Definitions

- A mobile robot is a structure capable to move and act (autonomously or remotely operated) in terrestrial, underwater or aerial environments
- Environments can be assumed to be
  - totally structured, partially structured or unstructured
  - totally known, partially known or unknown
- Structured environment = one knows the type and the geometric characteristics of the environment
  - office space: corridors, doors, chairs, tables, etc.
  - obstacles: static or dynamics or both
  - constant or slow-varying in time

Autonomy

- Autonomy is the ability to move independently from a human supervisor

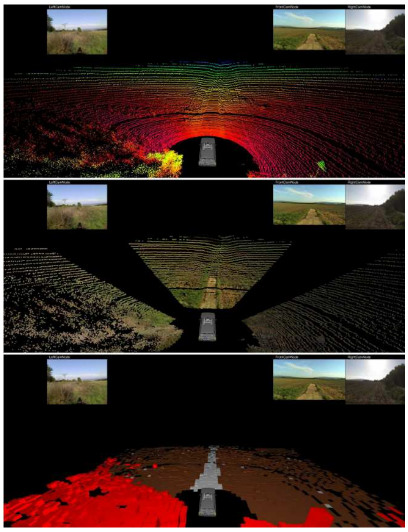
- It requires
  - Intelligence, i.e.,
    - CPU, algorithms, database
    - onboard or "in the cloud"
  - Sensors (for perceiving the environment)
  - Actuators (for motion and manipulation, etc.)
  - Energy source:
    - onboard generated
    - or supplied by an "umbilical cord"

Examples

structured

partially structured

unstructured



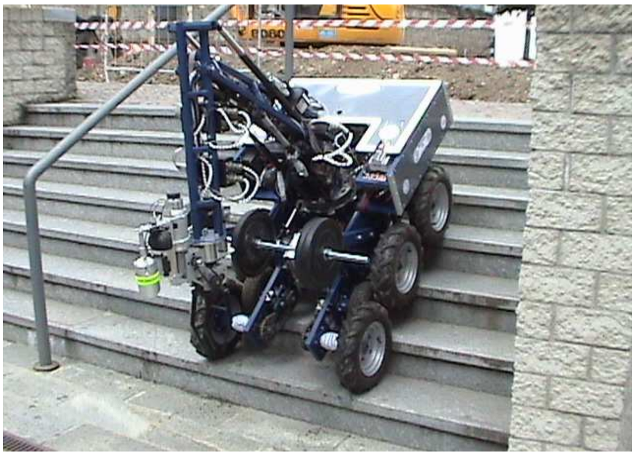
Fundamental problems in mobile robotics

- **Locomotion**: how the robot moves in the environment
- **Perception**: how the robot perceives the environment
- **Representation**: how the robot organizes the knowledge about the environment
- **Mapping**: how to build the map of the environment
- **Localization**: where is the robot in the map
- **Path planning/action planning**: what the robot shall do to go from here to there; what are the actions to be performed to complete a specified task
- **Supervision and control**: how are the command to actuators generated to perform simple or complex tasks. How to generate tasks

Locomotion

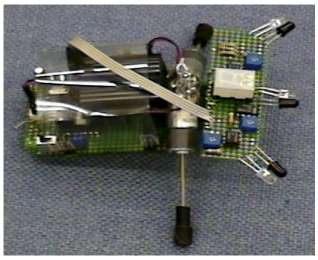
- Terrestrial robots
  - Wheeled
  - Legged
  - Mixed wheels/legs
  - Biped (humanoids)
  - Others (biomimetics = imitation of natural locomotion)
- Underwater robots
  - Propellers
  - Water jets
- Aerial robots
  - Fixed wings
  - Rotating wings (helicopters and quadcopters)
  - Airships and dirigibles
  - Flapping wings

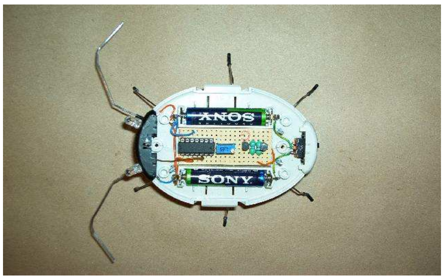
Terrestrial Robots – wheeled




Terrestrial Robots – legged

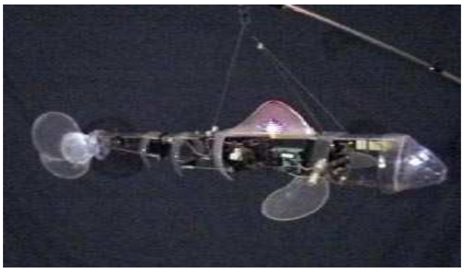


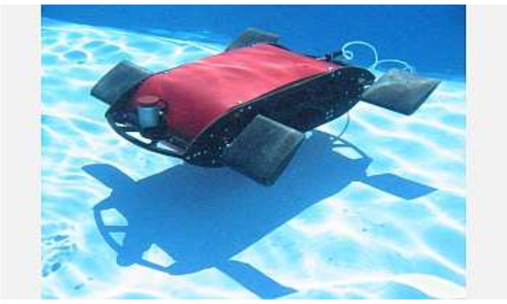


Terrestrial Robots – “humanoids”

Underwater Robots

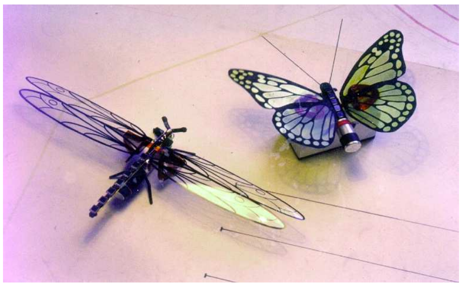
- To move they use
  - Propellers
  - Water jets
  - Fins or entire body motion



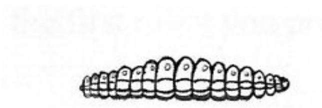


Aerial robots (UAV)

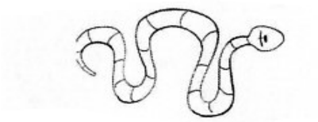
- Airship (lighter than air) or aircrafts (heavier than air)
- They use
  - Propellers
  - Rotating wings
  - Flapping wings



Natural Locomotion



Longitudinal waves



Transversal waves

Running

Jumping

Step

Biomimetic systems

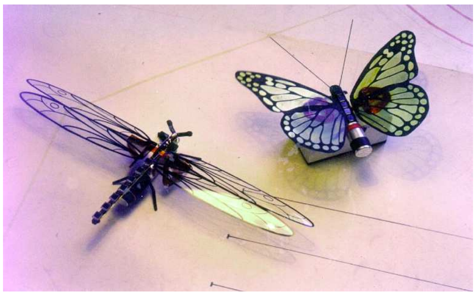
Nature imitation

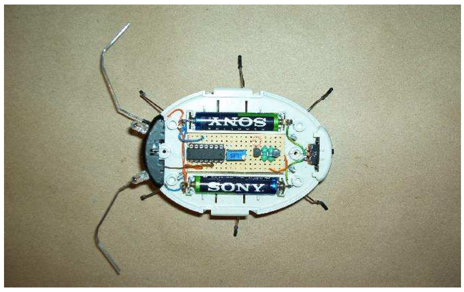
Longitudinal waves

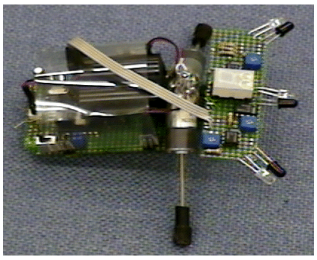
Transversal waves

Biomimesis

Copy from nature







Mobile Robots: topics treated

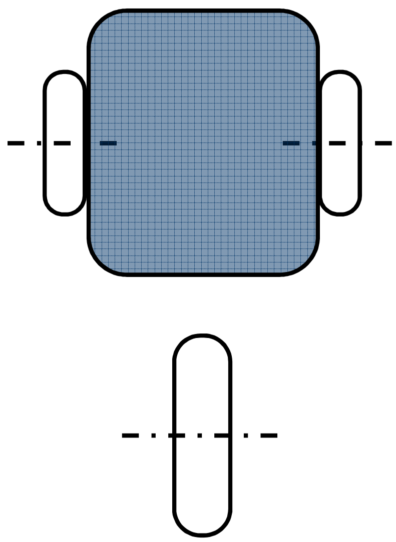
- Wheeled robots
- Kinematics
- Sensors
  - Absolute and relative position (odometry)
  - Speed
  - Proximity and distance
  - Active ranging
  - Vision
- Intelligence

Wheel Types

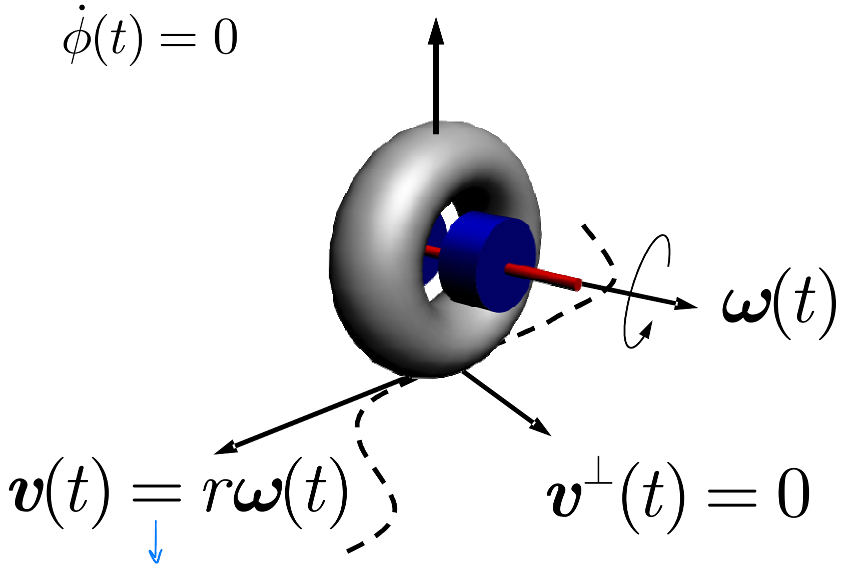
- Simple non steering wheels
- Simple steering wheels
- Castor wheel
- Omniwheel (omnidirectional wheel) or Swedish wheel
- Spherical omniwheel

- Wheel may be active or passive

Simple non steering wheel



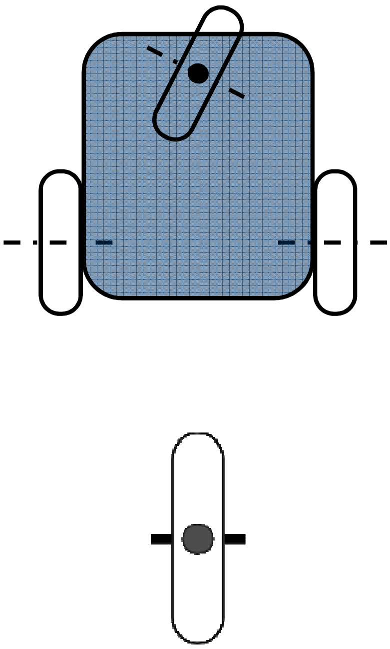
graphical scheme



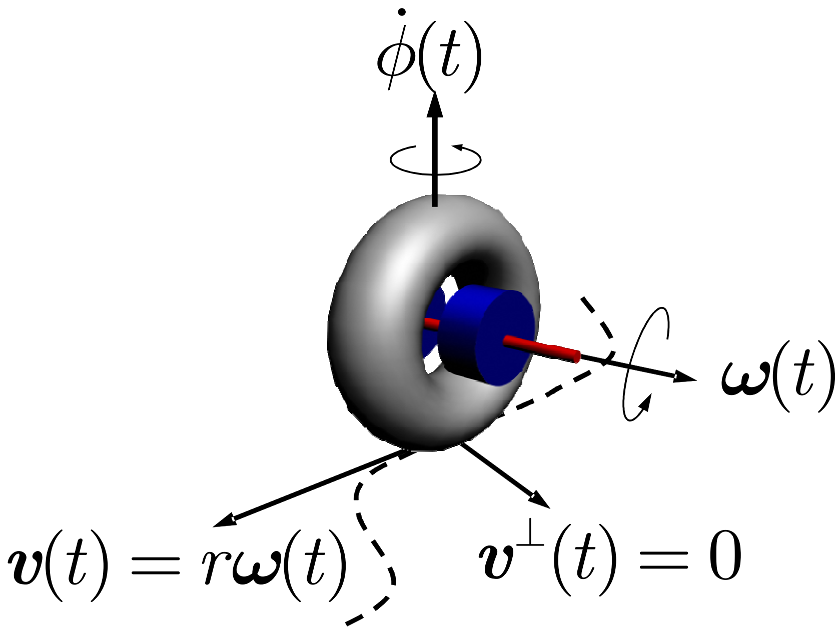
true only if no wheel slipping

Simple steering wheel

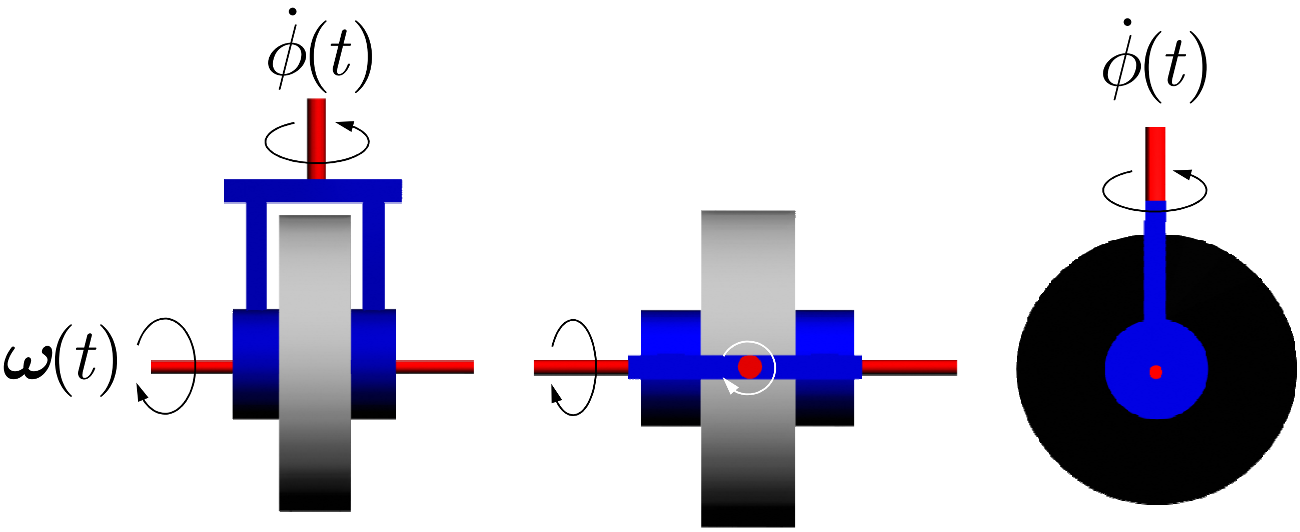
$$\dot{\phi}(t) \neq 0$$



graphical scheme



Simple steering wheel



front view

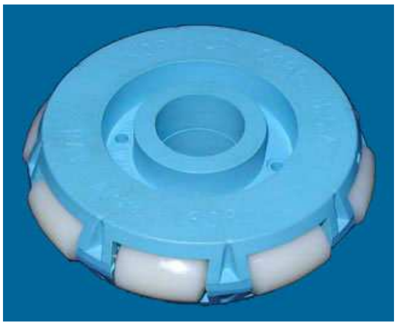
top view

side view

Castor Wheel



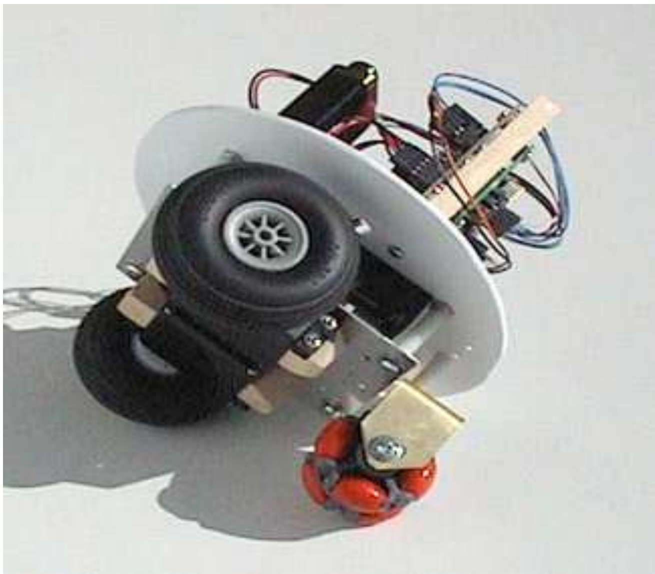

Omniwheel




Omniwheel

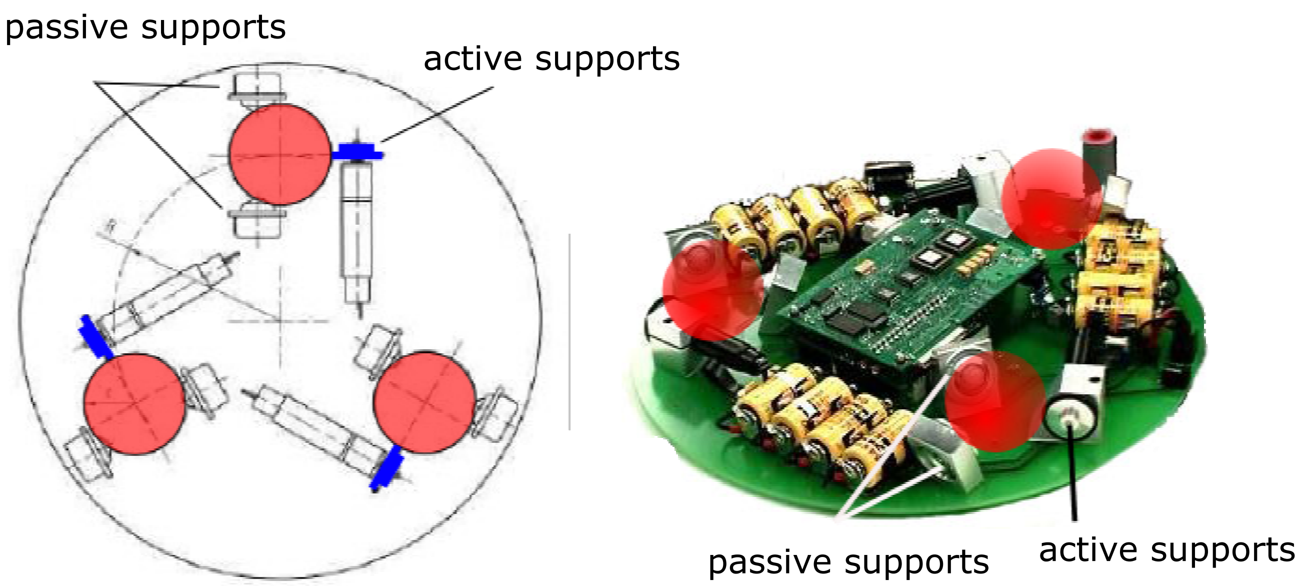
Industrial platforms
with omniwheels

Omniwheel

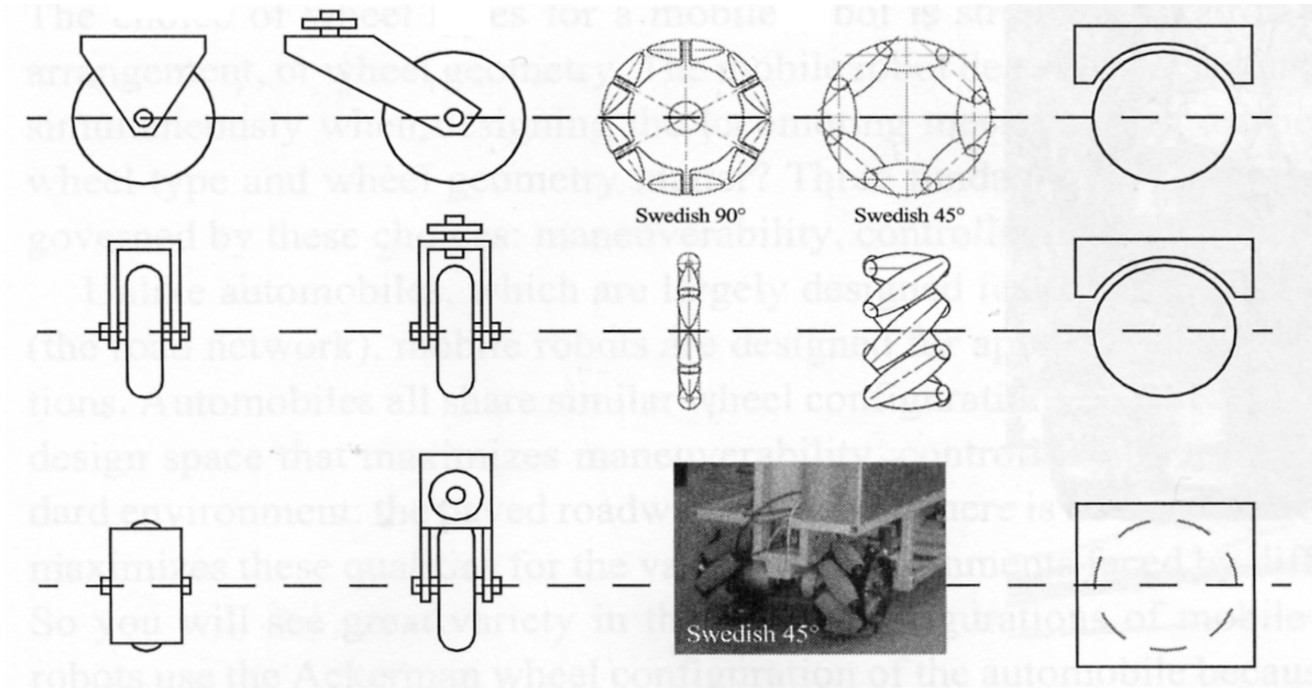


Omniwheel can also be used as a support in a differential drive robot

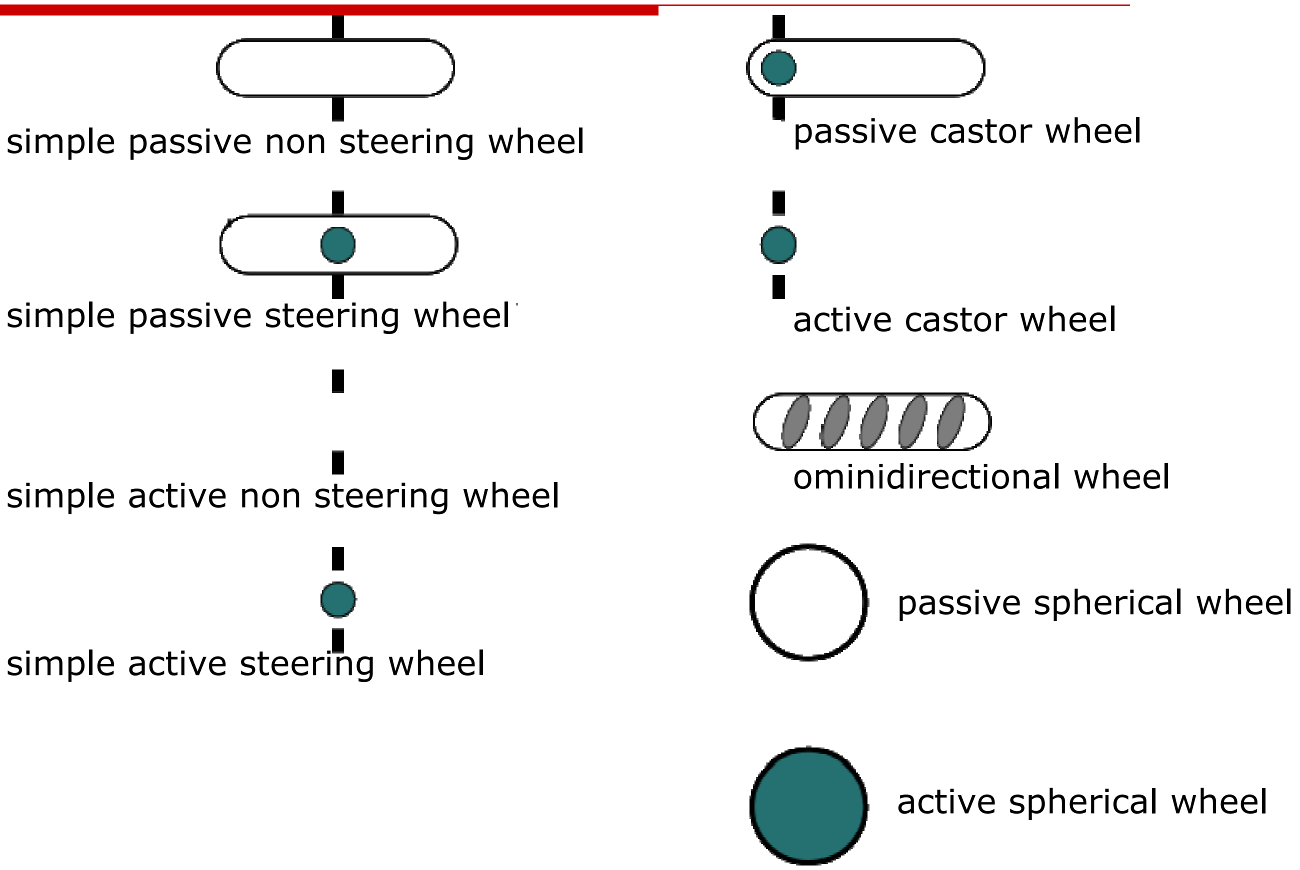
Omnidirectional Spherical Wheel



Wheels Symbols

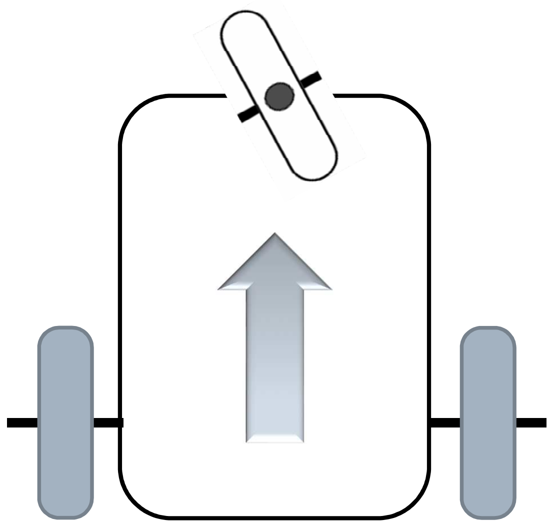


Wheels Symbols

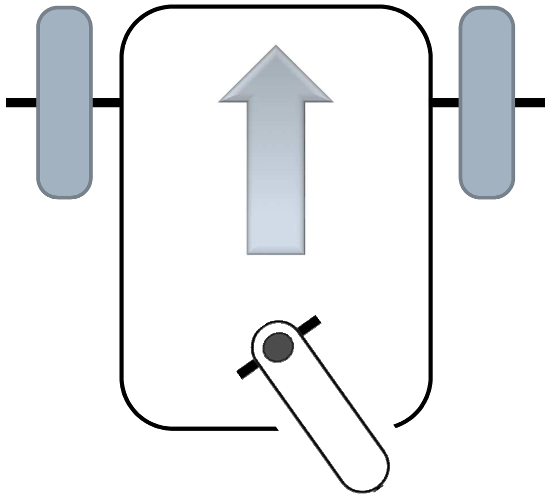


Typical structures

Active fixed wheels + steering wheel

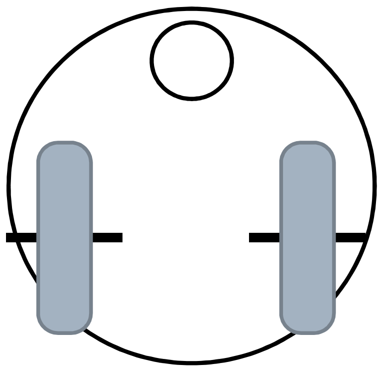


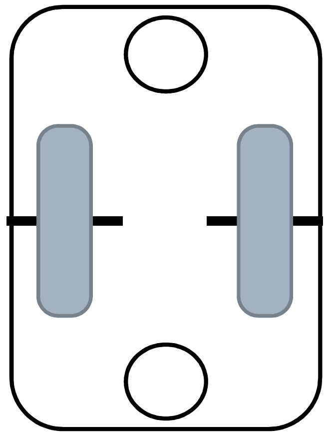
Active fixed wheels + castor passive wheel



Typical structures

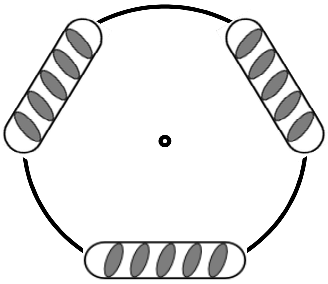
Differential wheels + passive spherical wheels



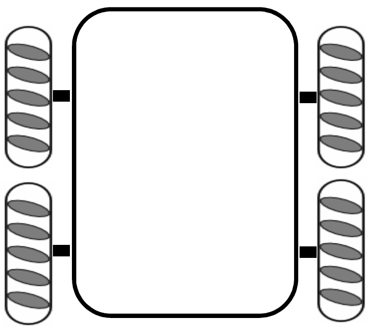


Typical structures

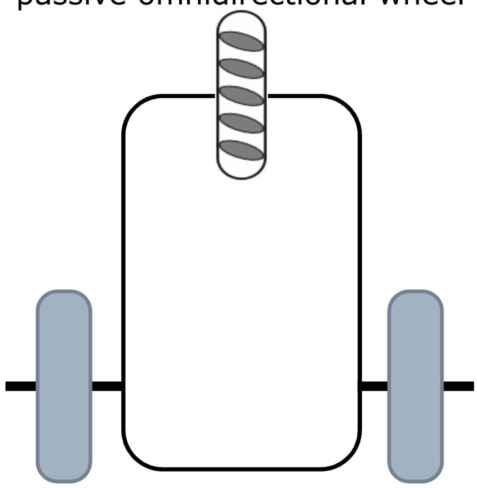
Active omnidirectional wheels



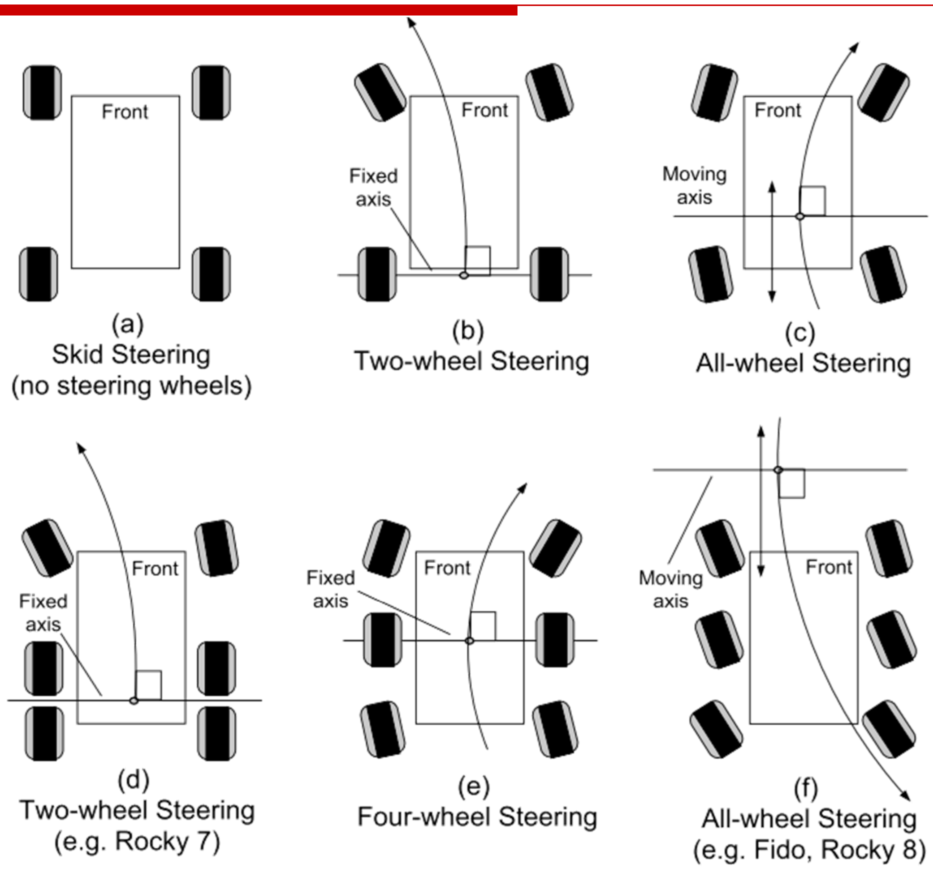
Omnidirectional active wheels



Fixed active wheel +
passive omnidirectional wheel



Steering



Steering

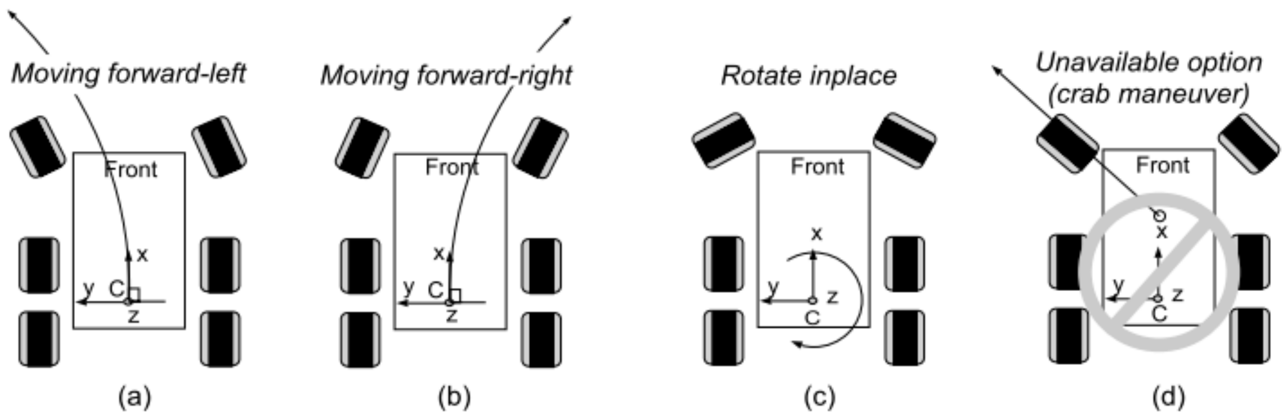


Fig. 2. The mobility of a six-wheel rover with two-wheel front steering (e.g. *Rocky 7*)

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Robotics

Mobile robots

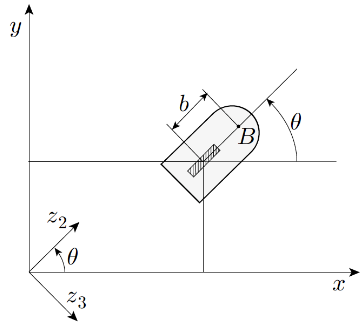
Part 1: Constraints and Kinematic Models

Kinematics and Constraints

- Wheeled vehicles are subject to constraints that limit the local mobility, without preventing (in general) the possibility of attaining arbitrary poses
  - *Think about parking your car: you can't move sideways, but you can do parallel parking with a suitable sequence of maneuvers!*

Constraints

- Let **q** ∈ R^n the vector generalized coordinates that describes the configuration of the robot and assume for the moment that the configuration space $\mathcal{C}$ (the set of all possible robot confgurations) coincides with R^n
  - For example in the case of a unicycle (single steered wheel) we have **q** = $[x\ y\ \theta]^T$, where (x, y) are the coordinate of the contact point of the wheel and θ the wheel angle w.r.t. an axis
- The robot motion is described by **q**(t) and can be subject to constraints



e.g. $x(t) - 2\theta(t) = 0$ → bilateral and scleronomic

$x(t) - 2\theta(t) + 3t = 0$ → bil. and rehonomic

Constraint Classification

i.e. time-variant vs time-invariant parameters

- Constraints can be expressed by
  - equalities → bilateral constraints =
  - inequalities → unilateral constraints <

  they can explicitly depend on time (rehonomic constraints) or being independent from it (scleronomic constraints)

- We will only deal with bilateral and scleronomic constraints, that is, represented by equalities in which the time does not explicitly appear
- We call holonomic (or integrable) those constraints that can be written as

$$h_i(\mathbf{q}) = 0, \quad i = 1, \ldots, k < n$$

e.g. if $x - 2\theta = 0$ (Holonomic) and $q = (x, y, \theta)$ ⇒ $q(x,y) = q(\theta, y)$ ⇒ one dim. is removed from q thanks to the constraint

Holonomic Constraints

- A system with all holonomic constraints is called a holonomic system
- The effect of holonomic constraints is to reduce the space of the accessible configurations to a subspace $\mathcal{C}$ with dimension n – k
  - The existence of k holonomic constraints leads to the elimination of k generalized coordinates, which can be expressed as functions of the other n – k (implicit function theorem) ⇒ Implicit function thm.
  - This procedure may lead to singularities. Alternatively, we can define a new, reduced set of n-k coordinates, directly defined on the accessible subspace

they produce limits on the configurations (e.g. rails of a train)

Kinematic constraints

e.g. $x + 2\dot{x} = 0 \Rightarrow \frac{dx}{dt} = -\frac{x}{2}$

limits the motion, NOT the config.

(Holonomic constr. limit the configuration)

- The constraints that involve both the generalized coordinates and their derivatives (velocities) are called kinematic constraints and, in general, they are expressed as

$$a_i(\mathbf{q}, \dot{\mathbf{q}}) = 0, \quad i = 1, \ldots, k < n$$

  - Kinematic constraints limit the admissible instantaneous motion of the system, constraining the set of generalized velocities that can be obtained in each configuration

- Kinematic constraints can be often written in Pfaffian form, that is, they are linear in the generalized velocities

e.g.

$$[3x + y]\,\dot{x} = 0$$

$$\mathbf{a}_i^T(\mathbf{q})\dot{\mathbf{q}} = 0, \quad i = 1, \ldots, k < n$$

(in matrix form) $\quad \mathbf{A}^T(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}$

this limits ONLY the motion (like a car that CANNOT move sideways)

Honolonomic and nonholonomic constraints

- The existence of k holonomic constraints implies the existence of k kinematic constraints → Derivi un holonomic constr. e ottieni un kinem. constr.

$$\frac{dh_i(\mathbf{q})}{dt} = \frac{dh_i(\mathbf{q})}{d\mathbf{q}}\dot{\mathbf{q}} = 0, \quad i = 1, \ldots, k$$

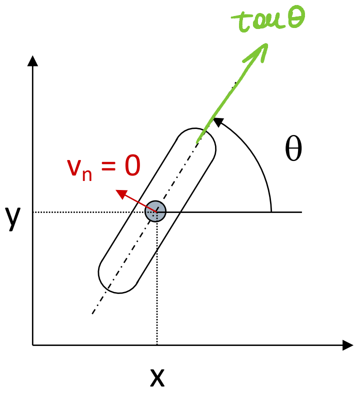
Abbiamo una derivata, vogliamo "tornare indietro"

- The contrary is not true in general, since kinematic constraints can be reduced to holonomic constraints only if they are integrable. If not, they are called nonholonomic or non-integrable
  - A system with at least a nonholonomic constraint is called nonholonomic system
  - Nonholonomic constraints do not imply any loss of accessibility in $\mathcal{C}$ (the number of generalized coordinates cannot be reduced), while velocities are constrained over a subspace of dimension n - k

Example of nonholonomic constraint

∅ HOLONOMIC CONSTR. ⇒ CAN be in ANY configuration

- A unicycle rolls without slippering on a horizontal plane



$$\mathbf{q} = [\mathrm{x} \quad \mathrm{y} \quad \theta]^T$$

(x, y) are the coordinates of the point of contact with the plane, θ is the wheel angle w.r.t. the x axis, the wheel rotation angle ϕ is not influent for our analysis

- The pure rolling constraint implies that the velocity of the contact point cannot have nonzero components along the normal direction to the wheel plane

$$\frac{dy}{dx} = tan\,\theta$$

i.e. it CAN ONLY Move Along the DIRECTION tanθ (because of the orientation of the wheel)

Example of nonholonomic constraint

CONT.

- Such a constraint can be expressed in Pfaffian form through:

chain rule

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{sin\,\theta}{cos\,\theta} \Rightarrow \dot{y}\,cos\,\theta = \dot{x}\,sin\,\theta$$

$$\Rightarrow \dot{x}\,sin\,\theta - \dot{y}\,cos\,\theta = 0 \Rightarrow [sin\,\theta \quad -cos\,\theta \quad 0]\,\dot{\mathbf{q}} = 0$$

$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$

→ NON INTEGRABLE

- The constraint is nonholonomic, since it does not imply any loss of accessibility in the wheel configuration space
  $\Rightarrow$ It is possible to displace the wheel from any initial configuration $\mathbf{q}_i$ to any final configuration $\mathbf{q}_f$ hrough a suitable sequence of motions that do not violate the pure rolling constraint

e.g. $\underbrace{[\cos\theta \ \sin\theta \ 0]}_{A^T} \dot{q} = 0$

Kinematic model

- Form the matrix formulation of the kinematic constraints

$$\mathbf{A}^{\mathrm{T}}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}$$

it is evident that the (n – k) admissible generalized velocities belong to the null space of $\mathbf{A}^{\mathrm{T}}(\mathbf{q})$ → i.e. All velocities $\dot{q}$ that satisfy our constraint $A^T(q)\dot{q} = 0$

$$\dot{\mathbf{q}} \in \mathrm{N}\left(\mathbf{A}^{\mathrm{T}}(\mathbf{q})\right)$$

- Denoting with $\{\mathbf{g}_1(\mathbf{q}), \ldots, \mathbf{g}_{n-k}(\mathbf{q})\}$ a base of $N(\mathbf{A}^{\mathrm{T}}(\mathbf{q}))$, it is possible to define the admissible trajectories as the solution of the nonlinear dynamical system

$$\dot{\mathbf{q}} = \sum_{j=1}^{m} \mathbf{g}_j(\mathbf{q})u_j = \mathbf{G}(\mathbf{q})\mathbf{u}, \quad m = n - k$$

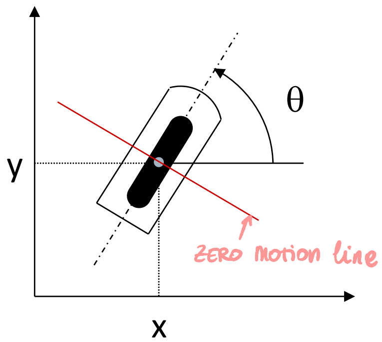
that represents the kinematic model of the constrained system

Kinematic model

- Vector **q** represents the state vector of the system, while $\mathbf{u} = [\,u_1 \ldots u_m]^{\mathrm{T}}$ is the input vector
- The obtained system is said to be driftless, since a null input causes $\dot{\mathbf{q}} = \mathbf{0}$
- The choice of the vectorial input fields $\mathbf{g}_j(\mathbf{q})$, is not unique and, consequently, also the choice of input **u** is not unique
  - The compontents of **u** can either have a physical interpretation and/or be related to the available control input, or not

Kinematic model of a unicycle

- Let us consider a unicycle, a vehicle with a single steerable wheel



seen in prev. example

The configuration is described by

$$\mathbf{q} = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$$

and the pure rolling constraint is

$$\dot{x}\,\sin\theta - \dot{y}\,\cos\theta = 0 \Rightarrow \begin{bmatrix} \sin\theta & -\cos\theta & 0 \end{bmatrix}\dot{\mathbf{q}} = 0$$

The line passing by the contact point of the wheel and orthogonal to the direction of motion is called zero motion line

- A base of the null space of the constraint matrix is given by:

$$\mathbf{g}_1(\mathbf{q}) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \end{bmatrix}^T, \; \mathbf{g}_2(\mathbf{q}) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

CONT.

Kinematic model of a unicycle

- Thus, we have:

$$\mathbf{G}(\mathbf{q}) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}$$

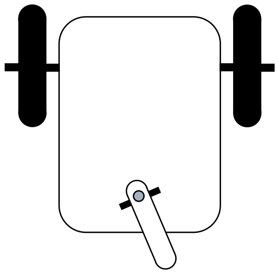
and, correspondingly, the unicycle kinematic model is given by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega$$

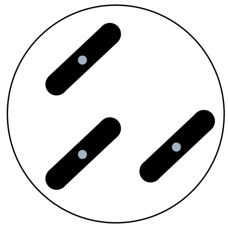
In this case the inputs have a physical meaning and are related to srtaightforward control inputs: *v* is the driving velocity (modulus of the velocity of the contact point) and ω is the steering velocity

Equivalence between the unicycle and other structures

- A unicycle would not be practically useful, due to its balancing problems. From a kinematic point of view, however, it is equivalent to more stable structures, such as
  - Differential drive vehicle



  - Synchro drive vehicle



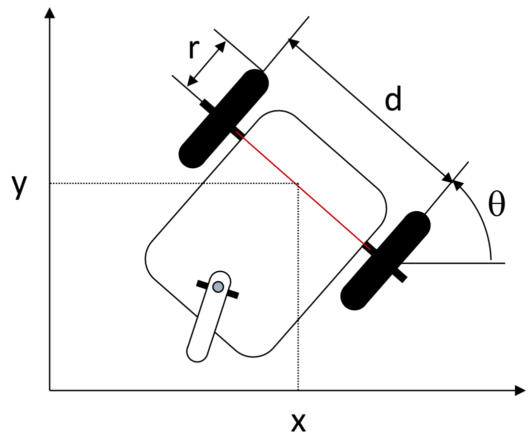
Equivalence with the differential drive

- The «physical» inputs are the rotation velocities of the two wheels, ω_R and ω_L
- There exists a direct correspondence between such input and the ones of the unicycle (drive velocity v and steering velocity ω)

$$v = \frac{r\left(\omega_R + \omega_L\right)}{2}$$

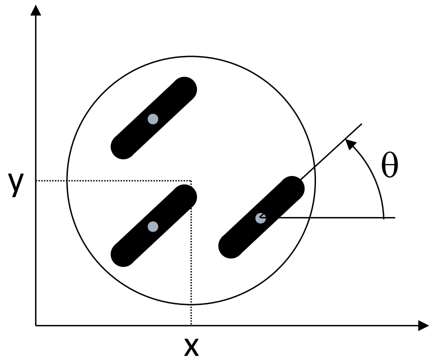
$$\omega = \frac{r\left(\omega_R - \omega_L\right)}{d}$$

(x, y) are the coordinates of the middle point of the wheel axis, θ the vehicle heading



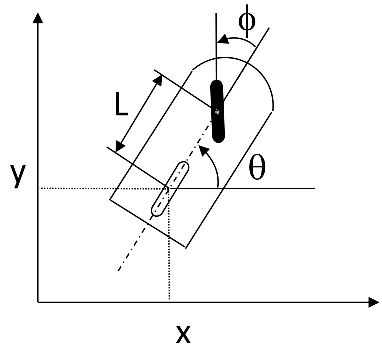
Equivalence with a synchro drive vehicle

- The equivalence is straightforward, since both drive and steer velocity are common to the three wheels
  - Coordinates (x, y) can represent here any point of the chassis, while θ is the wheels heading (the chassis has constant heading)



Kinematic model of a bicycle

- Let us consider a bicycle, that is a vehicle with a steered wheel and a fixed one, mounted at a distance L
  - Possible choice for the generalized coordinates:
  - (x, y) $\rightarrow$ coordinates of the contact point of the rear wheel
  - $\theta \rightarrow$ heading of the vehicle w.r.t. x
    $\phi \rightarrow$ steering angle of the front wheel



$$\mathbf{q} = \begin{bmatrix} x & y & \theta & \phi \end{bmatrix}^T$$

Kinematic model of a bicycle

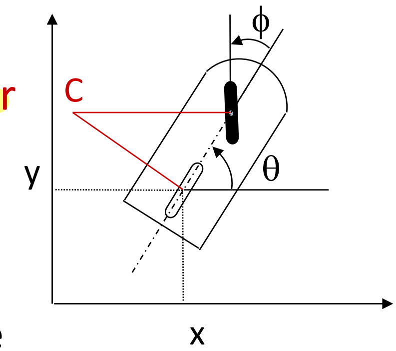
- There exist two pure rolling constraints, one for each wheel

front wheel → $\dot{x}_f \sin(\theta+\phi) - \dot{y}_f \cos(\theta+\phi) = 0$

rear wheel → $\dot{x} \sin\theta - \dot{y} \cos\theta = 0$ → same as unicycle

where (x_f, y_f) are the coordinates of the center of the rear wheel

- The two no motion lines intersect in a point C called instantaneous center of rotation
  - The position of C depends only on **q**
  - Every point of the chassis moves instantaneously along a circumference arc centered in C



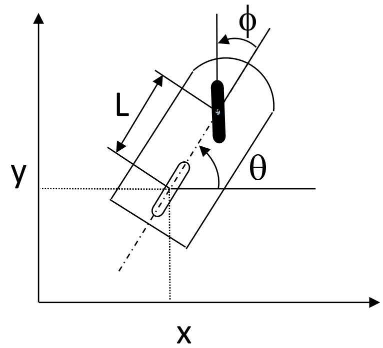
Kinematic model of a bicycle

- The coordinates (x_f, y_f) of the center of the front wheel can be expressed as a function of those of the rear wheel, using the rigid body constraint

$$x_f = x + L\cos\theta$$
$$y_f = y + L\sin\theta$$

- After some algebra, the first pure rolling constraint can be expressed as:

$$\dot{x}\sin(\theta+\phi) - \dot{y}\cos(\theta+\phi) - L\dot{\theta}\cos\phi = 0$$



- Matrix form of the constraint $\mathbf{A}^{\mathrm{T}}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}$

$$\mathbf{A}^{\mathrm{T}}(\mathbf{q}) = \begin{bmatrix} \sin\theta & -\cos\theta & 0 & 0 \\ \sin(\theta+\phi) & -\cos(\theta+\phi) & -\mathrm{L}\cos\phi & 0 \end{bmatrix}$$

- The rank of $\mathbf{A}^{\mathrm{T}}$ is 2 and, consequently, its null space has dimension 4 – 2 = 2. A possible base for N($\mathbf{A}^{\mathrm{T}}$($\mathbf{q}$)) is formed by the columns of

$$\mathbf{G}(\mathbf{q}) = \begin{bmatrix} \cos\theta\cos\phi & 0 \\ \sin\theta\cos\phi & 0 \\ \dfrac{\sin\phi}{\mathrm{L}} & 0 \\ 0 & 1 \end{bmatrix}$$

Kinematic model of a bicycle

- With this choice the kinematic model is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi \\ \sin\theta\cos\phi \\ \dfrac{\sin\phi}{\mathrm{L}} \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_2$$

where input u_2 is the steering velocity ω (provided by the front wheel)

- The expression of u_1 depends on the drive (front or back) of the vehicle
  - If the bycicle has front drive, we have directly u_1 = v, where is the driving velocity of the front wheel

Kinematic model of a bicycle

- If the vehicle is back drive, we can derive u_1 observing that the first two equations must coincide with those of the unicycle. This leads to set

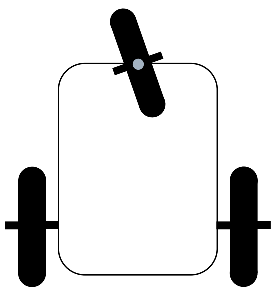
$$u_1 = \frac{v}{\cos\phi}$$

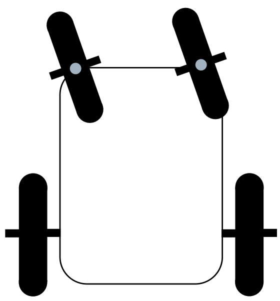
where v is the drive velocity of the back wheel, thus yielding the kinematic model

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ \dfrac{\tan\phi}{L} \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega$$

Equivalence of the bicycle with other structures

- Also the bicycle presents balance problems in practice, but also in this case we have kinematically equivalent structures
  - tricycle
  - automobile





In both cases we may have rear of front drive. The point (x, y) is the midpoint of the back wheel axis, θ is the vehicle heading, and ϕ is the steering angle