Details about the kernels in GP.

Radial Basis Function

In[1]:=
```
k[x_, y_, l_, A_] = A^2 E^(-(x-y)^2/(2 l^2));
AtoK = Solve[k[x, y, l, A] == K[x, y], A][[2]];

(* needed GP relations *)
K[x, y] == k[x, y, l, A]
∂x K[x, y] == ((∂x k[x, y, l, A]) /. AtoK // Simplify)
∂y K[x, y] == ((∂y k[x, y, l, A]) /. AtoK // Simplify)
∂x,y K[x, y] == ((∂x,y k[x, y, l, A]) /. AtoK // Simplify)
```

Out[3]= $K[x, y] == A^2 e^{-\frac{(x-y)^2}{2 l^2}}$

Out[4]= $K^{(1,0)}[x, y] == -\frac{(x-y) K[x, y]}{l^2}$

Out[5]= $K^{(0,1)}[x, y] == \frac{(x-y) K[x, y]}{l^2}$

Out[6]= $K^{(1,1)}[x, y] == \frac{\left(l^2 - (x-y)^2\right) K[x, y]}{l^4}$

Cauchy function

In[7]:=
```
k[x_, y_, l_, A_] = A^2 l^2/((x-y)^2 + l^2);
AtoK = Solve[k[x, y, l, A] == K[x, y], A][[2]];

(* needed GP relations *)
K[x, y] == k[x, y, l, A]
∂x K[x, y] == ((∂x k[x, y, l, A]) /. AtoK // Simplify)
∂y K[x, y] == ((∂y k[x, y, l, A]) /. AtoK // Simplify)
∂x,y K[x, y] == ((∂x,y k[x, y, l, A]) /. AtoK // Simplify)
```

Out[9]= $K[x, y] == \frac{A^2 l^2}{l^2 + (x-y)^2}$

Out[10]= $K^{(1,0)}[x, y] == -\frac{2 (x-y) K[x, y]}{l^2 + (x-y)^2}$

Out[11]= $K^{(0,1)}[x, y] == \frac{2 (x-y) K[x, y]}{l^2 + (x-y)^2}$

Out[12]= $K^{(1,1)}[x, y] == \frac{2 \left(l^2 - 3 (x-y)^2\right) K[x, y]}{\left(l^2 + (x-y)^2\right)^2}$

Matern 5/2 function

In[100]:=
```
d[x_, y_] = x - y;
r[x_, y_] = Abs[d[x, y]];
X[x_, y_, l_] = Sqrt[5] r[x, y]/l;
B[Y_] = 1 + Y + Y^2/3;
k[x_, y_, l_, A_] = A^2 B[X[x, y, l]] E^(-X[x,y,l]);
AtoK = Solve[k[x, y, l, A] == K[x, y], A][[2]];

(* needed GP relations *)
K[x, y] == k[x, y, l, A] /. Abs''[x - y] → 0 /. Abs'[x - y] → H[x - y] /. Abs[x_] :→ r[x]
∂x K[x, y] == ((∂x k[x, y, l, A]) /. AtoK /. Abs''[x - y] → 0 /. Abs'[x - y] → H[x - y] /.
    Abs[x_] :→ r[x] // Simplify)
∂y K[x, y] == ((∂y k[x, y, l, A]) /. AtoK /. Abs''[x - y] → 0 /. Abs'[x - y] → H[x - y] /.
    Abs[x_] :→ r[x] // Simplify)
∂x,y K[x, y] == ((∂x,y k[x, y, l, A]) /. AtoK /. Abs''[x - y] → 0 /. Abs'[x - y] → H[x - y] /.
    Abs[x_] :→ r[x] // Simplify)
```

Out[106]= $K[x, y] == A^2 e^{-\frac{\sqrt{5}\, r[x-y]}{l}} \left(1 + \frac{\sqrt{5}\, r[x-y]}{l} + \frac{5\, r[x-y]^2}{3\, l^2}\right)$

Out[107]= $K^{(1,0)}[x, y] == -\frac{5\, H[x-y] \times r[x-y] \left(l + \sqrt{5}\, r[x-y]\right) K[x, y]}{l \left(3\, l^2 + 3\sqrt{5}\, l\, r[x-y] + 5\, r[x-y]^2\right)}$

Out[108]= $K^{(0,1)}[x, y] == \frac{5\, H[x-y] \times r[x-y] \left(l + \sqrt{5}\, r[x-y]\right) K[x, y]}{l \left(3\, l^2 + 3\sqrt{5}\, l\, r[x-y] + 5\, r[x-y]^2\right)}$

Out[109]= $K^{(1,1)}[x, y] == \frac{5\, H[x-y]^2 \left(l^2 + \sqrt{5}\, l\, r[x-y] - 5\, r[x-y]^2\right) K[x, y]}{l^2 \left(3\, l^2 + 3\sqrt{5}\, l\, r[x-y] + 5\, r[x-y]^2\right)}$

Matern 7/2 function

In[111]:=
```
d[x_, y_] = x - y;
r[x_, y_] = Abs[d[x, y]];
R[x_, y_, l_] = r[x, y] / l;
X[x_, y_, l_] = Sqrt[7] R[x, y, l];
B[Y_] = 1 + Sqrt[7] Y + 14 Y^2/5 + 7 Sqrt[7] Y^3/15;
k[x_, y_, l_, A_] = A^2 B[R[x, y, l]] E^-X[x,y,l];
AtoK = Solve[k[x, y, l, A] == K[x, y], A][[2]];

(* needed GP relations *)
K[x, y] == k[x, y, l, A] /. Abs''[x - y] → 0 /. Abs'[x - y] → H[x - y] /. Abs[x_] :→ r[x]
∂x K[x, y] == ((∂x k[x, y, l, A]) /. AtoK /. Abs''[x - y] → 0 /. Abs'[x - y] → H[x - y] /.
    Abs[x_] :→ r[x] // FullSimplify)
∂y K[x, y] == ((∂y k[x, y, l, A]) /. AtoK /. Abs''[x - y] → 0 /. Abs'[x - y] → H[x - y] /.
    Abs[x_] :→ r[x] // FullSimplify)
∂x,y K[x, y] == ((∂x,y k[x, y, l, A]) /. AtoK /. Abs''[x - y] → 0 /. Abs'[x - y] → H[x - y] /.
    Abs[x_] :→ r[x] // FullSimplify)
```

Out[118]= $$K[x, y] == A^2 e^{-\frac{\sqrt{7}\, r[x-y]}{l}} \left(1 + \frac{\sqrt{7}\, r[x-y]}{l} + \frac{14\, r[x-y]^2}{5\, l^2} + \frac{7\sqrt{7}\, r[x-y]^3}{15\, l^3}\right)$$

Out[119]= $$K^{(1,0)}[x, y] == -\frac{7\, H[x-y] \times r[x-y] \left(3\, l^2 + 3\sqrt{7}\, l\, r[x-y] + 7\, r[x-y]^2\right) K[x, y]}{15\, l^4 + l\, r[x-y] \left(15\sqrt{7}\, l^2 + 7\, r[x-y] \left(6\, l + \sqrt{7}\, r[x-y]\right)\right)}$$

Out[120]= $$K^{(0,1)}[x, y] == \frac{7\, H[x-y] \times r[x-y] \left(3\, l^2 + 3\sqrt{7}\, l\, r[x-y] + 7\, r[x-y]^2\right) K[x, y]}{15\, l^4 + l\, r[x-y] \left(15\sqrt{7}\, l^2 + 7\, r[x-y] \left(6\, l + \sqrt{7}\, r[x-y]\right)\right)}$$

Out[121]= $$K^{(1,1)}[x, y] == \frac{7\, H[x-y]^2 \left(3\, l^3 + \sqrt{7}\, r[x-y] \left(3\, l^2 - 7\, r[x-y]^2\right)\right) K[x, y]}{15\, l^5 + l^2\, r[x-y] \left(15\sqrt{7}\, l^2 + 7\, r[x-y] \left(6\, l + \sqrt{7}\, r[x-y]\right)\right)}$$