-2),(2,-1),(2,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1)$ and $\mathcal{O}(2,-1)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1),\mathcal{O}(2,-1)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(2,-1),\mathcal{O}(-1,-1)) = \begin{cases} \mathbb{C}^{10} & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.22. **Polytope F.4D.0021.** Let P denote the polytope F.4D.0021 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & 0 & 1 & 0 & -1 \end{bmatrix}$$

with vertices given by the columns of:

$$\begin{pmatrix} -3 & 1 & -1 & 1 & 1 & 1 & -2 & 1 & 0 & 1 & 1 & 0 & 1 & -3 & 1 & -2 & 1 & -1 & 1 & 0 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 0 & -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(0,1,0,-1),\\(0,0,0,1),(1,-1,-1,1)\}$$

We use as presentation of the class group of X:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0.$$

There does not exist an ordering of the 25 line bundles

$\{(0,0,0,0,0),(-1,0,1,-1,-1),(-1,0,0,-1,0),(0,0,0,-1,-1),(0,-1,0,0,0),(0,-1,0,-1,0),(-1,-1,1,-1,0),$
$(0,0,0,0,-1),(0,-1,0,0,-1),(-1,0,1,-1,-2),(-1,0,1,-2,-1),(-1,0,0,-1,-1),(-1,-1,1,-1,-1),(0,-1,0,-1,-1),$
$(-1,-1,1,-2,0),(-1,-1,0,-1,0),(0,0,0,-1,-2),(-1,-1,1,-2,-1),(-1,0,1,-2,-2),(-1,-1,0,-1,-1),$
$(-1,-1,1,-1,-2),(0,-1,0,-1,-2),(-1,0,0,-1,-2),(-1,-1,0,-1,-2),(-1,-1,1,-2,-2)\}$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,0,0,-1,0)$ and $\mathcal{O}(-1,-1,0,-1,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,0,-1,0),\mathcal{O}(-1,-1,0,-1,-2)) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0,-1,-2),\mathcal{O}(-1,0,0,-1,0)) = \begin{cases} \mathbb{C}^7 & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.23. **Polytope F.4D.0022.** Let P denote the polytope F.4D.0022 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & -1 & 1 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & -1 & 1 & -1 & 1 & 0 & 1 & -3 & 1 & 0 & 1 & -3 & 1 & -2 & 1 \\ 1 & 1 & 0 & -2 & -2 & 0 & 0 & 1 & 1 & -2 & -2 & 1 & 1 & 0 & 0 & 1 & 1 \\ -1 & 0 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,1,-1,0),(0,0,1,0),(0,0,0,-1),(0,0,0,1),(1,-1,1,1)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 20 line bundles

$$\begin{aligned}\{&(0,0,0,0),(0,0,0,-1),(0,0,-1,-1),(0,-1,0,-1),(0,-1,0,0),\\ &(1,-1,-1,-1),(1,-1,0,-1),(0,-1,-1,-1),(0,-1,-1,-2),(0,0,-1,-2),\\ &(1,-2,-1,-1),(1,-1,-1,-2),(1,-2,0,-1),(1,-1,0,-2),(0,-1,0,-2),\\ &(1,-2,-1,-2),(1,-2,0,-2),(1,-1,-1,-3),(0,-1,-1,-3),(1,-2,-1,-3)\}\end{aligned}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,0,0,-1)$ and $\mathcal{O}(1,-2,-1,-1)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,0,0,-1),\mathcal{O}(1,-2,-1,-1) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(1,-2,-1,-1),\mathcal{O}(0,0,0,-1)) = \begin{cases} \mathbb{C}^2 & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.24. **Polytope F.4D.0023.** Let P denote the polytope F.4D.0023 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & -1 & -1 & 1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

vertices given by the columns of

$$\begin{pmatrix} 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 0 & 1 \\ -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & -4 & -2 & -2 & -2 & 1 & 1 & 1 \\ 0 & 0 & 0 & -4 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(1,1,-1,-1),(-1,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & -1 & 1 & 0 \\ -1 & 0 & -1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{15} \to S^{20} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 21 line bundles

$$\begin{aligned}&\{(0,0,0,0),(0,-1,-1,0),(-1,0,0,0),(0,0,1,-1),(-1,0,-1,0),\\&(0,0,0,-1),(0,-1,0,0),(-1,-1,-1,1),(0,-1,1,-1),(0,-1,0,-1),\\&(-1,-1,0,0),(-1,-1,-2,1),(-1,0,1,-1),(-1,0,0,-1),(0,0,1,-2),\\&(-1,-1,-1,0),(-1,-1,0,-1),(-1,-1,1,-1),(0,-1,1,-2),(-1,0,1,-2),\\&(-1,-1,1,-2)\}\end{aligned}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,-1,-1,0)$ and $\mathcal{O}(-1,-1,1,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(0,-1,-1,0),\mathcal{O}(-1,-1,1,-2)) = \begin{cases}\mathbb{C} & \text{in degree 2}\\ 0 & \text{else.}\end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,-1,1,-2),\mathcal{O}(0,-1,-1,0)) = \begin{cases}\mathbb{C}^7 & \text{in degree 0}\\ 0 & \text{else .}\end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.25. **Polytope F.4D.0024.** Let P denote the polytope F.4D.0024 in the polymake database, with half-space representation

$$\begin{bmatrix}1&0&0&1&0\\1&1&0&0&0\\1&-1&0&0&0\\1&-1&-1&1&1\\1&0&0&0&-1\\1&0&-1&0&0\\1&0&1&0&0\\1&0&0&-1&0\\1&1&1&-1&-1\end{bmatrix}$$

with vertices given by the columns of

$$\begin{pmatrix}-1&1&-1&1&-1&1&-1&-1&-1&1&-1&1&-1&1&-1&1&0&1&0&-1&1&0&1\\-1&-1&1&-1&1&1&-1&-1&-1&-1&1&-1&1&1&0&0&-1&-1&1&1&0&1&1\\-1&-1&-1&1&1&1&1&-1&1&1&1&0&0&0&-1&-1&-1&-1&-1&-1&1&1&1\\0&0&0&0&0&0&-4&-2&-2&-2&-2&1&1&1&1&1&1&1&1&1&1&1&1\end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(1,1,-1,-1),(-1,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix}1&1&0&0&0&0&0&0&0\\0&0&1&1&0&0&0&0&0\\0&0&0&0&1&1&0&0&0\\1&0&1&0&-1&0&-1&1&0\\-1&0&-1&0&1&0&1&0&1\end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{15} \to S^{20} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 33 line bundles

$$\begin{aligned}
&\{(0,0,0,0,0),(0,-1,0,-1,0),(-1,0,0,0,0),(0,0,-1,1,-1),(-1,0,-1,0,0),\\
&(-1,0,0,-1,0),(0,0,-1,0,-1),(0,0,0,0,-1),(0,-1,0,0,0),(0,-1,-1,0,0),\\
&(-1,-1,0,-1,1),(0,-1,-1,1,-1),(0,-1,-1,0,-1),(-1,-1,-1,0,0),\\
&(0,-1,-1,-1,0),(-1,-1,0,-2,1),(0,-1,0,0,-1),(-1,-1,-1,-1,1),\\
&(-1,0,-1,1,-1),(-1,0,0,0,-1),(-1,-1,0,0,0),(0,0,-1,1,-2),\\
&(-1,0,-1,0,-1),(-1,0,-1,-1,0),(-1,-1,0,-1,0),(-1,-1,-1,-1,0),\\
&(-1,-1,-1,0,-1),(-1,-1,-1,1,-1),(-1,-1,-1,-2,1),(0,-1,-1,1,-2),\\
&(-1,-1,0,0,-1),(-1,0,-1,1,-2),(-1,-1,-1,1,-2)\}
\end{aligned}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,-1,0,-1,0)$ and $\mathcal{O}(-1,-1,-1,1,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(0,-1,0,-1,0)),\mathcal{O}(1,-1,-1,1,-2)=\begin{cases}\mathbb{C} & \text{in degree 2}\\ 0 & \text{else.}\end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(1,-1,-1,1,-2),\mathcal{O}(0,-1,0,-1,0)))=\begin{cases}\mathbb{C}^4 & \text{in degree 0}\\ 0 & \text{else .}\end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

Furthermore, we verify that Bondal's numerical criterion fails for this variety, since the torus-invariant curve corresponding to the 3-cone with primitive ray generators $\{(0,-1,0,0),(-1,0,0,0),(0,0,-1,0)\}$ intersects each of the torus-invariant divisors corresponding to primitive ray generators $(-1,0,0,0)$ and $(0,-1,0,0)$ in -1.

1.26. **Polytope F.4D.0025.** Let P denote the polytope F.4D.0025 in the polymake database, with half-space representation given by

$$\begin{bmatrix}
1 & 1 & 0 & 0 & 0\\
1 & 0 & 0 & 1 & 0\\
1 & -1 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & -1\\
1 & -1 & -1 & 1 & 1\\
1 & 0 & 0 & -1 & 0\\
1 & 0 & -1 & 0 & 0\\
1 & 0 & 0 & 0 & 1\\
1 & 1 & 1 & -1 & -1\\
1 & 0 & 1 & 0 & 0
\end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix}
-1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 0 & -1 & -1 & 1 & 0 & 1 & 0 & -1 & 1 & -1 & 1 & -1 & 1 & 0 & 1 & 0 & -1 & 1 & 0 & 1\\
-1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 0 & -1 & -1 & 0 & 0 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & -1 & 0 & 1 & 1\\
-1 & -1 & -1 & 1 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(1,1,-1,-1),(0,0,0,1),(-1,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^{10}\xrightarrow{\begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0\\
1 & 0 & 1 & 0 & -1 & 0 & -1 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0\\
-1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 0 & 1
\end{pmatrix}}\mathbb{Z}^6$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{15} \to S^{20} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 50 line bundles

$$\{(0,0,0,0,0,0), (0,-1,0,-1,0,0), (-1,0,0,0,-1,0), (0,0,-1,1,-1,-1), (-1,0,-1,0,0,0),$$
$$(-1,0,0,-1,0,0), (0,0,-1,0,0,-1), (0,0,0,0,-1,-1), (0,-1,0,0,-1,0), (0,-1,-1,0,0,0),$$
$$(-1,-1,0,-1,0,1), (0,-1,-1,1,-1,-1), (0,-1,-1,0,0,-1), (-1,-1,0,-1,-1,1), (-1,-1,-1,0,0,0),$$
$$(0,-1,-1,-1,0,0), (-1,-1,0,-2,0,1), (0,0,-1,0,-1,-1), (0,-1,0,0,-1,-1), (-1,-1,-1,-1,0,1),$$
$$(-1,0,-1,1,-1,-1), (-1,0,0,0,-1,-1), (-1,0,-1,0,-1,0), (-1,-1,0,0,-1,0), (0,-1,-1,0,-1,0),$$
$$(0,0,-1,1,-1,-2), (0,-1,0,-1,-1,0), (-1,0,-1,0,0,-1), (-1,0,-1,-1,0,0), (-1,0,0,-1,-1,0),$$
$$(-1,-1,0,-1,0,0), (-1,0,-1,-1,-1,0), (-1,-1,0,0,-1,-1), (0,-1,-1,1,-1,-2), (-1,-1,-1,-2,0,1),$$
$$(-1,-1,0,-2,-1,1), (0,-1,-1,-1,-1,0), (-1,0,-1,1,-1,-2), (-1,-1,-1,-1,-1,1), (-1,-1,-1,1,-1,-1),$$
$$(-1,-1,-1,0,0,-1), (-1,-1,0,-1,-1,0), (0,-1,-1,0,-1,-1), (-1,-1,-1,0,-1,0), (-1,0,-1,0,-1,-1),$$
$$(-1,-1,-1,-1,0,0), (-1,-1,-1,1,-1,-2), (-1,-1,-1,-2,-1,1), (-1,-1,-1,0,-1,-1), (-1,-1,-1,-1,-1,0)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,0,0,-1)$ and $\mathcal{O}(1,-2,-1,-1)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,0,0,-1), \mathcal{O}(1,-2,-1,-1)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(1,-2,-1,-1), \mathcal{O}(0,0,0,-1)) = \begin{cases} \mathbb{C}^2 & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.27. **Polytope F.4D.0026.** Let P denote the polytope F.4D.0026 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & -1 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 \\ 1 & 0 & 1 & 1 & 1 & -3 & -3 & 1 & 1 & -1 & -1 & 1 & 1 \\ 0 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0), (0,-1,0,0), (0,0,-1,0), (0,0,1,0), (0,0,0,-1), (1,0,0,1), (0,1,-1,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 15 line bundles

$$\{(0,0,0),(0,0,-1),(0,-1,-1),(-1,-1,0),(-1,0,0),(0,-1,0),(-1,-1,-1),(-1,-2,0),$$
$$(-1,0,-1),(0,-2,-1),(0,-1,-2),(-1,-2,-1),(0,-2,-2),(-1,-1,-2),(-1,-2,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,0,-1)$ and $\mathcal{O}(-1,-2,0)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,-2,0)),\mathcal{O}(1,-2,-1) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,-2,0),\mathcal{O}(0,0,-1))) = \begin{cases} \mathbb{C}^3 & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.28. **Polytope F.4D.0027.** Let P denote the polytope F.4D.0027 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} -2 & 1 & -2 & 1 & 1 & 0 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -2 & 1 & -2 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & -2 & -2 & 1 & 1 & -2 & -2 & 1 & 1 \\ 1 & 1 & 1 & 1 & -2 & -1 & -1 & -2 & -2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays:

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(1,0,1,0),(0,0,0,-1),(0,0,1,-1),(0,1,0,1),(0,0,-1,1)\}$$

We use as presentation of the class group:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 19 line bundles

$$\{(0,0,0,0),(-1,1,-1,-1),(0,0,-1,-1),(-1,0,0,-1),(0,0,-1,0),(-1,1,0,-1),(-1,0,-1,-1),$$
$$(-1,1,-1,-2),(-1,1,-2,-1),(0,0,-2,-1),(-2,2,-1,-2),(-2,1,0,-2),(-2,1,-1,-2),(-1,1,-2,-2),$$
$$(-2,2,-2,-2),(-1,0,-2,-1),(-2,2,-1,-3),(-2,1,-2,-2),(-2,2,-2,-3)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,0,-1,0)$ and $\mathcal{O}(-2,2,-1,-3)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(0,0,-1,0)),\mathcal{O}(-2,2,-1,-3)=\begin{cases}\mathbb{C} & \text{in degree } 2\\ 0 & \text{else.}\end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-2,2,-1,-3),\mathcal{O}(0,0,-1,0)))=\begin{cases}\mathbb{C}^3 & \text{in degree } 0\\ 0 & \text{else .}\end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.29. **Polytope F.4D.0028.** Let P denote the polytope F.4D.0028 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 1\\ 1 & 0 & -1 & 0 & 0\\ 1 & 0 & 0 & -1 & 0\\ 1 & 0 & -1 & 0 & 1\\ 1 & 0 & 0 & 0 & -1\\ 1 & 1 & -2 & 1 & 1\\ 1 & 0 & 1 & 0 & 0\end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & -1 & 1 & -3 & 1 & 1 & 1 & -5 & 1 & -1 & 1\\ 1 & 1 & 1 & -1 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 1\\ 0 & 1 & 1 & -3 & -1 & 1 & 1 & 1 & 1 & -5 & -1 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(0,-1,0,1),(1,-2,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0\\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0\\ 1 & -2 & 0 & 1 & 1 & 0 & 0 & 1\end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{28} \to S^{35} \to S^{17} \to S^2 \to 0$$

There does not exist an ordering of the 18 line bundles

$$\{(0,0,0,0),(-1,0,0,1),(-1,-1,0,1),(-1,0,0,0),(0,0,0,-1),(-1,-1,0,0),(0,-1,-1,-1),$$
$$(-1,-1,-1,0),(-1,0,0,-1),(-1,-1,0,-1),(-1,-1,-1,-1),(0,-1,-1,-2),(0,0,0,-2),(-1,-1,-1,-2),$$
$$(-1,-1,0,-2),(-1,0,0,-2),(0,-1,-1,-3),(-1,-1,-1,-3)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,0,0,1)$ and $\mathcal{O}(-1,0,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,0,0,1)),\mathcal{O}(-1,0,0,-2))=\begin{cases}\mathbb{C} & \text{in degree } 2\\ 0 & \text{else.}\end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,0,0,-2),\mathcal{O}(-1,0,0,1)))=\begin{cases}\mathbb{C}^{10} & \text{in degree } 0\\ 0 & \text{else .}\end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.30. **Polytope F.4D.0029.** Let P denote the polytope F.4D.0029 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 1 & -2 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & -4 & 1 & 0 & 1 & 1 & 1 & -1 & 1 & -3 & 1 & 1 & 1 & -3 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & -4 & 1 & 1 & 1 & 1 & -3 & -1 & 1 & 1 & 1 & 1 & -3 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays:

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(0,0,0,1),(0,-1,0,1),(1,-2,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^{10} \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & -2 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{28} \to S^{35} \to S^{17} \to S^2 \to 0$$

There does not exist an ordering of the 21 line bundles

$$\{(0,0,0,0,0),(-1,-1,0,0,1),(-1,-1,-1,0,1),(-1,-1,0,0,0),(0,0,0,0,-1),(-1,0,-1,0,0),$$
$$(0,0,-1,-1,-1),(-1,0,-1,-1,0),(-1,-1,-1,0,0),(-1,0,-1,0,-1),(0,0,-1,-1,-2),(-1,0,-1,-1,-1),$$
$$(-1,-1,0,0,-1),(0,0,0,0,-2),(-1,-1,0,0,-2),(0,0,-1,-1,-3),(-1,0,-1,-1,-2),(-1,0,-1,0,-2),$$
$$(-1,-1,-1,0,-1),(-1,0,-1,-1,-3),(-1,-1,-1,0,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,0,0,1)$ and $\mathcal{O}(-1,-1,0,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,-1,0,0,1)),\mathcal{O}(-1,-1,0,0,-2) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,-1,0,0,-2),\mathcal{O}(-1,-1,0,0,1))) = \begin{cases} \mathbb{C}^{10} & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.31. **Polytope F.4D.0030.** Let P denote the polytope F.4D.0030, with half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 1 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -5 & -1 & 0 & -2 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & -2 & 0 & 1 & 1 & -2 & -1 & 1 & 1 & -2 & -1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -4 & -3 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -4 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays:

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(1,0,0,-1),(-1,0,0,1),(-1,1,0,1),(0,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{12} \to S^{39} \to S^{44} \to S^{19} \to S^2 \to 0$$

There does not exist an ordering of the 18 line bundles

$$\{(0,0,0,0),(0,-1,-1,0),(0,-1,-1,-1),(0,-1,-2,0),(0,0,-1,0),(0,0,0,-1),(-1,0,0,0),$$
$$(-1,0,0,-1),(0,-1,-2,-1),(0,-1,-1,-2),(-1,0,-1,0),(0,0,-1,-1),(0,0,0,-2),(0,-1,-2,-2),$$
$$(-1,0,0,-2),(-1,0,-1,-1),(0,0,-1,-2),(-1,0,-1,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,-1,-1,0)$ and $\mathcal{O}(-1,0,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,-1,0)),\mathcal{O}(-1,0,0,-2)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,0,-2)),\mathcal{O}(0,-1,-1,0)) = \begin{cases} \mathbb{C}^6 & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.32. **Polytope F.4D.0031.** Let P denote the polytope F.4D.0031 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & -3 & 1 & -4 & 1 & -2 & 1 & -1 & 1 & 0 & 1 \\ 1 & -1 & -1 & 1 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & -1 & 1 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & -4 & -3 & -2 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,1,-1,0),(0,0,1,0),(0,0,0,-1),(1,-1,-1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 & -1 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{28} \to S^{35} \to S^{17} \to S^2 \to 0$$

There does not exist an ordering of the 17 line bundles

$\{(0,0,0,0),(-1,0,-1,1),(-1,0,-1,0),(-1,1,-2,2),(-1,1,-1,1),(0,0,0,-1),(0,0,-1,0),$
$(-1,1,-2,1),(-1,1,-1,0),(0,0,-1,-1),(-1,0,-1,-1),(0,0,0,-2),(-1,1,-2,0),(-1,1,-1,-1),$
$(0,0,-1,-2),(-1,0,-1,-2),(-1,1,-2,-1)\}$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,0,-1,1)$ and $\mathcal{O}(-1,0,-1,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,-1,1)),\mathcal{O}(-1,0,-1,-2)) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,-1,-2)),\mathcal{O}(-1,0,-1,1)) = \begin{cases} \mathbb{C}^{10} & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.33. **Polytope F.4D.0032.** Let P denote the polytope F.4D.0032 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & -1 & 1 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & -1 & 1 & 0 & 1 & 1 & 1 & -5 & 1 & -2 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & -2 & 1 & -2 & -2 & 1 & 1 \\ 0 & -1 & 1 & 1 & 1 & 1 & -5 & -2 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(0,1,0,1),(1,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{28} \to S^{35} \to S^{17} \to S^2 \to 0$$

There does not exist an ordering of the 14 line bundles

$$\{(0,0,0),(-1,-1,0),(-1,-1,-1),(-1,-2,0),(0,-1,0),(0,0,-1),(-1,-2,-1),(-1,-1,-2),$$
$$(0,-1,-1),(0,0,-2),(-1,-2,-2),(-1,-1,-3),(0,-1,-2),(-1,-2,-3)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,0)$ and $\mathcal{O}(-1,-1,-3)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0)),\mathcal{O}(-1,-1,-3) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,-3),\mathcal{O}(-1,-1,0))) = \begin{cases} \mathbb{C}^{10} & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.34. **Polytope F.4D.0033.** Let P denote the polytope F.4D.0033 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & 0 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 0 & 0 & 1 & -2 & 0 & -2 & 1 & 1 & -2 & 0 & 1 & 1 & 0 & -2 & 1 & 1 \\ 0 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -4 & -2 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(-1,1,0,0),(0,0,-1,0),(0,0,0,-1),(-1,0,0,1),(0,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & 1 & 1 & 0 \end{pmatrix}} \mathbb{Z}^4$$

Free ranks which appear:

$$0 \to S^{10} \to S^{31} \to S^{32} \to S^{12} \to S^1 \to 0$$

There does not exist an ordering of the 19 line bundles

$$\{(0,0,0,0),(-1,0,0,-1),(0,0,-1,-1),(0,0,0,-1),(-1,0,0,0),(0,-1,0,0),(0,-1,-1,0),(0,-1,-1,-1),$$
$$(-1,-1,0,0),(-1,0,-1,-1),(-1,0,0,-2),(-1,-1,-1,0),(0,-1,0,-1),(0,0,-1,-2),(-1,-1,0,-1),(-1,0,-1,-2),$$
$$(-1,-1,-1,-1),(0,-1,-1,-2),(-1,-1,-1,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,0,0,-1)$ and $\mathcal{O}(-1,-1,-1,0)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(0,0,0,-1)),\mathcal{O}(-1,-1,-1,0) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,-1,-1,0),\mathcal{O}(0,0,0,-1))) = \begin{cases} \mathbb{C}^5 & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.35. **Polytope F.4D.0034.** Let P denote the polytope F.4D.0034 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & -1 & 1 & -1 & 1 & -1 & -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 & 0 & 1 & 1 & 1 & 1 & -1 & 1 & -3 & -1 & 1 & 1 & -1 & 1 & -3 & -2 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & -2 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,0,-1,0),(1,0,-1,0),(-1,0,1,0),(0,0,0,-1),(-1,0,0,1),(-1,1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^5 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^9$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{10} \to S^{31} \to S^{32} \to S^{12} \to S^{1} \to 0$$

There does not exist an ordering of the 24 line bundles

$\{(0,0,0,0,0),(0,0,0,-1,-1),(0,0,-1,-1,-1),(0,0,-1,0,-1),(0,0,0,0,-1),(-1,1,-1,0,-1),(-1,0,0,0,-1),(-1,0,0,0,0),\ldots$
$\ldots(-1,1,-1,0,-2),(0,0,-1,0,-2),(-1,0,-1,0,-1),(-1,0,0,-1,-1),(-1,0,0,0,-2),(0,0,-1,-1,-2),(-1,1,-1,-1,-2),\ldots$
$\ldots(0,0,0,-1,-2),(-1,0,-1,-1,-1),(-1,0,0,-1,-2),(-1,0,-1,0,-2),(-1,0,-1,-1,-2),(-1,1,-1,0,-3),\ldots$
$\ldots(0,0,-1,-1,-3),(-1,1,-1,-1,-3),(-1,0,-1,-1,-3)\}$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,0,-1,-1,-1)$ and $\mathcal{O}(-1,1,-1,0,-3)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(0,0,-1,-1,-1),\mathcal{O}(-1,1,-1,0,-3)) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,1,-1,0,-3),\mathcal{O}(0,0,-1,-1,-1))) = \begin{cases} \mathbb{C}^3 & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.36. **Polytope F.4D.0035.** Let P denote the polytope F.4D.0035 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 0 & -3 & -1 & 1 & 1 & 0 & -2 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & -3 & -1 & 1 & 1 \\ 0 & -1 & -1 & 1 & 1 & 0 & -1 & -1 & 1 & 1 & 0 & -1 & 1 & 0 & -1 & 1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -2 & -2 & -1 & -2 & -2 & -1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(-1,1,0,0),(0,0,-1,0),(1,-1,1,0),(0,0,0,-1),(0,0,0,1),(0,0,-1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 22 line bundles

$\{(0,0,0,0,0),(0,0,-1,-1,-1),(-1,0,0,0,0),(-1,-1,0,0,0),(0,0,0,-1,-1),(-1,-1,0,-1,0),\ldots$
$\ldots(0,0,-1,0,-1),(-1,-1,0,-1,-1),(-1,0,-1,0,-1),(0,0,-1,-1,-2),(-1,0,-1,-1,-1),(-1,0,0,-1,-1),\ldots$
$\ldots(0,0,-2,-1,-2),(-1,-1,-1,0,-1),(0,0,-2,0,-2),(-1,-1,-1,-1,-1),(-1,0,-1,-1,-2),(-1,-1,-1,-1,-2),\ldots$
$\ldots(0,0,-2,-1,-3),(-1,0,-2,-1,-2),(-1,0,-2,0,-2),(-1,0,-2,-1,-3)\}$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,0,-1,0)$ and $\mathcal{O}(-1,0,-2,-1,-3)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0,-1,0)),\mathcal{O}(-1,0,-2,-1,-3) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,-2,-1,-3),\mathcal{O}(-1,-1,0,-1,0))) = \begin{cases} \mathbb{C}^3 & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.37. **Polytope F.4D.0036.** Let P denote the polytope F.4D.0036 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & -1 & 1 & 1 & 1 & 1 & 0 & 1 & -2 & -2 & 0 & 1 & 1 & 1 & 1 & -2 & -1 & -2 & 1 \\ 1 & -2 & 0 & 1 & 0 & 1 & -1 & 0 & -1 & 1 & 1 & 1 & 0 & 0 & 1 & -2 & -2 & 1 & 1 \\ -2 & 1 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & 0 & -2 & -2 & -1 & 1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(-1,1,0,0),(0,0,-1,0),(0,1,1,0),(0,0,0,-1),(1,0,0,1),(1,0,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 & -1 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{14} \to S^{40} \to S^{38} \to S^{13} \to S^1 \to 0$$

There does not exist an ordering of the 21 line bundles

$$\{(0,0,0,0),(0,-1,-1,0),(-1,-1,0,1),(0,0,-1,-1),(-1,-1,0,0),(0,-1,-1,-1),$$
$$(-1,-1,-1,0),(0,-1,0,0),(0,-2,-1,0),(0,-2,-2,0),(0,0,-2,-2),(-1,-1,-1,-1),$$
$$(-1,-2,0,1),(-1,-2,-1,1),(0,-1,-2,-1),(-1,-2,-1,0),(-1,-1,-2,-1),(0,-2,-2,-1),$$
$$(0,-1,-2,-2),(-1,-2,-2,0),(-1,-2,-2,-1)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,0,1)$ and $\mathcal{O}(0,-1,-2,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0,1)),\mathcal{O}(0,-1,-2,-2) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,-2,-2),\mathcal{O}(-1,-1,0,1))) = \begin{cases} \mathbb{C}^3 & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.38. **Polytope F.4D.0037.** Let P denote the polytope F.4D.0037 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & -1 & 1 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} -2 & 1 & 0 & 1 & -2 & 1 & -1 & 0 & 1 & 1 & 1 & 1 & -1 & 1 & -2 & -1 & 1 & 1 & -1 & 1 & 0 & -2 & 1 \\ -1 & 0 & -1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & -2 & -2 & 1 & 0 & -2 & 0 & -1 & -1 & 1 \\ 0 & -2 & -2 & -1 & 1 & 1 & 1 & 0 & 0 & -1 & 0 & -2 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(-1,1,0,0),(0,0,-1,0),(1,-1,1,0),(0,0,0,-1),(1,-1,1,-1),(0,1,0,1),(0,1,-1,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{20} \to S^{54} \to S^{48} \to S^{15} \to S^1 \to 0$$

There does not exist an ordering of the 30 line bundles

$$\begin{aligned} &\{(0,0,0,0,0),(0,0,0,-1,-1),(-1,0,0,-1,-1),(0,-1,0,0,-1),(0,-1,1,-1,-2),\ldots \\ &\ldots(0,-1,0,-1,-1),(0,-1,1,-1,-1),(-1,0,0,-1,0),(-1,0,1,-1,-1),(0,-2,0,0,-2),\ldots \\ &\ldots(0,-1,0,-1,-2),(0,-2,1,-1,-2),(-1,0,0,-2,-1),(0,0,0,-2,-2),(-1,-1,0,-1,-1),\ldots \\ &\ldots(-1,0,1,-2,-2),(-1,-1,1,-1,-2),(-1,-1,1,-1,-1),(0,-1,1,-2,-2),(-1,0,1,-2,-1),\ldots \\ &\ldots(-1,-1,1,-2,-2),(-1,0,0,-2,-2),(0,-2,0,-1,-2),(0,-1,1,-2,-3),(0,-1,0,-2,-2),\ldots \\ &\ldots(0,-2,1,-1,-3),(-1,-1,0,-1,-2),(-1,-1,1,-2,-3),(-1,-1,0,-2,-2),(0,-2,1,-2,-3)\} \end{aligned}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,-1,1,-1,-1)$ and $\mathcal{O}(-1,0,0,-2,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,1,-1,-1)),\mathcal{O}(-1,0,0,-2,-2) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,0,-2,-2),\mathcal{O}(0,-1,1,-1,-1))) = \begin{cases} \mathbb{C}^1 & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.39. **Polytope F.4D.0038.** Let P denote the polytope F.4D.0038, with half-space representation

$$\begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 1 & -1 & -3 & -1 & -1 & 1 & 1 & 1 & -3 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 & 1 & 0 & -2 & -1 & 1 & 1 & 1 & 0 & 0 & -2 & -2 & 1 & 1 \\ 1 & 0 & 0 & -1 & 0 & -2 & 1 & 1 & 0 & 0 & -2 & -2 & -1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(-1,1,0,0),(0,0,-1,0),(0,1,1,0),(0,0,0,-1),(1,-1,0,1),(1,-1,-1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{14} \to S^{40} \to S^{38} \to S^{13} \to S^1 \to 0$$

There does not exist an ordering of the 20 line bundles

$$\{(0,0,0,0),(0,-1,0,-1),(0,0,-1,-1),(0,-1,-1,-1),(-1,0,-1,-1),(-1,0,-1,0),(0,-1,-1,-2),(0,-2,-1,-2),\ldots$$
$$\ldots,(0,0,-2,-2),(-1,-1,-1,-1),(-1,-1,-1,-2),(-1,0,-2,-2),(-1,0,-2,-1),(0,-1,-2,-2),(0,-2,0,-2),\ldots$$
$$\ldots(0,-2,-1,-3),(0,-1,-2,-3),(-1,-1,-2,-2),(-1,-1,-2,-3),(0,-2,-2,-3)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,0,-1,0)$ and $\mathcal{O}(0,-2,-1,-3)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,-1,0)),\mathcal{O}(0,-2,-1,-3) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-2,-1,-3),\mathcal{O}(-1,0,-1,0))) = \begin{cases} \mathbb{C}^3 & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.40. **Polytope F.4D.0039.** Let P denote the polytope F.4D.0039 in the polymake database, with the half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & 0 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -3 & 0 & -3 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & -2 & 1 & 1 & -2 & 1 & 1 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,0,-1,0),$$
$$(0,0,0,-1),(0,1,0,-1),(-1,0,0,1),(0,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{18} \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 19 line bundles

$\{(0,0,0,0),(0,0,-1,-1),(0,0,-1,0),(-1,0,0,0),(0,0,0,-1),(-1,0,0,-1),(0,-1,0,0),(-1,-1,0,0),\ldots$
$\ldots(-1,0,-1,0),(0,-1,-1,0),(-1,0,-1,-1),(0,0,-1,-2),(0,-1,0,-1),(-1,0,0,-2),(-1,0,-1,-2),(0,-1,-1,-1),\ldots$
$\ldots(-1,-1,0,-1),(-1,-1,-1,0),(-1,-1,-1,-1)\}$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,0,-1,0)$ and $\mathcal{O}(-1,0,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,0,-1,0)),\mathcal{O}(-1,0,0,-2) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,0,-2),\mathcal{O}(0,0,-1,0))) = \begin{cases} \mathbb{C}^3 & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.41. **Polytope F.4D.0040.** Let P denote the polytope F.4D.0040, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & -2 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & -4 & -2 & 1 \\ 1 & 0 & -1 & 0 & -1 & -1 & 1 & 1 & 0 & -1 & 1 & 0 & -1 & -1 & 1 & 1 \\ 0 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -3 & -3 & -2 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(-1,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(1,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0$$

Here there does not exist an ordering of the 17 line bundles $\mathcal{O}(a_1, ..., a_4)$ appearing on the left-hand side of the Hanlon-Hicks-Lazarev resolution for $(a_1, \ldots, a_4)$ in the set

$$\{(-1,0,-1,-3),(0,0,-1,-3),(-1,0,0,-2),(-1,-1,-1,-2),(-1,0,-1,-2),(0,0,0,-2),(-1,-1,0,-1),\ldots$$
$$\ldots(-1,0,-1,-1),(-1,0,0,-1),(-1,-1,-1,-1),(0,0,-1,-2),(-1,0,0,0),(-1,-1,-1,0),\ldots$$
$$\ldots(-1,-1,0,0),(0,0,-1,-1),(0,0,0,-1),(0,0,0,0)\}$$

which yields an exceptional collection. This is because the line bundles $\mathcal{O}(-1,-1,-1,0)$ and $\mathcal{O}(-1,0,-1,-3)$ have

$$Hom_{D^b(X)}(\mathcal{O}(-1,-1,-1,0),\mathcal{O}(-1,0,-1,-3)) = \begin{cases} \mathbb{C} & \text{in degree 2,} \\ 0 & \text{else} \end{cases}$$

and

$$Hom_{D^b(X)}(\mathcal{O}(-1,0,-1,-3),\mathcal{O}(-1,-1,-1,0)) = \begin{cases} \mathbb{C}^6 & \text{in degree 0,} \\ 0 & \text{else} \end{cases}$$

Furthermore, we verify that Bondal's numerical criterion fails for this variety, since the torus-invariant curve corresponding to the 3-cone with primtitive ray generators $\{(-1,0,0,0),(0,-1,0,0),(0,0,0,1)\}$ intersects the torus-invariant divisors corresponding to primitive ray generators $(0,-1,0,0)$ and $(0,0,0,1)$ each in -1.

1.42. **Polytope F.4D.0041.** Let P denote the polytope F.4D.0041 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 & -3 & -3 & 1 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 & 1 & -3 & 1 & 1 & -3 & -3 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 & -4 & -1 & -1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(-1,0,1,0),$$
$$(0,0,0,-1),(1,1,0,1),(1,0,-1,1)\}$$

We use as presentation of $Cl(X)$

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{10} \to S^{31} \to S^{32} \to S^{12} \to S^{1} \to 0$$

There does not exist an ordering of the 14 line bundles

$\{(-1,-3,-1),(0,-3,-2),(0,-3,-1),(-1,-2,-1),(-1,-2,0),(-1,-1,-1),(0,-2,0),(0,-2,-2),(0,-2,-1),(0,-1,0),\dots$
$(-1,0,0),(-1,-1,0),(0,-1,-1),(0,0,0)\}$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,0,0)$ and $\mathcal{O}(0,-3,-1)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,0,0)),\mathcal{O}(0,-3,-1) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(0,-3,-1),\mathcal{O}(-1,0,0))) = \begin{cases} \mathbb{C}^6 & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.43. **Polytope F.4D.0042.** Let P denote the polytope F.4D.0042 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} -3 & 0 & -3 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ -2 & 1 & 1 & 1 & 1 & -2 & -1 & 1 & 1 & -2 & -1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(1,0,0,-1),(-1,1,0,1),(0,0,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{8} \to S^{24} \to S^{25} \to S^{10} \to S^{1} \to 0$$

There does not exist an ordering of the 14 line bundles

$$\{(0,0,0),(0,0,-1),(0,-1,-1),(-1,0,0),(0,-1,0),(-1,0,-1),(0,-2,-1),(0,-1,-2),$$
$$(-1,-1,0),(0,0,-2),(-1,-1,-1),(0,-2,-2),(-1,0,-2),(-1,-1,-2)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(0,-1,0)$ and $\mathcal{O}(-1,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(0,-1,0),\mathcal{O}(-1,0,-2)) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,-2),\mathcal{O}(0,-1,0)) = \begin{cases} \mathbb{C}^6 & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.44. **Polytope F.4D.0043.** Let P denote the polytope F.4D.0043 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & -1 & 1 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 0 & 1 & 1 & 1 & -4 & 1 & -2 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\ 0 & -2 & 1 & 1 & 1 & 1 & -4 & -2 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(1,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{15} \to S^{20} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 13 line bundles

$$\{(0,0,0),(0,-1,-1),(0,0,-1),(-1,0,0),(-1,-1,0),(-1,0,-1),(0,-1,-2),(-1,-1,-1),\ldots$$
$$\ldots(0,0,-2),(0,-1,-3),(-1,-1,-2),(-1,0,-2),(-1,-1,-3)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,0)$ and $\mathcal{O}(0,0,-3)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0),\mathcal{O}(0,0,-3)) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,0,-3),\mathcal{O}(-1,-1,0)) = \begin{cases} \mathbb{C}^{10} & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.45. **Polytope F.4D.0044.** Let P denote the polytope F.4D.0044 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 1 & -1 & -1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & -3 & 1 & -3 & 1 & -2 & 1 & -2 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & -1 & -1 & -1 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 0 & -1 & -1 & 1 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & -3 & -3 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,1,0),(0,1,1,0),(0,0,0,-1),(1,-1,-1,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & -1 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 16 line bundles

$$\{(-1,-1,-2,-1),(-1,-1,-1,-2),(-1,0,-1,-2),(0,-1,-1,-2),(-1,-1,-1,-1),(-1,-1,-2,0),\dots$$
$$\dots(0,0,0,-2),(0,-1,-1,-1),(-1,-1,-1,0),(-1,-1,-2,1),(-1,0,-1,-1),(-1,-1,-1,1),(0,-1,-1,0),\dots$$
$$\dots(0,0,0,-1),(-1,0,-1,0),(0,0,0,0)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,-1,-1)$ and $\mathcal{O}(-1,-1,-1,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,-1,1),\mathcal{O}(-1,-1,-1,-2)) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,-1,-2),\mathcal{O}(-1,-1,-1,1))) = \begin{cases} \mathbb{C}^{10} & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.46. **Polytope F.4D.0045.** Let P denote the polytope F.4D.0045 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & -2 & 1 & 1 & 0 & 1 & 1 & 1 & -3 & 1 & -3 & 1 \\ -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\ -2 & 1 & 1 & 0 & 1 & 1 & -3 & -3 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to X, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,1),(1,0,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 13 line bundles

$$\{(0,0,0),(0,0,-1),(0,-1,-1),(-1,-1,-1),(-1,-1,0),(0,-1,-2),(0,0,-2),(-1,-2,-1),$$
$$(-1,-1,-2),(-1,-2,-2),(-1,-1,-3),(0,-1,-3),(-1,-2,-3)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,0)$ and $\mathcal{O}(-1,-1,-3)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,0),\mathcal{O}(-1,-1,-3)) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-1,-3),\mathcal{O}(-1,-1,0)) = \begin{cases} \mathbb{C}^{10} & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.47. **Polytope F.4D.0046.** Let P denote the polytope F.4D.0046 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & -3 & 1 & -3 & 1 \\ 1 & 1 & 1 & -2 & 1 & -2 & -2 & 1 & 1 \\ 0 & 1 & 1 & -3 & -3 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,1),(1,0,1,1)\}$$

We use as presentation of the class group of X

$$\mathbb{Z}^6 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^2$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 10 line bundles

$$\{(-2,-3),(-1,-3),(-2,-2),(-2,-1),(0,-2),(-1,-2),(-1,0),(-1,-1),(0,-1),(0,0)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,0)$ and $\mathcal{O}(-1,-3)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,0)),\mathcal{O}(-1,-3) = \begin{cases} \mathbb{C} & \text{in degree 2} \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,-3),\mathcal{O}(-1,0)) = \begin{cases} \mathbb{C}^{10} & \text{in degree 0} \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.48. **Polytope F.4D.0047.** Let P denote the polytope F.4D.0047 in the polymake database with half-space representation

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & -1 & 1 & 0 & 1 & -1 & 1 & -3 & 1 & -1 & 1 & -3 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 & -3 & -3 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(0,-1,0,1),(1,-1,0,1),(0,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{25} \to S^{27} \to S^{11} \to S^1 \to 0.$$

The collection of 20 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\{(-1,-1,-1,-2,-2),(-1,-1,-1,-1,-2),(-1,-1,-1,-2,-1),(-1,-1,-1,-1,-1),(-1,-1,0,-1,-1),$$
$$(-1,0,0,-1,-1),(0,-1,-1,-2,-2),(-1,-1,0,0,-1),(-1,0,0,0,-1),(-1,-1,0,-1,0),$$
$$(0,-1,-1,-2,-1),(-1,0,0,-1,0),(0,-1,-1,-1,-2),(-1,-1,0,0,0),(-1,0,0,0,0),$$
$$(0,-1,-1,-1,-1),(0,0,0,-1,-1),(0,0,0,-1,0),(0,0,0,0,-1),(0,0,0,0,0)\}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 2 & 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 5 & 3 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 3 & 5 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 3 & 5 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 3 & 6 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 5 & 3 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 6 & 3 & 2 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
13 & 8 & 8 & 5 & 4 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
13 & 8 & 8 & 5 & 0 & 4 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
17 & 10 & 10 & 6 & 4 & 4 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
18 & 10 & 10 & 6 & 5 & 5 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
26 & 14 & 18 & 10 & 8 & 8 & 6 & 3 & 5 & 5 & 4 & 3 & 3 & 2 & 2 & 2 & 2 & 1 & 0 & 0 \\
26 & 18 & 14 & 10 & 8 & 8 & 6 & 5 & 3 & 3 & 3 & 5 & 4 & 2 & 2 & 2 & 2 & 0 & 1 & 0 \\
38 & 26 & 26 & 18 & 13 & 13 & 9 & 8 & 8 & 8 & 6 & 8 & 6 & 5 & 5 & 4 & 4 & 2 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is

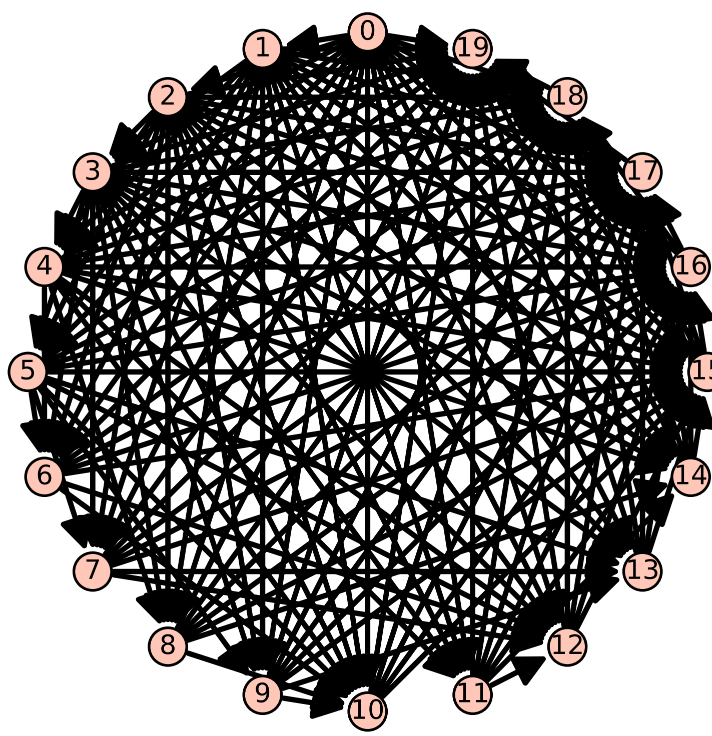

FIGURE 1. Directed graph showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

Direct calculation also shows that the toric variety X corresponding to F.4D.0047 satisfies Bondal's numerical criterion.

1.49. **Polytope F.4D.0048.** Let P denote the polytope F.4D.0048 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 0 & 1 & -2 & 1 & -2 & 1 & 0 & 1 & 0 & 1 & -1 & 1 & 0 & 1 & -1 & 1 & -2 & 1 & -1 & 1 & -2 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(0,0,0,1),(0,-1,0,1),(1,-1,0,1),(0,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^{10} \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^6$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{25} \to S^{27} \to S^{11} \to S^1 \to 0$$

The collection of 24 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\begin{aligned} &\{(-1,0,-1,-1,-2,-2),(-1,-1,-1,0,-1,-1),(-1,0,-1,0,-1,-1),(-1,0,-1,-1,-1,-2),\ldots \\ &\ldots(-1,0,-1,-1,-2,-1),(-1,-1,-1,0,0,-1),(-1,-1,0,0,-1,-1),(0,0,-1,-1,-2,-2),\ldots \\ &\ldots(-1,-1,-1,0,-1,0),(0,0,-1,-1,-2,-1),(0,0,-1,-1,-1,-2),(-1,-1,0,0,-1,0),\ldots \\ &\ldots(-1,-1,0,0,0,-1),(-1,0,-1,-1,-1,-1),(-1,-1,-1,0,0,0),(-1,0,-1,0,0,-1),\ldots \\ &\ldots(-1,0,-1,0,-1,0),(0,0,0,0,-1,-1),(0,0,-1,-1,-1,-1),(0,0,0,0,-1,0),\ldots \\ &\ldots(0,0,0,0,0,-1),(-1,0,-1,0,0,0),(-1,-1,0,0,0,0),(0,0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 2 & 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 1 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 5 & 3 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 3 & 5 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 5 & 3 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 3 & 5 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 2 & 3 & 6 & 2 & 2 & 2 & 1 & 0 & 2 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
13 & 0 & 8 & 8 & 5 & 4 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
13 & 0 & 8 & 8 & 5 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 2 & 6 & 3 & 2 & 2 & 2 & 0 & 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 4 & 4 & 4 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
9 & 4 & 4 & 4 & 2 & 1 & 1 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
14 & 6 & 6 & 9 & 4 & 2 & 2 & 4 & 3 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 0 & 0 & 0 \\
14 & 6 & 9 & 6 & 4 & 2 & 2 & 3 & 4 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 1 & 0 & 0 \\
17 & 4 & 10 & 10 & 6 & 4 & 4 & 2 & 2 & 4 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 2 & 1 & 0 & 0 & 0 & 1 & 0 \\
22 & 9 & 14 & 14 & 9 & 4 & 4 & 6 & 6 & 0 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 0 & 4 & 4 & 2 & 2 & 0 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

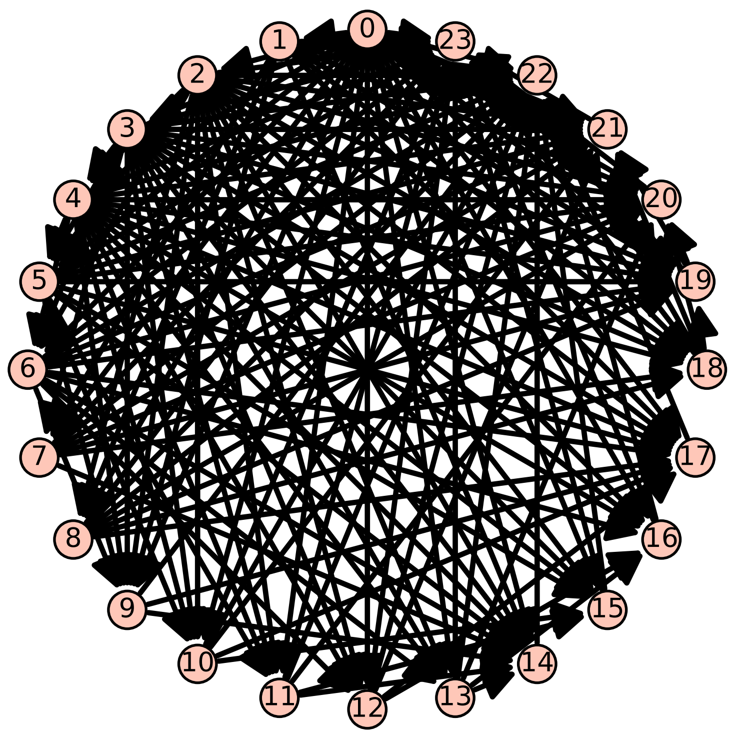

Figure 2. Directed graph showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $23 = |\mathcal{E}| - 1$.

1.50. **Polytope F.4D.0049.** Let P denote the polytope F.4D.0049 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & -2 & 1 & 0 & 1 & -2 & 1 & -2 & 1 & -1 & 1 & -2 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & -2 & -2 & 0 & 0 & -2 & -2 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(0,0,0,1),(0,-1,0,1),(1,-1,0,1),(0,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{25} \to S^{27} \to S^{11} \to S^1 \to 0.$$

The collection of 20 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\{(-1,-1,0,-1,-1),(-1,-1,0,0,-1),(0,-1,-1,-2,-2),(-1,0,0,-1,-1),(-1,-1,0,-1,0),$$
$$(0,-1,-1,-2,-1),(0,-1,-1,-1,-2),(-1,0,0,-1,0),(-1,0,0,0,-1),(-1,-1,0,0,0),$$
$$(0,-1,-1,-1,-1),(0,-1,0,-1,-1),(0,-1,0,0,-1),(0,-1,0,-1,0),(-1,0,0,0,0),$$
$$(0,0,0,-1,-1),(0,0,0,-1,0),(0,0,0,0,-1),(0,-1,0,0,0),(0,0,0,0,0)\}.$$

Here $Hom_{D^b_{Coh}(X)}(E_i,E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 4 & 4 & 4 & 2 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 4 & 4 & 4 & 2 & 1 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 5 & 4 & 4 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 5 & 3 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 3 & 5 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
14 & 6 & 9 & 6 & 4 & 2 & 3 & 4 & 2 & 2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
14 & 6 & 6 & 9 & 4 & 2 & 4 & 3 & 2 & 2 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
16 & 8 & 6 & 10 & 4 & 4 & 5 & 3 & 2 & 2 & 2 & 0 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
13 & 0 & 8 & 8 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
22 & 9 & 14 & 14 & 9 & 4 & 6 & 6 & 4 & 4 & 0 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 0 & 0 \\
16 & 8 & 10 & 6 & 4 & 4 & 3 & 5 & 2 & 2 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
26 & 13 & 16 & 16 & 10 & 8 & 8 & 8 & 5 & 4 & 4 & 4 & 4 & 2 & 2 & 2 & 2 & 1 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

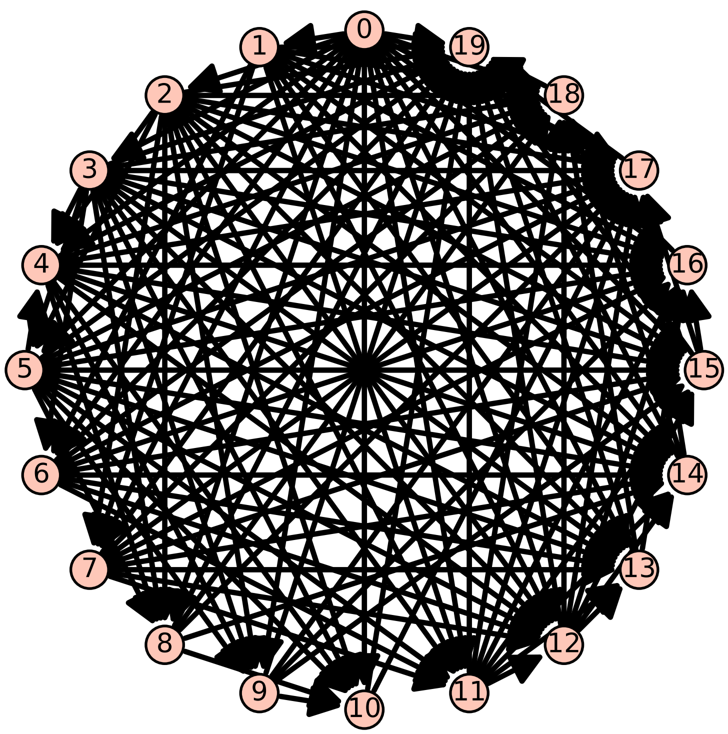

FIGURE 3. Directed graph showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.51. **Polytope F.4D.0050.** Let P denote the polytope F.4D.0050 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of:

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & -2 & -2 & 1 & 1 & -2 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{25} \to S^{27} \to S^{11} \to S^1 \to 0$$

16 line bundles which give exceptional collection:

$$\begin{aligned} \mathcal{E} = \{&(-1,-2,-2,-2),(-1,-2,-1,-2),(-1,-2,-2,-1),(-1,-1,-2,-2),(-1,-2,-1,-1),(-1,-1,-2,-1),\ldots \\ &\ldots(-1,-1,-1,-2),(-1,-1,-1,-1),(0,-1,-1,-1),(0,-1,0,-1),(0,-1,-1,0),(0,0,-1,-1),(0,-1,0,0),\ldots \\ &\ldots(0,0,-1,0),(0,0,0,-1),(0,0,0,0)\}. \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 4 & 4 & 4 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 4 & 4 & 4 & 2 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 14 & 9 & 6 & 6 & 4 & 3 & 4 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 14 & 6 & 9 & 6 & 4 & 4 & 3 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 14 & 6 & 6 & 9 & 3 & 4 & 4 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 22 & 14 & 14 & 9 & 9 & 6 & 6 & 4 & 4 & 2 & 2 & 0 & 1 & 0 & 0 & 0 \\ 22 & 9 & 14 & 14 & 6 & 9 & 6 & 4 & 4 & 0 & 2 & 2 & 0 & 1 & 0 & 0 \\ 22 & 14 & 9 & 14 & 6 & 6 & 9 & 4 & 4 & 2 & 0 & 2 & 0 & 0 & 1 & 0 \\ 35 & 22 & 22 & 22 & 14 & 14 & 14 & 9 & 8 & 4 & 4 & 4 & 2 & 2 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

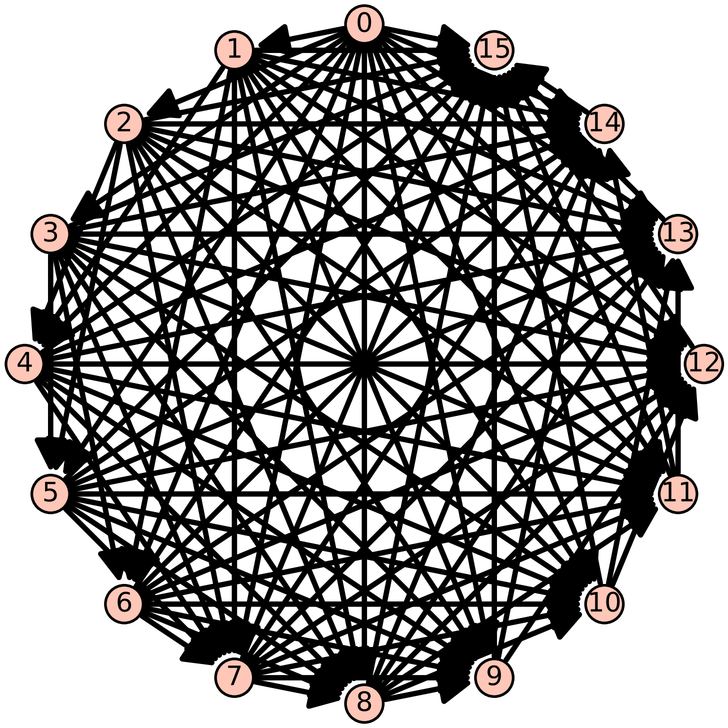

FIGURE 4. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.52. **Polytope F.4D.0051.** Let P denote the polytope F.4D.0051 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 0 & 1 & 0 & 1 & -3 & 1 & -1 & 1 & -1 & 1 & -3 & 1 & 0 & 1 & 0 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -2 & -2 & 0 & 0 & 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -3 & -3 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0), (0,-1,0,0), (0,1,0,0), (0,0,-1,0), (1,0,-1,0), (0,-1,1,0), (0,0,0,-1), (0,0,-1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} -1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & -1 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{25} \to S^{27} \to S^{11} \to S^1 \to 0$$

The collection of 16 line bundles $\mathcal{E}$ which appear on the left-hand side and give a full strong exceptional collection are

$$\{(1,2,-3,0),(0,1,-2,-1),(1,2,-3,1),(1,1,-2,0),(1,2,-2,0),(1,2,-2,1),(0,1,-2,0),(0,1,-1,-1),\ldots$$
$$\ldots(1,1,-2,1),(0,0,-1,-1),(1,1,-1,0),(0,0,0,-1),(1,1,-1,1),(0,1,-1,0),(0,0,-1,0),(0,0,0,0)\}$$

Here $Hom_{D^b_{Coh}(X)}(E_i,E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 3 & 5 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 3 & 6 & 2 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 5 & 3 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 6 & 3 & 2 & 2 & 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
13 & 8 & 8 & 5 & 4 & 0 & 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
17 & 10 & 10 & 6 & 4 & 4 & 2 & 2 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0\\
19 & 10 & 10 & 6 & 6 & 5 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0\\
28 & 19 & 14 & 10 & 10 & 8 & 3 & 3 & 6 & 5 & 2 & 2 & 2 & 1 & 0 & 0\\
28 & 14 & 19 & 10 & 10 & 8 & 6 & 5 & 3 & 3 & 2 & 2 & 2 & 0 & 1 & 0\\
42 & 28 & 28 & 19 & 17 & 13 & 10 & 8 & 10 & 8 & 6 & 5 & 4 & 2 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

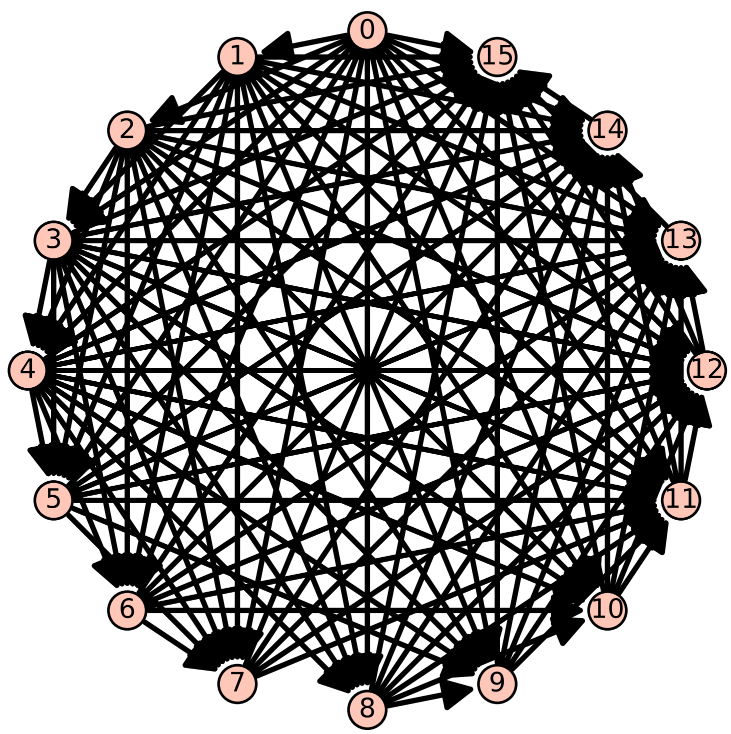

FIGURE 5. Quiver showing nonzero $Hom^0(E_i,E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}|-1$.

1.53. **Polytope F.4D.0052.** Let P denote the polytope F.4D.0052 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 0 & 1 & -2 & 1 & -2 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & -2 & 1 & -1 & 1 & -2 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(0,-1,0,1),(1,-1,0,1),(0,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^{10} \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^{6}.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{25} \to S^{27} \to S^{11} \to S^1 \to 0$$

The collection of 20 line bundles $\mathcal{E}$ which appear on the left-hand side and yield a full strong exceptional collection of line bundles are

$$\{(-1,0,-1,-2,-2),(-1,0,0,-1,-1),(-1,0,-1,-1,-2),(-1,0,-1,-2,-1),(-1,-1,0,-1,-1),(0,0,-1,-2,-2),\dots$$
$$\dots(0,0,-1,-2,-1),(0,0,-1,-1,-2),(-1,-1,0,-1,0),(-1,-1,0,0,-1),(-1,0,-1,-1,-1),(-1,0,0,0,-1),\dots$$
$$\dots(-1,0,0,-1,0),(0,0,0,-1,-1),(0,0,-1,-1,-1),(0,0,0,-1,0),(0,0,0,0,-1),(-1,0,0,0,0),(-1,-1,0,0,0),(0,0,0,0,0)\}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 1 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 0 & 5 & 3 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 2 & 6 & 3 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 0 & 3 & 5 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 2 & 3 & 6 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
13 & 0 & 8 & 8 & 5 & 4 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 4 & 4 & 4 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
10 & 5 & 4 & 4 & 2 & 2 & 1 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0\\
17 & 4 & 10 & 10 & 6 & 4 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 0 & 1 & 0 & 0 & 0\\
16 & 8 & 6 & 10 & 4 & 4 & 2 & 5 & 3 & 0 & 0 & 2 & 1 & 0 & 2 & 2 & 0 & 1 & 0 & 0\\
16 & 8 & 10 & 6 & 4 & 4 & 2 & 3 & 5 & 2 & 1 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 1 & 0\\
26 & 13 & 16 & 16 & 10 & 8 & 4 & 8 & 8 & 4 & 2 & 4 & 2 & 2 & 5 & 4 & 1 & 2 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

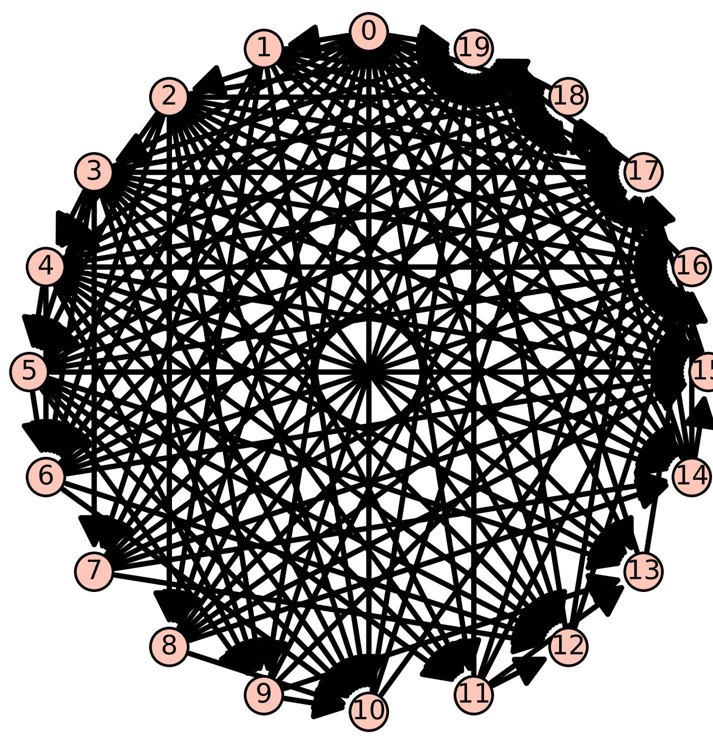

FIGURE 6. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.54. **Polytope F.4D.0053.** Let P denote the polytope F.4D.0053 in the polymake databse, with half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & -1 & 0 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & 1 & -3 & 1 & -3 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,0,-1,0),(-1,0,1,0),(0,0,0,-1),(0,0,0,1),(-1,0,0,1),(-1,1,0,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

The collection of 20 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-1,-1,-2),(-1,0,-1,-1,-2),(-1,-1,-1,-1,-1),(-1,-1,-1,0,-1),(-1,-1,0,0,-1),\ldots \\ &\ldots(-1,-1,-1,0,0),(-1,0,-1,-1,-1),(-1,-1,0,0,0),(-1,0,-1,0,-1),(0,-1,-1,-1,-2),\ldots \\ &\ldots(0,0,-1,-1,-2),(-1,0,0,0,-1),(-1,0,-1,0,0),(0,-1,-1,-1,-1),(0,0,-1,-1,-1),\ldots \\ &\ldots(-1,0,0,0,0),(0,-1,0,0,-1),(0,-1,0,0,0),(0,0,0,0,-1),(0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 2 & 3 & 1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
7 & 2 & 3 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 5 & 3 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 3 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 6 & 5 & 3 & 0 & 4 & 2 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
12 & 7 & 5 & 3 & 2 & 4 & 2 & 2 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 3 & 5 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
13 & 7 & 5 & 3 & 3 & 5 & 2 & 2 & 1 & 2 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
11 & 3 & 7 & 2 & 2 & 3 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
13 & 8 & 8 & 5 & 4 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
19 & 11 & 12 & 7 & 4 & 6 & 4 & 4 & 2 & 3 & 2 & 2 & 2 & 0 & 2 & 1 & 1 & 0 & 0 & 0\\
21 & 11 & 13 & 7 & 6 & 8 & 5 & 4 & 2 & 3 & 2 & 2 & 3 & 2 & 2 & 1 & 1 & 1 & 0 & 0\\
19 & 13 & 7 & 5 & 5 & 8 & 3 & 3 & 3 & 5 & 2 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0\\
31 & 21 & 19 & 13 & 10 & 13 & 8 & 6 & 6 & 8 & 5 & 4 & 5 & 4 & 3 & 3 & 2 & 2 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

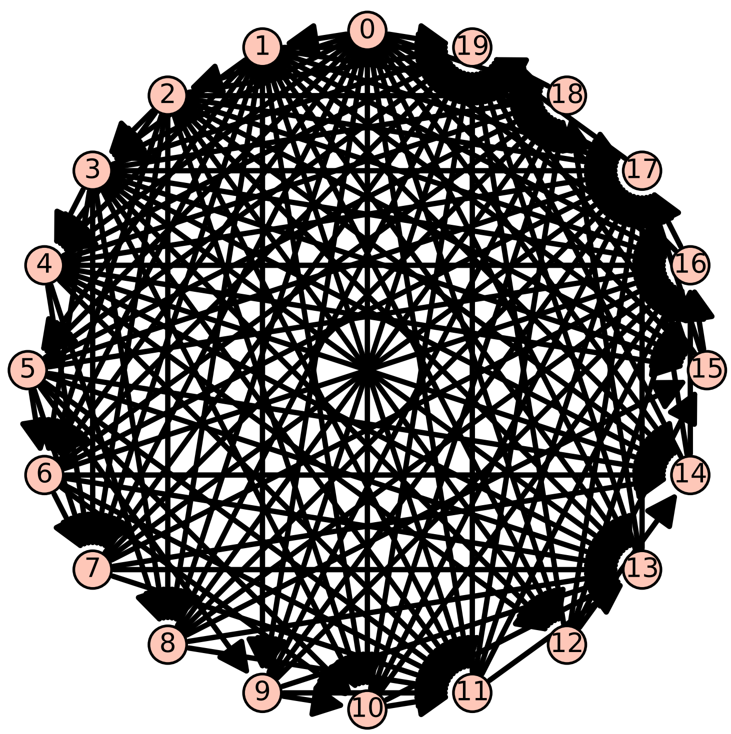

FIGURE 7. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.55. **Polytope F.4D.0054.** Let P denote the polytope F.4D.0054 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 0 & 0 & 0 & -1 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 1 & 0 & -2 & 1 & 1 & -1 & 1 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & 1 & -2 & 1 & -2 & -1 & 1 & 1 \\ 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,0,-1,0),(-1,0,1,0),(0,0,0,-1),(1,0,0,-1),(0,0,0,1),(-1,0,0,1),(-1,1,0,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^{10} \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^{6}$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

The collection of 24 line bundles $\mathcal{E}$ which appear on the left-hand side and give a full strong exceptional collection of line bundles are

$$\{(-1,-1,-1,-1,0,-1),(-1,-1,0,-1,-1,-2),(-1,-1,0,-1,0,-1),(0,-1,0,-1,-1,-2),(-1,-1,-1,0,0,-1),$$
$$(-1,-1,0,-1,-1,-1),(-1,0,-1,-1,0,-1),(-1,0,0,-1,-1,-2),(-1,-1,-1,-1,0,0),(-1,0,0,-1,0,-1),$$
$$(0,-1,0,-1,-1,-1),(0,-1,0,0,0,-1),(-1,-1,-1,0,0,0),(-1,0,0,-1,-1,-1),(-1,0,-1,-1,0,0),$$
$$(-1,0,-1,0,0,-1),(0,0,0,-1,-1,-2),(-1,-1,0,-1,0,0),(0,-1,0,0,0,0),(0,0,0,-1,-1,-1),$$
$$(-1,0,0,-1,0,0),(0,0,0,0,0,-1),(-1,0,-1,0,0,0),(0,0,0,0,0,0)\}$$

Here $Hom_{D^b_{Coh}(X)}(E_i,E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 3 & 2 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 3 & 1 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 3 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 2 & 3 & 1 & 1 & 1 & 2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 3 & 5 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 5 & 3 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 3 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 3 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 6 & 5 & 3 & 0 & 0 & 4 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 5 & 6 & 3 & 0 & 4 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
11 & 7 & 3 & 2 & 2 & 3 & 2 & 2 & 0 & 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
11 & 3 & 7 & 2 & 2 & 2 & 3 & 0 & 2 & 0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
12 & 7 & 5 & 3 & 2 & 2 & 4 & 1 & 2 & 0 & 2 & 0 & 1 & 2 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
12 & 5 & 7 & 3 & 2 & 4 & 2 & 2 & 1 & 2 & 0 & 1 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
13 & 8 & 8 & 5 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
19 & 12 & 11 & 7 & 4 & 6 & 4 & 4 & 2 & 4 & 0 & 2 & 2 & 2 & 3 & 1 & 2 & 2 & 0 & 0 & 2 & 1 & 1 & 0 \\
19 & 11 & 12 & 7 & 4 & 4 & 6 & 2 & 4 & 0 & 4 & 2 & 2 & 3 & 2 & 2 & 1 & 0 & 2 & 2 & 0 & 1 & 0 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

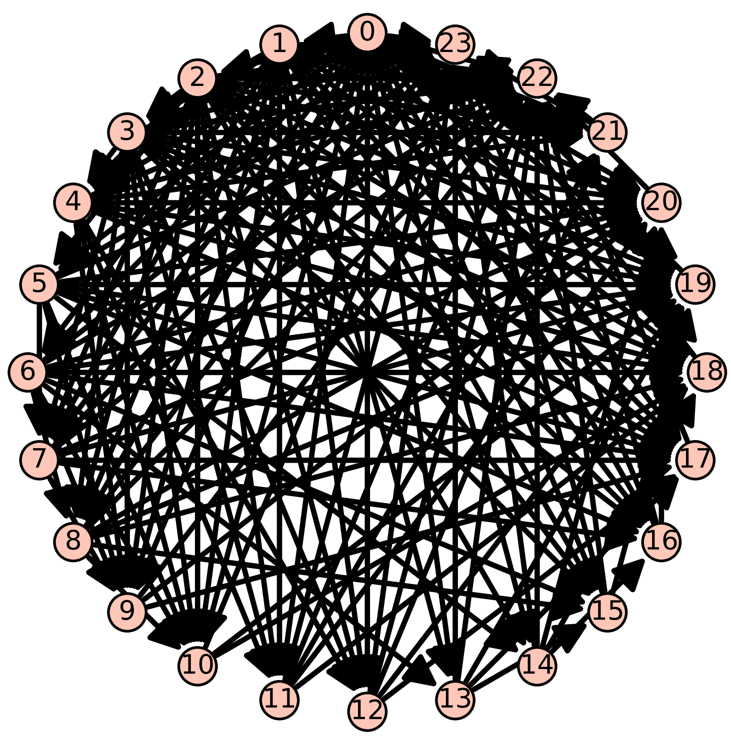

FIGURE 8. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $23 = |\mathcal{E}| - 1$.

1.56. **Polytope F.4D.0055.** Let P denote the polytope F.4D.0055 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -3 & -3 & 1 \\ 0 & -1 & 1 & 1 & 0 & -1 & 1 & 1 & 0 & -2 & 1 & 1 & 0 & -2 & 1 & 1 & 0 & -4 & 1 & 1 \\ -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(-1,1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(0,0,0,1),(0,0,1,1),(1,0,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

The collection of 20 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\{(-1,-1,-1,-2,-3),(-1,-1,-1,-1,-2),(-1,-1,0,-1,-2),(-1,0,-1,-1,-2),(0,-1,-1,-2,-3),$$
$$(-1,-1,-1,-2,-2),(-1,-1,-1,-1,-1),(0,-1,-1,-1,-2),(0,0,-1,-1,-2),(-1,0,0,0,-1),$$
$$(0,-1,-1,-2,-2),(-1,0,-1,-1,-1),(-1,-1,0,-1,-1),(0,-1,0,-1,-2),(0,0,0,0,-1),$$
$$(0,-1,-1,-1,-1),(0,-1,0,-1,-1),(-1,0,0,0,0),(0,0,-1,-1,-1),(0,0,0,0,0)\}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 3 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 3 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 3 & 3 & 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
7 & 4 & 2 & 2 & 1 & 1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 4 & 4 & 3 & 1 & 1 & 1 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
7 & 4 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 4 & 4 & 3 & 1 & 1 & 1 & 0 & 0 & 0 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
13 & 5 & 5 & 5 & 1 & 1 & 1 & 4 & 1 & 1 & 4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 6 & 5 & 5 & 3 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
14 & 9 & 7 & 5 & 4 & 3 & 2 & 0 & 0 & 0 & 5 & 3 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0\\
14 & 9 & 7 & 5 & 4 & 3 & 2 & 5 & 3 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0\\
21 & 13 & 9 & 9 & 5 & 5 & 2 & 7 & 4 & 2 & 7 & 4 & 2 & 2 & 1 & 1 & 1 & 1 & 0 & 0\\
25 & 13 & 13 & 12 & 5 & 5 & 5 & 9 & 4 & 4 & 9 & 4 & 4 & 3 & 1 & 1 & 1 & 1 & 1 & 0\\
37 & 25 & 21 & 19 & 13 & 12 & 9 & 14 & 9 & 7 & 14 & 9 & 7 & 5 & 5 & 4 & 4 & 3 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

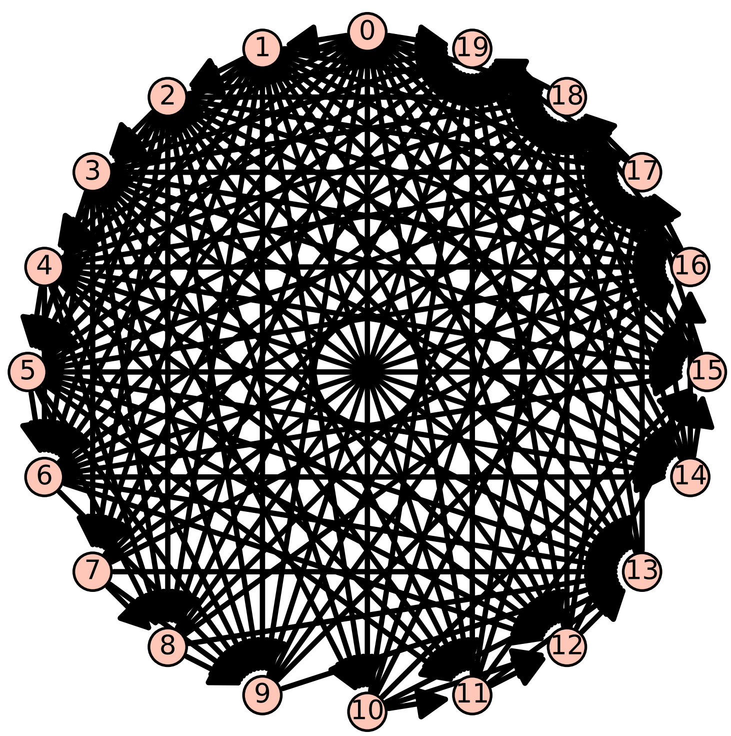

FIGURE 9. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.57. **Polytope F.4D.0056.** Let P denote the polytope F.4D.0056, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 0 & -2 & -2 & 1 & 1 & -2 & -2 & 1 \\ 0 & -2 & 1 & 1 & 0 & -2 & 1 & 1 & 0 & -1 & -3 & 1 & 1 & 0 & -3 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(-1,1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(0,0,1,-1),(0,0,0,1),(1,0,0,1),(0,0,-1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^{10} \xrightarrow{\begin{pmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^{6}$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

The collection of 24 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\begin{aligned}\mathcal{E} = \{&(-1,-1,0,-1,-2,-1),(-1,-1,-1,-1,-2,0),(-1,0,0,-1,-2,-1),(-1,-1,0,-1,-2,0),(0,-1,-1,-1,-2,0),\dots \\ &(-1,-1,-1,-1,-1,0),(-1,-1,-1,0,-1,0),(-1,-1,0,-1,-1,-1),(0,-1,0,-1,-2,-1),(0,-1,0,-1,-2,0),\dots \\ &0,0,0,-1,0),(0,0,0,-1,-2,-1),(0,-1,-1,-1,-1,0),(0,-1,0,-1,-1,-1),(-1,0,0,-1,-1,-1),\dots \\ &(-1,-1,0,-1,-1,0),(-1,-1,-1,0,0,0),(0,-1,-1,0,-1,0),(-1,0,0,0,0,0),(0,-1,0,-1,-1,0),\dots \\ &(0,0,0,0,-1,0),(0,-1,-1,0,0,0),(0,0,0,-1,-1,-1),(0,0,0,0,0,0)\}\end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&1&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&2&0&0&2&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
6&2&2&0&3&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
5&3&2&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
7&4&2&0&0&2&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
7&4&2&2&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
5&1&1&1&1&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
7&2&2&1&3&1&1&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
9&4&4&3&0&0&0&0&1&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0\\
9&4&4&0&0&3&0&0&1&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
9&5&2&2&2&2&1&0&1&1&1&2&0&0&0&1&0&0&0&0&0&0&0&0\\
12&5&5&3&3&3&1&1&1&1&1&3&0&1&1&1&1&0&0&0&0&0&0&0\\
10&6&4&0&5&0&3&2&2&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
12&7&4&2&5&2&3&2&2&1&1&0&2&0&0&0&0&1&1&0&0&0&0&0\\
15&7&7&3&6&3&3&3&2&1&1&0&3&1&1&0&0&1&1&1&0&0&0&0\\
14&9&7&0&0&5&0&0&4&3&0&0&0&0&2&0&0&0&0&0&1&0&0&0\\
14&9&7&5&0&0&0&0&4&0&3&0&0&2&0&0&0&0&0&0&0&1&0&0\\
23&15&12&5&9&5&6&5&7&3&3&0&5&2&2&0&0&3&3&2&1&1&1&0\\
19&12&9&5&5&5&3&2&5&3&3&5&0&2&2&3&2&1&0&0&1&1&0&1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

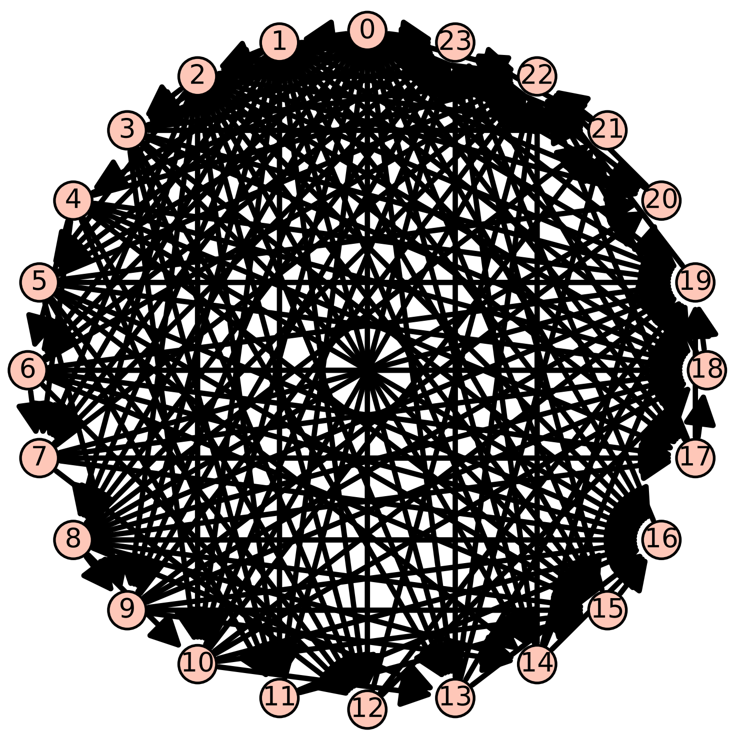

FIGURE 10. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $23 = |\mathcal{E}| - 1$.

1.58. **Polytope F.4D.0057.** Let P denote the polytope F.4D.0057 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & -1 & -1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & -2 & -2 & 1 & 1 & -2 & -2 & 1 & 1 \\ 0 & -2 & 1 & 1 & 0 & -1 & 1 & 1 & 0 & -1 & 1 & 1 & 0 & -3 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(-1,1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(0,0,0,1),(1,0,0,1),(0,0,-1,1)\}$$

We use as presentation of the class group:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

The collection of 20 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-1,-2,-1),(-1,-1,-1,-2,0),(-1,0,-1,-2,-1),(-1,-1,0,-1,0),(0,-1,-1,-2,-1),\ldots \\ &(-1,-1,-1,-1,-1),(0,0,-1,-2,-1),(0,-1,-1,-2,0),(-1,0,0,-1,0),(0,-1,-1,-1,-1),\ldots \\ &(-1,-1,0,0,0),(-1,-1,-1,-1,0),(-1,0,-1,-1,-1),(0,-1,0,-1,0),(0,-1,0,0,0),\ldots \\ &(0,-1,-1,-1,0),(-1,0,0,0,0),(0,0,0,-1,0),(0,0,-1,-1,-1),(0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 0 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 2 & 2 & 1 & 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 4 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 6 & 4 & 0 & 5 & 3 & 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
12 & 7 & 4 & 2 & 5 & 3 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 2 & 2 & 2 & 4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 4 & 4 & 3 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
15 & 7 & 7 & 3 & 6 & 3 & 3 & 3 & 2 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
14 & 8 & 4 & 4 & 7 & 4 & 2 & 2 & 2 & 2 & 1 & 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
18 & 8 & 8 & 6 & 9 & 4 & 4 & 3 & 2 & 2 & 1 & 1 & 3 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\
14 & 9 & 7 & 5 & 0 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\
23 & 15 & 12 & 5 & 9 & 6 & 5 & 5 & 7 & 3 & 3 & 3 & 0 & 2 & 2 & 0 & 0 & 1 & 1 & 0 \\
28 & 18 & 14 & 10 & 14 & 9 & 7 & 5 & 8 & 6 & 4 & 3 & 5 & 4 & 2 & 3 & 2 & 2 & 1 & 1
\end{pmatrix}
$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

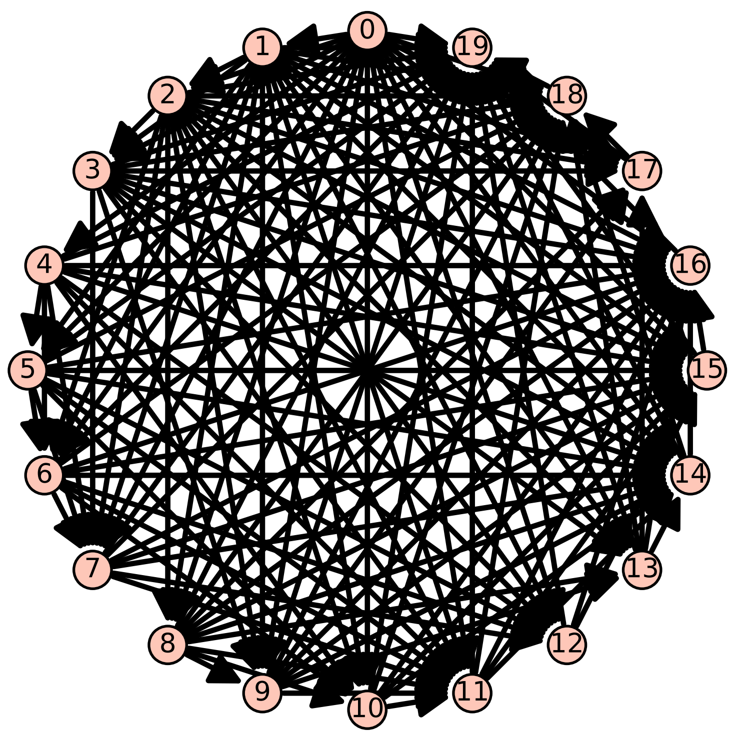

FIGURE 11. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.59. **Polytope F.4D.0058.** Let P denote the polytope F.4D.0058 in polymake, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 0 & 1 \\ 1 & -1 & -1 & 1 & -1 & 0 & 0 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 0 & 0 & -1 & -1 & 1 & 1 \\ 0 & -2 & 1 & 1 & -2 & -1 & 1 & 1 & 0 & 1 & 0 & 0 & -2 & -2 & -1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -2 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays:

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(1,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,-1,1,0),(0,0,0,-1),(-1,0,0,1)\}$$

We use as presentation of the class group:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0.$$

The collection of 20 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\begin{aligned}\mathcal{E} = \{&(-1,-1,-1,-1,-1),(-1,-1,-1,0,-1),(-1,-1,0,-1,-1),(0,0,-1,-1,-1),(-1,0,-1,-1,-1),\dots\\ &(-1,-1,-1,-1,0),(-1,0,-1,0,-1),(-1,0,-1,-1,0),(-1,-1,0,0,-1),(0,0,-1,-1,0),\dots\\ &(-1,-1,-1,0,0),(0,0,0,-1,-1),(-1,-1,0,-1,0),(0,0,-1,0,-1),(0,0,0,-1,0),\dots\\ &(-1,-1,0,0,0),(0,0,0,0,-1),(0,0,-1,0,0),(-1,0,-1,0,0),(0,0,0,0,0)\}\end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 3 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 3 & 1 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 3 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 3 & 3 & 3 & 3 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
11 & 7 & 2 & 3 & 3 & 2 & 2 & 2 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 5 & 0 & 6 & 4 & 3 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
12 & 5 & 2 & 7 & 4 & 3 & 2 & 2 & 1 & 0 & 0 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
10 & 6 & 4 & 5 & 0 & 3 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
15 & 9 & 6 & 5 & 5 & 3 & 3 & 2 & 2 & 2 & 3 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
19 & 12 & 4 & 11 & 6 & 7 & 4 & 4 & 2 & 0 & 2 & 3 & 2 & 2 & 2 & 1 & 0 & 1 & 0 & 0 \\
15 & 5 & 5 & 9 & 6 & 3 & 2 & 2 & 3 & 2 & 0 & 3 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\
25 & 15 & 10 & 15 & 10 & 9 & 6 & 4 & 6 & 4 & 5 & 5 & 2 & 3 & 2 & 3 & 2 & 1 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

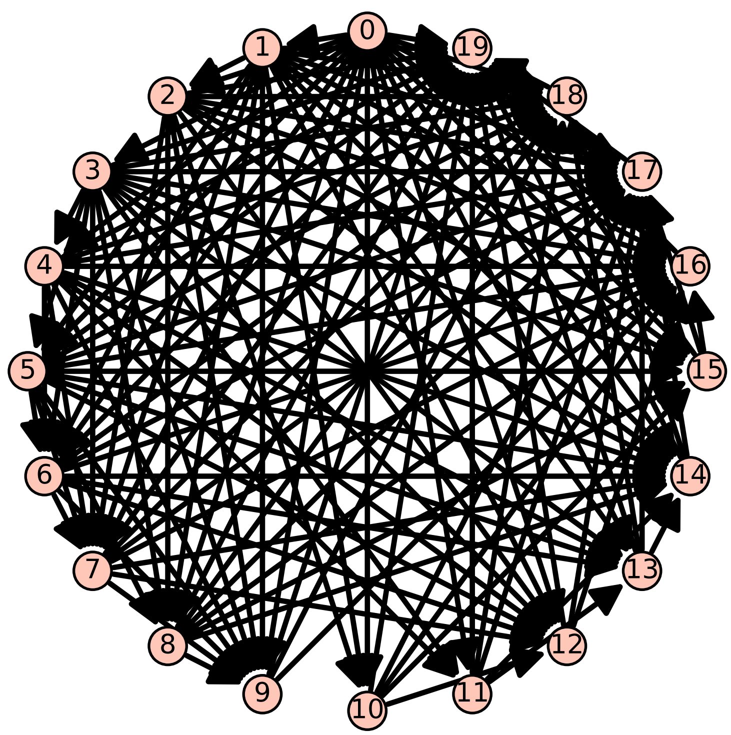

FIGURE 12. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.60. **Polytope F.4D.0059.** Let P denote the polytope F.4D.0059 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & -2 & -2 & -2 & 1 & -2 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & -2 & 1 & -2 & 1 & -2 & -2 & 1 & 1 \\ 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 & 0 & 0 & -3 & -3 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays:

$$\{((-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(-1,0,1,0),(0,0,0,-1),(0,0,0,1),(1,0,0,1),(0,1,0,1)\}$$

We use as presentation of the class group:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

The collection of 16 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\begin{aligned}\mathcal{E} = \{&(-1,-1,-2,-2),(-1,-1,-2,-1),(-1,0,-1,-1),(0,-1,-2,-2),(-1,-1,-1,-2),(-1,0,-1,0),\dots \\ &(0,-1,-2,-1),(0,-1,-1,-2),(-1,0,0,-1),(-1,-1,-1,-1),(0,0,-1,-1),(0,0,0,-1),\dots \\ &(-1,0,0,0),(0,0,-1,0),(0,-1,-1,-1),(0,0,0,0)\}\end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i,E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 2 & 2 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 2 & 2 & 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 3 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 6 & 4 & 5 & 3 & 2 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 12 & 7 & 4 & 5 & 3 & 2 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 11 & 3 & 3 & 7 & 2 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 15 & 7 & 7 & 6 & 3 & 3 & 3 & 2 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 19 & 11 & 6 & 12 & 7 & 4 & 4 & 3 & 2 & 2 & 2 & 0 & 1 & 0 & 0 & 0 \\ 24 & 11 & 11 & 15 & 7 & 7 & 6 & 3 & 2 & 2 & 3 & 2 & 1 & 1 & 0 & 0 \\ 23 & 15 & 12 & 9 & 6 & 5 & 5 & 7 & 3 & 3 & 0 & 2 & 0 & 0 & 1 & 0 \\ 37 & 24 & 19 & 23 & 15 & 12 & 10 & 11 & 7 & 6 & 5 & 4 & 3 & 2 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

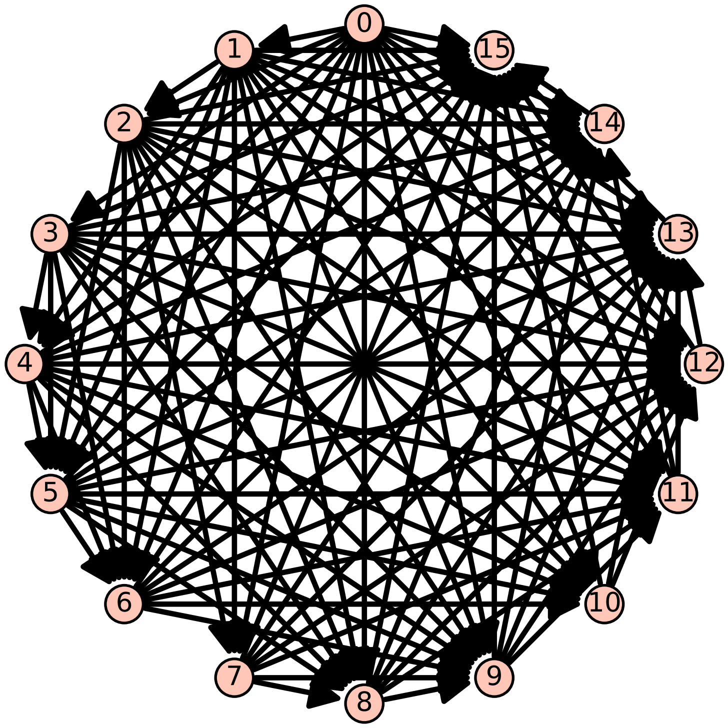

FIGURE 13. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.61. **Polytope F.4D.0060.** Let P denote the polytope F.4D.0060 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} -1 & 1 & -1 & 1 & 0 & -1 & 0 & -1 & 1 & 1 & 1 & 1 & 0 & -1 & 0 & -1 & -1 & 1 & -1 & 1 \\ -3 & -1 & 1 & 1 & 0 & -1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & -1 & 1 & 1 & -3 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -2 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,1,0),(-1,0,1,0),(-1,1,1,0),(0,0,0,-1),(0,0,-1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The collection $\mathcal{E}$ of 20 line bundles appearing on the left-hand side is

$\{(-1,0,-1,-1,-2),(-1,0,-1,0,-2),(-1,0,-1,-1,-1),(-1,-1,0,-1,-1),(0,0,-1,-1,-2),(-1,0,0,-1,-1),\ldots$
$\ldots(-1,-1,0,0,-1),(0,0,-1,0,-2),(-1,0,-1,0,-1),(-1,0,0,0,-1),(-1,-1,0,-1,0),(0,0,-1,-1,-1),\ldots$
$\ldots(-1,0,0,-1,0),(0,0,0,-1,-1),(-1,-1,0,0,0),(-1,0,0,0,0),(0,0,-1,0,-1),(0,0,0,0,-1),(0,0,0,-1,0),(0,0,0,0,0)\}$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 3 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 4 & 2 & 2 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 5 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 3 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
11 & 5 & 3 & 3 & 4 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 2 & 4 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 2 & 5 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 6 & 5 & 0 & 4 & 0 & 3 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
12 & 7 & 7 & 4 & 0 & 0 & 4 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
16 & 9 & 9 & 4 & 4 & 4 & 5 & 2 & 2 & 2 & 2 & 0 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\
17 & 11 & 5 & 5 & 7 & 3 & 3 & 4 & 3 & 2 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
19 & 9 & 11 & 6 & 7 & 4 & 5 & 2 & 2 & 2 & 4 & 2 & 3 & 2 & 1 & 1 & 1 & 0 & 1 & 0 \\
29 & 19 & 17 & 10 & 12 & 6 & 11 & 7 & 6 & 4 & 7 & 4 & 5 & 3 & 4 & 3 & 2 & 2 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

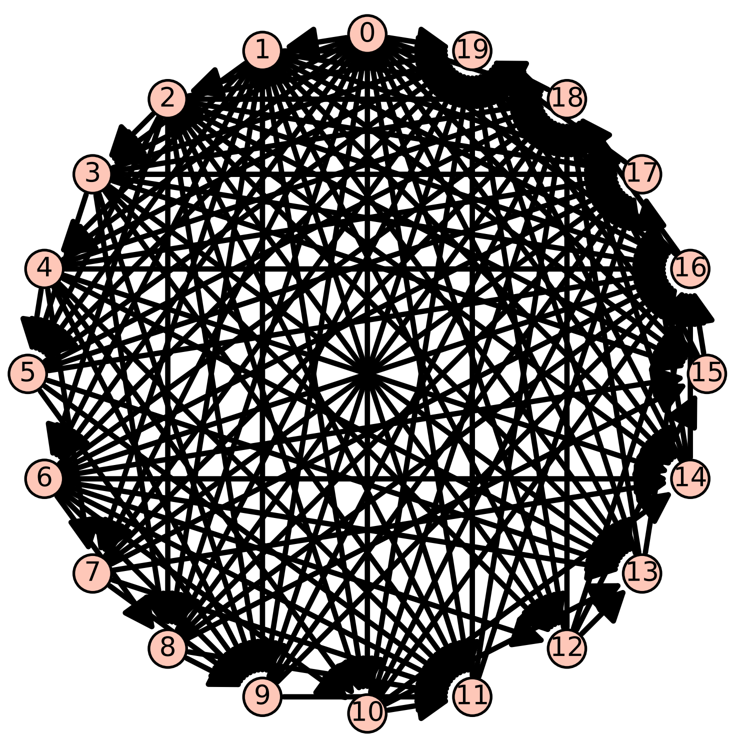

FIGURE 14. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.62. **Polytope F.4D.0061.** Let P denote the polytope F.4D.0061 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & -1 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & -1 & 0 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 \\ -2 & -1 & 1 & 1 & 0 & -1 & 1 & 1 & 0 & -2 & 1 & 1 & 0 & -2 & 1 & 1 & 0 & -1 & 1 & 1 & -2 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to X, with rays:

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,0,-1,0),(1,0,-1,0),(0,0,1,0),(-1,0,1,0),(-1,1,1,0),(0,0,0,-1),(0,0,-1,1)\}$$

We use as presentation of the class group:

$$\mathbb{Z}^{10} \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^{6}$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

The collection of 24 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\begin{aligned} \mathcal{E} = \{&(-1,0,-1,-1,-2,-1),(-1,1,-2,-1,-2,0),(0,0,-1,-1,-2,-1),(-1,0,-1,-1,-2,0),\ldots(-1,0,-1,-1,-1,-1),\ldots \\ &\ldots(-1,0,-1,0,-1,-1),(-1,1,-1,-1,-2,0),(-1,1,-2,-1,-2,1),(-1,1,-2,-1,-1,0),(-1,0,-1,0,0,-1),\ldots \\ &\ldots(-1,1,-1,-1,-2,1),(0,0,-1,-1,-1,-1),(0,0,-1,-1,-2,0),(-1,1,-2,-1,-1,1),(-1,0,-1,-1,-1,0),\ldots \\ &\ldots(-1,0,-1,0,-1,0),(0,0,0,0,-1,-1),(-1,1,-1,-1,-1,0),(0,0,0,0,-1,0),(-1,1,-1,-1,-1,1),\ldots \\ &\ldots(-1,0,-1,0,0,0),(0,0,0,0,0,-1),(0,0,-1,-1,-1,0),(0,0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&2&2&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
6&3&2&0&0&2&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
6&2&3&0&2&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
8&3&3&1&2&2&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
5&1&1&1&1&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
7&2&4&2&0&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
7&4&2&2&0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
5&0&3&0&0&2&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
5&3&0&0&2&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
9&5&2&2&2&2&1&1&0&0&2&0&1&0&1&1&0&0&0&0&0&0&0&0\\
9&2&5&2&2&2&1&0&1&0&2&1&0&1&0&0&1&0&0&0&0&0&0&0\\
10&5&6&0&0&4&3&2&0&0&0&0&0&2&0&0&0&1&0&0&0&0&0&0\\
10&6&5&0&4&0&3&0&2&0&0&0&0&0&2&0&0&0&1&0&0&0&0&0\\
13&8&5&2&4&3&3&2&2&2&0&0&1&0&2&0&0&0&1&1&0&0&0&0\\
13&5&8&2&3&4&3&2&2&2&0&1&0&2&0&0&0&1&0&0&1&0&0&0\\
12&7&7&4&0&0&4&0&0&0&0&2&2&0&0&0&0&0&0&0&0&1&0&0\\
16&9&9&4&4&4&5&2&2&0&4&2&2&2&2&2&2&1&1&0&0&1&1&0\\
21&13&13&4&6&6&8&4&4&4&0&2&2&3&3&0&0&2&2&2&2&1&0&1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

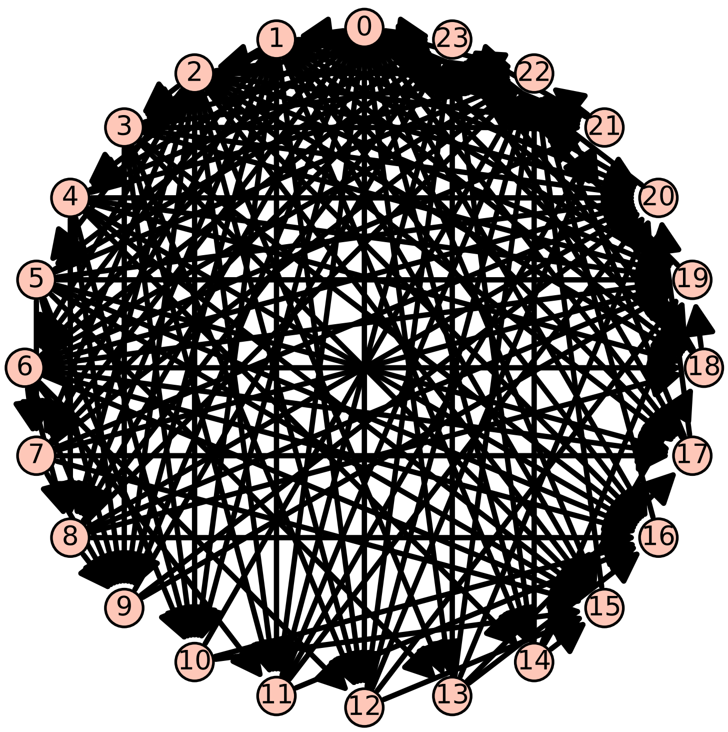

FIGURE 15. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $23 = |\mathcal{E}| - 1$.

1.63. **Polytope F.4D.0062.** Let P denote the polytope F.4D.0062 from the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & -1 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & -2 & 0 & -2 & 1 & 1 & 1 & 1 & 0 & -2 & 0 & -2 & 0 & 1 & 0 & 1 \\ -2 & -1 & 1 & 1 & 0 & -2 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & -2 & 1 & 1 & -2 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -2 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(1,0,-1,0),(0,0,1,0),(-1,0,1,0),(-1,1,1,0),(0,0,0,-1),(0,0,-1,1)\}$$

We use as presentation of the class group:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} -1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

The collection of 20 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\begin{aligned} \mathcal{E} = \{&(1,-2,-1,-2,0),(0,-1,-1,-2,-1),(0,-1,0,-1,-1),(1,-1,-1,-2,0),(1,-2,-1,-2,1),\dots \\ &\dots(1,-2,-1,-1,0),(0,-1,0,0,-1),(1,-1,-1,-2,1),(0,-1,-1,-1,-1),(0,-1,-1,-2,0),\dots \\ &\dots(1,-2,-1,-1,1),(0,-1,0,-1,0),(0,0,0,-1,-1),(1,-1,-1,-1,0),(0,0,0,-1,0),\dots \\ &\dots(1,-1,-1,-1,1),(0,-1,0,0,0),(0,0,0,0,-1),(0,-1,-1,-1,0),(0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 3 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 3 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 3 & 2 & 3 & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 3 & 3 & 3 & 3 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 5 & 0 & 6 & 4 & 3 & 2 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
13 & 5 & 3 & 8 & 4 & 3 & 2 & 2 & 2 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 6 & 4 & 5 & 0 & 3 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\
13 & 8 & 4 & 5 & 3 & 3 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 0 \\
15 & 9 & 6 & 5 & 5 & 3 & 3 & 2 & 2 & 2 & 0 & 0 & 0 & 3 & 1 & 1 & 1 & 0 & 0 & 0 \\
15 & 5 & 5 & 9 & 6 & 3 & 2 & 3 & 2 & 2 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
21 & 13 & 6 & 13 & 6 & 8 & 4 & 4 & 4 & 0 & 3 & 2 & 2 & 3 & 2 & 2 & 0 & 0 & 1 & 0 \\
25 & 15 & 10 & 15 & 10 & 9 & 6 & 6 & 4 & 4 & 5 & 3 & 2 & 5 & 3 & 2 & 2 & 2 & 1 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

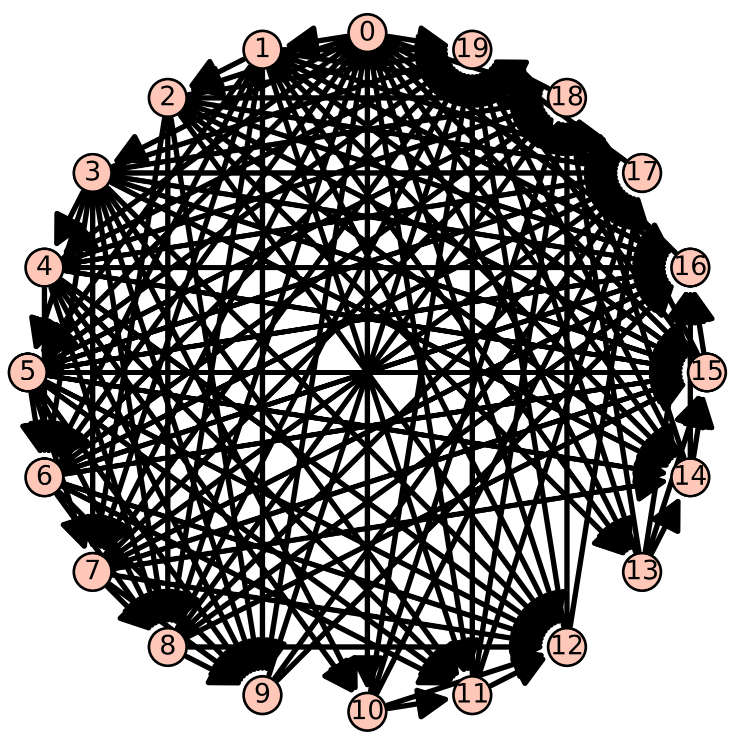

FIGURE 16. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.64. **Polytope F.4D.0063.** Let P denote the polytope F.4D.0063, with half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & -1 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} -1 & 1 & 1 & 0 & -1 & -1 & 1 & 1 & 0 & 1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 & -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & -2 & 1 & 0 & -1 & 1 & 1 & -2 & -2 & -1 & 1 & 1 \\ -1 & 1 & 0 & -1 & -1 & 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,1,0),(-1,0,1,0),(-1,1,1,0),(0,0,0,-1),(0,1,0,1)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & -1 & 1 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

There does not exist an ordering of the 24 line bundles $\mathcal{E}$

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-1,-2,0),(-1,-1,0,0,-2),(-1,-1,-1,-2,1),(-1,-1,0,-1,-1),(-1,0,0,-1,-1),\dots \\ &\dots(-1,-1,-1,-1,-1),(0,-1,-1,-2,0),(0,-1,-1,-1,-1),(-1,0,0,0,-2),(-1,0,0,-1,0),\dots \\ &\dots(-1,-1,-1,-1,1),(-1,-1,-1,-1,0),(0,-1,-1,-2,1),(0,0,0,-1,-1),(-1,-1,0,-1,0),\dots \\ &\dots(-1,-1,0,0,-1),(0,0,0,0,-1),(0,-1,-1,-1,1),(0,-1,-1,-1,0),(-1,-1,0,0,0),\dots \\ &\dots(-1,0,0,0,0),(0,0,0,-1,0),(-1,0,0,0,-1),(0,0,0,0,0)\} \end{aligned}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,0,0,0,0)$ and $\mathcal{O}(-1,-1,0,0,-2)$ with non-zero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,0,0,0,0)),\mathcal{O}(-1,-1,0,0,-2) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,-1,0,0,-2),\mathcal{O}(-1,0,0,0,0))) = \begin{cases} \mathbb{C}^3 & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.65. **Polytope F.4D.0064.** Let P denote the polytope F.4D.0064, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & -1 & 0 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 & -3 & -3 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(-1,0,0,1),(-1,0,1,1)\}$$

We use as presentation of the class group:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{13} \to S^{15} \to S^7 \to S^1 \to 0$$

The collection of 20 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-1,-1,-2),(-1,0,-1,-1,-2),(-1,-1,-1,0,-1),(-1,-1,0,0,-1),(0,-1,-1,-1,-2),\dots \\ &\dots(-1,-1,-1,-1,-1),(-1,-1,-1,0,0),(-1,-1,0,0,0),(0,-1,-1,-1,-1),(0,-1,0,0,-1),\dots \\ &\dots(-1,0,-1,0,-1),(-1,0,0,0,-1),(0,0,-1,-1,-2),(-1,0,-1,-1,-1),(-1,0,0,0,0),\dots \\ &\dots(0,0,-1,-1,-1),(0,-1,0,0,0),(-1,0,-1,0,0),(0,0,0,0,-1),(0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 3 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 3 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
7 & 4 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 4 & 3 & 3 & 2 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 2 & 2 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 2 & 0 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 2 & 2 & 2 & 2 & 4 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 6 & 4 & 0 & 0 & 5 & 2 & 0 & 0 & 0 & 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
10 & 6 & 0 & 4 & 0 & 5 & 0 & 2 & 0 & 0 & 3 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
14 & 8 & 4 & 4 & 4 & 7 & 2 & 2 & 2 & 0 & 4 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0\\
16 & 8 & 6 & 6 & 4 & 8 & 2 & 2 & 2 & 2 & 4 & 3 & 3 & 2 & 1 & 1 & 1 & 1 & 0 & 0\\
12 & 8 & 5 & 5 & 3 & 0 & 3 & 3 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
24 & 16 & 10 & 10 & 6 & 12 & 6 & 6 & 4 & 4 & 8 & 5 & 5 & 3 & 3 & 3 & 2 & 2 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

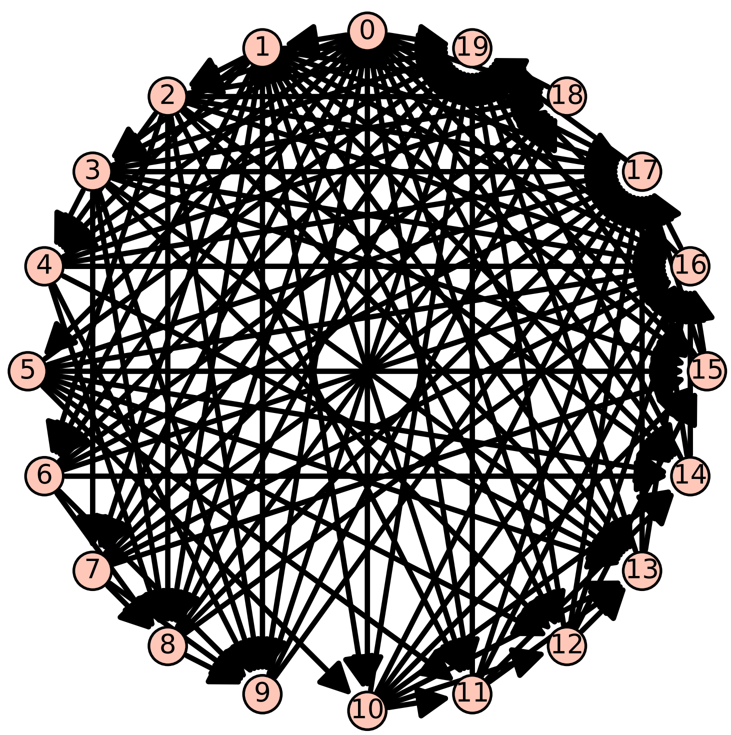

FIGURE 17. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.66. **Polytope F.4D.0065.** Let P denote the polytope F.4D.0065 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 0 & 0 & -1 & -1 & 0 & -1 & 0 & -1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & -1 & 0 & -1 & 0 & -1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(1,0,0,-1),(0,0,0,1),(-1,0,0,1),(-1,0,1,1)\}$$

We use as presentation of the class group:

$$\mathbb{Z}^{10} \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^{6}$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{13} \to S^{15} \to S^7 \to S^1 \to 0$$

The collection of 24 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-1,-1,0,-1),(-1,-1,0,-1,-1,-2),(-1,0,0,-1,-1,-2),(-1,-1,0,-1,0,-1),(-1,-1,-1,0,0,-1),\ldots \\ &\ldots(-1,-1,-1,-1,0,0),(-1,-1,0,-1,-1,-1),(0,-1,0,-1,-1,-2),(-1,0,-1,-1,0,-1),(-1,-1,0,-1,0,0),\ldots \\ &\ldots(-1,0,0,-1,0,-1),(-1,-1,-1,0,0,0),(0,-1,0,0,0,-1),(0,-1,0,-1,-1,-1),(-1,0,-1,0,0,-1),\ldots \\ &\ldots(0,0,0,-1,-1,-2),(-1,0,-1,-1,0,0),(-1,0,0,-1,-1,-1),(-1,0,-1,0,0,0),(0,0,0,0,0,-1),\ldots \\ &\ldots(-1,0,0,-1,0,0),(0,-1,0,0,0,0),(0,0,0,-1,-1,-1),(0,0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&2&0&0&2&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
5&2&1&0&2&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&1&1&0&1&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
5&3&2&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
5&3&0&0&0&2&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&2&0&2&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
6&2&2&3&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0\\
6&2&0&3&0&2&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0\\
4&0&0&2&2&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
7&4&2&0&2&2&1&0&2&1&1&0&0&0&0&1&0&0&0&0&0&0&0&0\\
10&6&4&5&0&0&0&0&0&2&0&3&2&0&0&0&1&0&0&0&0&0&0&0\\
8&4&0&4&4&0&2&0&0&0&0&2&0&0&2&0&0&1&0&0&0&0&0&0\\
10&4&2&5&4&2&2&2&0&0&0&2&1&1&2&0&0&1&1&0&0&0&0&0\\
8&2&2&4&2&2&0&0&2&0&0&1&1&1&1&0&0&0&0&1&0&0&0&0\\
10&6&0&5&0&4&0&0&0&0&2&3&0&2&0&0&0&0&0&0&1&0&0&0\\
8&5&2&0&3&2&2&2&0&1&1&0&0&0&0&0&0&0&0&0&0&1&0&0\\
16&10&4&8&6&4&4&4&0&2&2&5&2&2&3&0&1&2&2&0&1&2&1&0\\
14&8&4&7&4&4&2&0&4&2&2&4&2&2&2&2&1&1&0&2&1&0&0&1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

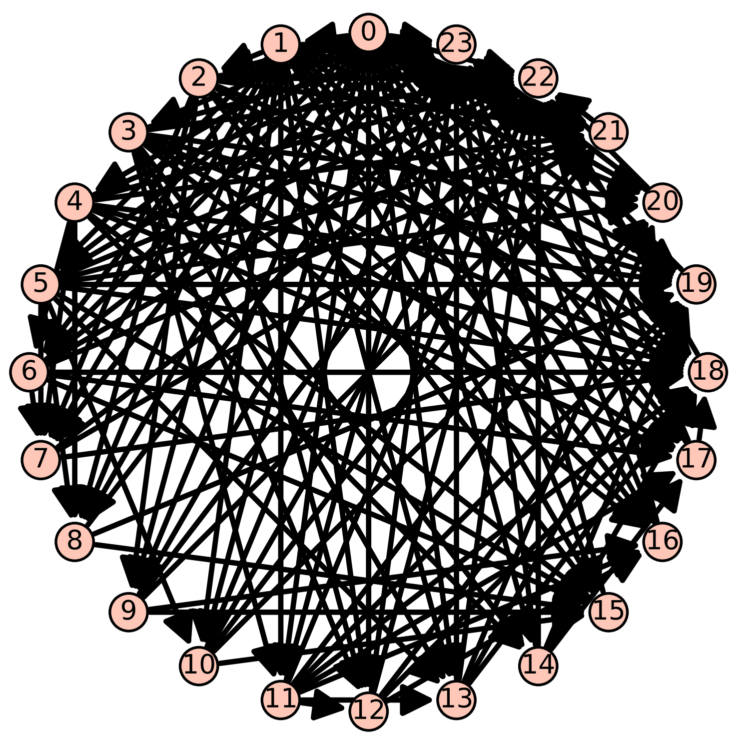

FIGURE 18. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $23 = |\mathcal{E}| - 1$.

1.67. **Polytope F.4D.0066.** Let P denote the polytope F.4D.0066 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 1 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 1 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 0 & 0 & -1 & -1 & 0 & -1 & 0 & -1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ -2 & 1 & 0 & 1 & 0 & -2 & 1 & 1 & -2 & 1 & -1 & 1 & -2 & -1 & 1 & 1 & 0 & 1 & -1 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(1,0,0,-1),(1,1,0,-1),(0,0,0,1),(-1,0,0,1),(-1,0,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^{10} \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^{6}$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{25} \to S^{27} \to S^{11} \to S^1 \to 0$$

The collection of 24 line bundles $\mathcal{E}$ which appear on the left-hand side are

$$\begin{aligned} \mathcal{E} = \{&(-1,0,-1,-1,-1,-2),(-1,-1,-2,-1,0,-1),(-1,0,-1,-1,0,-1),(-1,-1,-2,0,0,-1),(0,0,-1,-1,-1,-2),\ldots \\ &\ldots(-1,-1,-2,-1,0,0),(-1,0,0,-1,-1,-2),(-1,-1,-1,-1,0,-1),(-1,0,-1,-1,-1,-1),(-1,-1,-1,0,0,-1),\ldots \\ &\ldots(0,0,0,-1,-1,-2),(0,0,-1,-1,-1,-1),(-1,-1,-2,0,0,0),(-1,0,0,-1,-1,-1),(-1,-1,-1,-1,0,0),\ldots \\ &\ldots(0,0,-1,0,0,-1),(-1,0,-1,-1,0,0),(-1,0,0,-1,0,-1),(0,0,-1,0,0,0),(-1,0,0,-1,0,0),\ldots \\ &\ldots(0,0,0,0,0,-1),(0,0,0,-1,-1,-1),(-1,-1,-1,0,0,0),(0,0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 2 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 1 & 2 & 1 & 1 & 2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 2 & 4 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 2 & 4 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 4 & 2 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 4 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 4 & 4 & 0 & 0 & 4 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
11 & 4 & 6 & 2 & 2 & 4 & 2 & 2 & 2 & 0 & 2 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
11 & 6 & 4 & 2 & 2 & 4 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 6 & 2 & 2 & 2 & 3 & 2 & 0 & 1 & 0 & 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
10 & 2 & 6 & 2 & 2 & 3 & 0 & 2 & 1 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
12 & 7 & 7 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
12 & 7 & 7 & 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
18 & 10 & 11 & 4 & 4 & 6 & 3 & 4 & 6 & 4 & 0 & 2 & 2 & 2 & 2 & 2 & 0 & 2 & 0 & 2 & 1 & 1 & 1 & 0 \\
18 & 11 & 10 & 4 & 4 & 6 & 4 & 3 & 6 & 0 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 2 & 0 & 1 & 1 & 0 & 1
\end{pmatrix}
$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

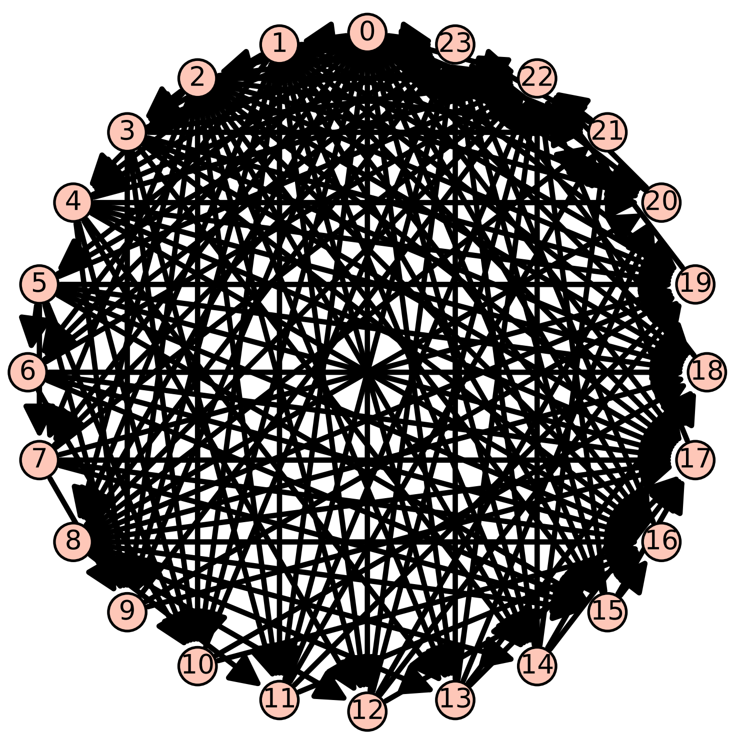

FIGURE 19. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $23 = |\mathcal{E}| - 1$.

1.68. **Polytope F.4D.0067.** Let P denote the polytope F.4D.0067 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & -2 & -2 & 0 & -2 & 0 & -2 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(1,0,0,-1),(0,0,0,1),(-1,0,0,1),(-1,0,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{13} \to S^{15} \to S^7 \to S^1 \to 0$$

20 line bundles appearing on left-hand side which do give an FSEC of line bundles are:

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-1,0,-1),(-1,0,-1,0,-1),(-1,-1,0,0,-1),(-1,-1,-1,0,0),(-1,0,-1,-1,-2),(0,-1,-1,0,-1),\dots \\ &\dots(-1,0,-1,0,0),(0,0,-1,0,-1),(-1,-1,0,0,0),(-1,0,0,0,-1),(-1,0,-1,-1,-1),(0,-1,0,0,-1),\dots \\ &\dots(0,0,-1,-1,-2),(0,-1,-1,0,0),(0,-1,0,0,0),(0,0,0,0,-1),(0,0,-1,0,0),(-1,0,0,0,0),\dots \\ &\dots(0,0,-1,-1,-1),(0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 3 & 2 & 2 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 0 & 3 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 3 & 5 & 2 & 2 & 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 0 & 2 & 2 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 5 & 6 & 4 & 3 & 2 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 4 & 4 & 0 & 2 & 0 & 4 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
10 & 4 & 4 & 2 & 2 & 2 & 5 & 0 & 0 & 0 & 2 & 1 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0\\
10 & 0 & 6 & 4 & 0 & 0 & 5 & 0 & 2 & 0 & 3 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
16 & 6 & 10 & 4 & 4 & 4 & 8 & 0 & 2 & 2 & 5 & 2 & 3 & 0 & 2 & 2 & 1 & 1 & 0 & 0\\
12 & 6 & 4 & 4 & 2 & 2 & 6 & 2 & 0 & 0 & 2 & 2 & 3 & 0 & 1 & 1 & 0 & 0 & 1 & 0\\
20 & 10 & 12 & 8 & 6 & 4 & 10 & 4 & 4 & 2 & 6 & 4 & 5 & 2 & 3 & 2 & 2 & 1 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

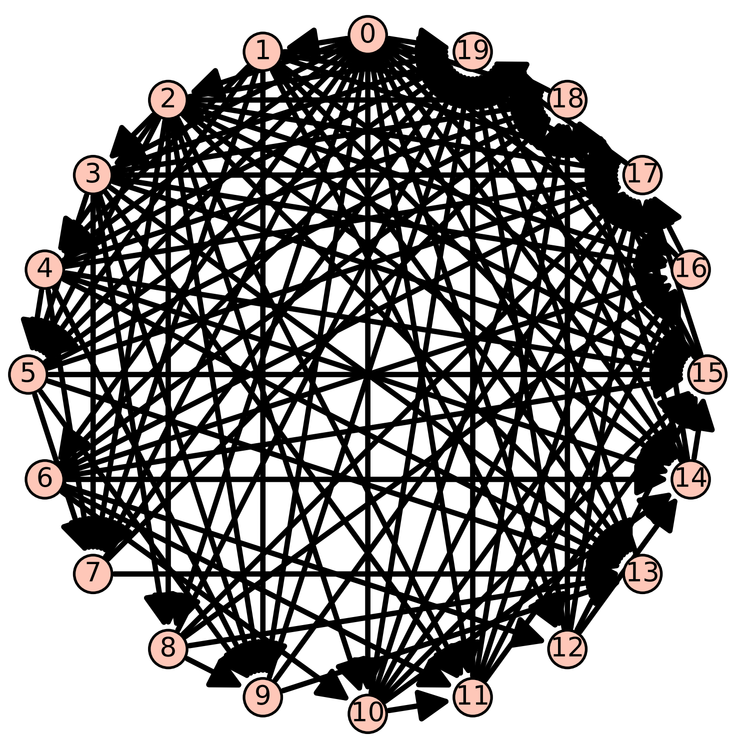

FIGURE 20. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.69. **Polytope F.4D.0068.** Let P denote the polytope F.4D.0068 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 1 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & -2 & -2 & -2 & 0 & 0 & -2 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ -2 & 1 & -2 & 1 & -2 & 1 & 0 & 1 & 0 & -2 & 1 & 1 & 0 & 1 & -1 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(1,0,0,-1),(1,1,0,-1),(0,0,0,1),(-1,0,0,1),(-1,0,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{25} \to S^{27} \to S^{11} \to S^1 \to 0$$

The 20 line bundles appearing on left-hand side which do give a full strong exceptional collection of line bundles are:

$$\begin{aligned} \mathcal{E} = \{&(-1,-2,-1,0,-1),(0,-1,-1,0,-1),(-1,-2,0,0,-1),(0,-1,-1,-1,-2),(-1,-2,-1,0,0),(-1,-1,-1,0,-1),\ldots \\ &\ldots(-1,-1,0,0,-1),(0,0,-1,-1,-2),(0,-1,-1,-1,-1),(-1,-2,0,0,0),(-1,-1,-1,0,0),(0,-1,0,0,-1),\ldots \\ &\ldots(0,-1,-1,0,0),(0,0,-1,0,-1),(0,-1,0,0,0),(0,0,-1,0,0),(0,0,0,0,-1),(0,0,-1,-1,-1),\ldots \\ &\ldots(-1,-1,0,0,0),(0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 1 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 4 & 2 & 2 & 1 & 2 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 0 & 4 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 4 & 4 & 4 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
11 & 4 & 6 & 4 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 0 & 2 & 4 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 3 & 2 & 6 & 2 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
14 & 7 & 4 & 8 & 4 & 4 & 2 & 2 & 2 & 2 & 0 & 1 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 0 \\
14 & 7 & 8 & 4 & 2 & 4 & 2 & 4 & 2 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
12 & 0 & 7 & 7 & 0 & 4 & 0 & 0 & 4 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\
18 & 6 & 10 & 11 & 4 & 4 & 4 & 3 & 6 & 0 & 2 & 2 & 2 & 2 & 2 & 0 & 0 & 1 & 1 & 0 \\
24 & 12 & 14 & 14 & 7 & 8 & 4 & 7 & 8 & 4 & 4 & 4 & 2 & 4 & 2 & 2 & 2 & 2 & 1 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

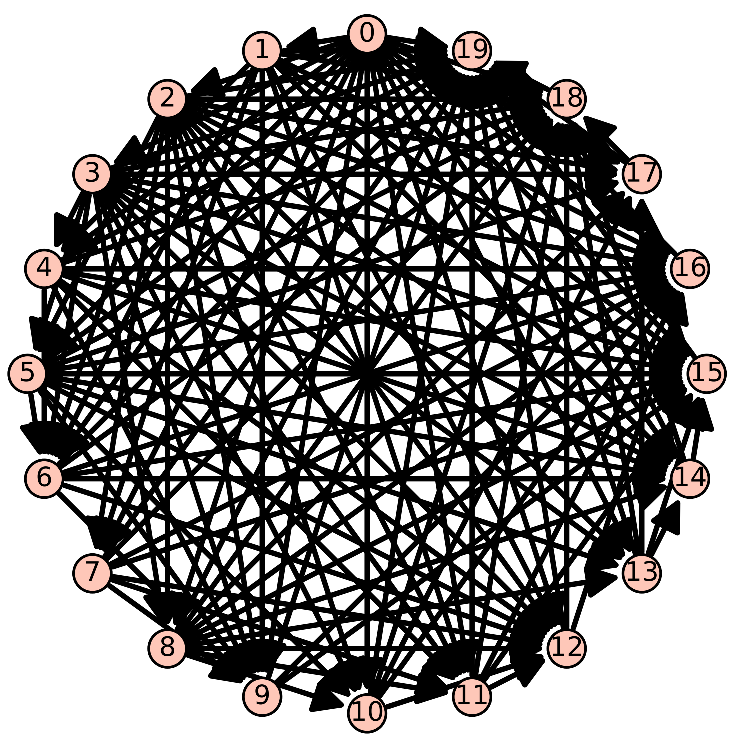

FIGURE 21. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.70. **Polytope F.4D.0069.** Let P denote the polytope F.4D.0069, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and vertices given by the columns of:

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & -3 & 0 & -3 & -1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 & 0 & 1 & 0 & -3 & 1 & 1 & -3 & -1 & 1 & 1 \\ 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,1,0),(-1,0,1,0),(-1,1,1,0),(0,0,0,-1),(1,0,0,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^4$$

free ranks which appear in Hanlon-Hicks-Lazarev resolution:

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

16 line bundles appearing on left-hand side which do give an FSEC of line bundles are:

$$\begin{aligned} \mathcal{E} = \{&(0,-1,-1,-2),(0,0,-1,-2),(0,-1,0,-2),(0,-1,-1,-1),(-1,0,-1,-2),(-1,0,0,-2),\ldots \\ &\ldots(0,-1,-1,0),(0,0,0,-2),(-1,0,-1,-1),(0,-1,0,-1),(0,0,-1,-1),(-1,0,0,-1),\ldots \\ &\ldots(0,0,-1,0),(0,0,0,-1),(0,-1,0,0),(0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i,E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 4 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 5 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 11 & 5 & 3 & 4 & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 4 & 4 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 14 & 5 & 5 & 4 & 4 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 17 & 11 & 5 & 7 & 3 & 3 & 4 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 16 & 10 & 7 & 0 & 0 & 4 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 23 & 14 & 9 & 7 & 7 & 5 & 4 & 4 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 \\ 26 & 14 & 11 & 10 & 7 & 5 & 4 & 4 & 4 & 3 & 2 & 1 & 1 & 1 & 1 & 0 \\ 38 & 26 & 17 & 16 & 10 & 11 & 10 & 7 & 7 & 5 & 3 & 4 & 3 & 2 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

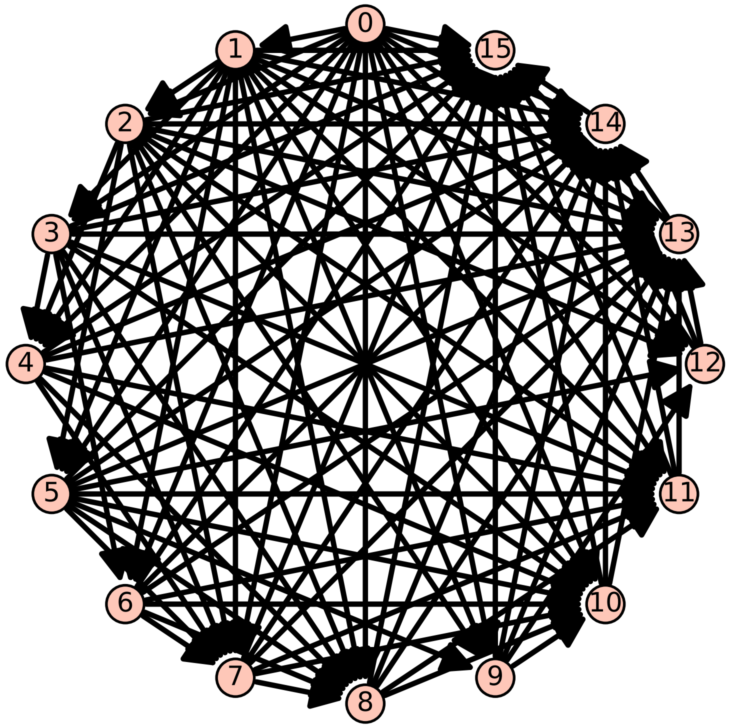

FIGURE 22. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.71. **Polytope F.4D.0070.** Let P denote the polytope F.4D.0070 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & -1 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and vertices given by the columns of:

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & -3 & 0 & -3 & -1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 & 0 & 1 & 0 & -3 & 1 & 1 & -3 & -1 & 1 & 1 \\ 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,1,0),(-1,0,1,0),(-1,1,1,0),(0,0,0,-1),(1,0,-1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^4$$

free ranks which appear in Hanlon-Hicks-Lazarev resolution:

$$0 \to S^8 \to S^{25} \to S^{27} \to S^{11} \to S^1 \to 0$$

16 line bundles appearing on left-hand side which do give an FSEC of line bundles are:

$$\begin{aligned}\mathcal{E} = \{&(-1,0,-1,-2),(-1,-1,-2,-1),(0,0,-1,-2),(-1,0,0,-2),(-1,0,-1,-1),(-1,-1,-2,0),\ldots\\ &\ldots(-1,-1,-1,-1),(0,0,0,-2),(0,0,-1,-1),(-1,0,0,-1),(-1,0,-1,0),(0,0,-1,0),\ldots\\ &\ldots(-1,0,0,0),(-1,-1,-1,0),(0,0,0,-1),(0,0,0,0)\}\end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i,E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 2 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 2 & 2 & 4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
7 & 4 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 4 & 4 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 6 & 2 & 3 & 2 & 2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
14 & 8 & 4 & 7 & 4 & 2 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\
17 & 8 & 6 & 7 & 4 & 4 & 2 & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0\\
20 & 8 & 8 & 10 & 4 & 4 & 4 & 2 & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 0\\
16 & 10 & 7 & 0 & 0 & 0 & 0 & 4 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
26 & 17 & 10 & 10 & 7 & 7 & 3 & 6 & 2 & 4 & 2 & 2 & 0 & 1 & 1 & 0\\
32 & 20 & 14 & 16 & 10 & 7 & 7 & 8 & 4 & 4 & 4 & 2 & 2 & 2 & 1 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

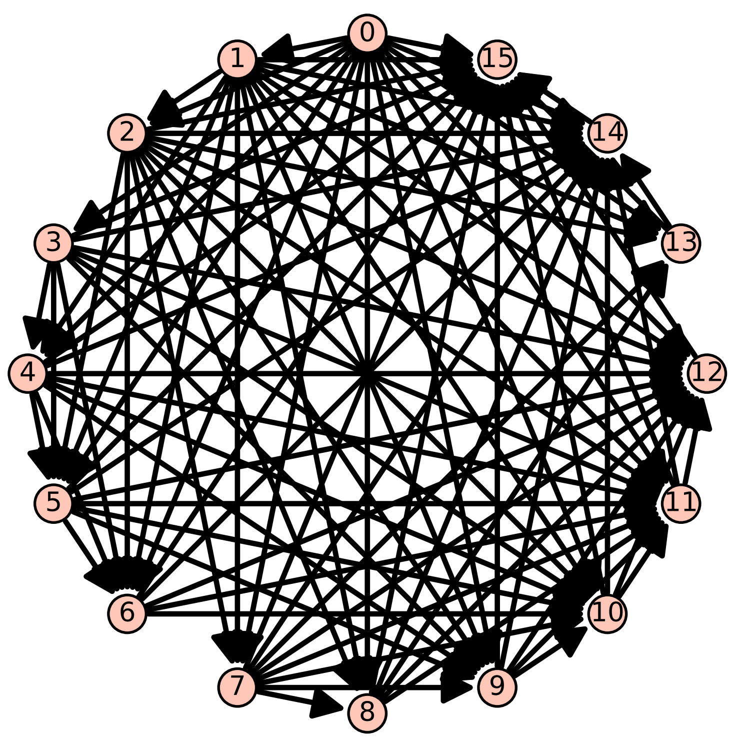

FIGURE 23. Quiver showing nonzero $Hom^0(E_i,E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.72. **Polytope F.4D.0071.** Let P denote the polytope F.4D.0071 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(1,0,0,1),(0,0,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{13} \to S^{15} \to S^7 \to S^1 \to 0$$

The 16 line bundles appearing on the left-hand side which do give a full strong exceptional collection of line bundles are:

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-2,-2),(-1,0,-1,-1),(-1,-1,-1,-2),(-1,-1,-2,-1),(0,-1,-2,-2),(-1,0,-1,0),\dots \\ &\dots(-1,0,0,-1),(0,0,-1,-1),(-1,-1,-1,-1),(0,-1,-2,-1),(0,-1,-1,-2),(0,0,0,-1),\dots \\ &\dots(-1,0,0,0),(0,0,-1,0),(0,-1,-1,-1),(0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i,E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 3 & 5 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 5 & 3 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 4 & 4 & 2 & 0 & 4 & 0 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 10 & 4 & 4 & 2 & 2 & 5 & 0 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 \\ 13 & 8 & 8 & 5 & 4 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 16 & 10 & 6 & 4 & 4 & 8 & 0 & 2 & 3 & 5 & 2 & 2 & 0 & 1 & 0 & 0 \\ 16 & 6 & 10 & 4 & 4 & 8 & 2 & 0 & 5 & 3 & 2 & 2 & 0 & 0 & 1 & 0 \\ 26 & 16 & 16 & 10 & 8 & 13 & 4 & 4 & 8 & 8 & 5 & 4 & 2 & 2 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

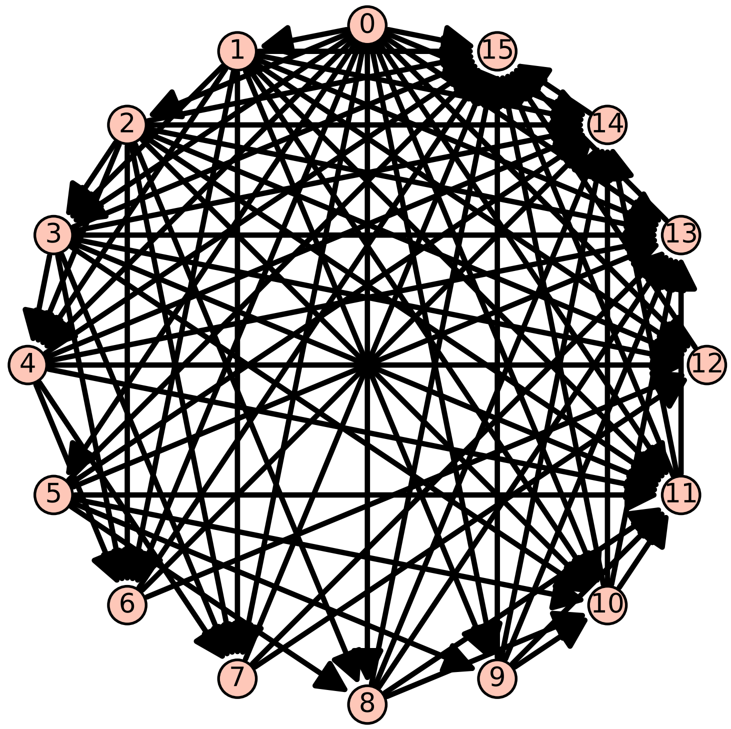

FIGURE 24. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.73. **Polytope F.4D.0072.** Let P denote the polytope F.4D.0072, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 \\ -2 & -2 & 1 & 1 & -2 & -2 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(0,0,0,1),(1,0,0,1),(0,0,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & -1 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{25} \to S^{27} \to S^{11} \to S^1 \to 0$$

The 16 line bundles appearing on the left-hand side which do give a full strong exceptional collection of line bundles are:

$$\begin{aligned}\mathcal{E} = \{&(1,-2,-3,-3),(1,-1,-2,-2),(1,-2,-2,-3),(1,-2,-3,-2),(0,-1,-2,-2),(1,-2,-2,-2),\dots\\ &\dots(0,-1,-1,-2),(0,-1,-2,-1),(0,0,-1,-1),(1,-1,-2,-1),(1,-1,-1,-2),(0,0,-1,0),\dots\\ &\dots(0,0,0,-1),(1,-1,-1,-1),(0,-1,-1,-1),(0,0,0,0)\}\end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 1 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 2 & 3 & 6 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 2 & 6 & 3 & 2 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 4 & 4 & 4 & 2 & 0 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
11 & 6 & 4 & 4 & 2 & 2 & 2 & 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0\\
18 & 10 & 11 & 6 & 4 & 4 & 6 & 3 & 0 & 2 & 2 & 2 & 1 & 0 & 0 & 0\\
18 & 10 & 6 & 11 & 4 & 4 & 3 & 6 & 2 & 0 & 2 & 2 & 0 & 1 & 0 & 0\\
17 & 4 & 10 & 10 & 6 & 4 & 2 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 0\\
30 & 17 & 18 & 18 & 11 & 8 & 10 & 10 & 4 & 4 & 6 & 4 & 2 & 2 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

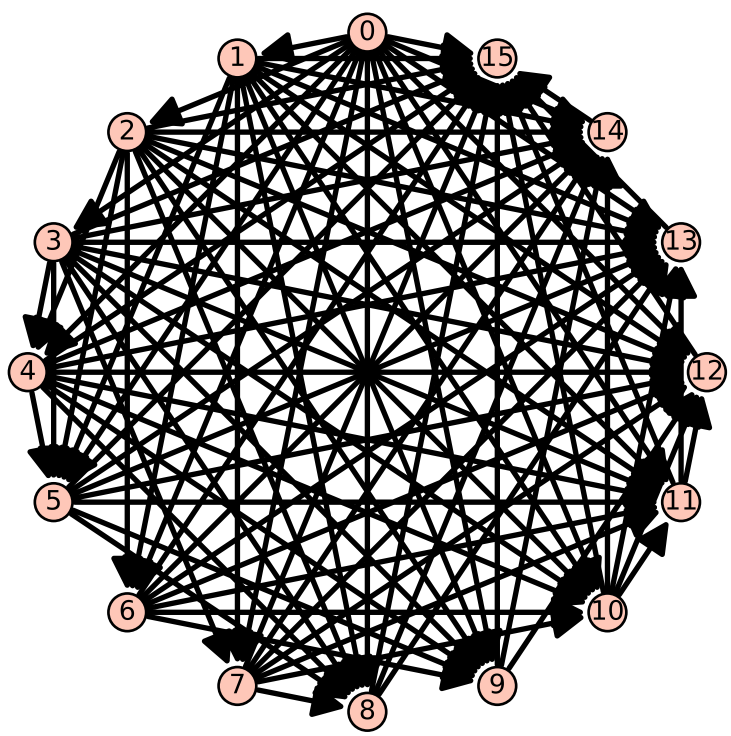

FIGURE 25. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.74. **Polytope F.4D.0073.** Let P denote the polytope F.4D.0073 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & -3 & 1 & -3 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -2 & -2 & -2 & -2 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & -3 & -3 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(1,-1,0,0),(0,0,-1,0),(0,-1,1,0),(0,0,0,-1),(0,1,0,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^3$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{25} \to S^{27} \to S^{11} \to S^1 \to 0$$

The 12 line bundles appearing on the left-hand side which do give a full strong exceptional collection of line bundles are:

$$\mathcal{E} = \{(1,0,-3),(0,-1,-2),(1,1,-3),(1,0,-2),(0,0,-2),(0,-1,-1),(1,1,-2),\dots$$
$$\dots(1,0,-1),(0,-1,0),(1,1,-1),(0,0,-1),(0,0,0)\}$$

Here $Hom_{D^b_{Coh}(X)}(E_i,E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 6 & 3 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 3 & 6 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 17 & 10 & 10 & 6 & 4 & 2 & 2 & 1 & 0 & 0 & 0 & 0 \\ 20 & 10 & 10 & 6 & 6 & 2 & 2 & 1 & 1 & 0 & 0 & 0 \\ 30 & 20 & 14 & 10 & 10 & 6 & 3 & 2 & 2 & 1 & 0 & 0 \\ 30 & 14 & 20 & 10 & 10 & 3 & 6 & 2 & 2 & 0 & 1 & 0 \\ 46 & 30 & 30 & 20 & 17 & 10 & 10 & 6 & 4 & 2 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

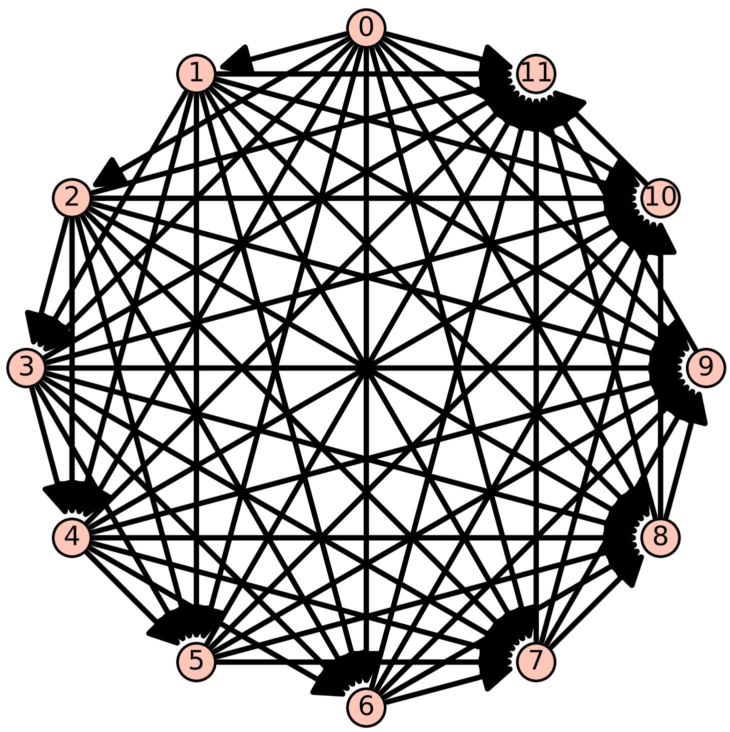

FIGURE 26. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $11 = |\mathcal{E}| - 1$.

1.75. **Polytope F.4D.0074.** Let P denote the polytope F.4D.0074 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 0 & -1 & -1 & 1 & 1 & 0 & -1 & -1 & 1 & 1 & 0 & -1 & -1 & 1 & 1 & 0 & -1 & -1 & 1 & 1 & 0 & -1 & -1 & 1 \\ 0 & -1 & -1 & 1 & 1 & 0 & -1 & -1 & 1 & 1 & 0 & -1 & -1 & 1 & 1 & 0 & -1 & -1 & 1 & 1 & 0 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety, with rays in $\Sigma(1)$:

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(-1,1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(0,0,0,1),(0,0,1,1)\}$$

We use as presentation of $Cl(X_\Sigma)$:

$$\mathbb{Z}^{10} \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^6$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{12} \to S^{13} \to S^6 \to S^1 \to 0$$

The collection $\mathcal{E}$ of 25 line bundles which appear on one side of the Hanlon-Hicks-Lazarev resolution of the diagonal yields a full strong exceptional collection

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-1,-1,-1,-2), (-1,-1,0,-1,-1,-2), (0,-1,-1,-1,-1,-2), (-1,-1,-1,-1,-1,-1), \ldots \\ &\ldots (-1,-1,-1,0,-1,-1), (-1,0,0,-1,-1,-2), (-1,-1,-1,-1,0,-1), (-1,-1,0,-1,-1,-1), \ldots \\ &\ldots (0,-1,-1,-1,-1,-1), (-1,-1,0,0,-1,-1), (-1,0,0,-1,-1,-1), (-1,0,0,0,-1,-1), \ldots \\ &\ldots (0,-1,-1,0,-1,-1), (-1,-1,0,-1,0,-1), (0,-1,-1,-1,0,-1), (-1,0,0,-1,0,-1), \ldots \\ &\ldots (-1,-1,-1,0,0,0), (0,0,0,-1,-1,-2), (0,0,0,-1,-1,-1), (0,0,0,0,-1,-1), \ldots \\ &\ldots (0,0,0,-1,0,-1), (-1,-1,0,0,0,0), (0,-1,-1,0,0,0), (-1,0,0,0,0,0), (0,0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 2 & 3 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 3 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 3 & 0 & 2 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 3 & 3 & 3 & 3 & 3 & 3 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 4 & 4 & 2 & 4 & 0 & 0 & 0 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 4 & 0 & 0 & 4 & 4 & 2 & 2 & 0 & 2 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
8 & 4 & 4 & 2 & 0 & 4 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
12 & 6 & 6 & 3 & 4 & 4 & 4 & 0 & 3 & 2 & 2 & 1 & 2 & 2 & 2 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
8 & 0 & 4 & 0 & 4 & 4 & 2 & 2 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
12 & 4 & 4 & 4 & 6 & 6 & 3 & 3 & 0 & 2 & 2 & 2 & 2 & 2 & 1 & 2 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
16 & 8 & 8 & 4 & 8 & 8 & 4 & 4 & 4 & 4 & 4 & 2 & 4 & 4 & 2 & 2 & 2 & 1 & 2 & 2 & 2 & 1 & 2 & 1 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

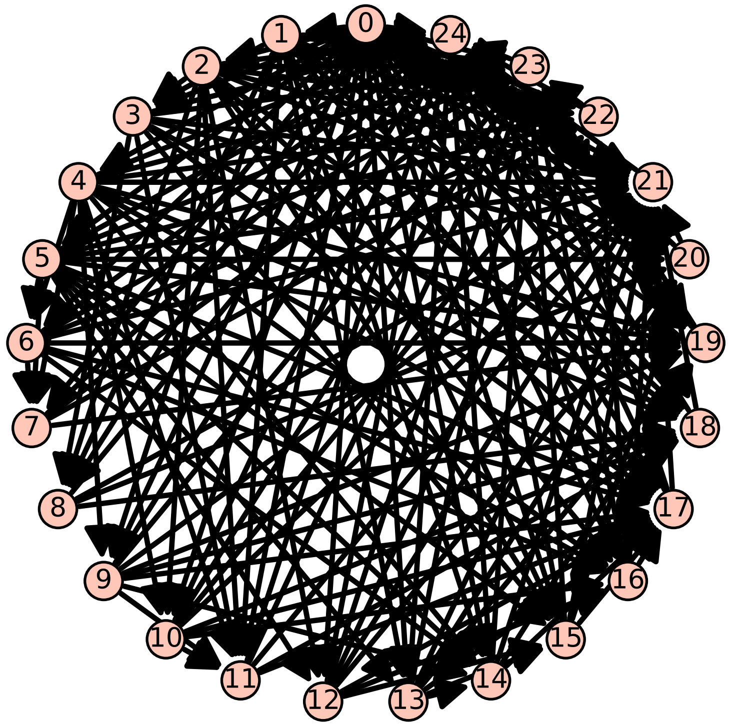

FIGURE 27. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $24 = |\mathcal{E}| - 1$.

1.76. **Polytope F.4D.0075.** Let P denote the polytope F.4D.0075 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} -1 & 1 & 0 & -1 & 0 & 1 & -1 & 1 & 0 & -1 & 0 & 1 & -1 & 1 & 0 & -1 & 0 & 1 & -1 & 1 & 0 & -1 & 0 & 1 & -1 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(1,-1,0,0),(0,1,0,0),(-1,1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(0,0,0,1),(0,0,1,1)\}$$

We use as presentation of $Cl(X_\Sigma)$:

$$\mathbb{Z}^{11} \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^7$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{12} \to S^{13} \to S^6 \to S^1 \to 0$$

The collection $\mathcal{E}$ of 30 line bundles yields a full strong exceptional collection

$$\{(-1,0,-1,-1,-1,-1,-2),(-1,-1,-1,0,-1,-1,-2),(-1,-1,0,0,-1,-1,-2),(-1,0,-1,0,-1,-1,-2),\dots$$
$$\dots(-1,-1,-1,0,-1,-1,-1),(-1,0,-1,-1,-1,-1,-1),(-1,-1,-1,0,0,-1,-1),(0,0,-1,-1,-1,-1,-2),\dots$$
$$\dots(-1,0,-1,-1,-1,0,-1),(-1,0,-1,-1,0,-1,-1),(-1,-1,-1,0,-1,0,-1),(-1,-1,0,0,-1,-1,-1),\dots$$
$$\dots(-1,-1,0,0,-1,0,-1),(0,0,-1,-1,-1,-1,-1),(0,0,-1,-1,0,-1,-1),(0,0,-1,-1,-1,0,-1),\dots$$
$$\dots(-1,0,-1,0,-1,-1,-1),(-1,0,-1,0,0,-1,-1),(-1,-1,0,0,0,-1,-1),(-1,0,-1,0,-1,0,-1),\dots$$
$$\dots(-1,-1,-1,0,0,0,0),(-1,0,-1,-1,0,0,0),(0,0,0,0,-1,-1,-2),(0,0,0,0,-1,-1,-1),\dots$$
$$\dots(0,0,0,0,0,-1,-1),(-1,-1,0,0,0,0,0),(0,0,0,0,-1,0,-1),(0,0,-1,-1,0,0,0),\dots$$
$$\dots(-1,0,-1,0,0,0,0),(0,0,0,0,0,0,0)\}$$

Here $Hom_{D^b_{Coh}(X)}(E_i,E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&0&0&1&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&0&0&2&0&2&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&1&1&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&1&1&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&2&0&2&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&0&0&0&2&2&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&2&0&0&0&2&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
6&3&0&2&0&2&2&0&0&0&1&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&0&2&2&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&0&2&0&0&2&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
6&0&3&2&0&2&2&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&0&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
6&0&0&2&3&2&2&0&0&0&0&1&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0\\
6&2&2&3&2&0&0&0&0&2&1&0&0&0&1&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0\\
6&2&2&0&2&3&0&0&2&0&0&1&1&0&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
6&2&2&3&2&0&0&0&2&0&1&0&0&0&1&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0\\
8&0&4&4&0&4&2&2&0&0&0&0&0&0&2&2&1&0&0&0&0&0&1&0&0&0&0&0&0&0\\
6&2&2&0&2&3&0&0&0&2&0&1&1&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
9&3&3&3&3&3&3&0&3&0&1&1&1&1&1&1&1&1&1&0&1&1&0&0&1&0&0&0&0&0\\
9&3&3&3&3&3&3&0&0&3&1&1&1&1&1&1&1&1&1&1&0&0&0&1&0&1&0&0&0&0\\
8&4&0&4&0&4&2&2&0&0&2&0&2&1&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
8&0&0&4&4&4&2&2&0&0&0&2&0&0&0&0&0&2&1&0&0&0&0&0&0&0&0&1&0&0\\
12&4&4&6&4&6&3&3&0&4&2&2&2&1&2&2&1&2&1&2&0&0&1&2&0&1&1&1&1&0\\
12&4&4&6&4&6&3&3&4&0&2&2&2&1&2&2&1&2&1&0&2&2&1&0&1&0&1&1&0&1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

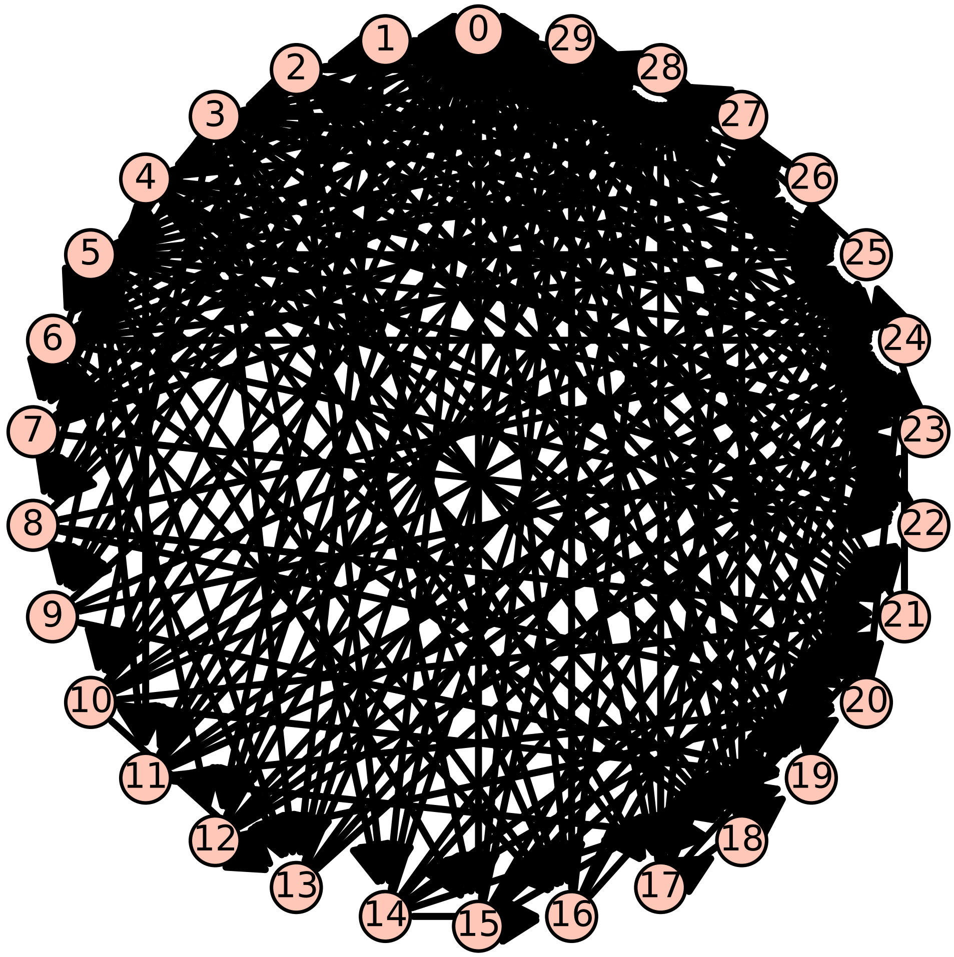

FIGURE 28. Directed graph showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $29 = |\mathcal{E}| - 1$.

1.77. **Polytope F.4D.0076.** Let P denote the polytope F.4D.0076 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices of P given by the columns which are continued through the next two matrices due length:

$$\begin{pmatrix} -1 & 1 & 0 & -1 & 0 & 1 & -1 & 1 & 0 & -1 & 0 & 1 & -1 & 1 & 0 & -1 & 0 & 1 & -1 & 1 & 0 & -1 & 0 & 1 & -1 & 1 & 0 & -1 & 0 & 1\dots \\ 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & 1 & 1\dots \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0\dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1\dots \end{pmatrix}$$

$$\begin{pmatrix} \dots-1 & 1 & 0 & -1 & 0 & 1 \\ \dots 0 & 0 & -1 & -1 & 1 & 1 \\ \dots 1 & 1 & 1 & 1 & 1 & 1 \\ \dots 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\begin{aligned}&\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(1,-1,0,0),(0,1,0,0),(-1,1,0,0),\dots\\&\dots(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(0,0,1,-1),(0,0,0,1),(0,0,-1,1)\}\end{aligned}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^{12} \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^{8}$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{12} \to S^{13} \to S^6 \to S^1 \to 0$$

The 36 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution and yield a full strong exceptional collection of line bundles are:

$$\begin{aligned}\mathcal{E} = \{&(-1,0,-1,-1,-1,0,-1,-1),(-1,0,-1,-1,-1,-1,-1,0),(-1,-1,-1,0,-1,0,-1,-1),(-1,-1,-1,0,-1,-1,-1,0),\dots\\&\dots(-1,-1,-1,0,-1,-1,0,0),(-1,-1,-1,0,0,0,-1,-1),(-1,0,-1,-1,-1,0,-1,0),(-1,-1,-1,0,-1,0,-1,0),\dots\\&\dots(-1,0,-1,-1,0,0,-1,-1),(-1,0,-1,-1,-1,-1,0,0),(0,0,-1,-1,-1,0,-1,-1),(-1,-1,0,0,-1,0,-1,-1),\dots\\&\dots(-1,0,-1,0,-1,0,-1,-1),(-1,0,-1,0,-1,-1,-1,0),(-1,-1,0,0,-1,-1,-1,0),(0,0,-1,-1,-1,-1,-1,0),\dots\\&\dots(0,0,0,0,-1,0,-1,-1),(-1,0,-1,-1,0,0,0,0),(-1,0,-1,0,-1,0,-1,0),(-1,0,-1,0,-1,-1,0,0),\dots\\&\dots(-1,0,-1,0,0,0,-1,-1),(-1,-1,0,0,-1,0,-1,0),(0,0,-1,-1,-1,-1,0,0),(-1,-1,0,0,-1,-1,0,0),\dots\\&\dots(-1,-1,0,0,0,0,-1,-1),(0,0,-1,-1,-1,0,-1,0),(0,0,-1,-1,0,0,-1,-1),(0,0,0,0,-1,-1,-1,0),\dots\\&\dots(-1,-1,-1,0,0,0,0,0),(0,0,-1,-1,0,0,0,0),(-1,-1,0,0,0,0,0,0),(-1,0,-1,0,0,0,0,0),\dots\\&\dots(0,0,0,0,0,0,-1,-1),(0,0,0,0,-1,-1,0,0),(0,0,0,0,-1,0,-1,0),(0,0,0,0,0,0,0,0)\}\end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&1&1&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&0&0&2&0&0&2&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&2&0&0&0&0&2&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&0&0&2&0&2&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&0&2&0&0&2&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&0&2&0&0&0&2&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&2&0&0&0&2&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&0&0&2&2&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&0&2&0&2&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
4&2&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&0&0&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
6&2&2&2&0&0&3&0&2&1&1&0&0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
6&2&2&2&0&3&0&0&2&0&0&1&1&0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
6&2&2&2&3&0&0&0&2&0&0&0&0&0&0&1&1&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
6&2&2&2&3&0&0&0&0&0&0&0&0&0&0&1&1&1&0&2&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
6&2&2&2&0&3&0&0&0&0&0&1&1&0&1&0&0&0&0&2&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
6&2&2&2&0&0&3&0&0&1&1&0&0&1&0&0&0&0&0&2&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
6&0&3&0&2&2&2&0&0&0&0&0&1&1&0&0&1&0&2&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
6&0&0&3&2&2&2&0&0&1&0&1&0&0&0&1&0&0&2&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
6&3&0&0&2&2&2&2&0&0&1&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
6&3&0&0&2&2&2&0&0&0&1&0&0&0&1&0&0&1&2&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
6&0&0&3&2&2&2&2&0&1&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
6&0&3&0&2&2&2&2&0&0&0&0&1&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
9&3&3&3&3&3&3&3&3&1&1&1&1&1&1&1&1&1&0&0&1&1&1&0&0&0&0&0&1&0&1&1&1&0&0&0\\
9&3&3&3&3&3&3&3&0&1&1&1&1&1&1&1&1&1&0&3&0&0&0&1&1&1&0&0&1&0&1&1&0&1&0&0\\
9&3&3&3&3&3&3&0&3&1&1&1&1&1&1&1&1&1&3&0&1&1&1&0&0&0&1&1&0&1&0&0&0&0&1&0\\
9&3&3&3&3&3&3&0&0&1&1&1&1&1&1&1&1&1&3&3&0&0&0&1&1&1&1&1&0&1&0&0&0&0&0&1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

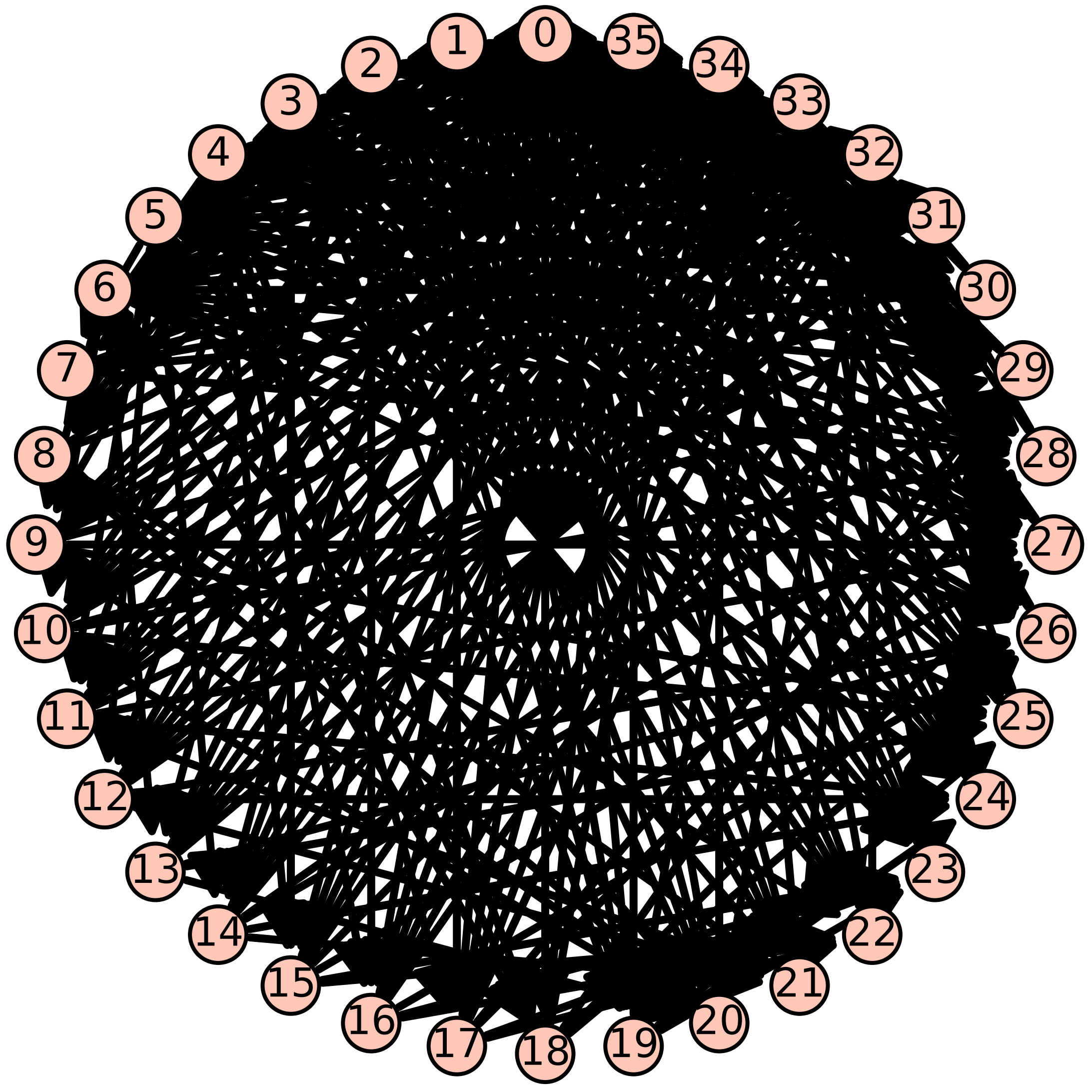

FIGURE 29. Directed graph showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $35 = |\mathcal{E}| - 1$.

1.78. **Polytope F.4D.0077.** Let P denote the polytope F.4D.0077 in the polymake database, with half-space representation given by

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} -1 & 1 & -1 & 1 & 0 & -1 & 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \\ 0 & 0 & -2 & -2 & 0 & 0 & -2 & -2 & 0 & -2 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 2 & 2 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,-1,1,0),(1,0,0,-1),(0,0,0,1),(-1,0,0,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^2 \to S^7 \to S^9 \to S^5 \to S^1 \to 0$$

The 10 line bundles which appear on the left-hand side which do give a full strong exceptional collection of line bundles are:

$$\mathcal{E} = \{(-1,-1,-2,1), (-1,-1,-1,1), (-1,-1,-1,0), (0,-1,-1,0), (-1,0,-2,1), (-1,0,-1,0), \ldots$$
$$\ldots (0,0,-1,0), (0,-1,0,0), (-1,0,-1,1), (0,0,0,0)\}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 0 \\ 4 & 0 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 6 & 2 & 3 & 2 & 2 & 0 & 1 & 1 & 1 & 0 \\ 8 & 4 & 4 & 4 & 2 & 2 & 2 & 2 & 1 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

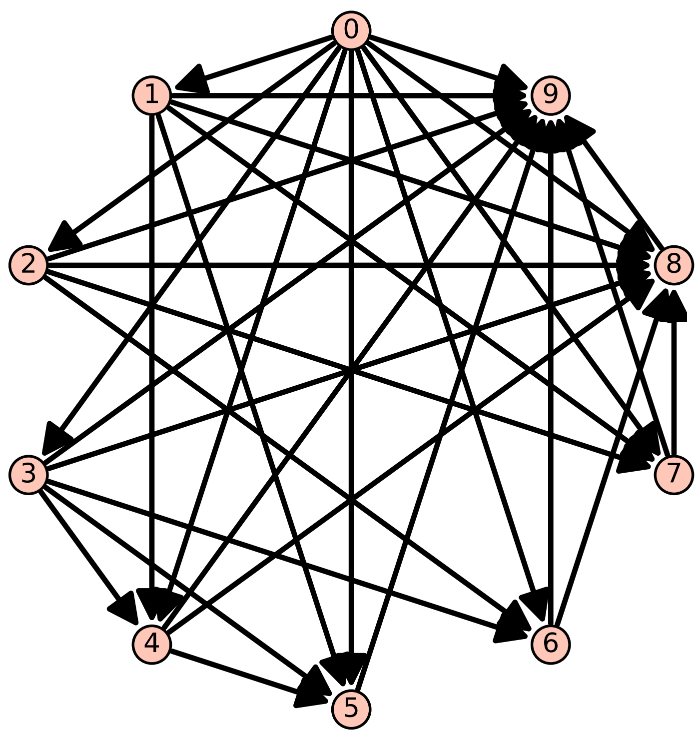

FIGURE 30. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $9 = |\mathcal{E}| - 1$.

1.79. **Polytope F.4D.0078.** Let P denote the polytope F.4D.0078 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 & 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 & 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,-1,1,0),(0,0,0,-1),(1,0,0,-1),(0,0,0,1),(-1,0,0,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^{10} \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^6$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{12} \to S^{13} \to S^6 \to S^1 \to 0$$

There are 24 line bundles appearing on the left-hand side which do give a full strong exceptional collection of line bundles:

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-1,0,-1,-1),(-1,-1,-1,-1,-1,0),(-1,-1,-1,0,-1,0),(-1,0,-1,0,-1,-1),\dots \\ &\dots(0,-1,-1,0,-1,-1),(-1,0,-1,-1,-1,0),(-1,-1,0,0,-1,-1),(-1,-1,0,-1,-1,0),\dots \\ &\dots(-1,-1,-1,-1,0,0),(0,0,-1,0,-1,-1),(-1,0,-1,-1,0,0),(0,-1,0,0,-1,-1),\dots \\ &\dots(-1,0,-1,0,-1,0),(-1,-1,0,-1,0,0),(-1,-1,0,0,-1,0),(-1,0,0,-1,-1,0),\dots \\ &\dots(-1,0,0,0,-1,-1),(0,-1,-1,0,0,0),(0,0,0,0,-1,-1),(0,0,-1,0,0,0),\dots \\ &\dots(-1,0,0,-1,0,0),(-1,0,0,0,-1,0),(0,-1,0,0,0,0),(0,0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 3 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 3 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 6 & 4 & 0 & 5 & 0 & 2 & 0 & 0 & 0 & 0 & 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 3 & 0 & 2 & 2 & 2 & 0 & 0 & 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 3 & 3 & 3 & 3 & 3 & 0 & 0 & 3 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 3 & 0 & 2 & 2 & 2 & 0 & 2 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
9 & 3 & 3 & 3 & 3 & 3 & 0 & 3 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
10 & 6 & 4 & 0 & 0 & 5 & 2 & 0 & 0 & 0 & 0 & 0 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
10 & 6 & 4 & 5 & 0 & 0 & 2 & 0 & 0 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
15 & 9 & 6 & 5 & 5 & 5 & 3 & 0 & 5 & 3 & 2 & 3 & 2 & 3 & 2 & 1 & 3 & 2 & 0 & 0 & 1 & 1 & 1 & 0 \\
15 & 9 & 6 & 5 & 5 & 5 & 3 & 5 & 0 & 3 & 2 & 3 & 2 & 3 & 2 & 1 & 0 & 0 & 3 & 2 & 1 & 1 & 0 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

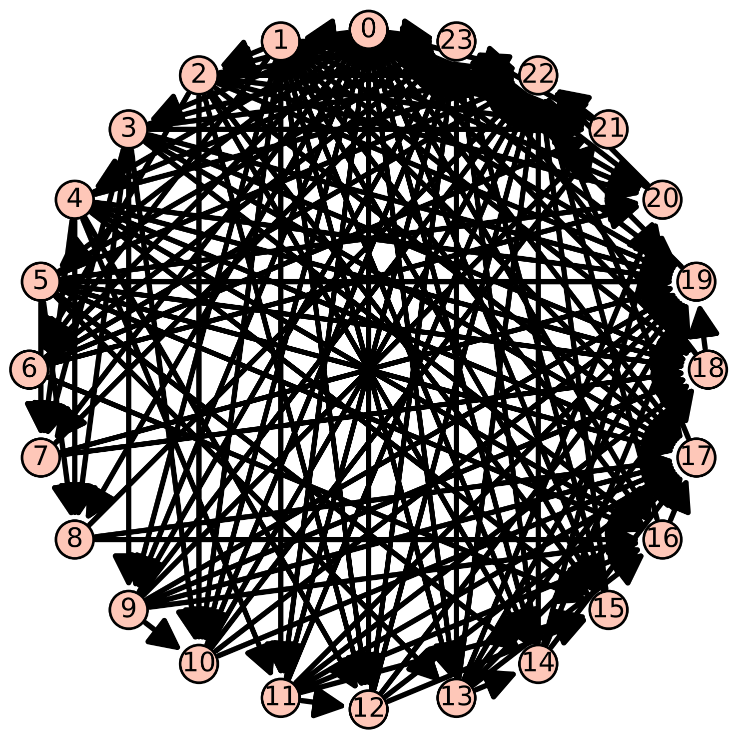

FIGURE 31. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $23 = |\mathcal{E}| - 1$.

1.80. **Polytope F.4D.0079.** Let P denote the polytope F.4D.0079 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and vertices of P given by the columns of

$$\begin{pmatrix} 1 & 1 & 0 & -1 & 0 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 0 & -1 & 0 & -1 & -1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & -2 & 1 & -2 & -2 & 1 & 1 & -2 & -2 & 1 & 1 \\ 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,1,0),(-1,0,1,0),(0,0,0,-1),(0,0,0,1),(0,1,0,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{12} \to S^{13} \to S^6 \to S^1 \to 0.$$

The 20 line bundles appearing on left-hand side which do give a full strong exceptional collection of line bundles are:

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-1,-1,-2),(-1,-1,-1,0,-1),(-1,0,0,-1,-2),(0,-1,-1,-1,-2),(-1,-1,0,-1,-2),\dots \\ &\dots(-1,-1,-1,-1,-1),(0,-1,-1,0,-1),(-1,-1,0,0,-1),(-1,0,0,0,-1),(0,-1,-1,-1,-1),\dots \\ &\dots(-1,-1,0,-1,-1),(-1,0,0,-1,-1),(0,0,0,-1,-2),(-1,-1,-1,0,0),(-1,0,0,0,0),\dots \\ &\dots(0,0,0,0,-1),(0,-1,-1,0,0),(-1,-1,0,0,0),(0,0,0,-1,-1),(0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 3 & 0 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 0 & 3 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 3 & 3 & 3 & 3 & 3 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 4 & 0 & 4 & 4 & 2 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
12 & 4 & 4 & 6 & 6 & 3 & 3 & 2 & 2 & 1 & 2 & 2 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
10 & 6 & 4 & 5 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\
10 & 6 & 4 & 0 & 5 & 0 & 0 & 3 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\
15 & 9 & 6 & 5 & 5 & 5 & 0 & 3 & 3 & 3 & 2 & 2 & 2 & 3 & 0 & 0 & 1 & 1 & 1 & 0 \\
20 & 12 & 8 & 10 & 10 & 5 & 5 & 6 & 6 & 3 & 4 & 4 & 2 & 4 & 3 & 2 & 2 & 2 & 1 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

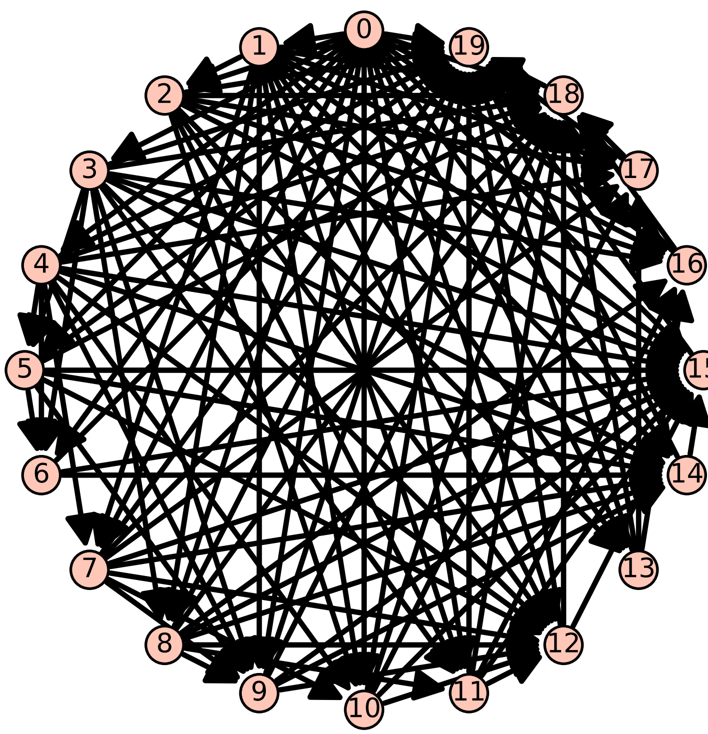

FIGURE 32. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.81. **Polytope F.4D.0080.** Let P denote the polytope F.4D.0080 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

and vertices of P given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & -2 & 0 & 0 & -2 & -2 & 1 & -2 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & -3 & 1 & 0 & -2 & 1 & 1 & 0 & -3 & 1 & 1 \\ 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to X, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,1,0),(-1,0,1,0),(0,0,0,-1),(1,0,0,1),(1,1,0,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

16 line bundles appearing on left-hand side which do give an FSEC of line bundles are:

$$\begin{aligned} \mathcal{E} = \{&(0,-1,-2,-3),(0,-1,-2,-2),(-1,0,-2,-3),(0,0,-2,-3),(0,-1,-1,-2),(-1,0,-2,-2),\dots \\ &\dots(0,0,-2,-2),(0,0,-1,-2),(-1,0,-1,-2),(0,-1,-1,-1),(0,-1,0,-1),(0,0,0,-1),\dots \\ &\dots(0,-1,0,0),(-1,0,-1,-1),(0,0,-1,-1),(0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i,E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 4 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 0 & 4 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 2 & 5 & 2 & 2 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 4 & 4 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 14 & 4 & 5 & 5 & 4 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 14 & 7 & 8 & 4 & 2 & 4 & 2 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 16 & 0 & 10 & 7 & 0 & 0 & 0 & 4 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 23 & 7 & 14 & 9 & 7 & 4 & 2 & 5 & 4 & 2 & 2 & 1 & 1 & 1 & 0 & 0 \\ 20 & 10 & 8 & 8 & 4 & 4 & 4 & 2 & 1 & 2 & 1 & 1 & 0 & 0 & 1 & 0 \\ 32 & 16 & 20 & 14 & 7 & 10 & 7 & 8 & 4 & 4 & 2 & 4 & 2 & 1 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

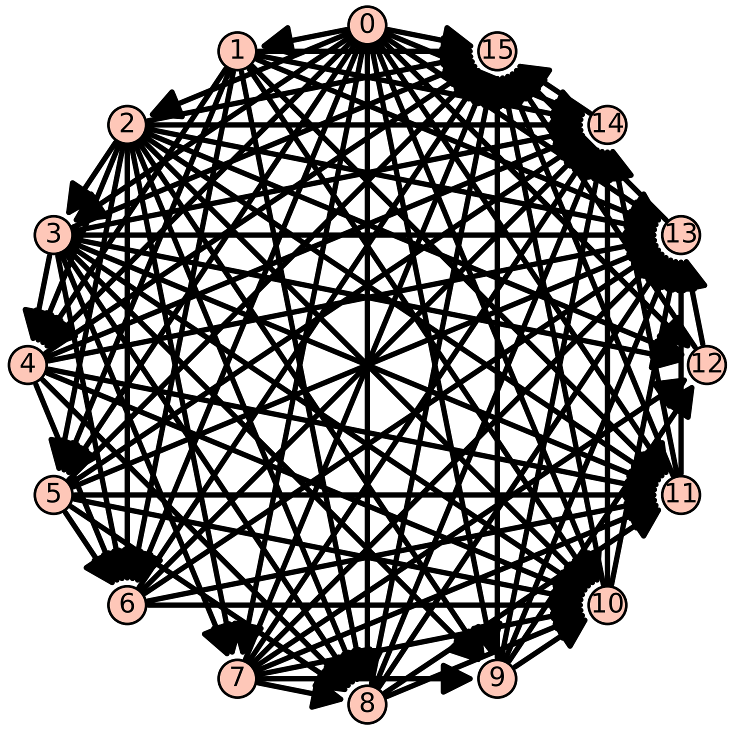

FIGURE 33. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.82. **Polytope F.4D.0081.** Let P denote the polytope F.4D.0081 in the polymake datbase, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

and vertices of P given by the columns of

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & -2 & -2 & -2 & 1 & -2 & 1 \\ 0 & 1 & 0 & 1 & -2 & -1 & 1 & 1 & 0 & 1 & 0 & 1 & -4 & -1 & 1 & 1 \\ 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 & 0 & 0 & -3 & -3 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(-1,0,1,0),(-1,1,1,0),(0,0,0,-1),(0,0,0,1),(1,0,0,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

There are 16 line bundles appearing on left-hand side which give a full strong exceptional collection of line bundles:

$$\begin{aligned}\mathcal{E} = \{&(-1,-1,-1,-2),(-1,0,-1,-2),(-1,-1,0,-1),(-1,-1,-1,-1),(0,-1,-1,-2),(-1,-1,0,0),\ldots\\ &\ldots(0,0,-1,-2),(-1,0,0,-1),(0,-1,0,-1),(-1,0,-1,-1),(0,-1,-1,-1),(0,0,-1,-1),\ldots\\ &\ldots(0,-1,0,0),(0,0,0,-1),(-1,0,0,0),(0,0,0,0)\}\end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
7 & 4 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 5 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 4 & 4 & 3 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
12 & 5 & 5 & 3 & 3 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
14 & 5 & 5 & 5 & 4 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\
14 & 9 & 7 & 5 & 0 & 1 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
19 & 12 & 9 & 5 & 5 & 4 & 3 & 3 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 0\\
23 & 14 & 9 & 9 & 7 & 5 & 5 & 4 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0\\
28 & 14 & 14 & 12 & 9 & 5 & 5 & 4 & 5 & 4 & 3 & 1 & 1 & 1 & 1 & 0\\
42 & 28 & 23 & 19 & 14 & 14 & 12 & 9 & 7 & 7 & 5 & 5 & 4 & 3 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

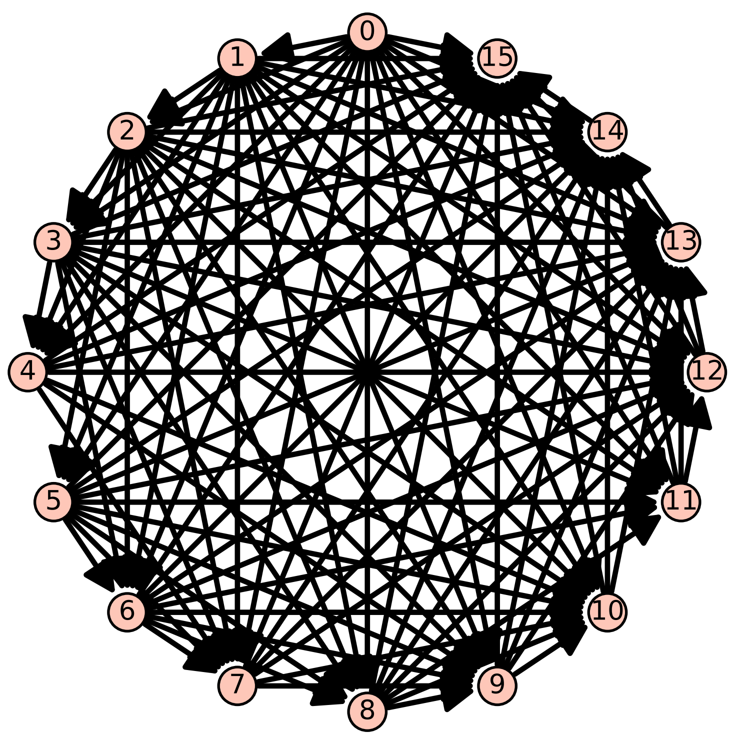

FIGURE 34. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.83. **Polytope F.4D.0082.** Let P denote the polytope F.4D.0082 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

with vertices of P given by the columns of

$$\begin{pmatrix} -2 & 1 & 0 & 1 & -2 & 1 & 0 & 1 & -2 & 1 & 0 & 1 & -2 & 1 & 0 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(1,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(0,0,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{12} \to S^{13} \to S^6 \to S^1 \to 0$$

There are 16 line bundles appearing on left-hand side which do give a full strong exceptional collection of line bundles:

$$\begin{aligned} \mathcal{E} = \{&(1,-2,-1,-2),(1,-1,-1,-2),(1,-2,-1,-1),(0,-1,-1,-2),(1,-2,0,-1),(1,-1,0,-1),\dots \\ &\dots(0,-1,-1,-1),(1,-1,-1,-1),(0,-1,0,-1),(1,-2,0,0),(0,0,-1,-2),(0,0,-1,-1),\dots \\ &\dots(1,-1,0,0),(0,0,0,-1),(0,-1,0,0),(0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 3 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 2 & 2 & 3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 0 & 2 & 2 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 3 & 3 & 3 & 3 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 10 & 6 & 4 & 5 & 0 & 0 & 2 & 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 10 & 5 & 0 & 6 & 4 & 2 & 0 & 3 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 15 & 5 & 5 & 9 & 6 & 3 & 0 & 3 & 3 & 2 & 2 & 0 & 1 & 1 & 0 & 0 \\ 15 & 9 & 6 & 5 & 5 & 0 & 3 & 3 & 2 & 3 & 2 & 1 & 0 & 0 & 1 & 0 \\ 25 & 15 & 10 & 15 & 10 & 5 & 5 & 9 & 6 & 6 & 4 & 3 & 3 & 2 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

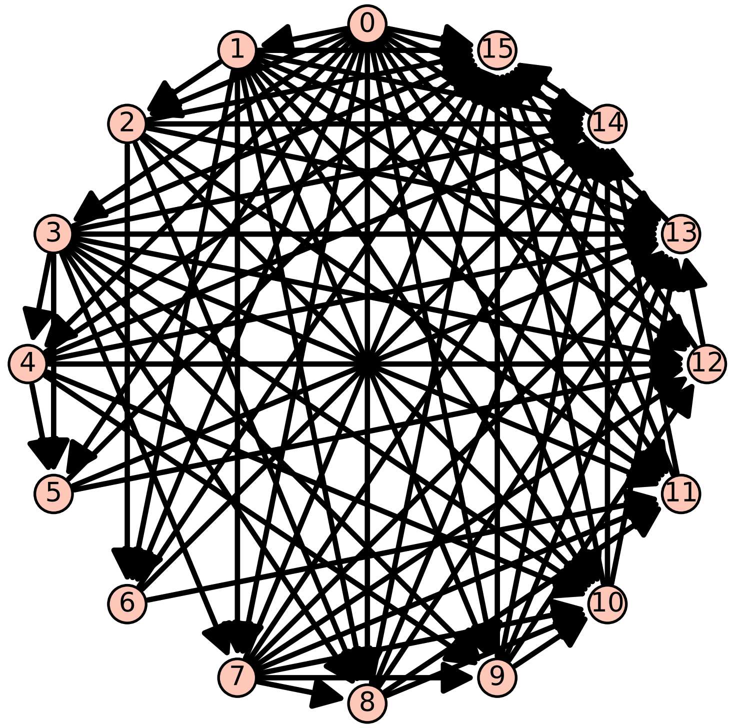

FIGURE 35. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.84. **Polytope F.4D.0083.** Let P denote the polytope F.4D.0083 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \end{bmatrix}$$

and vertices of P given by the columns of

$$\begin{pmatrix} -2 & 1 & 0 & 1 & -1 & 1 & 0 & 1 & -2 & 1 & -2 & 1 & -1 & 1 & 0 & 1 & -2 & 1 & 0 & 1 \\ -1 & -1 & 1 & 1 & 0 & 0 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(1,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,1,0),(0,1,1,0),(0,0,0,-1),(0,0,-1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

There are 20 line bundles appearing on the left-hand side which do give a full strong exceptional collection of line bundles:

$$\begin{aligned}\mathcal{E} = \{&(1,-2,2,-3,-4),(1,-2,1,-2,-3),(0,-1,1,-2,-3),(1,-2,2,-3,-3),(1,-1,1,-2,-3),(1,-2,2,-2,-3),\ldots\\ &\ldots(0,-1,0,-1,-2),(1,-2,1,-2,-2),(0,-1,1,-2,-2),(1,-2,2,-2,-2),(0,0,0,-1,-2),(1,-1,1,-1,-2),\ldots\\ &\ldots(0,-1,1,-1,-2),(1,-1,1,-2,-2),(0,0,0,0,-1),(0,-1,1,-1,-1),(1,-1,1,-1,-1),(0,0,0,-1,-1),\ldots\\ &\ldots(0,-1,0,-1,-1),(0,0,0,0,0)\}\end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 0 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 3 & 0 & 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 2 & 2 & 3 & 2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 0 & 0 & 3 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 3 & 2 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 3 & 3 & 3 & 3 & 1 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 5 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 2 & 5 & 5 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 6 & 5 & 5 & 4 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
10 & 5 & 4 & 6 & 5 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
15 & 9 & 10 & 5 & 6 & 0 & 0 & 2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
15 & 5 & 6 & 9 & 4 & 1 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
16 & 9 & 5 & 9 & 10 & 1 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
25 & 9 & 9 & 15 & 6 & 3 & 1 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

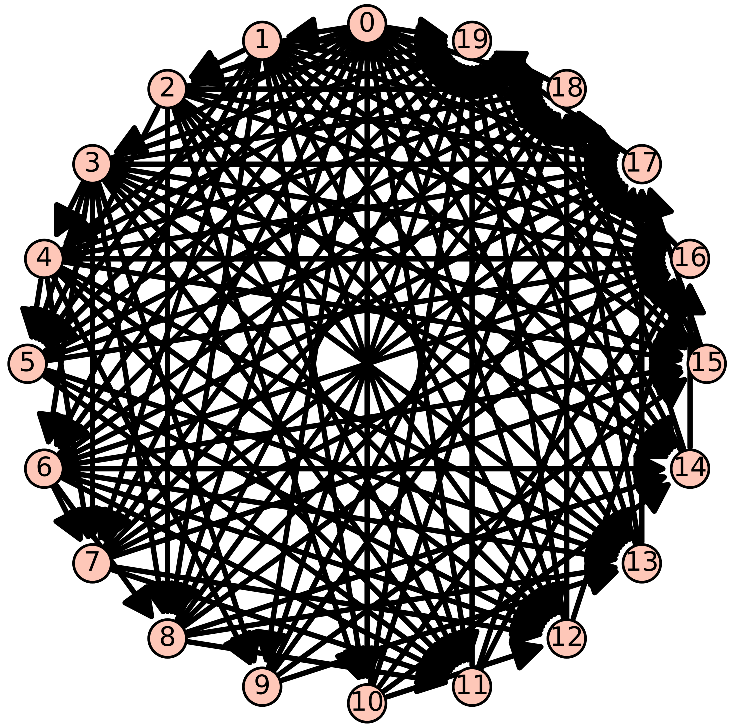

FIGURE 36. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.85. **Polytope F.4D.0084.** Let P denote the polytope F.4D.0084 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & -1 & 0 & 0 \end{bmatrix}$$

and vertices of P given by the columns of

$$\begin{pmatrix} -2 & 1 & -2 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & -2 & 1 & -2 & 1 & 0 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & -3 & -3 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(1,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,1),(0,1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

There are 16 line bundles appearing on the left-hand side which do give an FSEC of line bundles as follows:

$$\begin{aligned}\mathcal{E} = \{&(1,-2,-3,-4),(1,-2,-3,-3),(0,-1,-2,-3),(1,-1,-2,-3),(1,-2,-2,-3),(0,-1,-2,-2),\dots\\ &\dots(0,0,-1,-2),(1,-2,-2,-2),(1,-1,-1,-2),(1,-1,-2,-2),(0,-1,-1,-2),(0,0,0,-1),\dots\\ &\dots(1,-1,-1,-1),(0,0,-1,-1),(0,-1,-1,-1),(0,0,0,0)\}\end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 3 & 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 5 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 2 & 2 & 3 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 2 & 2 & 5 & 2 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
12 & 5 & 5 & 3 & 3 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\
10 & 6 & 4 & 5 & 0 & 3 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
16 & 9 & 4 & 9 & 4 & 5 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
19 & 12 & 9 & 5 & 5 & 3 & 5 & 3 & 2 & 2 & 1 & 0 & 0 & 1 & 0 & 0\\
21 & 9 & 9 & 12 & 6 & 5 & 2 & 3 & 3 & 2 & 0 & 0 & 0 & 0 & 1 & 0\\
33 & 21 & 16 & 19 & 10 & 12 & 9 & 5 & 5 & 4 & 2 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

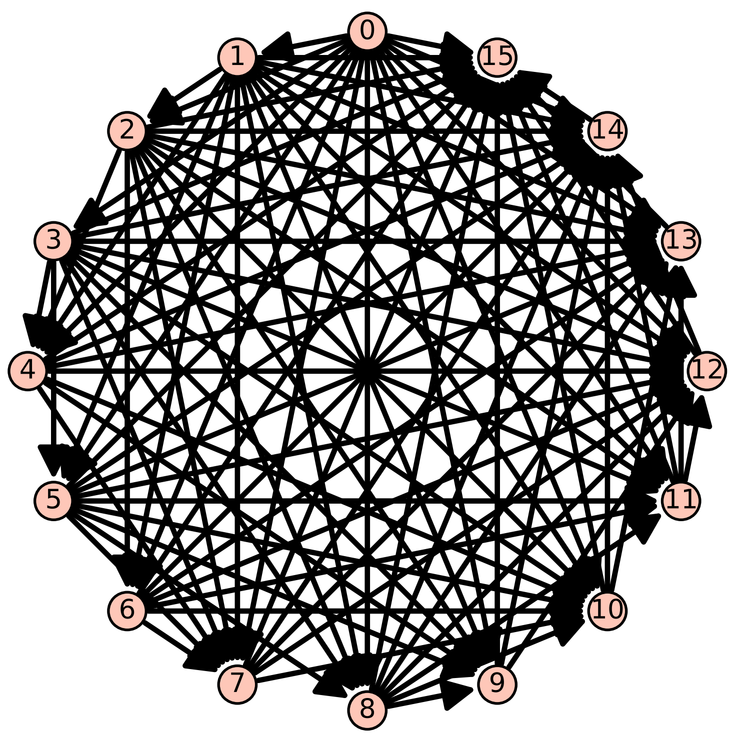

FIGURE 37. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.86. **Polytope F.4D.0085.** Let P denote the polytope F.4D.0085 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

and vertices of P given by the columns of:

$$\begin{pmatrix} 0 & 1 & 0 & 1 & -3 & 1 & 0 & 1 & 0 & 1 & -3 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -2 & -2 & -2 & -2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -3 & -3 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,1,1,0),(0,0,0,-1),(0,1,0,1),(1,1,0,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

There are 12 line bundles appearing on left-hand side which give a full strong exceptional collection of line bundles as follows:

$$\begin{aligned} \mathcal{E} = \{&(-1,-2,-3),(-1,-2,-2),(0,-2,-3),(-1,-1,-2),(0,-2,-2),(0,-1,-2),\dots \\ &\dots(-1,-1,-1),(-1,0,-1),(0,0,-1),(-1,0,0),(0,-1,-1),(0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 2 & 5 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 5 & 2 & 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 15 & 5 & 5 & 5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 16 & 9 & 9 & 4 & 5 & 2 & 2 & 0 & 1 & 0 & 0 & 0 \\ 25 & 15 & 9 & 9 & 5 & 2 & 5 & 2 & 1 & 1 & 0 & 0 \\ 25 & 9 & 9 & 9 & 5 & 5 & 2 & 2 & 1 & 0 & 1 & 0 \\ 41 & 25 & 25 & 16 & 15 & 9 & 9 & 4 & 5 & 2 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

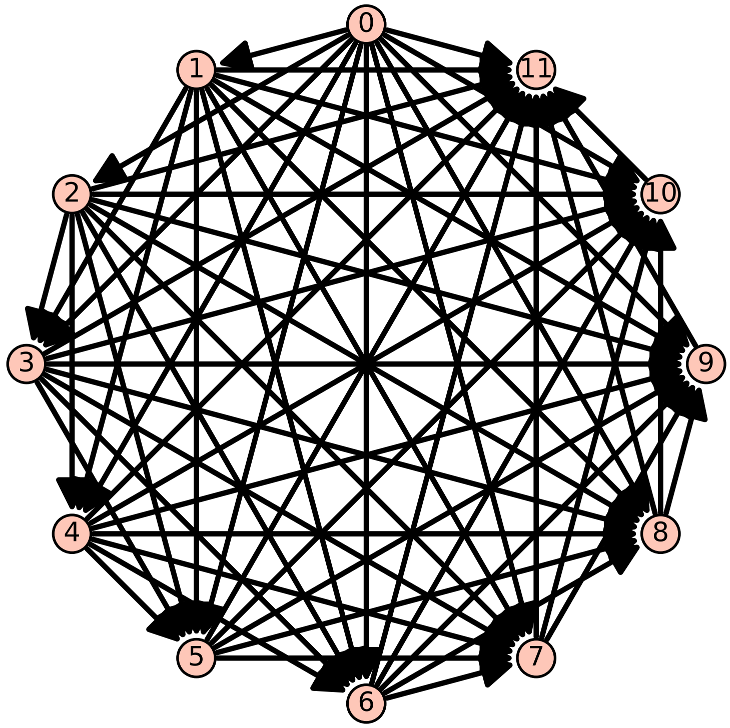

FIGURE 38. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $11 = |\mathcal{E}| - 1$.

1.87. **Polytope F.4D.0086.** Let P denote the polytope F.4D.0086 in polymake, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

and vertices of P given by the columns of:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & -2 & -2 & 0 & 1 & -1 & -2 & -2 & 1 & -1 & 1 \\ 1 & 0 & -1 & 0 & -2 & 1 & 1 & 0 & -1 & 1 & 1 & 1 & 0 & -2 & -2 & 1 & 1 \\ -2 & 0 & 0 & 1 & 1 & -1 & 1 & 0 & 0 & -2 & -2 & -1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,1,1,0),(0,0,0,-1),(1,0,0,1),(1,-1,0,1),(1,0,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{12} \to S^{34} \to S^{33} \to S^{12} \to S^1 \to 0$$

There does not exist an ordering for the 18 line bundles:

$$\begin{aligned}\mathcal{E} = \{&(-2,-2,-2,-3),(-1,-2,-2,-3),(-2,-1,-1,-2),(-2,-2,-1,-3),(-2,-2,-1,-2),(-2,0,0,-1),\dots\\ &\dots(0,-2,-2,-2),(-1,-2,-2,-2),(-2,-1,0,-2),(-1,-2,-1,-2),(-2,-1,0,-1),(-1,-1,-1,-2),\dots\\ &\dots(-1,0,0,0),(-1,0,0,-1),(-1,-1,0,-1),(-1,-1,-1,-1),(0,-1,-1,-1),(0,0,0,0)\}\end{aligned}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,0,0,0)$ and $\mathcal{O}(-1,-2,-2,-3)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,0,0,0),\mathcal{O}(-1,-2,-2,-3)) = \begin{cases}\mathbb{C} & \text{in degree 2}\\ 0 & \text{else.}\end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(-1,-2,-2,-3),\mathcal{O}(-1,0,0,0)) = \begin{cases}\mathbb{C}^6 & \text{in degree 0}\\ 0 & \text{else .}\end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.88. **Polytope F.4D.0087.** Let P denote the polytope F.4D.0087 in the polymake database, with half-space representation

$$\begin{bmatrix}1 & -1 & 0 & 0 & 0\\ 1 & 0 & 1 & 0 & -1\\ 1 & 0 & -1 & 0 & 0\\ 1 & 0 & -1 & 1 & 1\\ 1 & 0 & 0 & -1 & 0\\ 1 & 0 & -1 & 0 & 1\\ 1 & 0 & 0 & 0 & -1\\ 1 & 1 & 0 & 0 & 1\end{bmatrix}$$

and vertices of P given by the columns of :

$$\begin{pmatrix}-1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1\\ 1 & 1 & 1 & 1 & -1 & -3 & -3 & -1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1\\ 0 & 0 & 1 & 1 & 0 & -2 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(1,0,0,1),(0,-1,0,1),(0,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix}0 & 1 & 0 & -1 & 1 & 0 & 0 & 0\\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0\\ 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0\\ 0 & -1 & 1 & 1 & 0 & 0 & 0 & 1\end{pmatrix}} \mathbb{Z}^4.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0.$$

The 16 line bundles appearing on the left-hand side of the Hanlon-Hicks-Lazarev resolution which do give a full strong exceptional collection of line bundles are:

$$\begin{aligned}\mathcal{E} = \{&(-1,-2,0,-1),(-1,-2,0,0),(0,-2,0,-1),(-1,-1,0,-1),(0,-2,-1,-2),(0,-2,0,0),\dots\\ &\dots(-1,0,0,-1),(0,-1,-1,-2),(0,-2,-1,-1),(0,-1,0,-1),(-1,-1,0,0),(-1,0,0,0),\dots\\ &\dots(0,0,0,-1),(0,-1,-1,-1),(0,-1,0,0),(0,0,0,0)\}.\end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i,E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 2 & 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 3 & 3 & 1 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
12 & 6 & 6 & 3 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
15 & 7 & 6 & 3 & 3 & 3 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 3 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
11 & 3 & 7 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
15 & 5 & 9 & 3 & 6 & 2 & 2 & 0 & 0 & 0 & 3 & 1 & 1 & 0 & 0 & 0 \\
24 & 11 & 15 & 7 & 6 & 6 & 2 & 3 & 2 & 2 & 3 & 3 & 1 & 1 & 0 & 0 \\
18 & 9 & 6 & 3 & 6 & 3 & 3 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
30 & 15 & 18 & 9 & 12 & 6 & 6 & 5 & 3 & 2 & 6 & 3 & 3 & 1 & 2 & 1
\end{pmatrix}
$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

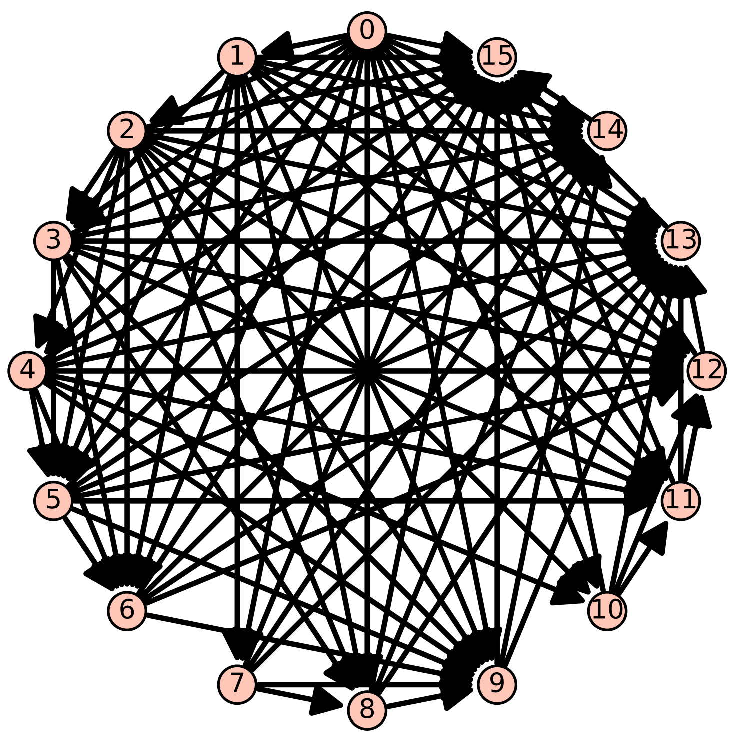

FIGURE 39. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.89. **Polytope F.4D.0088.** Let P denote the polytope F.4D.0088 in the polymake database, with half-space representation

$$
\begin{bmatrix}
1 & 0 & 0 & -1 & 0 \\
1 & 0 & 1 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 \\
1 & 0 & -1 & 1 & 0 \\
1 & 0 & 0 & -1 & 1 \\
1 & -1 & 0 & 0 & 0 \\
1 & 1 & -1 & 1 & 0 \\
1 & 0 & 0 & 0 & -1
\end{bmatrix}
$$

and vertices of P given by the columns of:

$$\begin{pmatrix} -3 & 1 & -1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & -3 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -2 & -2 & 0 & 0 & 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -3 & -3 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,-1,1,0),(1,-1,1,0),(0,0,0,-1),(0,0,-1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

The 16 line bundles appearing on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal which do give a full strong exceptional collection of line bundles are:

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-1,-2),(-1,-1,0,-2),(0,-1,-1,-2),(-1,-1,-1,-1),(-1,0,-1,-1),(0,-1,-1,-1),\dots \\ &\dots(0,-1,0,-2),(-1,-1,0,-1),(-1,0,0,-1),(-1,0,-1,0),(0,0,-1,-1),(0,0,0,-1),\dots \\ &\dots(0,0,-1,0),(-1,0,0,0),(0,-1,0,-1),(0,0,0,0)\}. \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 4 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 5 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 2 & 4 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 2 & 5 & 2 & 2 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 14 & 5 & 5 & 5 & 4 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 12 & 7 & 7 & 4 & 0 & 4 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 16 & 9 & 9 & 4 & 4 & 5 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 \\ 23 & 14 & 9 & 9 & 7 & 5 & 5 & 4 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 \\ 23 & 9 & 14 & 9 & 7 & 5 & 2 & 2 & 5 & 4 & 2 & 1 & 1 & 0 & 1 & 0 \\ 37 & 23 & 23 & 16 & 12 & 14 & 9 & 7 & 9 & 7 & 4 & 5 & 4 & 2 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

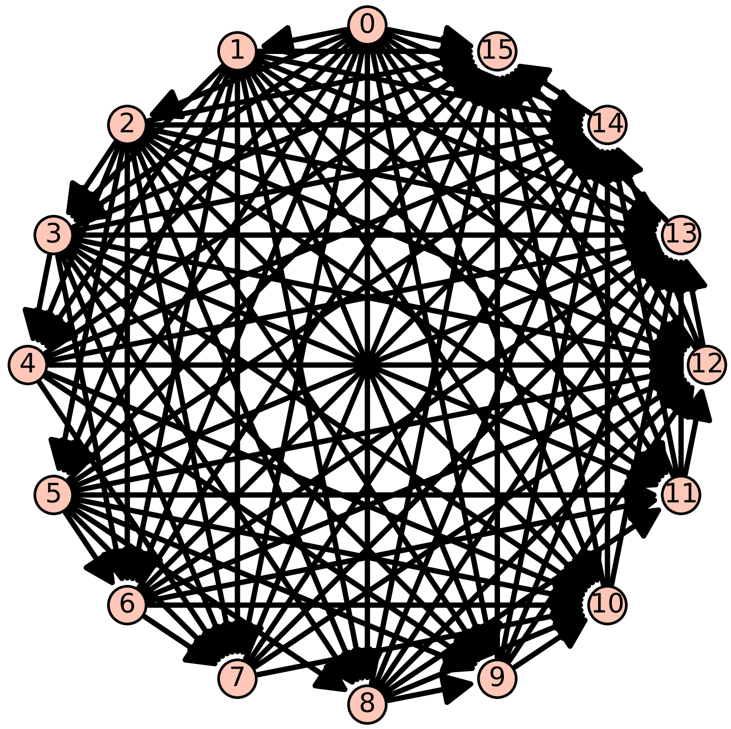

FIGURE 40. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.90. **Polytope F.4D.0089.** Let P denote the polytope F.4D.0089 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & -1 & 0 \end{bmatrix}$$

and vertices of P given by the columns of

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -4 & -2 & -1 & 0 & 1 & 0 & 1 \\ -2 & 1 & 1 & 1 & 0 & -1 & 0 & -2 & 1 & 1 & 1 & 0 & -2 & -1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & -3 & -3 & -2 & 1 & 1 & 1 & 1 \\ 0 & -3 & -3 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(1,0,-1,0),(-1,0,1,0),(-1,1,1,0),(0,0,0,-1),(0,1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} -1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^4.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^{10} \to S^{31} \to S^{32} \to S^{12} \to S^1 \to 0$$

There does not exist an ordering for the 17 line bundles:

$$\mathcal{E} = \{(1,-1,-2,-4),(0,-1,-2,-3),(0,-1,-1,-3),(0,0,-1,-3),(1,-1,-2,-3),(0,-1,-2,-2),\dots$$
$$\dots(1,-1,-2,-2),(0,0,0,-2),(1,-1,-1,-3),(0,0,-1,-2),(0,-1,-1,-2),(1,-1,-1,-2),\dots$$
$$\dots(0,0,-1,-1),(1,-1,-1,-1),(0,0,0,-1),(0,-1,-1,-1),(0,0,0,0)\}$$

which appear on one side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(1,-1,-1,-1)$ and $\mathcal{O}(0,-1,-1,-3)$ with non-zero Hom's in both directions. That is,

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(1,-1,-1,-1),\mathcal{O}(0,-1,-1,-3)) = \begin{cases} \mathbb{C} & \text{in degree } 2 \\ 0 & \text{else.} \end{cases}$$

and

$$Hom^\bullet_{D^b(X)}(\mathcal{O}(0,-1,-1,-3),\mathcal{O}(1,-1,-1,-1)) = \begin{cases} \mathbb{C}^6 & \text{in degree } 0 \\ 0 & \text{else .} \end{cases}$$

Therefore, there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.91. **Polytope F.4D.0090.** Let P denote the polytope F.4D.0090 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

and vertices of P given by the columns of:

$$\begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -2 & -2 & 0 & 0 & 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -3 & -3 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,-1,1,0),(0,0,0,-1),(0,0,-1,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{13} \to S^{15} \to S^7 \to S^1 \to 0$$

16 line bundles appearing on left-hand side which do give an FSEC of line bundles are:

$$\mathcal{E} = \{(-1,-1,-1,-2),(0,-1,-1,-2),(-1,-1,-1,-1),(-1,-1,0,-1),(-1,0,-1,-2),(-1,-1,0,0),(-1,0,-1,-1),\dots$$
$$\dots(-1,0,0,-1),(0,-1,-1,-1),(0,-1,0,-1),(0,0,-1,-2),(-1,0,0,0),(0,-1,0,0),(0,0,-1,-1),\dots$$
$$\dots(0,0,0,-1),(0,0,0,0)\}$$

Here $Hom_{D^b_{Coh}(X)}(E_i,E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
7 & 4 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 4 & 4 & 3 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 2 & 2 & 0 & 3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 2 & 2 & 2 & 4 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\
10 & 6 & 4 & 0 & 5 & 2 & 0 & 0 & 3 & 2 & 0 & 1 & 0 & 0 & 0 & 0\\
14 & 8 & 4 & 4 & 7 & 2 & 2 & 0 & 4 & 2 & 2 & 1 & 1 & 0 & 0 & 0\\
18 & 8 & 8 & 6 & 9 & 2 & 2 & 2 & 4 & 4 & 3 & 1 & 1 & 1 & 0 & 0\\
14 & 9 & 7 & 5 & 0 & 4 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
28 & 18 & 14 & 10 & 14 & 8 & 6 & 4 & 9 & 7 & 5 & 4 & 3 & 2 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

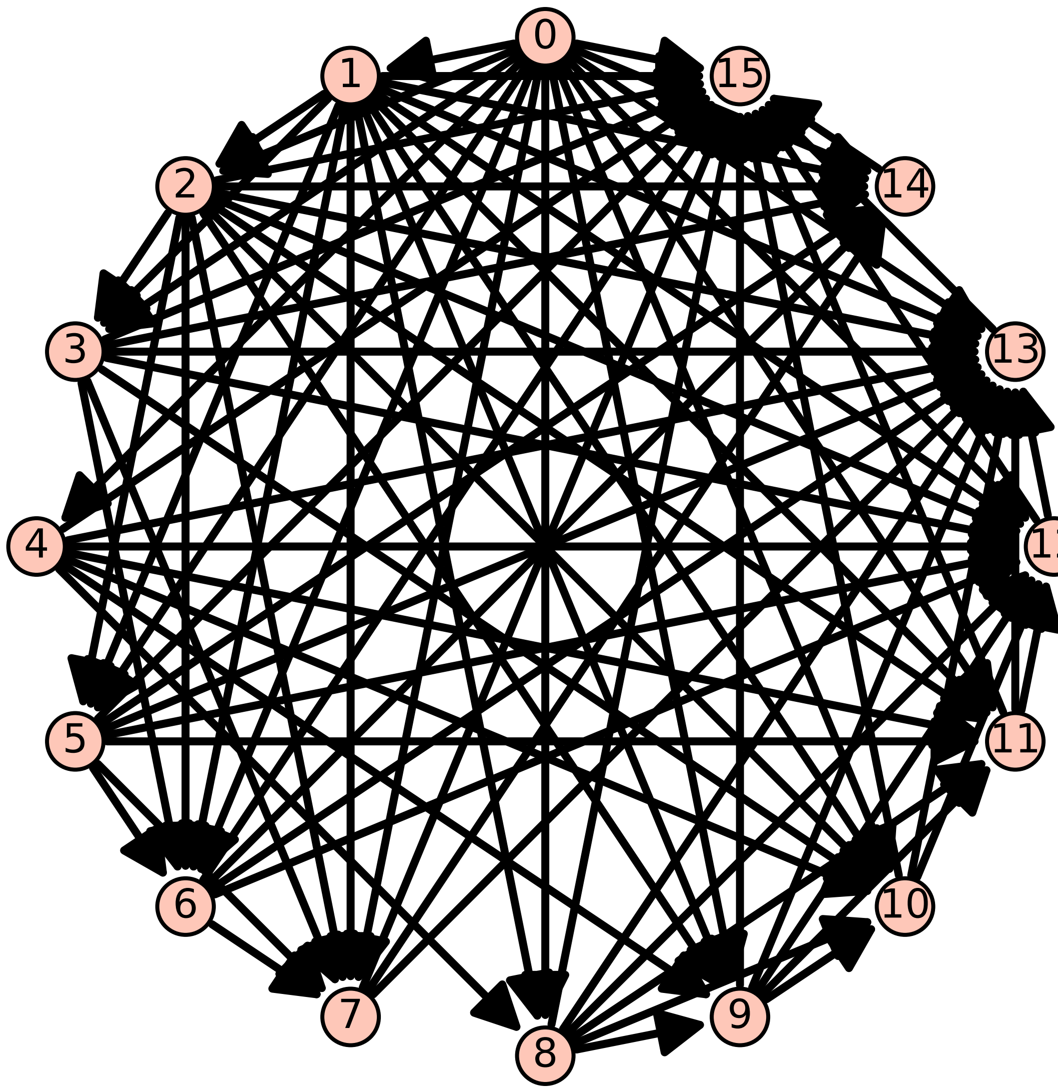

FIGURE 41. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.92. **Polytope F.4D.0091.** Let P denote the polytope F.4D.0091 in the polymake datbase, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices of P given by the columns of

$$\begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & -2 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(0,-1,0,1),(0,-1,1,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5.$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{13} \to S^{15} \to S^7 \to S^1 \to 0$$

The 20 line bundles appearing on the left-hand side which do give a full strong exceptional collection of line bundles are:

$$\begin{aligned}\mathcal{E} = \{&(-1,-1,0,-1,-2),(0,-1,0,-1,-2),(-1,-1,0,0,-1),(-1,-1,-1,0,-1),(-1,0,0,-1,-2),(-1,-1,0,-1,-1),\dots \\ &\dots(-1,-1,-1,0,0),(-1,-1,0,0,0),(-1,0,0,0,-1),(0,-1,0,0,-1),(-1,0,0,-1,-1),(0,-1,0,-1,-1),\dots \\ &\dots(0,-1,-1,0,-1),(0,0,0,-1,-2),(0,0,0,0,-1),(0,-1,0,0,0),(0,-1,-1,0,0),(-1,0,0,0,0),\dots \\ &\dots(0,0,0,-1,-1),(0,0,0,0,0)\}\end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 3 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 3 & 2 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 4 & 2 & 2 & 2 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 0 & 2 & 0 & 3 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 2 & 2 & 2 & 2 & 4 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 4 & 4 & 0 & 0 & 4 & 0 & 2 & 0 & 0 & 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
10 & 6 & 0 & 4 & 0 & 5 & 2 & 0 & 0 & 0 & 3 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
14 & 8 & 4 & 4 & 4 & 7 & 2 & 2 & 0 & 2 & 4 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\
12 & 4 & 6 & 4 & 2 & 6 & 0 & 2 & 2 & 0 & 2 & 2 & 3 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
10 & 6 & 5 & 4 & 2 & 0 & 2 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
20 & 12 & 10 & 8 & 4 & 10 & 4 & 6 & 4 & 2 & 6 & 4 & 5 & 2 & 3 & 2 & 1 & 2 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

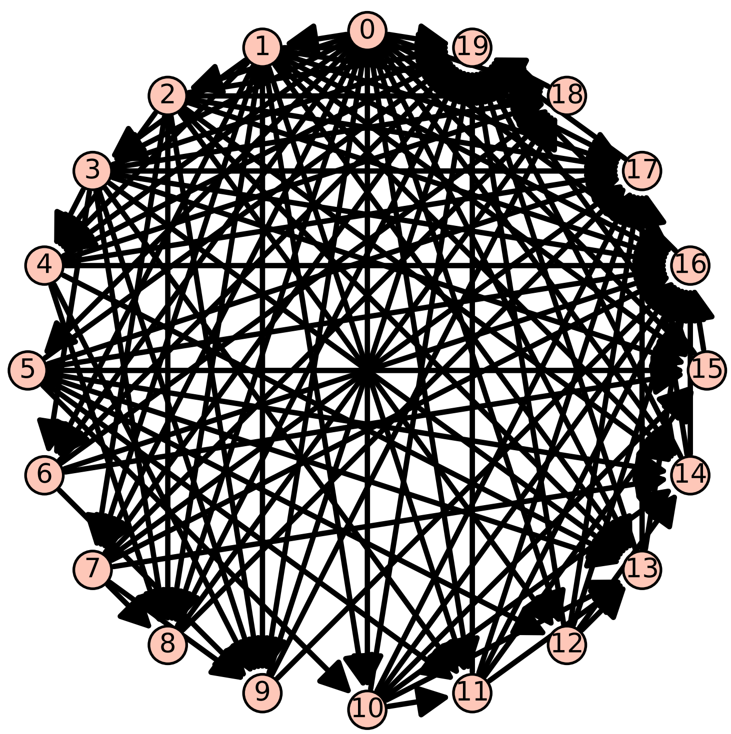

FIGURE 42. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.93. **Polytope F.4D.0092.** Let P denote the polytope F.4D.0092, with half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 1 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

and vertices of P given by the columns of

$$\begin{pmatrix} -2 & 1 & -2 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -2 & 1 & -1 & 1 & -2 & 1 \\ 1 & 1 & 1 & 1 & -2 & -2 & -2 & -2 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & -2 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -3 & -3 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(1,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(0,-1,0,1),(0,-1,1,1)\}$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^{16} \xrightarrow{\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

The 16 line bundles appearing on the left-hand side which do give a full strong exceptional collection of line bundles are:

$$\begin{aligned} \mathcal{E} = \{&(-2,2,-3,-4),(-2,1,-2,-3),(-2,2,-3,-3),(-1,1,-2,-3),(-2,2,-2,-3),(0,0,-1,-2),\dots \\ &\dots(-1,0,-1,-2),(-2,1,-2,-2),(-2,2,-2,-2),(-1,1,-2,-2),(-1,1,-1,-2),(0,0,-1,-1),\dots \\ &\dots(-1,1,-1,-1),(0,0,0,-1),(-1,0,-1,-1),(0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 3 & 3 & 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 12 & 3 & 5 & 3 & 3 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 5 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 12 & 6 & 6 & 0 & 0 & 3 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 15 & 9 & 5 & 6 & 2 & 3 & 2 & 0 & 0 & 3 & 1 & 0 & 1 & 0 & 0 & 0 \\ 18 & 6 & 9 & 6 & 3 & 3 & 3 & 3 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 21 & 12 & 9 & 6 & 6 & 5 & 2 & 2 & 2 & 3 & 3 & 1 & 1 & 0 & 1 & 0 \\ 30 & 18 & 15 & 12 & 6 & 9 & 6 & 5 & 2 & 6 & 3 & 3 & 3 & 2 & 1 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

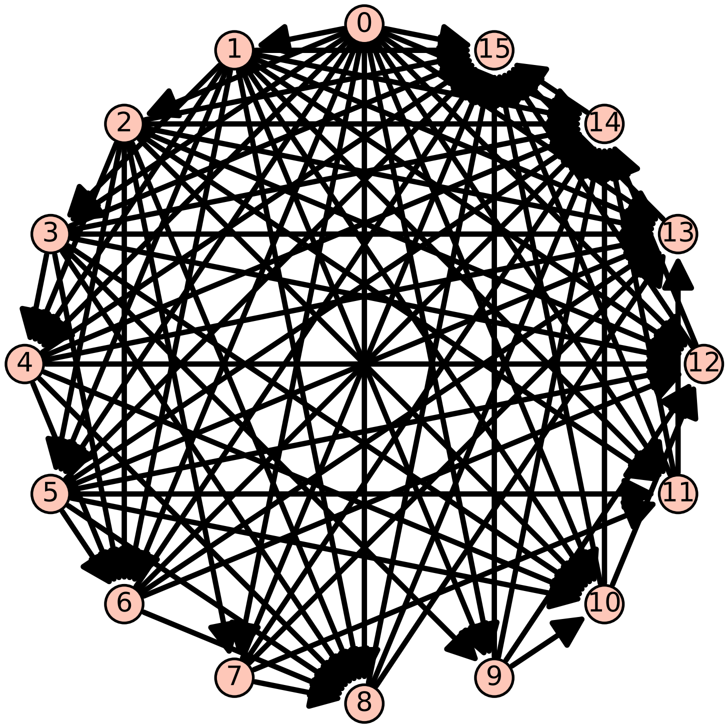

FIGURE 43. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$.

1.94. **Polytope F.4D.0093.** Let P denote the polytope F.4D.0093 in the polymake database, with half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

and vertices of P given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -2 & 1 & 1 & -2 & -2 & 1 & -2 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & -3 & 0 & 0 & -3 & 1 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 & -4 & -1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,-1,1,0),(0,0,0,-1),(1,0,0,1),(1,1,0,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

The collection of 12 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution yields the full strong exceptional collection

$$\begin{aligned}\mathcal{E} &= \{(-1,-2,-3),(0,-2,-3),(-1,-2,-2),(-1,-1,-2),(0,-2,-2),(0,-1,-2),(-1,-1,-1),(-1,0,-1),\ldots \\ &= \ldots,(0,-1,-1),(0,0,-1),(-1,0,0),(0,0,0)\}.\end{aligned}$$

Here, $Hom_{D^b(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 3 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 5 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 12 & 5 & 5 & 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 15 & 5 & 5 & 5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 19 & 12 & 9 & 5 & 5 & 3 & 2 & 0 & 1 & 0 & 0 & 0 \\ 25 & 15 & 9 & 9 & 5 & 5 & 2 & 2 & 1 & 1 & 0 & 0 \\ 31 & 15 & 15 & 12 & 5 & 5 & 5 & 3 & 1 & 1 & 1 & 0 \\ 47 & 31 & 25 & 19 & 15 & 12 & 9 & 5 & 5 & 3 & 2 & 1 \end{pmatrix}$$

The fact F is lower-triangular shows that $\mathcal{E}$ is exceptional. The quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$ for $i \neq j$ is then given by the directed graph in Figure 44. Starting from vertex 0 in $\mathcal{Q}$ (which corresponds to $\mathcal{O}_X(-1,-2,-3)$) and moving counter-clockwise, no arrows appear with vertex label of the head less than the vertex label of the tail.

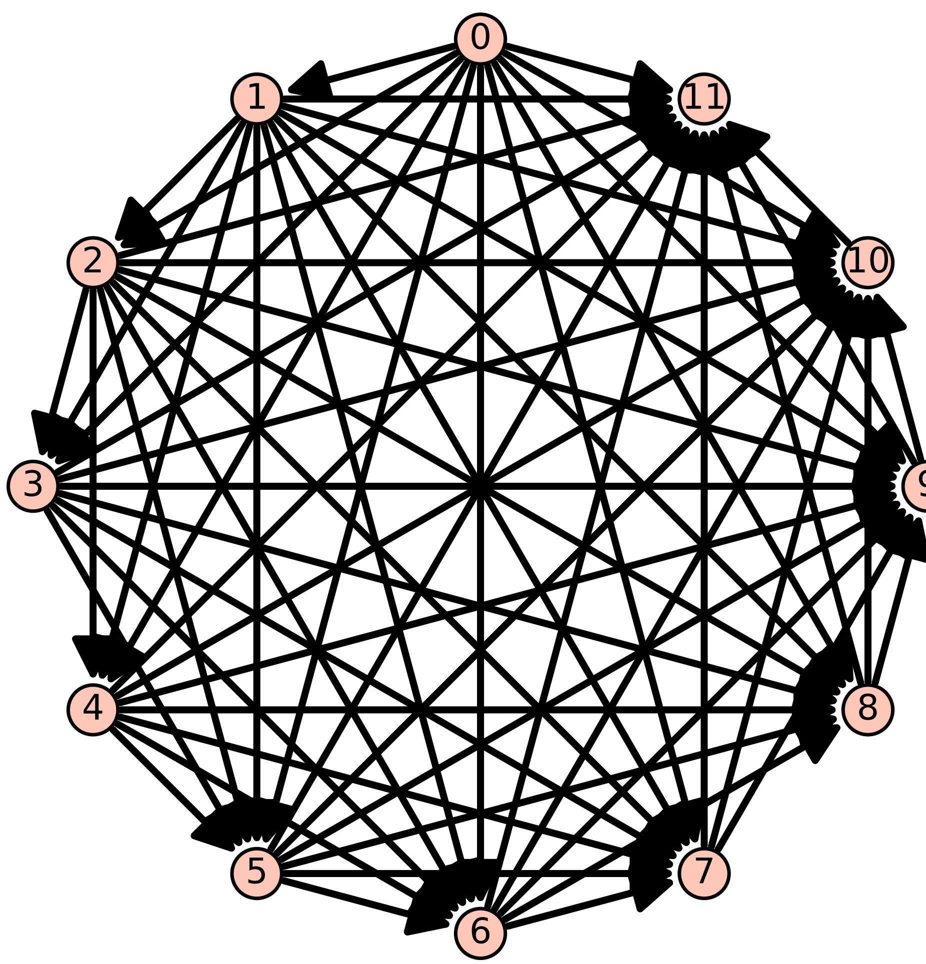

FIGURE 44. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $11 = |\mathcal{E}| - 1$.

1.95. **Polytope F.4D.0094.** The polytope P (denoted as F.4D.0094 in polymake) has polymake half-space representation:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The vertices of P are given by the columns of:

$$\begin{pmatrix} 1 & -1 & 1 & 1 & 1 & -1 & 1 & -2 & 1 & 1 & 1 & -2 & 1 & -4 & 1 \\ -1 & -1 & -1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -2 & -1 & 1 & 1 & 1 & 1 & -4 & -2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated with P. The rays of the fan Σ for X are:

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(0,1,0,1),(1,1,1,1)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1$$

The collection of 15 line bundles which appear on the left hand side of the Hanlon-Hicks-Lazarev resolution yield the full strong exceptional collection:

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-2,-4),(-1,-1,-2,-3),(-1,-1,-1,-3),(0,-1,-1,-3),(-1,0,-1,-3),(-1,-1,-2,-2),\ldots \\ &\ldots(-1,-1,-1,-2),(-1,0,-1,-2),(0,-1,-1,-2),(0,0,0,-2),(-1,-1,-1,-1),(0,-1,-1,-1),\ldots \\ &\ldots(-1,0,-1,-1),(0,0,0,-1),(0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 4 & 0 & 3 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 4 & 3 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 12 & 5 & 3 & 3 & 3 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 13 & 5 & 4 & 4 & 3 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 16 & 9 & 0 & 6 & 0 & 0 & 4 & 0 & 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 16 & 9 & 0 & 6 & 0 & 4 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 22 & 12 & 6 & 6 & 6 & 5 & 3 & 3 & 3 & 0 & 1 & 1 & 1 & 0 & 0 \\ 25 & 13 & 9 & 9 & 6 & 5 & 4 & 4 & 3 & 3 & 1 & 1 & 1 & 1 & 0 \\ 41 & 25 & 16 & 16 & 10 & 13 & 9 & 9 & 6 & 6 & 4 & 4 & 3 & 3 & 1 \end{pmatrix}$$

The fact F is lower-triangular shows that $\mathcal{E}$ is exceptional. The quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$ for $i \neq j$ is then given by the directed graph in Figure 44. Starting from vertex 0 in $\mathcal{Q}$ (which corresponds to $\mathcal{O}_X(-1,-2,-3)$) and moving counter-clockwise, no arrows appear with vertex label of the head less than the vertex label of the tail.

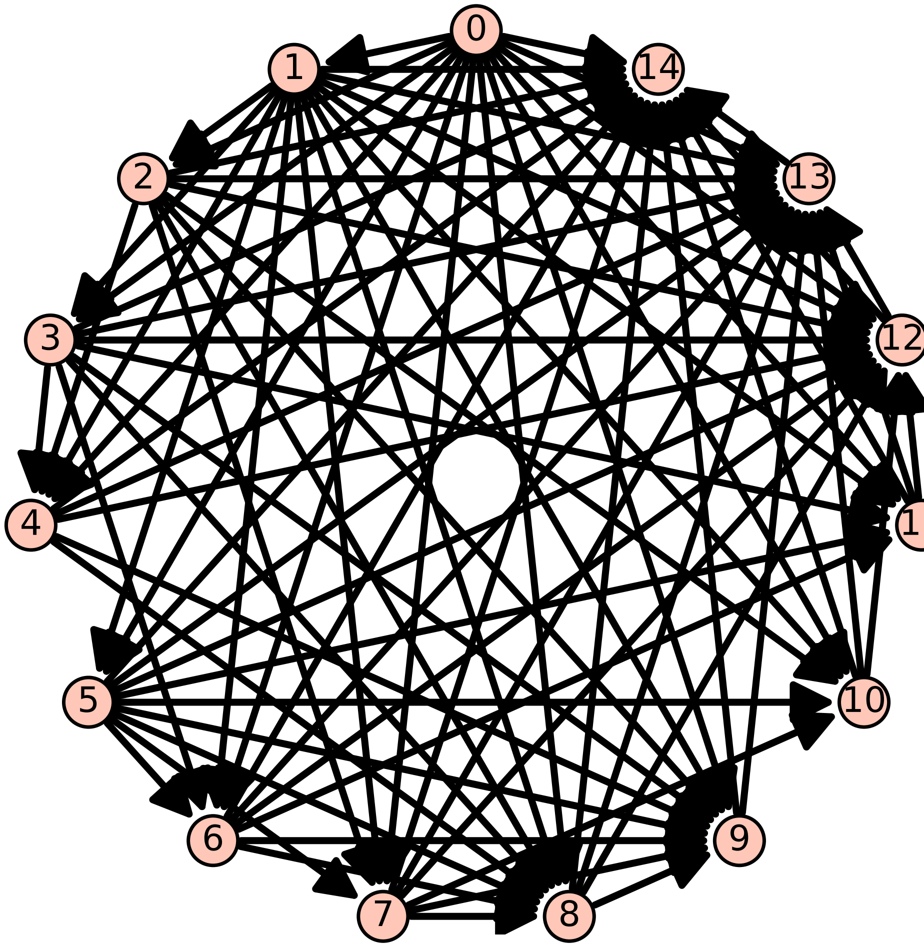

FIGURE 45. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $14 = |\mathcal{E}| - 1$.

1.96. **Polytope F.4D.0095.** The polytope P (denoted as F.4D.0095 in polymake) has polymake half space representation:

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The vertices of P are given by the columns of:

$$\begin{pmatrix} 1 & 1 & -2 & 1 & -2 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -3 & 1 & -3 & 1 \\ -1 & 1 & -1 & -1 & 1 & 1 & 0 & -1 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & 0 & 1 & 1 \\ -2 & -2 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -3 & -3 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated with P. The rays of the fan Σ for X are:

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(0,0,0,1),(0,-1,0,1),(1,0,1,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^9 \xrightarrow{\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}} \mathbb{Z}^5$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0$$

The collection of 18 unique line bundles which appear on the left hand side of the Hanlon Hicks resolution yield the full strong exceptional collection:

$$\mathcal{E} = \{(-1,0,-1,-1,-3), (-1,-1,-1,0,-3), (0,0,-1,-1,-3), (-1,0,-1,0,-3), (-1,-1,-1,0,-2), (-1,-1,0,0,-2), \ldots$$
$$\ldots(-1,0,-1,-1,-2), (-1,-1,-1,0,-1), (-1,-1,0,0,-1), (0,0,-1,-1,-2), (-1,0,-1,0,-2), (0,0,0,0,-2), \ldots$$
$$\ldots(-1,0,-1,-1,-1), (-1,-1,0,0,0), (0,0,-1,-1,-1), (-1,0,-1,0,-1), (0,0,0,0,-1), (0,0,0,0,0)\}$$

Here $Hom_{D^b(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$F = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 4 & 3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 4 & 0 & 3 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 2 & 1 & 1 & 3 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
12 & 5 & 3 & 3 & 3 & 3 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
12 & 6 & 0 & 0 & 6 & 0 & 2 & 0 & 0 & 3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
15 & 7 & 3 & 3 & 6 & 0 & 2 & 1 & 1 & 3 & 3 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
16 & 9 & 6 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
16 & 9 & 0 & 6 & 0 & 0 & 4 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
26 & 15 & 6 & 6 & 10 & 0 & 7 & 3 & 3 & 6 & 6 & 0 & 3 & 3 & 1 & 1 & 1 & 0 \\
22 & 12 & 6 & 6 & 6 & 6 & 5 & 3 & 3 & 3 & 0 & 3 & 1 & 0 & 1 & 1 & 0 & 1
\end{pmatrix}$$

The fact F is lower-triangular shows that $\mathcal{E}$ is exceptional. The quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$ for $i \neq j$ is then given by the directed graph in Figure 44. Starting from vertex 0 in $\mathcal{Q}$ (which corresponds to $\mathcal{O}_X(-1, -2, -3)$) and moving counter-clockwise, no arrows appear with vertex label of the head less than the vertex label of the tail.

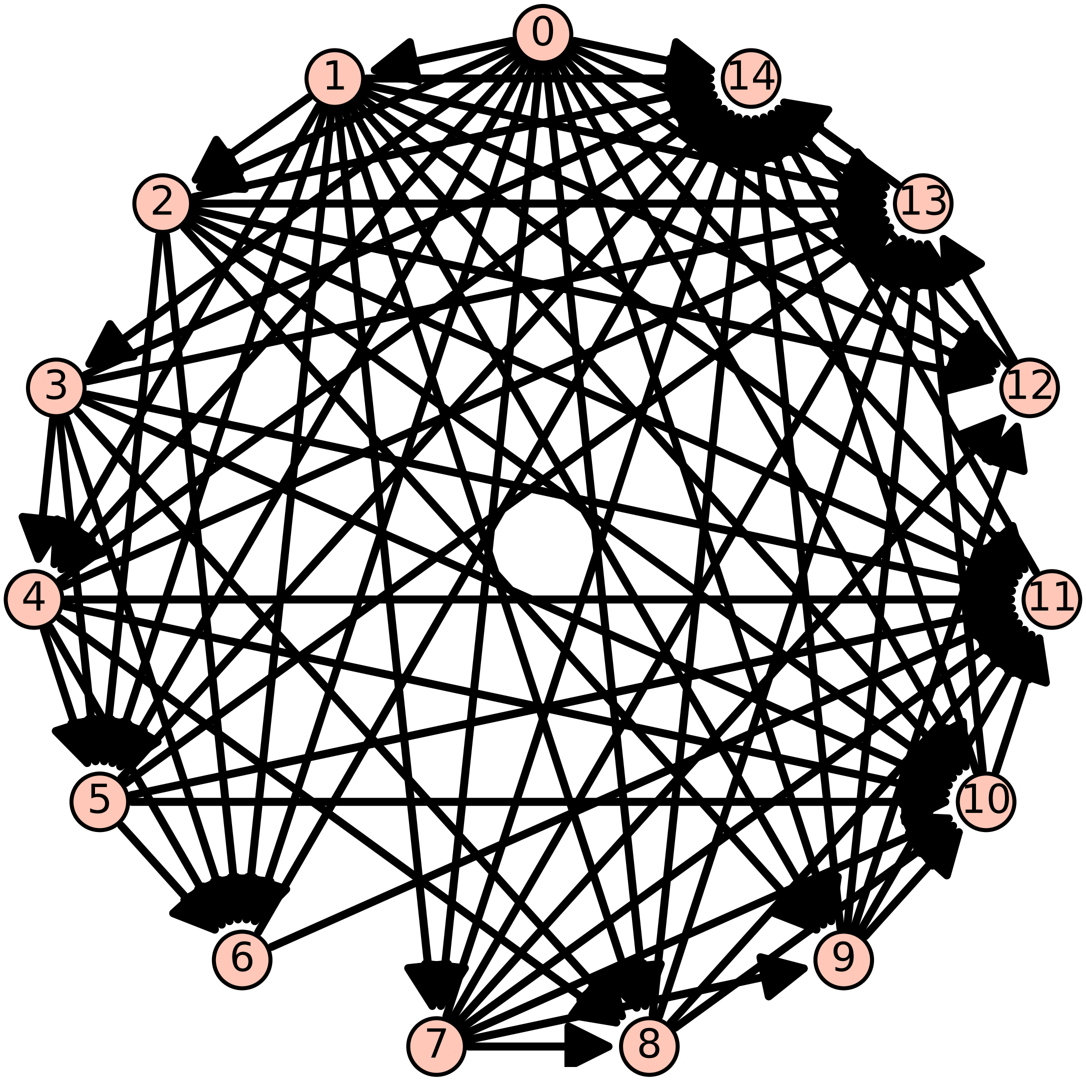

FIGURE 46. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $14 = |\mathcal{E}| - 1$.

1.97. **Polytope F.4D.0096.** The polytope P (denoted as F.4D.0096 in polymake) has polymake half space representation:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The vertices of P are given by the columns of:

$$\begin{pmatrix} 1 & -2 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -3 & 1 & -3 & 1 \\ 1 & 1 & 1 & 0 & -1 & 0 & 0 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 \\ -2 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -3 & -3 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated with P. The rays of the fan Σ for X are:

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(0,-1,0,1),(1,0,1,1)\}$$

The presentation of the class group of X is:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1$$

The collection of 15 line bundles which appear on the left hand side of the Hanlon-Hicks-Lazarev resolution yield the full strong exceptional collection:

$$\begin{aligned} \mathcal{E} = \{&(-1,0,-1,-1,-3), (-1,-1,-1,0,-3), (0,0,-1,-1,-3), (-1,0,-1,0,-3), (-1,-1,-1,0,-2), (-1,-1,0,0,-2), \dots \\ &\dots(-1,0,-1,-1,-2), (-1,-1,-1,0,-1), (-1,-1,0,0,-1), (0,0,-1,-1,-2), (-1,0,-1,0,-2), (0,0,0,0,-2), \dots \\ &\dots(-1,0,-1,-1,-1), (-1,-1,0,0,0), (0,0,-1,-1,-1), (-1,0,-1,0,-1), (0,0,0,0,-1), (0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 2 & 0 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 2 & 1 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 2 & 2 & 4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 12 & 6 & 0 & 6 & 3 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 4 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 15 & 7 & 3 & 6 & 3 & 3 & 0 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 18 & 8 & 6 & 9 & 4 & 3 & 3 & 2 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 16 & 9 & 6 & 0 & 0 & 0 & 0 & 4 & 0 & 3 & 0 & 0 & 1 & 0 & 0 \\ 26 & 15 & 6 & 10 & 6 & 6 & 0 & 7 & 3 & 3 & 3 & 0 & 1 & 1 & 0 \\ 32 & 18 & 12 & 16 & 9 & 6 & 6 & 8 & 4 & 6 & 3 & 3 & 2 & 1 & 1 \end{pmatrix}$$

The fact F is lower-triangular shows that $\mathcal{E}$ is exceptional. The quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$ for $i \neq j$ is then given by the directed graph in Figure 44. Starting from vertex 0 in $\mathcal{Q}$ (which corresponds to $\mathcal{O}_X(-1,-2,-3)$) and moving counter-clockwise, no arrows appear with vertex label of the head less than the vertex label of the tail.

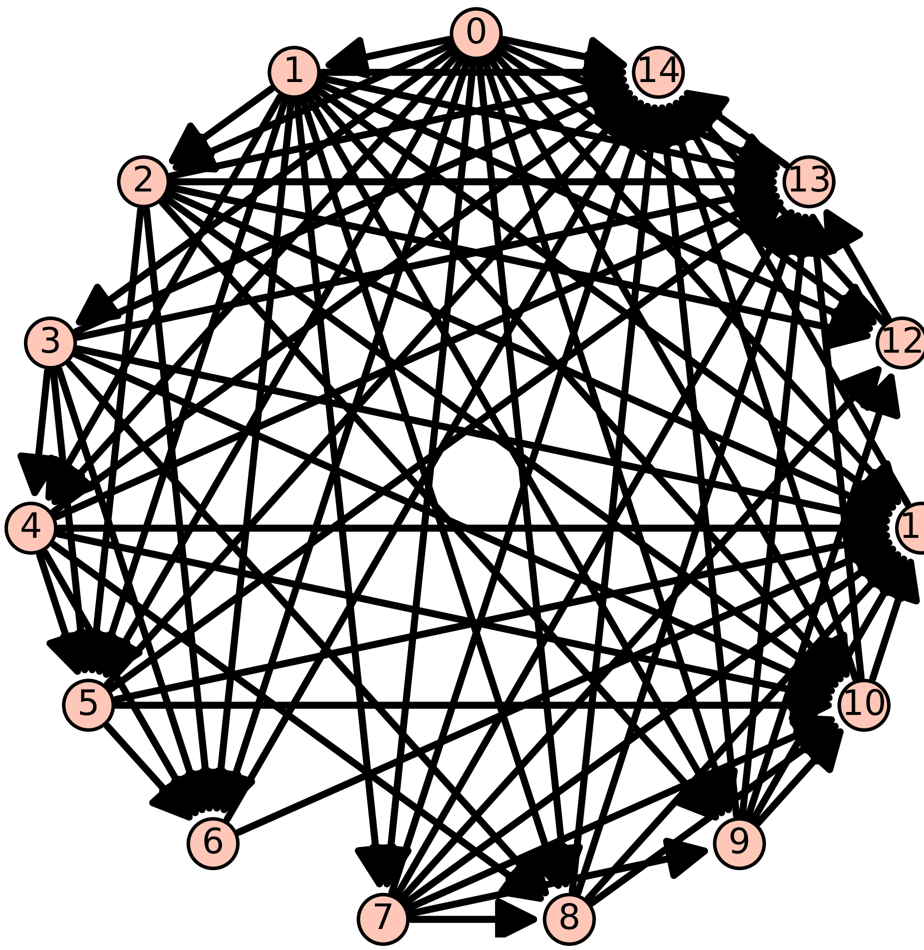

FIGURE 47. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.98. **Polytope F.4D.0097.** The polytope P (denoted as F.4D.0097 in polymake) has polymake half space representation:

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The vertices of P are given by the columns of:

$$\begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P. The rays of the fan Σ of X are:

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(0,0,0,1),(0,0,1,1)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^5$$

The free ranks that appear in the Hanlon-Hicks-Lazarev resolution for $\mathcal{O}_\Delta$ for $\Delta \subset X \times X$:

$$0 \to S^2 \to S^7 \to S^9 \to S^5 \to S^1 \to 0$$

The collection of 20 line bundles which appear on the left hand side of the Hanlon-Hicks-Lazarev resolution yield the full strong exceptional collection:

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-1,-1,-2), (-1,0,-1,-1,-2), (0,-1,-1,-1,-2), (-1,-1,-1,-1,-1), (-1,-1,0,-1,-1), (-1,-1,-1,0,-1), \ldots \\ &\ldots (0,-1,-1,-1,-1), (-1,0,-1,-1,-1), (-1,0,-1,0,-1), (-1,-1,0,0,0), (0,-1,-1,0,-1), (0,-1,0,-1,-1), \ldots \\ &\ldots, (-1,0,0,-1,-1), (0,0,-1,-1,-2), (0,-1,0,0,0), (-1,0,0,0,0), (0,0,-1,-1,-1), (0,0,-1,0,-1), \ldots \\ &\ldots (0,0,0,-1,-1), (0,0,0,0,0)\}. \end{aligned}$$

Here $Hom_{D^b(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$F = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 2 & 3 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 2 & 2 & 0 & 3 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 4 & 0 & 4 & 4 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
8 & 4 & 0 & 0 & 4 & 4 & 0 & 2 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
12 & 4 & 4 & 4 & 6 & 6 & 0 & 2 & 2 & 2 & 3 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\
8 & 4 & 4 & 2 & 0 & 4 & 2 & 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
8 & 4 & 4 & 2 & 4 & 0 & 2 & 2 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
16 & 8 & 8 & 4 & 8 & 8 & 4 & 4 & 4 & 4 & 4 & 4 & 2 & 2 & 2 & 2 & 1 & 2 & 2 & 1
\end{pmatrix}$$

The fact F is lower-triangular shows that $\mathcal{E}$ is exceptional. The quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$ for $i \neq j$ is then given by the directed graph in Figure 44. Starting from vertex 0 in $\mathcal{Q}$ (which corresponds to $\mathcal{O}_X(-1,-2,-3)$) and moving counter-clockwise, no arrows appear with vertex label of the head less than the vertex label of the tail.

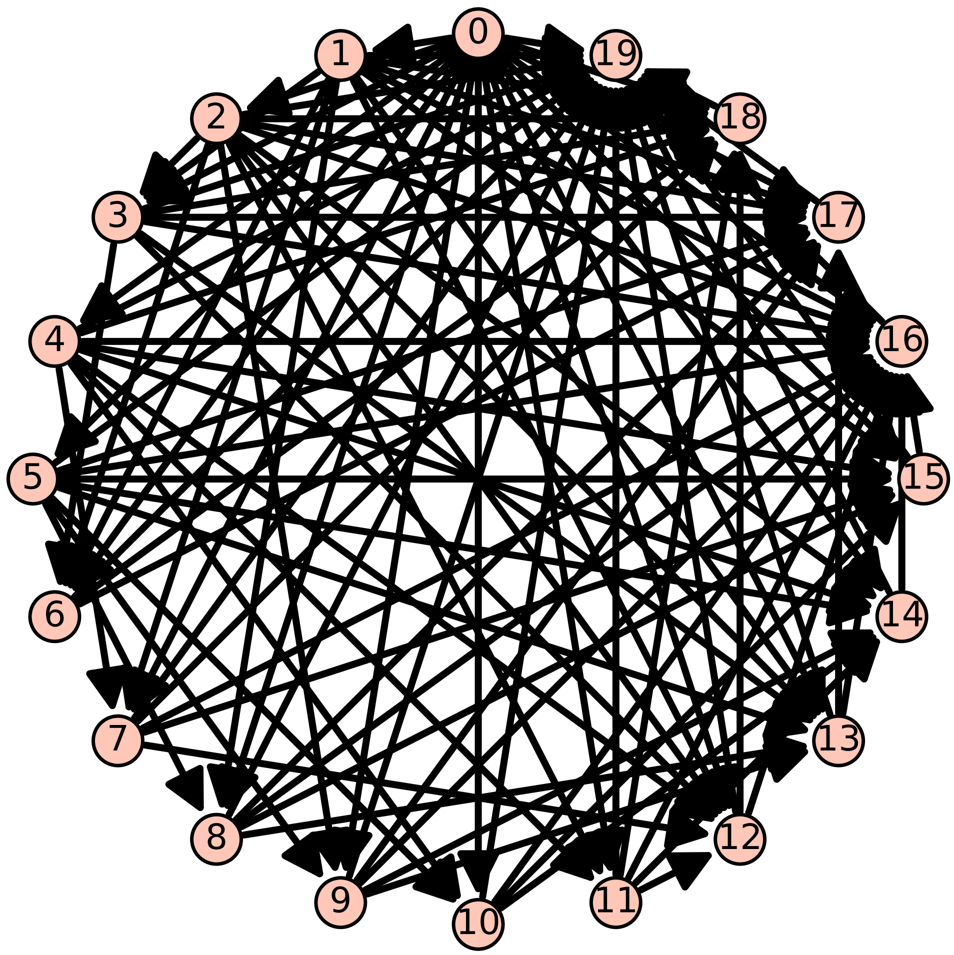

FIGURE 48. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $19 = |\mathcal{E}| - 1$.

1.99. **Polytope F.4D.0098.** The polytope P (denoted as F.4D.0098 in polymake) has half-space representation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

The vertices of P are given by the columns of:

$$\begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P. The rays of the fan Σ for X are:

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(0,0,1,-1),(0,0,0,1),(0,0,-1,1)\}.$$

We use as presentation of the class group of X is:

$$\mathbb{Z}^{10} \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^{6}$$

The free ranks that appear in the Hanlon-Hicks-Lazarev resolution for $\mathcal{O}_\Delta$ for $\Delta \subset X \times X$ is:

$$0 \to S^2 \to S^7 \to S^9 \to S^5 \to S^1 \to 0$$

The collection of 24 line bundles which appear on the left hand side of the Hanlon-Hicks-Lazarev resolution yield the full strong exceptional collection:

$$\begin{aligned} \mathcal{E} = \{ & (-1,-1,-1,0,-1,-1), (-1,-1,-1,-1,-1,0), (-1,0,-1,-1,-1,0), (-1,0,-1,0,-1,-1), (0,-1,-1,-1,-1,0), \dots \\ & \dots (0,-1,-1,0,-1,-1), (-1,-1,-1,-1,0,0), (-1,-1,0,0,-1,-1), (-1,-1,-1,0,-1,0), (0,-1,-1,-1,0,0), \dots \\ & \dots (-1,0,0,0,-1,-1), (-1,-1,0,0,0,0), (-1,0,-1,0,-1,0), (-1,0,-1,-1,0,0), (0,-1,0,0,-1,-1), \dots \\ & \dots (0,-1,-1,0,-1,0), (0,0,-1,-1,-1,0), (0,0,-1,0,-1,-1), (0,0,0,0,-1,-1), (0,-1,0,0,0,0), \dots \\ & \dots (0,0,-1,0,-1,0), (-1,0,0,0,0,0), (0,0,-1,-1,0,0), (0,0,0,0,0,0)\} \end{aligned}$$

Here $Hom_{D^b_{Coh}(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following matrix:

$$F = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 4 & 4 & 4 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 4 & 0 & 4 & 4 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 4 & 4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 0 & 2 & 3 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
6 & 2 & 3 & 2 & 0 & 2 & 0 & 2 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
6 & 2 & 3 & 2 & 0 & 2 & 0 & 2 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
12 & 4 & 6 & 4 & 6 & 4 & 0 & 4 & 2 & 2 & 2 & 2 & 3 & 2 & 2 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 1 & 0 \\
12 & 4 & 6 & 4 & 6 & 4 & 4 & 0 & 2 & 2 & 2 & 2 & 3 & 2 & 2 & 1 & 1 & 1 & 2 & 0 & 2 & 0 & 0 & 1
\end{pmatrix}$$

The fact F is lower-triangular shows that $\mathcal{E}$ is exceptional. The quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$ for $i \neq j$ is then given by the directed graph in Figure 44. Starting from vertex 0 in $\mathcal{Q}$ (which corresponds to $\mathcal{O}_X(-1,-2,-3)$) and moving counter-clockwise, no arrows appear with vertex label of the head less than the vertex label of the tail.

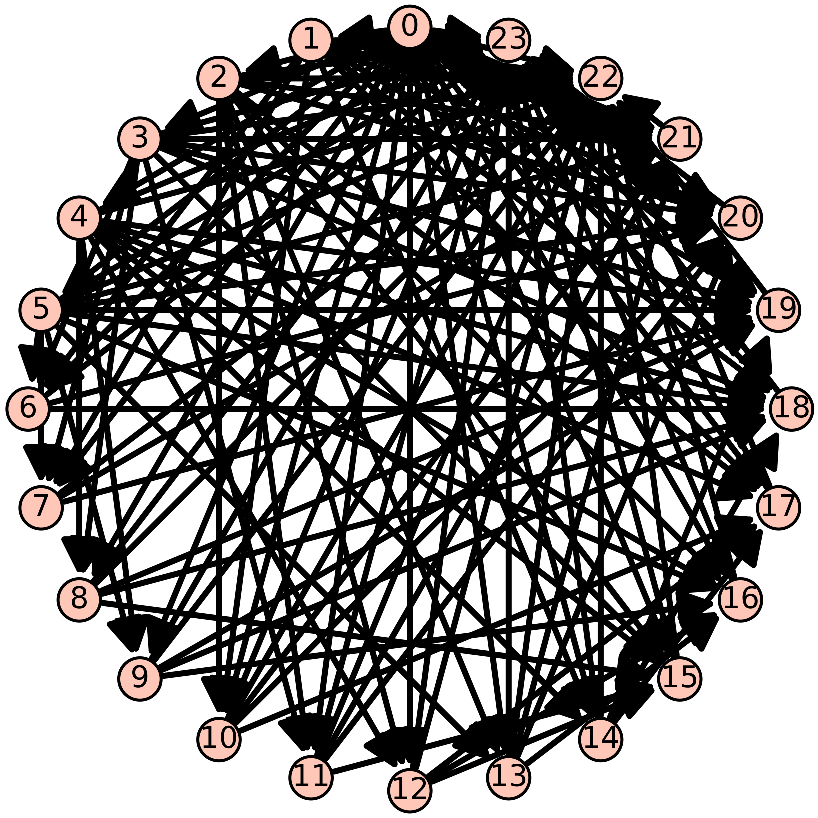

FIGURE 49. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $23 = |\mathcal{E}| - 1$.

1.100. **Polytope F.4D.0099.** The polytope P (denoted as F.4D.0099 in polymake) has half-space representation:

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

The vertices of P are given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 \\ -1 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated with P. The primitive ray generators of the fan Σ for X are:

$$\{(-1,0,0,0), (0,-1,0,0), (0,1,0,0), (0,0,-1,0), (0,1,-1,0), (0,0,1,0), (0,-1,1,0), (0,0,0,-1), (1,0,0,1)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^9 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^5.$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{12} \to S^{13} \to S^6 \to S^1$$

The collection of 18 line bundles which appear on the left hand side of the Hanlon-Hicks-Lazarev resolution yield the full strong exceptional collection:

$$\begin{aligned}
&\{(-1,0,-1,-1,-2),(-1,-1,-1,0,-2),(-1,-1,0,0,-2),(0,0,-1,-1,-2),(-1,0,-1,0,-2),\dots\\
&\quad\dots(-1,0,-1,-1,-1),(-1,-1,-1,0,-1),(0,0,-1,-1,-1),(-1,0,-1,0,-1),(-1,-1,0,0,-1),\dots\\
&\quad\dots(0,0,0,0,-2),(-1,-1,-1,0,0),(-1,0,-1,-1,0),(0,0,-1,-1,0),(-1,0,-1,0,0),\dots\\
&\quad\dots(-1,-1,0,0,0),(0,0,0,0,-1),(0,0,0,0,0)\}
\end{aligned}$$

Here $Hom_{D^b(X)}(E_i, E_j)$ is concentrated in degree 0 for all i and j, with the $Hom^0(E_i, E_j)$ given by the (i,j) entry of the following matrix:

$$F = \begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
2&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
3&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\
3&0&1&1&1&0&1&0&0&0&0&0&0&0&0&0&0&0\\
6&3&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
6&2&3&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
6&2&0&3&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
6&2&0&0&3&0&0&0&0&0&1&0&0&0&0&0&0&0\\
9&3&3&3&3&0&3&0&1&1&1&1&0&0&0&0&0&0\\
9&3&3&3&3&3&0&0&1&1&1&0&1&0&0&0&0&0\\
12&6&0&6&0&0&0&2&0&3&0&0&0&1&0&0&0&0\\
12&6&0&0&6&0&0&2&0&0&3&0&0&0&1&0&0&0\\
12&6&6&0&0&0&0&2&3&0&0&0&0&0&0&1&0&0\\
18&9&6&6&6&0&6&3&3&3&3&3&0&1&1&1&1&0\\
18&9&6&6&6&6&0&3&3&3&3&0&3&1&1&1&0&1
\end{pmatrix}$$

The fact F is lower-triangular shows that $\mathcal{E}$ is exceptional. The quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$ for $i \neq j$ is then given by the directed graph in Figure 44. Starting from vertex 0 in $\mathcal{Q}$ (which corresponds to $\mathcal{O}_X(-1,-2,-3)$) and moving counter-clockwise, no arrows appear with vertex label of the head less than the vertex label of the tail.

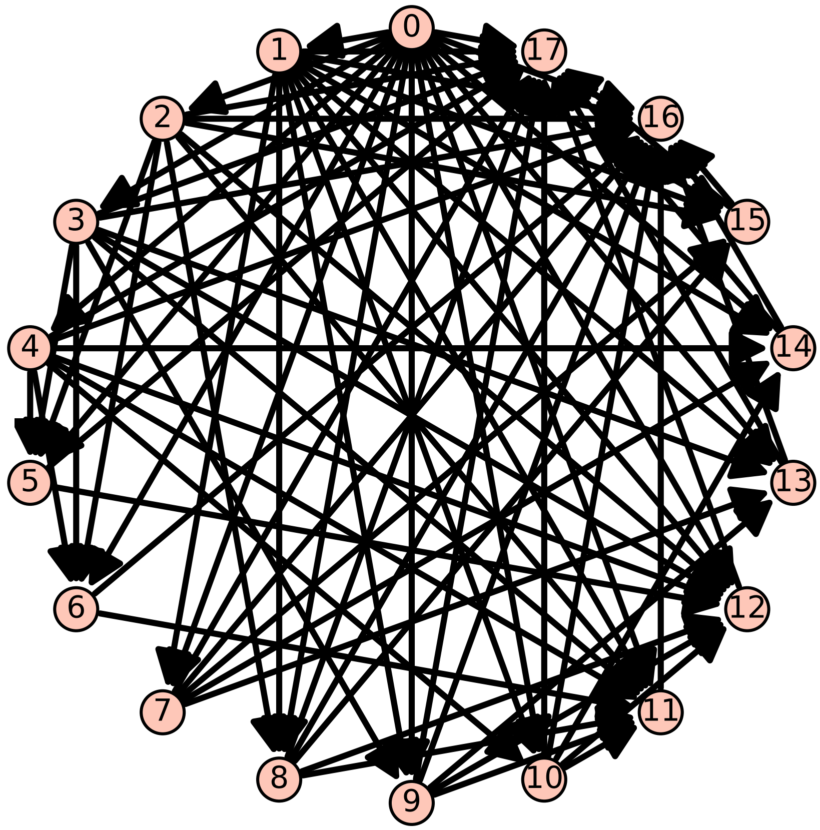

FIGURE 50. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $17 = |\mathcal{E}| - 1$.

1.101. **Polytope F.4D.0100.** The polytope P (denoted as F.4D.0100 in polymake) has half-space representation:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

and vertices given by the columns of:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & -3 & 1 & -2 & 1 & -1 & 1 & -3 & 1 & -1 & 1 \\ -1 & -1 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -3 & -3 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated with P with primitive ray generators:

$$\{(-1,0,0,0), (0,-1,0,0), (0,1,0,0), (0,0,-1,0), (0,0,1,0), (0,1,1,0), (0,0,0,-1), (1,-1,0,1)\}.$$

We use as presentation of $Cl(X)$

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^4.$$

The Hanlon-Hicks-Lazarev resolution of the diagonal yields the free ranks:

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0$$

and the full strong exceptional collection of 15 line bundles $\mathcal{O}_X(a_1, a_2, a_3, a_4)$ for (a_1, a_2, a_3, a_4) in:

$$\begin{aligned}\mathcal{E} = \{&(-1,-1,-2,-2),(-1,-1,-1,-2),(-1,0,-1,-2),(-1,-1,-2,-1),(0,-1,-1,-2),(-1,-1,-1,-1),\ldots\\ &\ldots(-1,0,-1,-1),(0,0,0,-2),(-1,-1,-2,0),(0,-1,-1,-1),(-1,-1,-1,0),(0,-1,-1,0),\ldots\\ &\ldots(-1,0,-1,0),(0,0,0,-1),(0,0,0,0)\}\end{aligned}$$

The full quiver is given by the fact that for the above ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0 with the rank of $Hom(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 2 & 0 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 2 & 2 & 4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 4 & 3 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
12 & 5 & 3 & 3 & 3 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\
12 & 6 & 0 & 6 & 0 & 3 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
18 & 8 & 6 & 9 & 3 & 4 & 3 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0\\
16 & 9 & 6 & 0 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 & 1 & 0 & 0\\
22 & 12 & 6 & 6 & 6 & 3 & 0 & 5 & 3 & 3 & 1 & 0 & 1 & 1 & 0\\
32 & 18 & 12 & 16 & 6 & 9 & 6 & 8 & 6 & 3 & 4 & 3 & 2 & 1 & 1
\end{pmatrix}$$

i.e., the (i, j) entry of this matrix holds the rank of $Hom^0(E_j, E_i)$. The quiver $\mathcal{Q}$ showing nonzero $Hom(E_j, E_i)$ for $i \neq j$ is given by the directed graph:

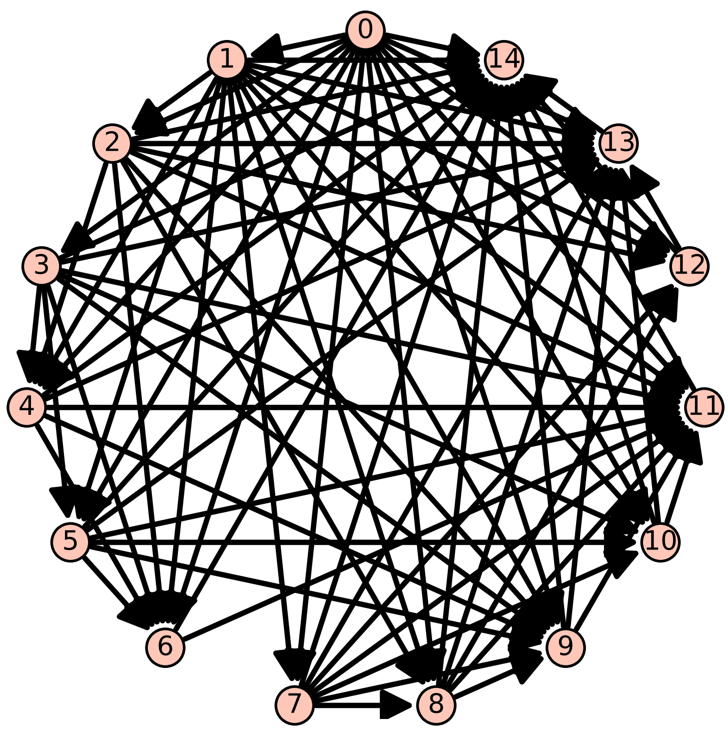

FIGURE 51. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $14 = |\mathcal{E}| - 1$

Here, the property that $\mathcal{E}$ is exceptional is visible from the fact that F is lower triangular.

1.102. **Polytope F.4D.0101.** The polytope P (denoted as F.4D.0101 in polymake) has half-space representation:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

with vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 \\ -1 & 0 & 1 & -1 & 1 & -1 & -1 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 \\ 0 & -1 & -1 & 1 & 1 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated with P, with primitive ray generators

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,1,0),(0,1,1,0),(0,0,0,-1),(1,0,0,1)\}.$$

We use as presentation of the class group of X:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^4.$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{12} \to S^{13} \to S^6 \to S^1 \to 0$$

and the full strong exceptional collection of 15 line bundles $\mathcal{O}_X(a_1,a_2,a_3,a_4)$ for (a_1,a_2,a_3,a_4) in:

$$\begin{aligned} \mathcal{E} = \{&(-1,-1,-2,-2),(-1,-1,-1,-2),(0,-1,-1,-2),(-1,0,-1,-2),(-1,-1,-2,-1),(-1,-1,-1,-1),\dots \\ &\dots(-1,0,-1,-1),(0,-1,-1,-1),(0,0,0,-2),(-1,-1,-2,0),(-1,-1,-1,0),(-1,0,-1,0),\dots \\ &\dots(0,-1,-1,0),(0,0,0,-1),(0,0,0,0)\} \end{aligned}$$

The full quiver is given by the fact that via the above ordering on $\mathcal{E}$, $Hom(E_i,E_j)$ is concentrated in degree 0 and the rank of $Hom(E_j,E_i)$ is given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 2 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 3 & 3 & 3 & 3 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 12 & 4 & 6 & 6 & 3 & 3 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 \\ 12 & 6 & 0 & 6 & 0 & 0 & 2 & 0 & 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 12 & 6 & 6 & 0 & 0 & 0 & 2 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 18 & 9 & 6 & 6 & 6 & 0 & 3 & 3 & 3 & 3 & 0 & 1 & 1 & 1 & 0 \\ 24 & 12 & 12 & 12 & 6 & 6 & 4 & 6 & 6 & 3 & 3 & 2 & 2 & 1 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom(E_j,E_i)$ for $i \neq j$ is given by the directed graph:

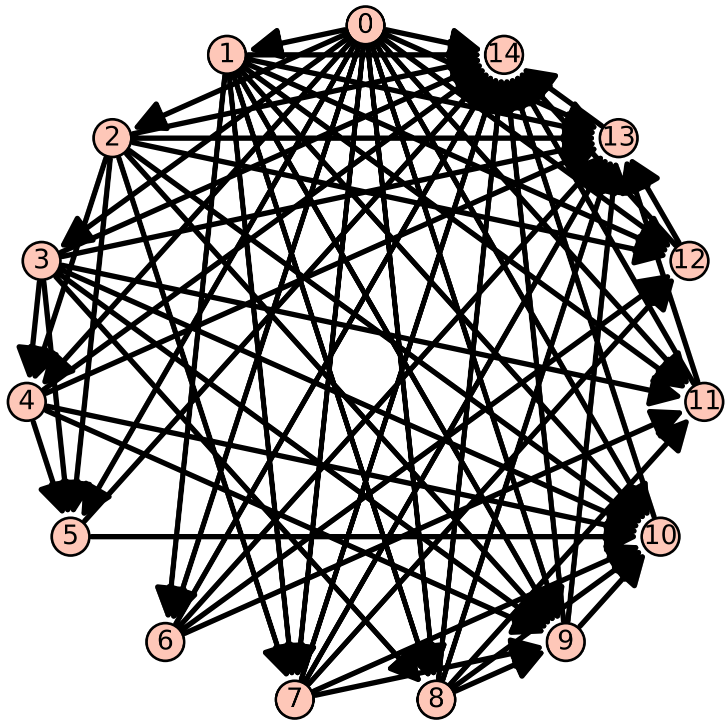

FIGURE 52. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $14 = |\mathcal{E}| - 1$

Here, the property that $\mathcal{E}$ is exceptional is visible from the fact that F is lower triangular.

1.103. **Polytope F.4D.0102.** The polytope P (denoted as F.4D.0102 in polymake) has polymake half-space representation:

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

with vertices given by the columns of

$$\begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -2 & -2 & -1 & -1 & 1 & 1 & -2 & -2 & 1 & 1 \\ 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated with P. Then X has primitive ray generators:

$$\{(-1,0,0,0), (1,0,0,0), (0,-1,0,0), (0,0,-1,0), (0,0,1,0), (0,0,0,-1), (0,1,0,1), (0,1,1,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution of the diagonal yields the free ranks

$$0 \to S^4 \to S^{13} \to S^{15} \to S^7 \to S^1 \to 0$$

and the full strong exceptional collection of 16 line bundles
$\mathcal{O}_X(a_1, a_2, a_3, a_4)$ for (a_1, a_2, a_3, a_4) in

$$\begin{aligned}\mathcal{E} = \{&(-1,-1,-2,-3),(0,-1,-2,-3),(-1,-1,-2,-2),(0,-1,-2,-2),(-1,-1,-1,-2),(0,-1,-1,-2),\dots\\ &\dots(-1,-1,-1,-1),(0,-1,-1,-1),(-1,0,-2,-2),(0,0,-2,-2),(-1,0,-1,-1),(0,0,-1,-1),\dots\\ &\dots(-1,-1,0,-1),(0,-1,0,-1),(-1,0,0,0),(0,0,0,0)\}\end{aligned}$$

The full quiver is given by the fact that via the above ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0, with the rank of $Hom(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 3 & 2 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 0 & 3 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 3 & 6 & 3 & 2 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 0 & 4 & 0 & 2 & 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
12 & 6 & 8 & 4 & 4 & 2 & 2 & 1 & 6 & 3 & 2 & 1 & 0 & 0 & 0 & 0\\
6 & 0 & 3 & 0 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
12 & 6 & 6 & 3 & 6 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0\\
12 & 0 & 9 & 0 & 6 & 0 & 4 & 0 & 6 & 0 & 3 & 0 & 2 & 0 & 1 & 0\\
24 & 12 & 18 & 9 & 12 & 6 & 8 & 4 & 12 & 6 & 6 & 3 & 4 & 2 & 2 & 1
\end{pmatrix}$$

i.e, the (i, j) entry of this matrix holds the rank of $Hom^0(E_j, E_i)$. The directed quiver $\mathcal{Q}$ showing nonzero $Hom(E_j, E_i)$ for $i \neq j$ is given by:

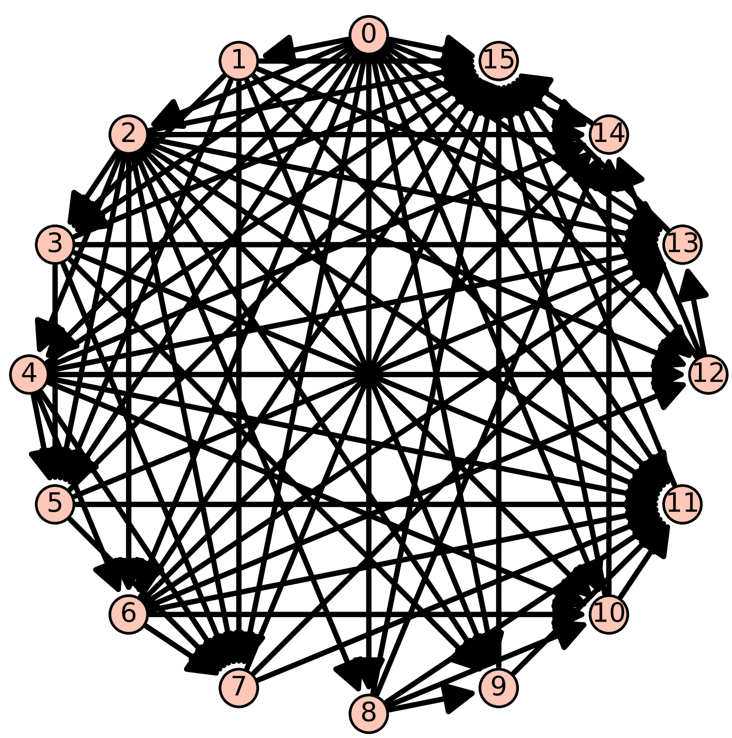

FIGURE 53. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$

Here, the property that $\mathcal{E}$ is exceptional is visible from the fact that F is lower triangular.

1.104. **Polytope F.4D.0103.** The polytope P (denoted as F.4D.0103 in polymake) has polymake half-space representation:

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

with vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -3 & 1 & -2 & 1 & 1 & -3 & 1 \\ 1 & 1 & 1 & -3 & 1 & -2 & 1 & 1 & -3 & 1 & 1 \\ 0 & 1 & -1 & 0 & 0 & -1 & -1 & -1 & 1 & 1 & 1 \\ -3 & -3 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated with P with primitive ray generators:

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(1,1,0,1),(1,1,1,1)\}$$

We use as presentation of $Cl(X)$

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The Hanlon-Hicks-Lazarev resolution of the diagonal yields the free ranks

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0$$

and the full strong exceptional collection of 11 line bundles $\mathcal{E} = \{\mathcal{O}_X(a_1,a_2,a_3)\}$ for (a_1,a_2,a_3) in

$$\begin{aligned}\mathcal{E} = \{&(-1,-3,-4),(-1,-3,-3),(0,-3,-3),(-1,-2,-3),(-1,-2,-2),(0,-2,-2),(-1,-1,-2),(0,-1,-1),\dots \\ &\dots(-1,0,-1),(-1,-1,-1),(0,0,0)\}\end{aligned}$$

The full quiver is given by the fact that via the above ordering on $\mathcal{E}$, $Hom(E_i,E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 4 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 4 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 14 & 4 & 5 & 4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 20 & 10 & 8 & 4 & 4 & 2 & 1 & 1 & 0 & 0 & 0 \\ 20 & 0 & 10 & 0 & 0 & 4 & 0 & 0 & 1 & 0 & 0 \\ 30 & 10 & 14 & 10 & 4 & 5 & 4 & 1 & 1 & 1 & 0 \\ 40 & 20 & 20 & 10 & 10 & 8 & 4 & 4 & 2 & 1 & 1 \end{pmatrix}$$

The quiver $\mathcal{Q}$ showing nonzero $Hom(E_j,E_i)$ for $i \neq j$ is given by:

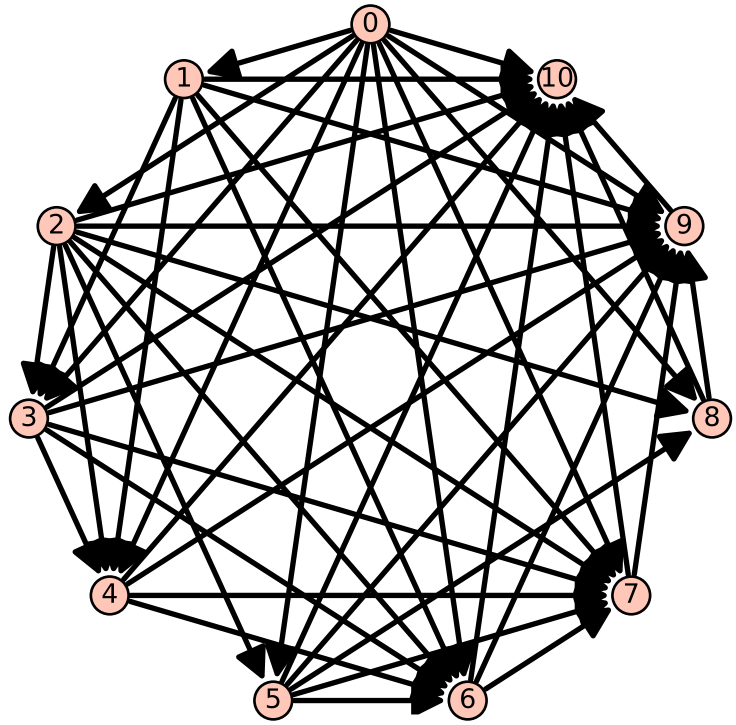

FIGURE 54. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $10 = |\mathcal{E}| - 1$

1.105. **Polytope F.4D.0104.** The polytope P (denoted as F.4D.0104 in polymake) has polymake half-space representation

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & -2 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} -1 & -1 & -1 & -1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & -4 & 1 & 1 & 1 & 1 & 1 & -3 & -1 & 1 & 1 \\ -4 & 1 & 1 & 1 & 1 & -3 & -1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -5 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated with P, with primitive ray generators:

$$\{(-1,0,0,0), (1,0,0,0), (0,-1,0,0), (0,0,-1,0), (0,0,0,-1), (1,0,0,-1), (-2,1,1,1)\}$$

and presentation of the class group given by

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ -2 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{28} \to S^{35} \to S^{17} \to S^2 \to 0.$$

There does not exist an ordering of the 12 line bundles:

$\{(-1,0,-3),(-1,-1,-2),(-1,0,-2),(0,0,-3),(-1,-1,-1),(0,0,-2),(-1,0,-1),(-1,0,0),(0,0,-1),(-1,-1,0),\ldots$
$\ldots(-1,-1,1),(0,0,0)\}$

which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution for which the collection is exceptional, since there are a pair of line bundles $\mathcal{O}(-1,-1,1)$ and $\mathcal{O}(-1,0,-3)$ with nonzero Hom's in both directions. That is,

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,-1,1),\mathcal{O}(-1,0,-3)) = \begin{cases} \mathbb{C}^1 & \text{in degree } 3 \\ 0 & \text{else} \end{cases}$$

and

$$Hom^{\bullet}_{D^b(X)}(\mathcal{O}(-1,0,-3),\mathcal{O}(-1,-1,1)) = \begin{cases} \mathbb{C}^{20} & \text{in degree } 0 \\ 0 & \text{else} \end{cases}$$

Therefore there does not exist an ordering for $\mathcal{E}$ under which $\mathcal{E}$ is exceptional.

1.106. **Polytope F.4D.0105.** Let P denote the polytope F.4D.0105 in the polymake database with the half-space representation:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -3 & 1 & -3 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & -2 & 1 & -2 & -2 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 & -3 & -3 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with rays:

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(0,1,0,1),(1,0,1,1)\}$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0$$

The 12 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal give a full strong exceptional collection from the ordering

$\mathcal{E} = \{(-1,-2,-3),(-1,-1,-3),(-1,-2,-2),(-1,-1,-2),(-1,-2,-1),(-1,-1,-1),(0,-1,-2),(0,-1,-1),(0,-1,0),\ldots$
$\ldots(0,0,-2),(0,0,-1),(0,0,0)\}$

The full quiver is given by the fact that via the above ordering on $\mathcal{E}$, $Hom(E_i,E_j)$ is concentrated in degree 0, with the rank of $Hom(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 12 & 6 & 6 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 3 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 15 & 6 & 7 & 3 & 2 & 1 & 3 & 1 & 0 & 0 & 0 & 0 \\ 26 & 10 & 15 & 6 & 7 & 3 & 6 & 3 & 1 & 0 & 0 & 0 \\ 11 & 7 & 3 & 2 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\ 24 & 15 & 11 & 7 & 3 & 2 & 6 & 2 & 0 & 3 & 1 & 0 \\ 42 & 26 & 24 & 15 & 11 & 7 & 12 & 6 & 2 & 6 & 3 & 1 \end{pmatrix}$$

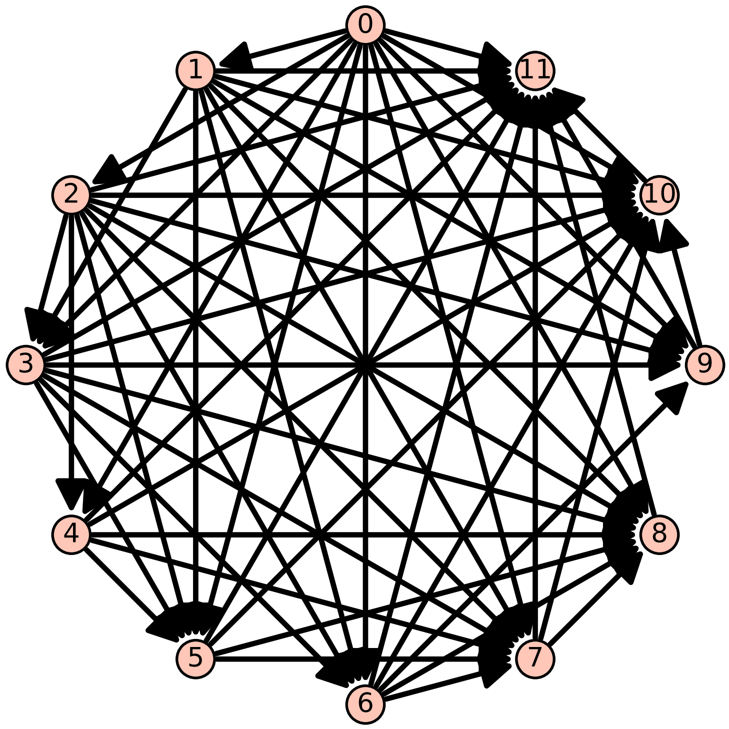

FIGURE 55. Directed quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $11 = |\mathcal{E}| - 1$

1.107. **Polytope F.4D.0106.** The polytope P (denoted F.4D.0106 in polymake) has half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -3 & 1 & -3 & 1 \\ 0 & 1 & 0 & -2 & 1 & 1 & 0 & 1 & 0 & -4 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 & -3 & -3 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators:

$$\{(-1,0,0,0), (0,-1,0,0), (0,0,-1,0), (0,0,0,-1), (0,0,0,1), (1,0,1,1), (1,1,1,1)\}$$

and presentation of the class group given by:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0.$$

The collection of 12 line bundles appearing on the left-hand side yields the full strong exceptional collection $\mathcal{O}_X(a_1, a_2, a_3)$ for (a_1, a_2, a_3) in

$$\mathcal{E} = \{(-1,-3,-4),(-1,-3,-3),(-1,-2,-3),(0,-2,-3),(-1,-2,-2),(0,-2,-2),(-1,-1,-2),(0,-1,-2),\ldots$$
$$\ldots(-1,-1,-1),(0,-1,-1),(0,0,-1),(0,0,0)\}.$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0, with the rank of $Hom(E_j, E_i)$ given by the (i, j) entry of the following matrix

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
7 & 4 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 5 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 4 & 4 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
14 & 5 & 5 & 4 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0\\
16 & 10 & 7 & 0 & 4 & 0 & 2 & 0 & 1 & 0 & 0 & 0\\
23 & 14 & 9 & 7 & 5 & 4 & 2 & 2 & 1 & 1 & 0 & 0\\
30 & 14 & 14 & 10 & 5 & 4 & 5 & 4 & 1 & 1 & 1 & 0\\
46 & 30 & 23 & 16 & 14 & 10 & 9 & 7 & 5 & 4 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

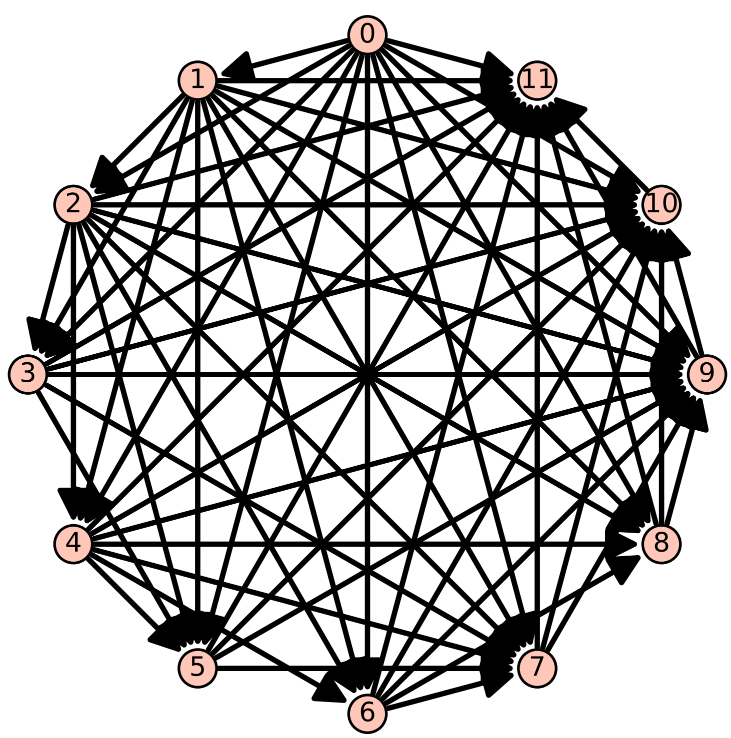

FIGURE 56. Directed quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $11 = |\mathcal{E}| - 1$

Here, the property that $\mathcal{E}$ is exceptional is visible from the fact that the above matrix is lower triangular.

1.108. **Polytope F.4D.0107.** Let P denote the polytope F.4D.0107 in the polymake database with the half-space representation:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & -1 & 1 & -2 & 1 & 1 & 1 & -1 & 1 & -4 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & -2 & -2 & -2 & 1 & 1 \\ -2 & -1 & 1 & 1 & 1 & 1 & -4 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated with P, with rays:

$$\{(-1,0,0,0),(0,-1,0,0),(-1,1,0,0),(0,0,-1,0),(0,1,1,0),(0,0,0,-1),(1,-1,0,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The exceptional collection of 13 line bundles appearing on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal is

$$\mathcal{E} = \{(0,-2,-2),(-1,-2,-1),(0,-1,-2),(0,-2,-1),(-1,-1,-1),(0,0,-2),(-1,-2,0),(0,-2,0),\ldots$$
$$\ldots(0,-1,-1),(-1,-1,0),(0,0,-1),(0,-1,0),(0,0,0)\}$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i,E_j)$ is concentrated in degree 0, with the rank of $Hom(E_j,E_i)$ given by the (i,j) entry of the following matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 3 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 12 & 5 & 3 & 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 12 & 3 & 5 & 3 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 18 & 9 & 6 & 3 & 3 & 3 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 18 & 6 & 9 & 3 & 3 & 0 & 0 & 3 & 1 & 0 & 1 & 0 & 0 \\ 27 & 12 & 12 & 9 & 5 & 3 & 3 & 3 & 3 & 1 & 1 & 1 & 0 \\ 36 & 18 & 18 & 9 & 9 & 6 & 3 & 6 & 3 & 3 & 3 & 1 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

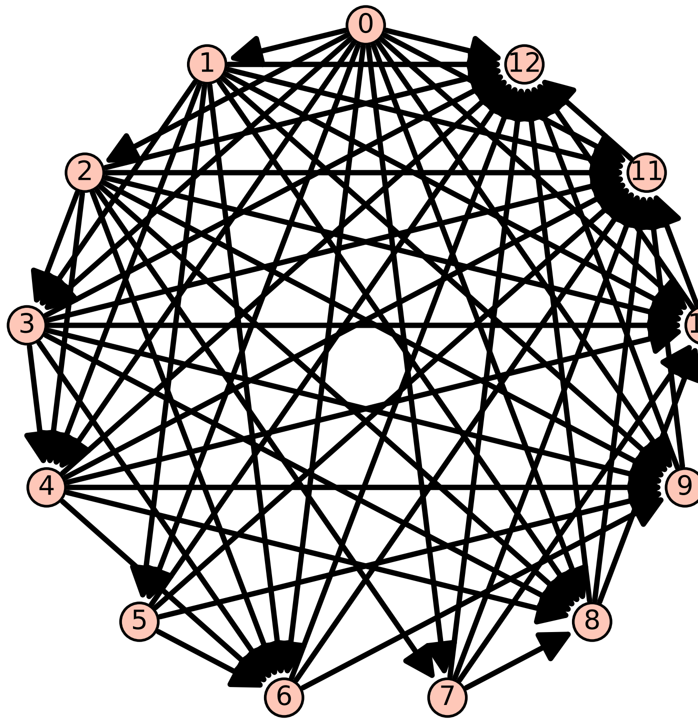

FIGURE 57. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $12 = |\mathcal{E}| - 1$

1.109. **Polytope F.4D.0108.** Let P denote the polytope F.4D.0108 in the polymake database with the half-space representation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of:

$$\begin{pmatrix} 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & -4 & -1 & -1 & 1 \\ 0 & 0 & -2 & 1 & 1 & 1 & 1 & 0 & 0 & -2 & -2 & 1 & 1 \\ -1 & 1 & 1 & -2 & 1 & -2 & -2 & -1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators

$$\{(-1,0,0,0),(0,-1,0,0),(-1,1,0,0),(0,0,-1,0),(0,1,1,0),(0,0,0,-1),(1,-1,0,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^8 \to S^{24} \to S^{25} \to S^{10} \to S^1 \to 0$$

The 13 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal are

$$\mathcal{E} = \{(0,-2,-2), (-1,-2,-1), (0,-1,-2), (0,-2,-1), (-1,-1,-1), (0,0,-2), (-1,-2,0), (0,-2,0), \ldots$$
$$\ldots(0,-1,-1), (-1,-1,0), (0,0,-1), (0,-1,0), (0,0,0)\}.$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0, with the rank of $Hom(E_j, E_i)$ given by the (i, j) entry of the following matrix

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 3 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
12 & 5 & 3 & 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
12 & 3 & 5 & 3 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
18 & 9 & 6 & 3 & 3 & 3 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
18 & 6 & 9 & 3 & 3 & 0 & 0 & 3 & 1 & 0 & 1 & 0 & 0 \\
27 & 12 & 12 & 9 & 5 & 3 & 3 & 3 & 3 & 1 & 1 & 1 & 0 \\
36 & 18 & 18 & 9 & 9 & 6 & 3 & 6 & 3 & 3 & 3 & 1 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

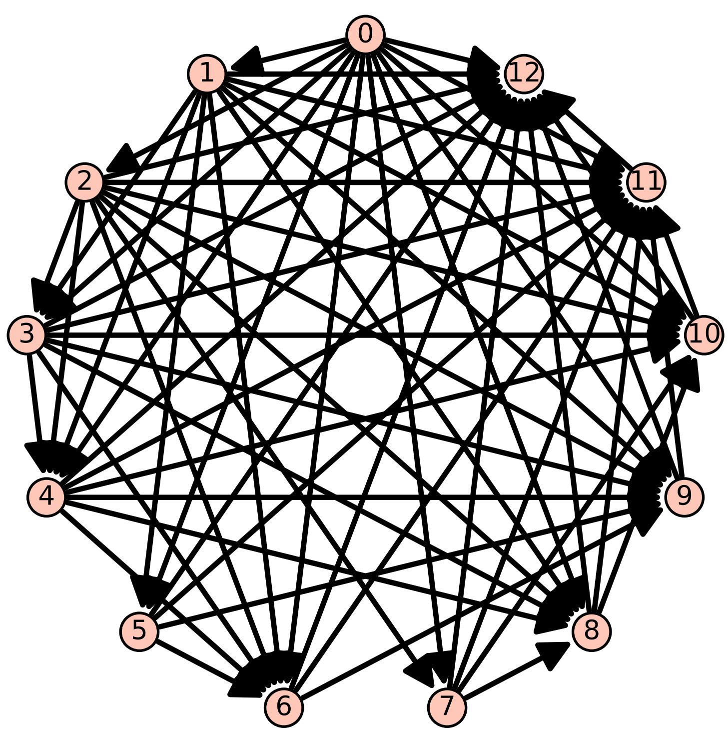

FIGURE 58. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $12 = |\mathcal{E}| - 1$

1.110. **Polytope F.4D.0109.** Let P denote the polytope F.4D.0109 in the polymake database with the half space representation:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

with vertices given by the columns of:

$$\begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let X denote the complete toric variety associated to P, with primitive ray generators:

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(0,0,1,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^2 \to S^7 \to S^9 \to S^5 \to S^1 \to 0.$$

The 16 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal give a full strong exceptional collection

$$\{(-1,-1,-1,-2),(-1,0,-1,-2),(0,-1,-1,-2),(-1,-1,-1,-1),(-1,-1,0,-1),(-1,-1,0,0),\ldots$$
$$\ldots(0,-1,-1,-1),(0,-1,0,-1),(-1,0,-1,-1),(-1,0,0,-1),(0,0,-1,-2),(0,-1,0,0),\ldots$$
$$\ldots(-1,0,0,0),(0,0,-1,-1),(0,0,0,-1),(0,0,0,0)\}$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0, with the rank of $Hom(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 2 & 2 & 3 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 2 & 2 & 0 & 3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 8 & 4 & 0 & 4 & 4 & 0 & 2 & 0 & 2 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 12 & 4 & 4 & 6 & 6 & 0 & 2 & 2 & 2 & 2 & 3 & 1 & 1 & 0 & 0 & 0 \\ 10 & 6 & 4 & 0 & 5 & 2 & 0 & 0 & 3 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 10 & 6 & 4 & 5 & 0 & 2 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 20 & 12 & 8 & 10 & 10 & 4 & 6 & 4 & 6 & 4 & 5 & 3 & 2 & 2 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

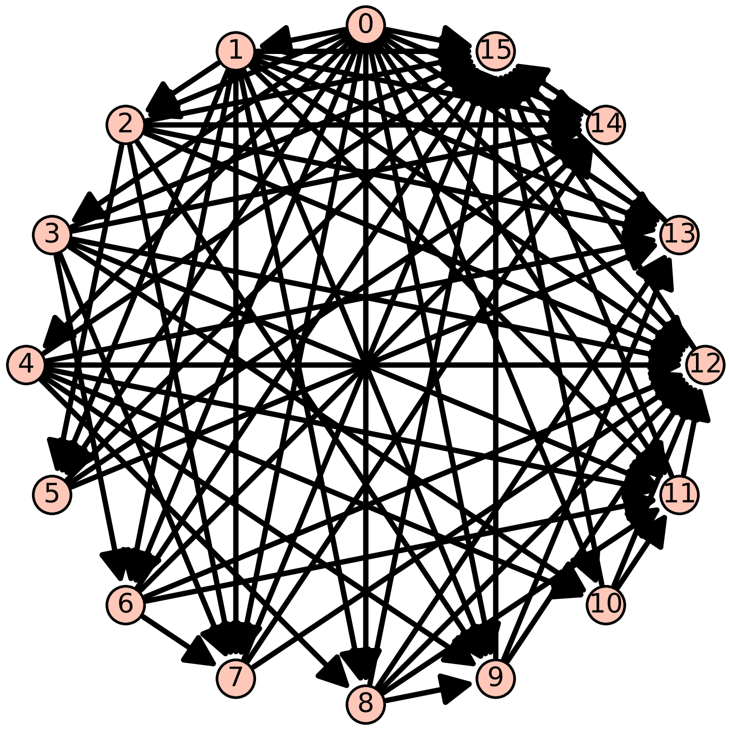

FIGURE 59. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$

1.111. **Polytope F.4D.0110.** Let P denote the polytope F.4D.0110 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ -2 & -2 & 1 & 1 & -2 & -2 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated with P, with primitive ray generators:

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,1,0,-1),(0,0,0,1),(0,0,1,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^4$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{13} \to S^{15} \to S^7 \to S^1 \to 0.$$

The 16 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution and which give a full strong exceptional collection are

$$\begin{aligned}\mathcal{E} = \{&(-1,1,-2,-3),(0,1,-2,-3),(-1,1,-2,-2),(-1,0,-1,-2),(-1,1,-1,-2),(-1,0,-1,-1),\dots\\ &\dots(-1,0,0,-1),(-1,1,-1,-1),(0,0,-1,-2),(0,1,-2,-2),(0,1,-1,-2),(-1,0,0,0),\dots\\ &\dots(0,0,-1,-1),(0,1,-1,-1),(0,0,0,-1),(0,0,0,0)\}.\end{aligned}$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
7 & 2 & 4 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
7 & 4 & 2 & 2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 2 & 2 & 2 & 4 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\
8 & 4 & 4 & 0 & 4 & 2 & 0 & 0 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0\\
14 & 8 & 4 & 4 & 7 & 2 & 0 & 2 & 2 & 4 & 2 & 1 & 1 & 0 & 0 & 0\\
14 & 4 & 8 & 4 & 7 & 2 & 2 & 0 & 4 & 2 & 2 & 1 & 0 & 1 & 0 & 0\\
12 & 7 & 7 & 4 & 0 & 4 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
24 & 14 & 14 & 8 & 12 & 8 & 4 & 4 & 7 & 7 & 4 & 4 & 2 & 2 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

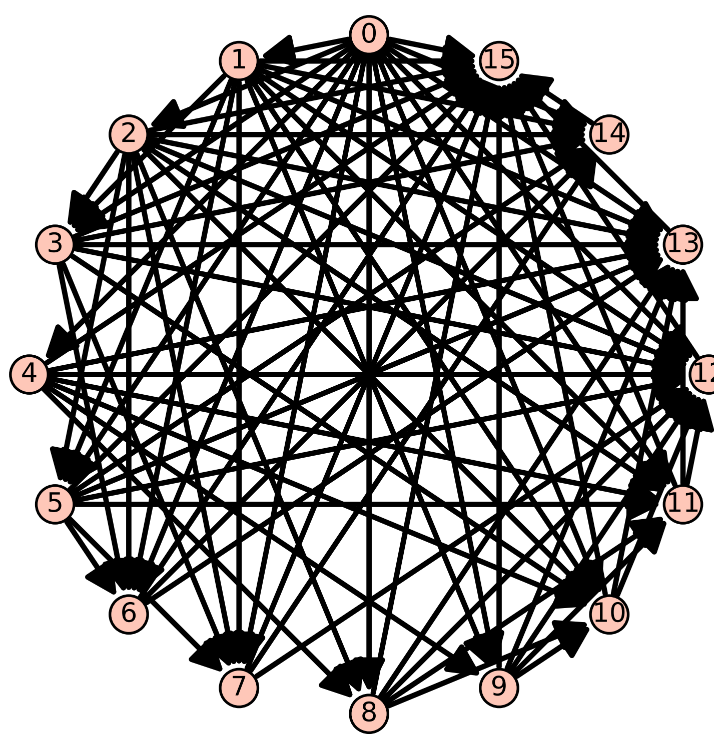

FIGURE 60. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$

1.112. **Polytope F.4D.0111.** Let P denote the polytope F.4D.0111 in the polymake database with the half-space representation:

$$\begin{bmatrix} 1 & 0 & 0 & 0-1 & \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of:

$$\begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -2 & -2 & -2 & -2 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & -3 & -3 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators:

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,-1,1,0),(0,0,0,-1),(0,1,0,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}} \mathbb{Z}^3.$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{13} \to S^{15} \to S^7 \to S^1 \to 0.$$

The 12 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ give the full strong exceptional collection:

$$\begin{aligned}\mathcal{E} = \{&(-1,-1,-2),(0,-1,-2),(-1,0,-2),(-1,-1,-1),(-1,0,-1),(-1,-1,0),(0,0,-2),(0,-1,-1),\dots \\ &\dots(-1,0,0),(0,0,-1),(0,-1,0),(0,0,0)\}\end{aligned}$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i,E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 4 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 4 & 4 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 8 & 2 & 2 & 4 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 14 & 8 & 4 & 7 & 2 & 0 & 4 & 2 & 1 & 0 & 0 & 0 \\ 20 & 8 & 8 & 10 & 2 & 2 & 4 & 4 & 1 & 1 & 0 & 0 \\ 16 & 10 & 7 & 0 & 4 & 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 32 & 20 & 14 & 16 & 8 & 4 & 10 & 7 & 4 & 2 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

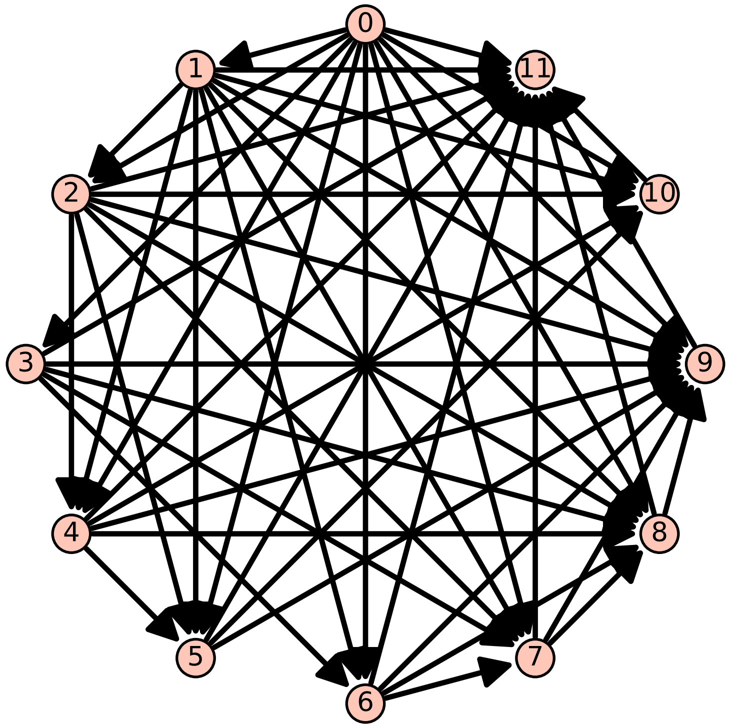

FIGURE 61. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $11 = |\mathcal{E}| - 1$

1.113. **Polytope F.4D.0112.** Let P denote the polytope F.4D.0112 in the polymake database with the half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 \\ 0 & 1 & -2 & 1 & 0 & 0 & 1 & 1 & -2 & -2 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,1,0),(0,1,1,0),(0,0,0,-1),(1,0,0,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}} \mathbb{Z}^3.$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^4 \to S^{12} \to S^{13} \to S^6 \to S^1 \to 0.$$

The 12 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution and which give a full strong exceptional collection are

$$\{(-1,-2,-2),(-1,-1,-2),(0,-1,-2),(-1,-2,-1),(-1,-1,-1),(0,-1,-1),(0,0,-2),(-1,-2,0),\dots$$
$$\dots(-1,-1,0),(0,-1,0),(0,0,-1),(0,0,0)\}$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i,E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
5 & 0 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 2 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
9 & 3 & 3 & 3 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0\\
15 & 5 & 9 & 6 & 3 & 0 & 3 & 2 & 1 & 0 & 0 & 0\\
12 & 6 & 6 & 0 & 0 & 2 & 3 & 0 & 0 & 1 & 0 & 0\\
18 & 9 & 6 & 6 & 0 & 3 & 3 & 3 & 0 & 1 & 1 & 0\\
30 & 15 & 18 & 12 & 6 & 5 & 9 & 6 & 3 & 3 & 2 & 1
\end{pmatrix}.$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

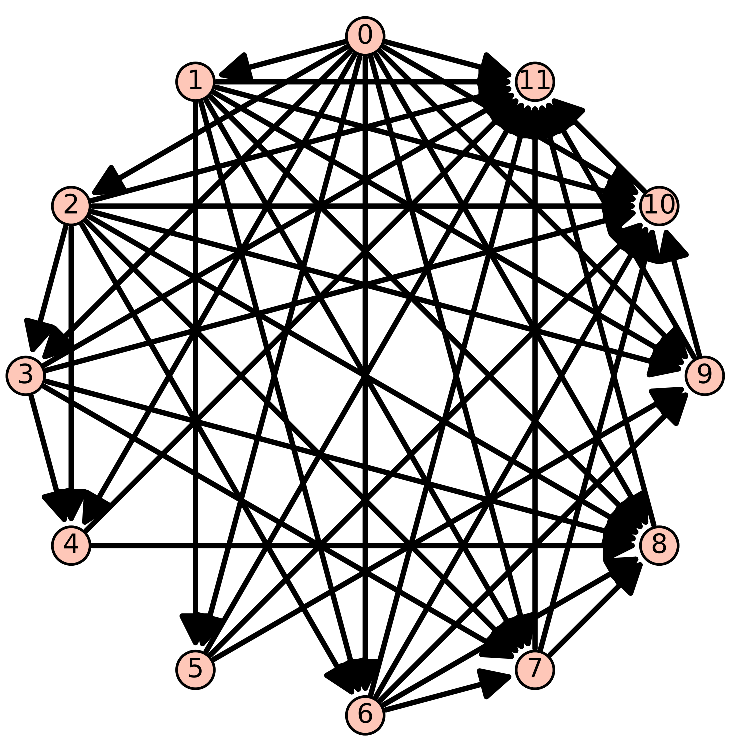

FIGURE 62. Quiver showing nonzero $Hom^0(E_i,E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $11 = |\mathcal{E}| - 1$

1.114. **Polytope F.4D.0113.** Let P denote the polytope F.4D.0113 in the polymake database with half-space representation:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 & 1 \end{bmatrix}$$

The vertices for P are given by the columns of:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -3 & 1 & -3 & 1 & -1 & 1 & -1 & 1 \\ 0 & 1 & -2 & 1 & 0 & 0 & 1 & 1 & -2 & -2 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -3 & -3 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X be the complete toric variety associated with P, with primitive ray generators:

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,1,0),(0,1,1,0),(0,0,0,-1),(1,0,-1,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The free ranks given by the Hanlon-Hicks-Lazarev resolution are as follows:

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0.$$

The 12 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution and which give a full strong exceptional collection are

$$\{(-1,-2,-2),(-1,-1,-2),(-1,-2,-1),(0,-1,-2),(-1,-1,-1),(-1,-2,0),(0,0,-2),(0,-1,-1),\ldots$$
$$\ldots(-1,-1,0),(0,0,-1),(0,-1,0),(0,0,0)\}$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i,E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 5 & 2 & 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 12 & 3 & 5 & 3 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 12 & 6 & 6 & 0 & 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 21 & 12 & 9 & 6 & 5 & 2 & 3 & 2 & 1 & 1 & 0 & 0 \\ 22 & 6 & 12 & 6 & 3 & 5 & 0 & 3 & 1 & 0 & 1 & 0 \\ 38 & 22 & 21 & 12 & 12 & 9 & 6 & 6 & 5 & 3 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

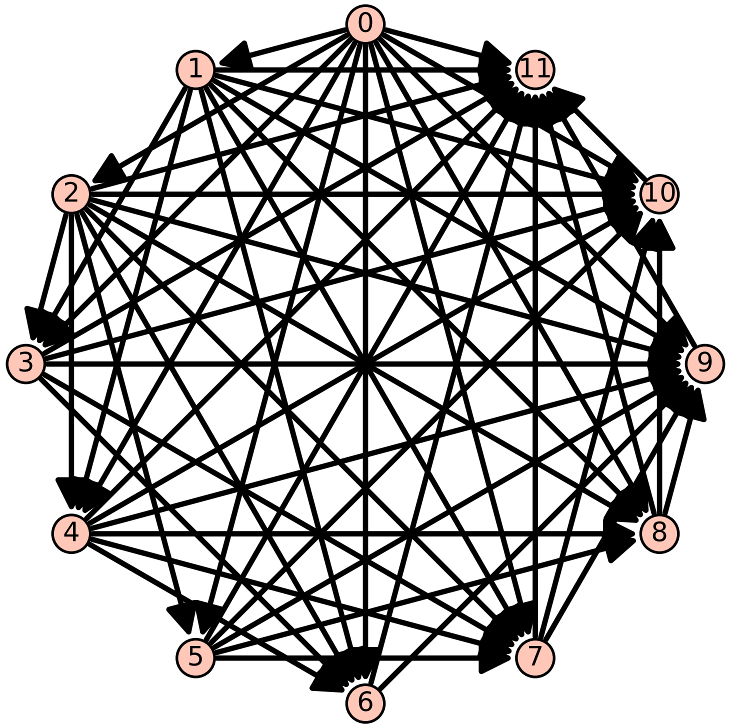

FIGURE 63. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $11 = |\mathcal{E}| - 1$

1.115. **Polytope F.4D.0114.** Let P denote the polytope F.4D.0114 in the polymake database with the half-space representation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 1 & 0 \end{bmatrix}$$

and vertices given by the columns of:

$$\begin{pmatrix} 1 & 1 & -3 & 1 & 1 & 1 & -3 & 1 \\ 1 & 1 & 1 & 1 & -3 & -3 & 1 & 1 \\ 0 & 1 & 0 & 0 & -4 & 1 & 1 & 1 \\ -3 & -3 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,-1,1,0),(0,0,0,-1),(1,1,0,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^6 \xrightarrow{\begin{pmatrix} 0 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^2.$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0.$$

The 8 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution and which give a full strong exceptional collection are

$$\mathcal{E} = \{(-1,-3),(0,-3),(-1,-2),(0,-2),(-1,-1),(-1,0),(0,-1),(0,0)\}.$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 9 & 5 & 2 & 1 & 0 & 0 & 0 & 0 \\ 15 & 5 & 5 & 1 & 1 & 0 & 0 & 0 \\ 25 & 15 & 9 & 5 & 2 & 1 & 0 & 0 \\ 35 & 15 & 15 & 5 & 5 & 1 & 1 & 0 \\ 55 & 35 & 25 & 15 & 9 & 5 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

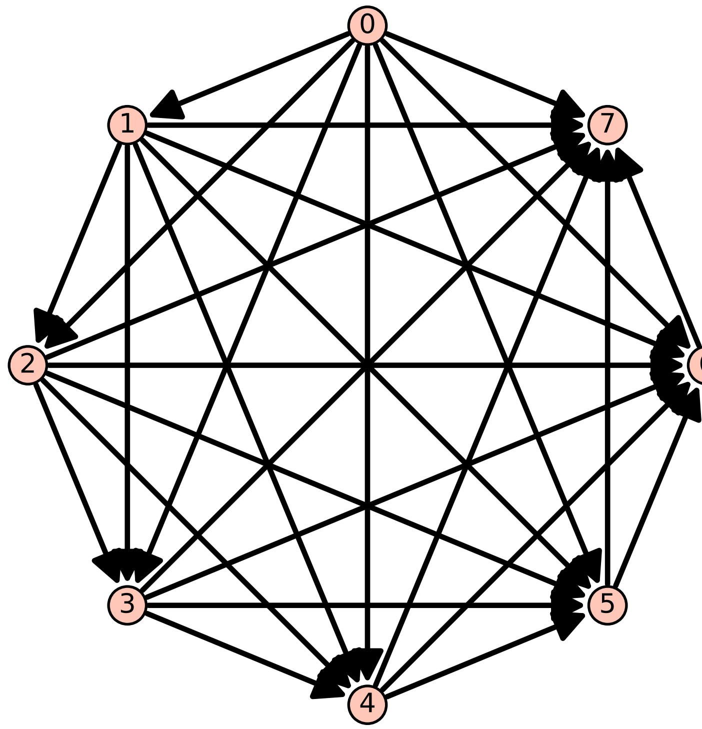

FIGURE 64. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $7 = |\mathcal{E}| - 1$

1.116. **Polytope F.4D.0115.** Let P denote the polytope F.4D.0115 in the polymake database with the half-space representation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

with vertices given by the columns of:

$$\begin{pmatrix} 1 & 1 & -1 & 1 & 1 & 1 & -5 & 1 \\ 1 & -1 & 1 & 1 & 1 & -5 & 1 & 1 \\ -1 & 1 & 1 & 1 & -5 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators:

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(1,1,1,2)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^6 \xrightarrow{\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 2 & 0 & 1 \end{pmatrix}} \mathbb{Z}^2.$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^5 \to S^{19} \to S^{26} \to S^{14} \to S^2 \to 0.$$

The 9 line bundles appearing on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal are:

$$\{(-1,-5),(0,-3),(-1,-4),(0,-2),(-1,-3),(0,-1),(-1,-2),(-1,-1),(0,0)\}.$$

There does not exist an ordering of this collection for which it is exceptional, since for the pair of line bundles $\mathcal{O}_X(-1,-1)$ and $\mathcal{O}_X(-1,-5)$, we have

$$\mathrm{Hom}_{D^b(X)}(\mathcal{O}_X(-1,-1),\mathcal{O}_X(-1,-5)) = \begin{cases} \mathbb{C} & \text{in degree 3,} \\ 0 & \text{else} \end{cases}$$

and

$$\mathrm{Hom}_{D^b(X)}(\mathcal{O}_X(-1,-5),\mathcal{O}_X(-1,-1)) = \begin{cases} \mathbb{C}^{35} & \text{in degree 0,} \\ 0 & \text{else} \end{cases}.$$

1.117. **Polytope F.4D.0116.** Let P denote the polytope F.4D.0116 in the polymake database, with the half-space representation:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -3 & 1 & -3 & 1 \\ -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 & -3 & -3 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators:

$$\{(-1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(1,0,1,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The Hanlon-Hicks-Lazarev resolution of $\mathcal{O}_\Delta$ yields the free ranks (written as an ungraded resolution of S-modules, for S the homogeneous coordinate ring of $Y \cong X \times X$):

$$0 \to S^3 \to S^{11} \to S^{14} \to S^7 \to S^1 \to 0.$$

The 12 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal which yield a full strong exceptional collection are

$$\mathcal{E} = \{(-1,-1,-3),(0,-1,-3),(-1,-1,-2),(-1,0,-2),(-1,-1,-1),(-1,0,-1),(0,-1,-2),(0,0,-2),(-1,0,0),\dots$$
$$\dots(0,0,-1),(0,-1,-1),(0,0,0)\}.$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i,E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following lower-triangular matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 0 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
9 & 0 & 4 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 3 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
8 & 4 & 2 & 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0\\
16 & 0 & 9 & 6 & 4 & 3 & 0 & 0 & 1 & 0 & 0 & 0\\
12 & 6 & 6 & 0 & 2 & 0 & 3 & 0 & 0 & 1 & 0 & 0\\
18 & 9 & 8 & 6 & 2 & 2 & 4 & 3 & 0 & 1 & 1 & 0\\
32 & 16 & 18 & 12 & 8 & 6 & 9 & 6 & 2 & 4 & 3 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is then given by:

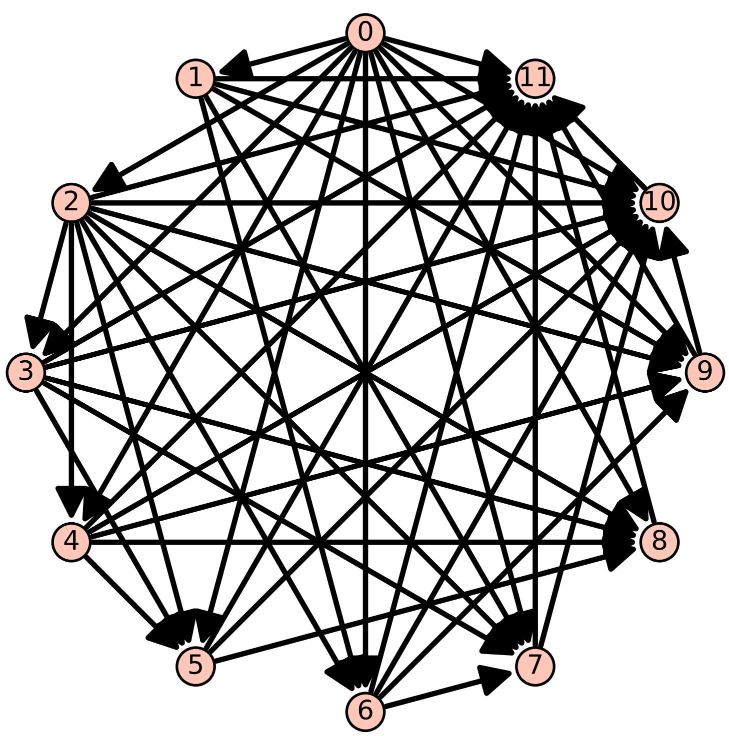

FIGURE 65. Quiver showing nonzero $Hom^0(E_i,E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $11 = |\mathcal{E}|-1$

1.118. **Polytope F.4D.0117.** Let P denote the polytope F.4D.0117 in polymake with the half-space representation:

$$\begin{bmatrix}
1 & 0 & -1 & 0 & 0\\
1 & 0 & 0 & 0 & -1\\
1 & 1 & -1 & 1 & 0\\
1 & -1 & 0 & 0 & 0\\
1 & 0 & 0 & -1 & 0\\
1 & 0 & 1 & 0 & 1
\end{bmatrix}$$

and vertices given by the columns of:

$$\begin{pmatrix} 1 & -1 & 1 & 1 & 1 & -4 & 1 & -1 & 1 \\ 1 & 1 & 1 & -2 & 1 & -2 & -2 & 1 & 1 \\ -1 & 1 & 1 & -4 & -1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(1,-1,1,0),(0,0,0,-1),(0,1,0,1)\}.$$

We use as presentation of the class group

$$\mathbb{Z}^6 \xrightarrow{\begin{pmatrix} -1 & 1 & -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}} \mathbb{Z}^2.$$

The free ranks appearing in the Hanlon-Hicks-Lazarev resolution of the diagonal are:

$$0 \to S^6 \to S^{20} \to S^{23} \to S^{10} \to S^1 \to 0.$$

The 9 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal which yield a full strong exceptional collection are

$$\{(2,-4),(1,-3),(2,-3),(2,-2),(1,-2),(0,-2),(0,-1),(1,-1),(0,0)\}$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following lower-triangular matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 12 & 5 & 3 & 1 & 1 & 0 & 0 & 0 & 0 \\ 15 & 5 & 5 & 1 & 1 & 1 & 0 & 0 & 0 \\ 22 & 12 & 6 & 5 & 3 & 0 & 1 & 0 & 0 \\ 31 & 15 & 12 & 5 & 5 & 3 & 1 & 1 & 0 \\ 53 & 31 & 22 & 15 & 12 & 6 & 5 & 3 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is then given by:

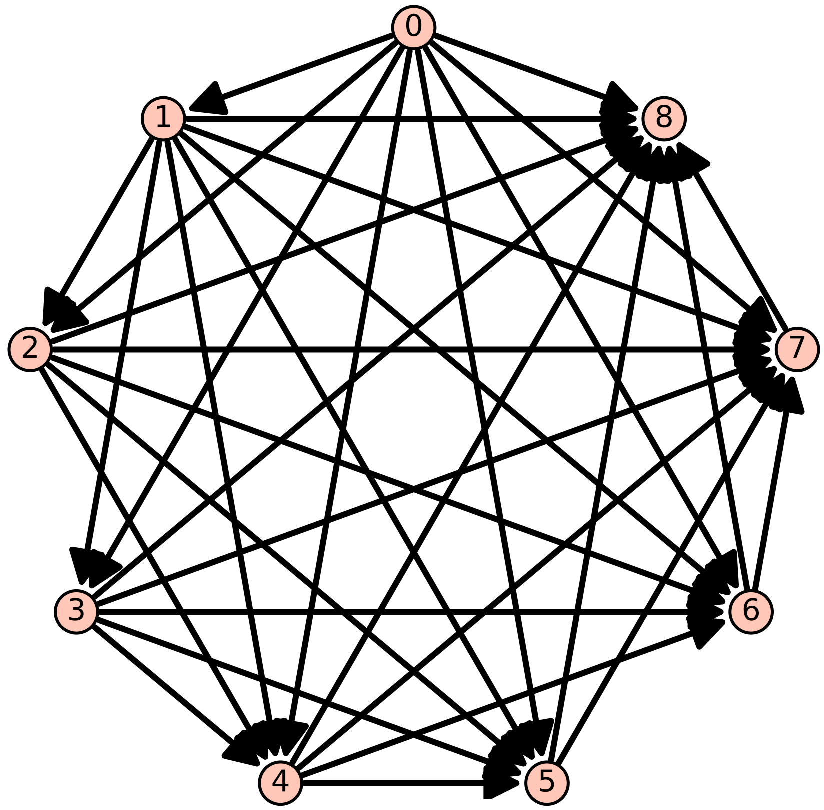

FIGURE 66. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $8 = |\mathcal{E}| - 1$

1.119. **Polytope F.4D.0118.** Let P denote the polytope F.4D.0118 in polymake with the half-space representation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and vertices given by the columns of:

$$\begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(0,0,0,1)\}.$$

We use as presentation of $Cl(X)$:

$$\mathbb{Z}^8 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^4.$$

The free ranks appearing in the Hanlon-Hicks-Lazarev resolution of the diagonal are:

$$0 \to S^1 \to S^4 \to S^6 \to S^4 \to S^1 \to 0$$

The 16 line bundles appearing on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal yield the full strong exceptional collection:

$$\{(-1,-1,-1,-1),(-1,0,-1,-1),(-1,-1,-1,0),(-1,-1,0,-1),(0,-1,-1,-1),(-1,0,-1,0),\ldots$$
$$\ldots(0,-1,0,-1),(-1,-1,0,0),(0,0,-1,-1),(-1,0,0,-1),(0,-1,-1,0),(0,0,-1,0),\ldots$$
$$\ldots(-1,0,0,0),(0,0,0,-1),(0,-1,0,0),(0,0,0,0)\}$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following lower-triangular matrix:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 0 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 0 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
8 & 4 & 4 & 0 & 4 & 2 & 0 & 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0\\
8 & 4 & 4 & 4 & 0 & 0 & 2 & 0 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0\\
8 & 4 & 0 & 4 & 4 & 2 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0\\
8 & 0 & 4 & 4 & 4 & 0 & 2 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0\\
16 & 8 & 8 & 8 & 8 & 4 & 4 & 4 & 4 & 4 & 4 & 2 & 2 & 2 & 2 & 1
\end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is given in Figure 67.

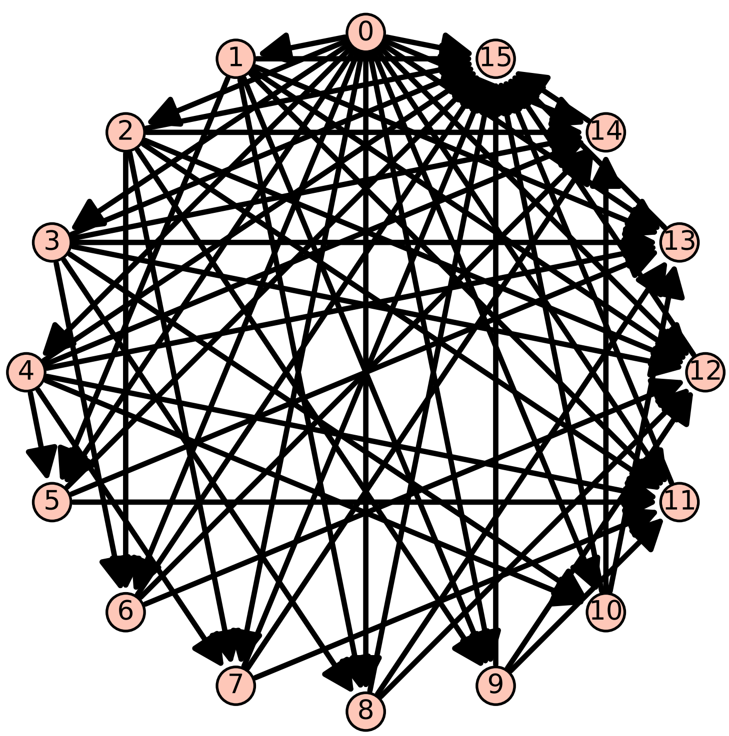

FIGURE 67. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $15 = |\mathcal{E}| - 1$

1.120. **Polytope F.4D.0119.** Let P denote the polytope F.4D.0119 in polymake with the half-space representation:

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

with vertices given by the columns of:

$$\begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -2 & -2 & 1 & 1 & -2 & -2 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators

$$\{(-1,0,0,0),(1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,1,0),(0,0,0,-1),(0,1,0,1)\}.$$

We use the presentation of the class group:

$$\mathbb{Z}^7 \xrightarrow{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^3.$$

The 9 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal which yield a full strong exceptional collection are

$$\{(-1,-1,-2),(1,0,2),(0,-1,-2),(-1,-1,-1),(-1,-1,0),(-1,0,-1),(0,-1,-1),(0,0,-2),(0,-1,0),(-1,0,0),\ldots$$
$$\ldots(0,0,-1),(0,0,0)\}.$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following lower-triangular matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 2 & 0 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 2 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 12 & 4 & 6 & 6 & 0 & 2 & 2 & 3 & 1 & 0 & 0 & 0 \\ 12 & 6 & 0 & 6 & 2 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 12 & 6 & 6 & 0 & 2 & 0 & 3 & 0 & 0 & 0 & 1 & 0 \\ 24 & 12 & 12 & 12 & 4 & 6 & 6 & 6 & 3 & 2 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is given in Figure 68.

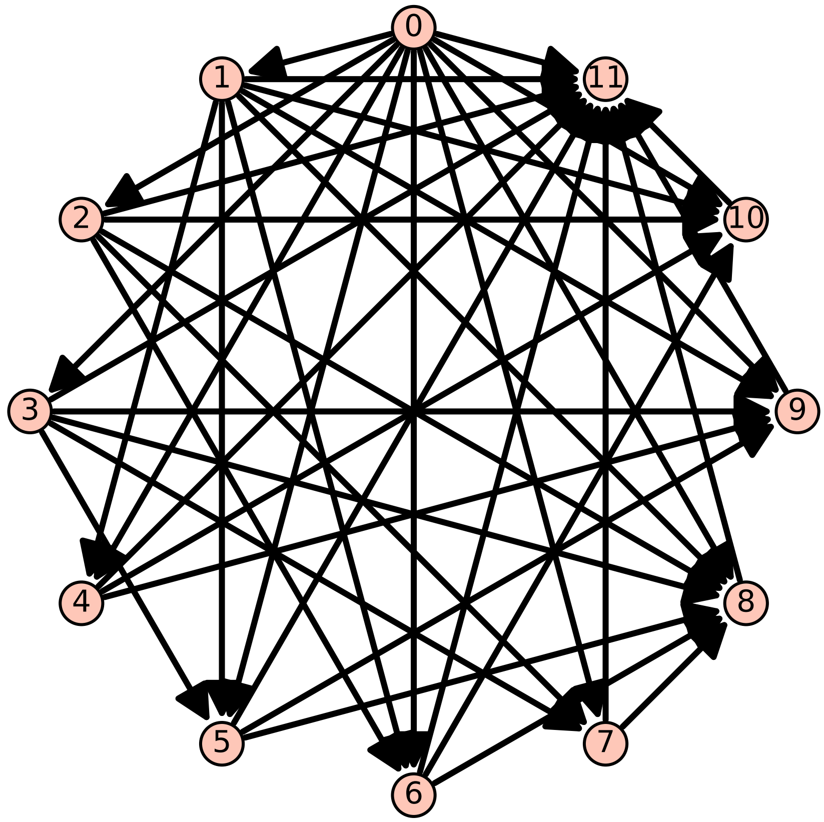

FIGURE 68. Quiver showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $11 = |\mathcal{E}| - 1$

1.121. **Polytope F.4D.0120.** Let P denote the polytope F.4D.0120 in polymake with the half-space representation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & -4 & 1 \\ 1 & -2 & 1 & 1 & 1 & -4 & 1 & 1 \\ -2 & 1 & 1 & 1 & -4 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(0,0,0,1),(1,1,1,1)\}.$$

We use the presentation of $Cl(X)$:

$$\mathbb{Z}^6 \xrightarrow{\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^2.$$

The free ranks appearing in the Hanlon-Hicks-Lazarev resolution of the diagonal are:

$$0 \to S^4 \to S^{15} \to S^{20} \to S^{10} \to S^1 \to 0$$

The 8 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal which yield a full strong exceptional collection are

$$\mathcal{E} = \{(-1,-4),(0,-3),(-1,-3),(-1,-2),(0,-2),(0,-1),(-1,-1),(0,0)\}.$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j, E_i)$ given by the (i, j) entry of the following lower-triangular matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 10 & 4 & 0 & 1 & 0 & 0 & 0 & 0 \\ 14 & 5 & 4 & 1 & 1 & 0 & 0 & 0 \\ 20 & 10 & 0 & 4 & 0 & 1 & 0 & 0 \\ 30 & 14 & 10 & 5 & 4 & 1 & 1 & 0 \\ 55 & 30 & 20 & 14 & 10 & 5 & 4 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is

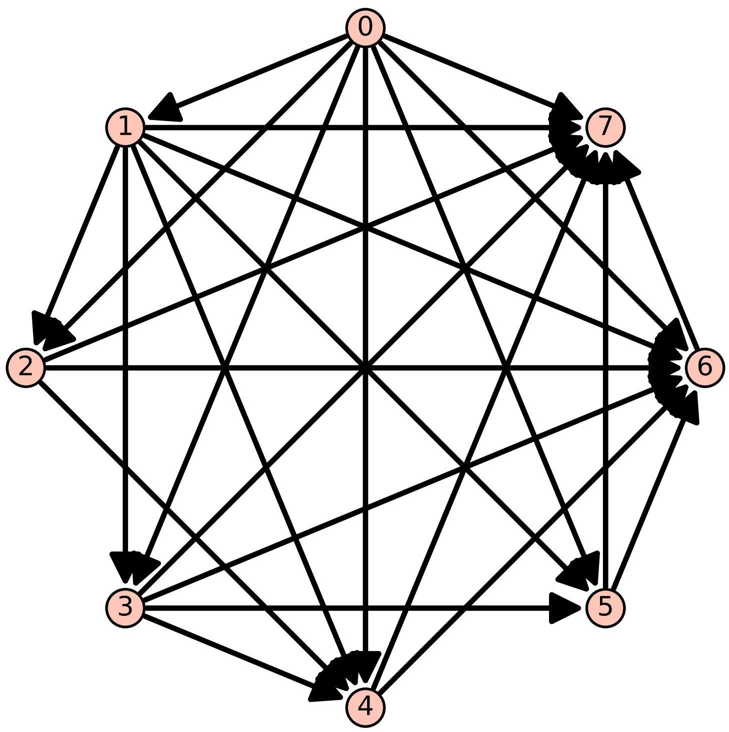

FIGURE 69. Directed graph showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $7 = |\mathcal{E}| - 1$.

1.122. **Polytope F.4D.0121.** Let P denote the polytope F.4D.0121 in polymake with the half-space representation:

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & -3 & 1 & 1 & -3 & 1 \\ 1 & 1 & -3 & 1 & 1 & -3 & 1 & 1 \\ -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 \\ -3 & -3 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators

$$\{(-1, 0, 0, 0), (0, -1, 0, 0), (0, 0, -1, 0), (0, 0, 1, 0), (0, 0, 0, -1), (1, 1, 0, 1)\}$$

We use the presentation of the class group:

$$\mathbb{Z}^6 \xrightarrow{\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^2.$$

The free ranks appearing in the Hanlon-Hicks-Lazarev resolution of the diagonal are:

$$0 \to S^3 \to S^{11} \to S^{14} \to S^7 \to S^1 \to 0$$

The 8 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal which yield a full strong exceptional collection are

$$\mathcal{E} = \{(0,0),(1,0),(0,1),(0,2),(0,3),(1,1),(1,2),(1,3)\}$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following lower-triangular matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 8 & 2 & 0 & 4 & 1 & 0 & 0 & 0 \\ 20 & 8 & 2 & 10 & 4 & 1 & 0 & 0 \\ 20 & 10 & 4 & 0 & 0 & 0 & 1 & 0 \\ 40 & 20 & 8 & 20 & 10 & 4 & 2 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is

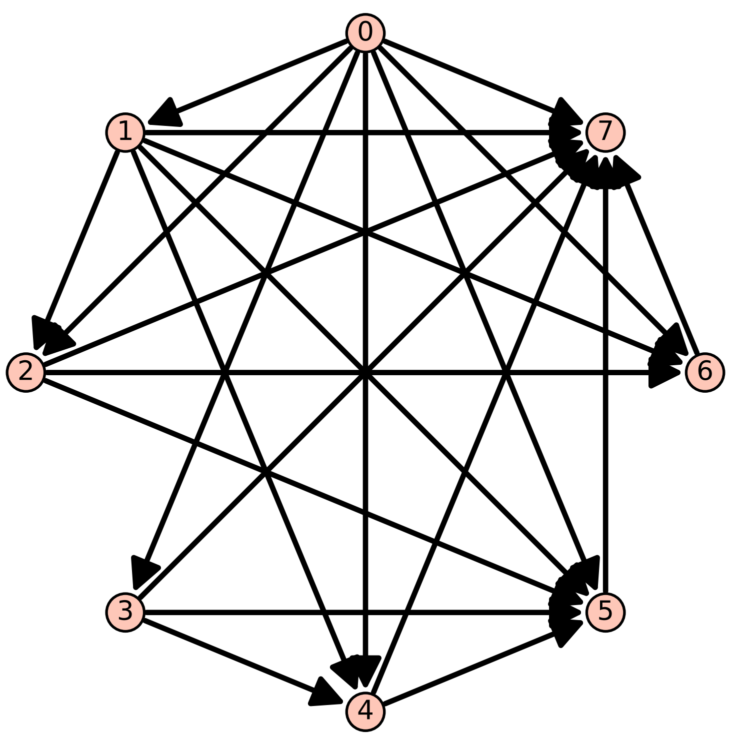

FIGURE 70. Directed graph showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $7 = |\mathcal{E}| - 1$.

1.123. **Polytope F.4D.0122.** Let P denote the polytope F.4D.0122 in polymake with the half space representation:

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & -2 & 1 & 1 & 1 & -2 & 1 & -2 & 1 \\ 1 & 1 & 1 & -2 & 1 & -2 & -2 & 1 & 1 \\ -2 & 1 & 1 & -2 & -2 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let X denote the complete toric variety associated to P, with primitive ray generators

$$\{(-1,0,0,0),(0,-1,0,0,),(0,0,-1,0),(1,0,1,0),(0,0,0,-1),(0,1,0,1)\}.$$

We use as presentation of the class group:

$$\mathbb{Z}^6 \xrightarrow{\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}} \mathbb{Z}^2.$$

The free ranks appearing in the Hanlon-Hicks-Lazarev resolution of the diagonal are:

$$0 \to S^4 \to S^{12} \to S^{13} \to S^6 \to S^1 \to 0.$$

The 9 line bundles which appear on the left-hand side of the Hanlon-Hicks-Lazarev resolution of the diagonal which yield a full strong exceptional collection are

$$\{(-2,-2),(-2,-1),(-1,-2),(-1,-1),(-2,0),(0,-2),(-1,0),(0,-1),(0,0)\}.$$

The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i,E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j,E_i)$ given by the (i,j) entry of the following lower-triangular matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 3 & 0 & 1 & 0 & 0 & 0 & 0 \\ 9 & 3 & 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 18 & 9 & 6 & 3 & 0 & 3 & 1 & 0 & 0 \\ 18 & 6 & 9 & 0 & 3 & 3 & 0 & 1 & 0 \\ 36 & 18 & 18 & 6 & 6 & 9 & 3 & 3 & 1 \end{pmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j,E_i)$, for $i \neq j$ is

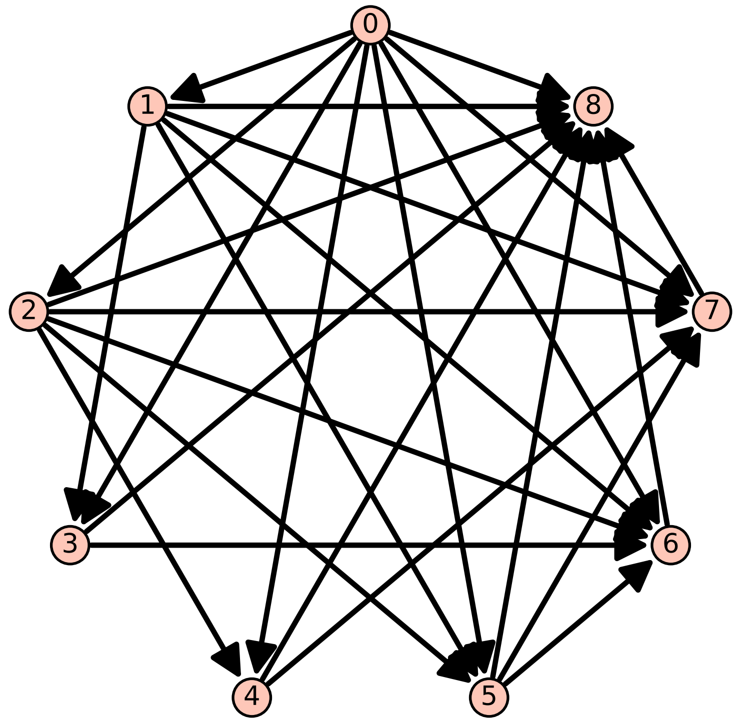

FIGURE 71. Directed graph showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $8 = |\mathcal{E}| - 1$.

1.124. **Polytope F.4D.0123.** Let P be the polytope F.4D.0123 in polymake with the half-space representation

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

and vertices given by the columns of

$$\begin{pmatrix} 1 & 1 & 1 & -4 & 1 \\ 1 & 1 & -4 & 1 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ -4 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Let $X \cong \mathbb{P}^4$ denote the complete toric variety associated to P, with primitive ray generators

$$\{(-1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1),(1,1,1,1)\}.$$

We use as presentation of the class group:

$$\mathbb{Z}^5 \xrightarrow{\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \end{pmatrix}} \mathbb{Z}^1.$$

The free ranks appearing in the Hanlon-Hicks-Lazarev resolution of the diagonal are:

$$0 \to S^4 \to S^{15} \to S^{20} \to S^{10} \to S^1 \to 0.$$

Here, we recover the Beilinson collection $\mathcal{E} = \{-4,-3,-2,-1,0\}$. The directed quiver is given by the fact that via this ordering on $\mathcal{E}$, $Hom(E_i, E_j)$ is concentrated in degree 0, with the rank of $Hom^0(E_j, E_i)$ given by the (i,j) entry of the following lower-triangular matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 & 0 \\ 15 & 5 & 1 & 0 & 0 \\ 35 & 15 & 5 & 1 & 0 \\ 70 & 35 & 15 & 5 & 1 \end{bmatrix}$$

The directed quiver $\mathcal{Q}$ showing nonzero $Hom^0(E_j, E_i)$, for $i \neq j$ is

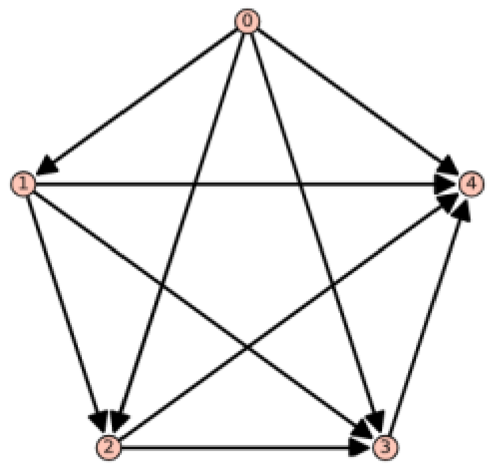

FIGURE 72. Directed graph showing nonzero $Hom^0(E_i, E_j)$ for $i \neq j$ in $\mathcal{E}$. Indexing on vertices from 0 to $7 = |\mathcal{E}| - 1$.