MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.

   (a) Governments issue bonds as a way to borrow money from investors to raise funds to finance projects.

   (b) Slowing economic growth can also cause the yield curve to flatten ass investors become less optimistic about the future, they may demand a lower yield on long-term bonds, causing the spread between long-term and short-term yields to narrow.

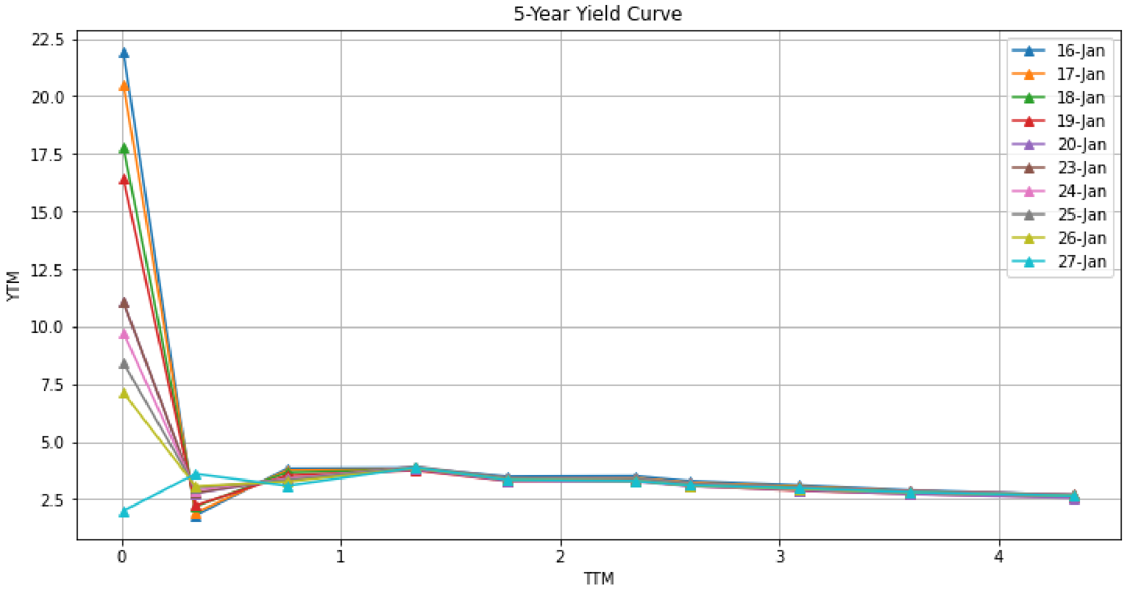
   (c) QE is a monetary policy to increase the money supply and stimulate economic growth, where the Fed has purchased large amounts of T-bonds and mortgage-backed securities to inject liquidity into the market.

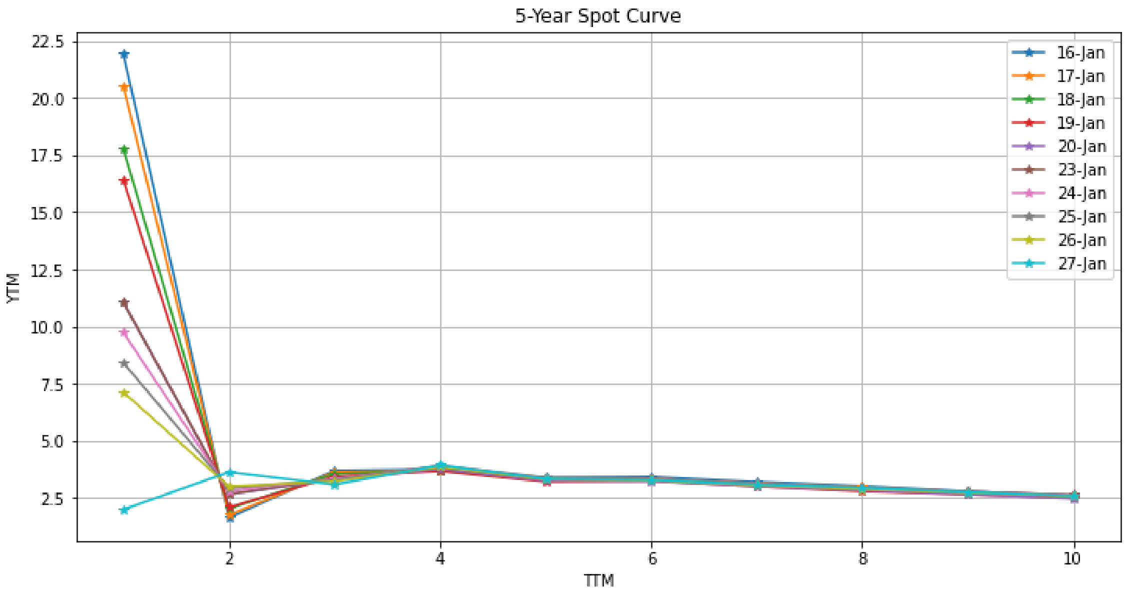
2. Data has been scraped from the web from 16 Jan to 27 Jan. I scraped data off the given website using a webscrapping library known as BeautifulSoup. The 10 bonds (as seen in the DataFrame below) are chosen because these 10 bonds are the most evenly distributed bonds among the 41 bonds withiin a 5-year window and that the coupon rates are not drastically different from one another.

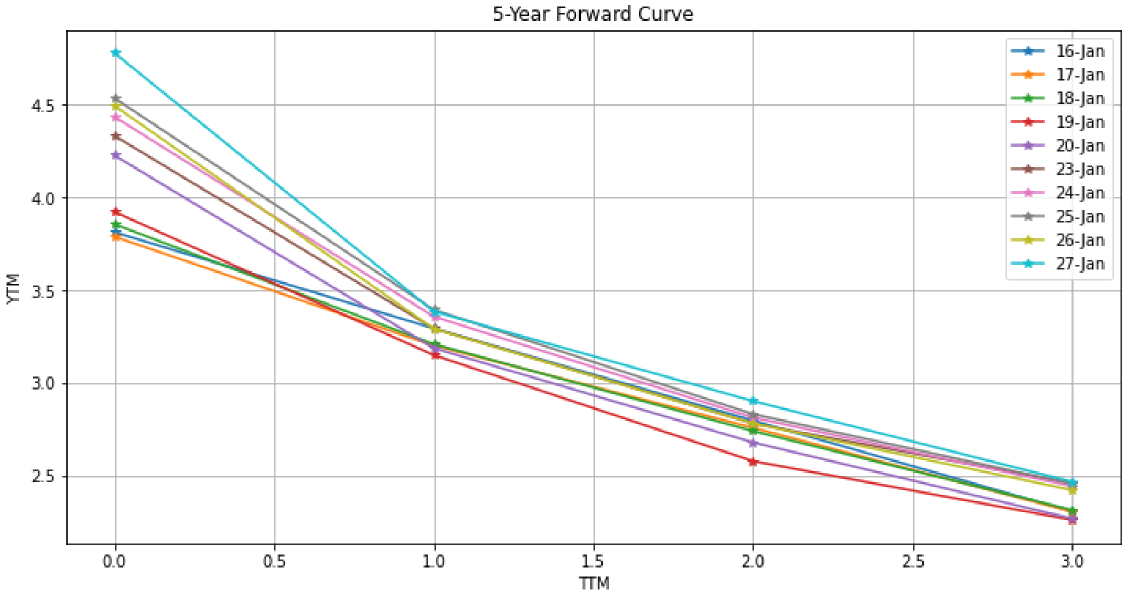
3. The eigenvalues associated with the covariance matrix of these stochastic processes tells us about the characteristics of the yield curve. The eigenvalues tells us about how much of the variance can be explained by its corresponding eigenvector. The largest eigenvalue will account for the largest explained variance and the second eigenvalue will account for the second largest amount of explained variance and so on.

Empirical Questions - 75 points

4. These are the assumptions made in the calculations. 1 year = 365 day. Today's date = 30 January 2023.


5-Year Yield Curve
YTM
TTM
16-Jan
17-Jan
18-Jan
19-Jan
20-Jan
23-Jan
24-Jan
25-Jan
26-Jan
27-Jan


5-Year Spot Curve
YTM
TTM
16-Jan
17-Jan
18-Jan
19-Jan
20-Jan
23-Jan
24-Jan
25-Jan
26-Jan
27-Jan



I believe that the yield curve is as such because a recession may be on the horizon due to the aftermath of the COVID-19 pandemic as well as a series of Quantitative Easing done by many governments around the world, more significantly by the United States. This caused inflation to eventually spiral out of control in 2022 and as the government started raising interest rates to curb inflation, it led the the front end of the curve flipping up. Since future expectations of interest rates (and thus bond yields) are expected to be lower than what they currently are, the tail end of the curve inevitably will flatten out and be lower than the front end of the curve.

(a) I chose to interpolate the yield curve using the Cubic Spine Interpolation technique because this method of interpolation can avoid the problem of Runge's phenomenon. This method also gives an interpolating polynomial that is smoother and has smaller error than some other interpolating polynomials such as Lagrange polynomial and Newton polynomial.

(b) Because Treasury notes and bonds are generally issued as coupon bonds, their prices cannot simply be used to construct the spot rate curve or to calculate forward rates. Instead, a theoretical spot rate curve and implied forward rates are constructed through the process of bootstrapping which calculates the forward rates by considering the value of the zero coupon bonds equivalent to the Treasury bonds. The calculated forward rates can then construct the spot-rate curve by adding the yields for each term to the desired maturity.
The bootstrapping technique is based on the price-yield equation using different rates for each of the 6-month terms, as determined by market prices:

$$\frac{C}{(1+rate)^1} + \frac{C}{(1+rate)^2} + \ldots + \frac{C}{(1+rate)^i} = 100$$

(c) Forward Rate, f can be derived through the following formula:

$$f = \frac{(r_a+1)^{t_a}}{(r_b+1)^{t_b}} - 1$$

where r_a = the spot rate for the bond of term t_a periods and r_b = the spot rate for the bond with a shorter term of t_b periods.
After obtaining the 1y forward points from the formua via the spot rate obtained previously, I can get the estimate of 1y-1y, 1y-2y, 1y-3y annd 1y-4y forward rates via interpolation. The graph will be as follows:

5. Covariance Matrix of daily log-returns of Yield.

	0	1	2	3	4
0	0.005933	0.000201	-0.000292	0.000190	-0.000202
1	0.000201	0.000057	0.000048	0.000100	0.000023
2	-0.000292	0.000048	0.000115	0.000146	0.000083
3	0.000190	0.000100	0.000146	0.000325	0.000142
4	-0.000202	0.000023	0.000083	0.000142	0.000087

```
The eigenvalues for the log-returns of yield are:
              0
0  5.967300e-03
1  4.981160e-04
2  3.595666e-05
3  1.603801e-05
4  1.749428e-20

The eigenvectors for the log-returns of yield are:
          0         1         2         3         4
0  0.997120 -0.000718 -0.013898  0.074538  0.001238
1  0.033867  0.243791  0.719319 -0.325918  0.561940
2 -0.049175  0.414188  0.428307  0.746536 -0.292006
3  0.032111  0.793829 -0.206815 -0.454731 -0.345329
4 -0.033983  0.372625 -0.506130  0.352315  0.692605
```

Covariance Matrix of daily log-returns of Forward Rates.

	0	1	2	3
0	0.000069	0.000006	-0.000102	-0.000033
1	0.000006	0.000119	0.000140	0.000045
2	-0.000102	0.000140	0.000404	0.000154
3	-0.000033	0.000045	0.000154	0.000065

```
The eigenvalues for the log-returns of the 1-yr forward rates are:
click to scroll output; double click to hide
0  5.388632e-04
1  9.876483e-05
2  1.984349e-05
3 -2.489969e-20

The eigenvectors for the log-returns of the 1-yr forward rates are:
          0         1         2         3
0  0.207589  0.615750 -0.653265 -0.388591
1 -0.319859  0.776449  0.446047  0.309613
2 -0.864618 -0.110870 -0.093059 -0.481127
3 -0.327158 -0.075411 -0.604671  0.722256
```

6. The eigenvector that is associated with the largest eigenvalue tell us how much variance can be explained by its associated eigenvector and it represents the dataset's largest variation after orthogonal decomposition is in that associated eigenvector's direction.

References and GitHub Link to Code

https://github.com/reggyiseggy/Assignment1