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EXERCISE AND SPORT SCIENCES REVIEWS

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Determining Muscle's Force and Action in Multi-Articular Movement

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INTRODUCTION

Knowledge of muscle forces and their action on the body is fundamental for improving the diagnosis and treatment of persons with movement disabilities and analyzing the techniques used by Olympic-caliber athletes to achieve exceptional performance. Muscle forces can sometimes be recorded directly or can be computed from electromyographic (EMG) signals. Alternatively, muscle forces can be estimated using models of multijoint dynamics, either alone or together with kinematic and reaction force data. Once muscle force is determined, the next step is to determine the body motion caused by the muscle force.

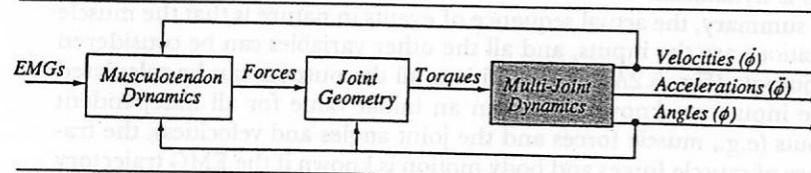
The direction of the torque exerted by a muscle, determined by the muscle's anatomical location, is often used to infer the muscle's action on movement of the body. For example, a muscle that develops a flexor torque is assumed to flex the spanned joint. For tasks in which only one joint of the body is free to rotate in a single direction, any muscle spanning the joint and exerting a flexor torque will indeed act to flex the joint.

For multijoint movement, however, extreme caution should be exercised when anatomy alone is used to infer the action of a muscle, since a muscle acts to accelerate all joints, whether spanned or not. For example, soleus exerts only an ankle extensor torque, yet it can accelerate the knee into extension with more vigor than it accelerates the ankle into extension. Biarticular muscles can have a multitude of actions. For example, gastrocnemius, which exerts both ankle-extensor and knee-flexor torques, can act to (a) extend the ankle and flex the knee, (b) flex the ankle (*sic*) and flex the knee, or (c) extend the ankle and extend the knee (*sic*). Since a muscle's action depends on the position of the body and on the muscle's interaction with external objects (such as the ground), it can vary among motor tasks, and even during a single motor task.

Classified either by its action on joint rotation or by the direction of its torque(s), a muscle can be said to assist (or hinder) the motor task and can also be said to work in concert with (or in opposition to) other muscles. At times, a muscle is called an *agonist* if its torque is in the same

FIGURE 6.1

System dynamics. The excitation of muscles received from the nervous system is manifested in their electromyograms (EMGs), the “inputs” to the dynamic system. The forces developed by muscles and transmitted to their tendons (forces) depend on the integrated dynamical properties of the muscles and tendons (musculotendon dynamics) and on the joint angles (ϕ) and velocities ($\dot{\phi}$). Forces are linear in torques by the relations among the muscles and the joint centers of rotation (joint geometry), which depend on joint angles. Torques cause rotary motion of the joints, which are the joint angular accelerations ($\ddot{\phi}$) and velocities ($\dot{\phi}$), and joint angles (ϕ).



direction as the net torque that muscles must develop to produce the motion. Two muscles can also be called an *agonist–antagonist pair* if their torques are in opposite directions. Alternatively, the actions caused by a muscle can be used to determine agonist–antagonist relationships. Because muscles that work together need not function identically, it may be best to study how muscles work as *synergists* rather than as *agonists*.

MUSCLE FORCE DURING MOVEMENT

Muscle forces can sometimes be measured directly. Often this is not possible, and instead muscle forces are estimated by processing the recorded EMG signals. Another indirect approach is to apply static optimization to estimate what each muscle force must be to produce the required torques, which are computed from measurements of body motion and external reaction forces. Muscle forces can also be estimated without measurement of EMG signals, body motion, or reaction forces by applying dynamic optimization techniques to a model of the system dynamics (Fig. 6.1) and the task being performed.

Recording Tendon Force

Tendon force has been recorded in animals during a variety of motor tasks [117, 118]. One method is to measure the strain, which is proportional to force, in a metal transducer shaped as a buckle and through which the tendon passes [11, 117, 118]. Another method is to use a liquid metal strain gauge placed in parallel with the tendon [18, 106]. With either technique, the net force delivered to the tendon from all of the

muscle's motor units is recorded. This net force is usually assumed to be the sum of the forces generated by the motor units, though it may not be if some units absorb mechanical energy from others. Obviously, only those muscles with well-defined external tendons are candidates, and the technique has limited potential in humans.

Estimating Force from the Electromyogram

The estimation of force from an EMG signal [58] is most often based on a Hill-type model of muscle dynamics [55, 125]. In such models, muscle force depends on the neural excitation (EMG signal) and the length and velocity (rate of shortening or lengthening) of the muscle fibers. As Hill noted long ago, tendon stretch affects the length and velocity of muscle fibers. Therefore, muscle force must be computed from a model that includes both muscle and tendon (Figs. 6.1 and 6.2A). The model must be a dynamic one because force depends on its past trajectory [125]. With a model of the attachment of muscle and tendon to the body, muscle-fiber length and velocity can be calculated from the joint angles and velocities. Therefore, the model has EMG activity, joint angles, and joint velocities as inputs, and force as the output (Figs. 6.1 and 6.2A).

A consequence of this type of model for muscle dynamics is the well-known, steady-state proportional relation between the force and the smoothed EMG signal observed in isometric contractions [77]. The estimation of muscle forces from EMG signals has been used to gain insight into muscle's “active state” [43], neural control of human locomotion [104, 105], and energy storage in series musculotendon elastic structures during human walking [56] and jumping [13]. Forces in muscles during walking, estimated from EMG signals, have been shown to compare favorably with estimates computed from measurements of body motion and external reaction forces, as described below [57, 93, 105].

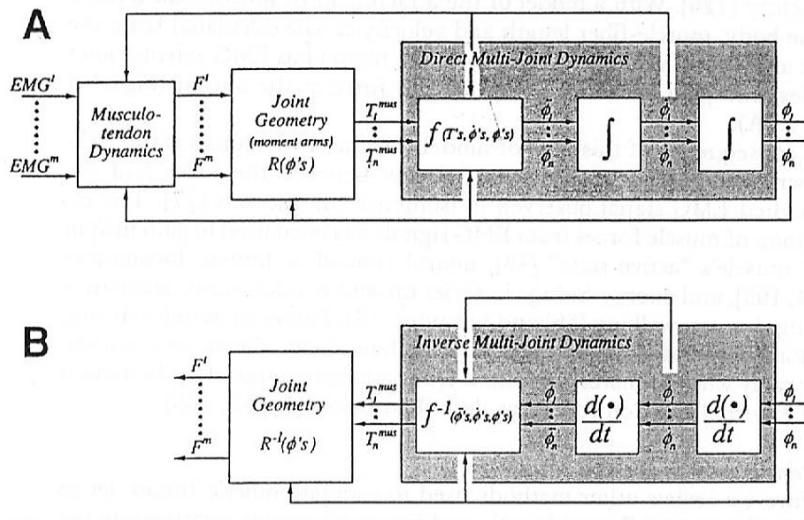
Systems Dynamics and Net Muscle Torques

Before we review other methods used to estimate muscle forces, let us review the *natural* flow of kinetic and kinematic events occurring in the body (i.e., the system dynamics; Fig. 6.1). The nervous system excites the muscular system, and this excitation is manifested in the EMG signals. Excitation of the muscular system leads to the development of forces, which are related to torques at the joints by the juxtaposition of the muscles to the joints. Torques cause angular accelerations of the joints which, after time, cause the joint angular velocities and angles to change. Thus, the natural flow of events is efferent: from the nervous system, to the muscular system to the skeletal system.

To scrutinize the natural flow of events further, let us assume that m muscles are producing movement of n joints, where $m > n$ (Fig. 6.2A).

FIGURE 6.2

Comparison of direct (A) and inverse (B) system dynamics. A, Muscle excitations are the inputs, body motions are the outputs, and thus the flow of events is natural. If all the elements are modeled (i.e., musculotendon dynamics, moment arms $R(\phi's)$, and multijoint dynamics), and if the criterion by which performance is judged is specified mathematically, dynamic optimization can be applied to find all the “inputs” ($EMG^1 \dots EMG^m$), and thus all the “outputs” (i.e., $F^1 \dots F^m$; $T_1^{mus} \dots T_n^{mus}$; $\ddot{\phi}_1 \dots \ddot{\phi}_n$; $\dot{\phi}_1 \dots \dot{\phi}_n$; $\phi_1 \dots \phi_n$). B, Body positions are the inputs, and thus the flow of events is inverted. Some variation of the inverse dynamics approach is used to find the net muscle torques ($T_1^{mus} \dots T_n^{mus}$) derived from $f^{-1}(\phi s, \dot{\phi}s, \ddot{\phi}s)$, measurements of $\phi_1 \dots \phi_n$ and computations of $\dot{\phi}_1 \dots \dot{\phi}_n$ and $\ddot{\phi}_1 \dots \ddot{\phi}_n$ (see text). Individual muscle forces are not uniquely determined from net muscle torques and static optimization is used to find a “best” set of muscle forces (see text).



Thus, m EMG signals ($EMG^1 \dots EMG^m$) generate m muscle forces ($F^1 \dots F^m$), which are trajectory dependent (i.e., dynamic, given by musculotendon dynamics). Each muscle force generates a torque about all the joints it spans. Each torque is the product of muscle force and its *moment arm* (the muscle's shortest distance from the joint). The sum of the torques developed by all muscles spanning a joint is the *net muscle torque*; so for n joints there are n net muscle torques ($T_1^{mus} \dots T_n^{mus}$) (see Appendix A for discussion of muscle force, moment of muscle force, moment arm, and torque).

These net muscle torques produce motion of the joints, which is given by the direct multijoint dynamics of the system (Fig. 6.2A). The reason the dynamics is referred to as “direct dynamics” is that it specifies the *natural sequence* in which motion occurs. Thus, the net muscle torques, in combination with torques due to gravity and current motion of the joints (not shown in Fig. 6.2A; see figure legend), accelerate the joints of the body instantly ($\ddot{\phi}_1 \dots \ddot{\phi}_n$). However, joint angular velocities ($\dot{\phi}_1 \dots \dot{\phi}_n$) and joint angles ($\phi_1 \dots \phi_n$) do not change instantly, but develop over time (Fig. 6.2A, integral symbols). Thus, motion of the joints is dynamical.

In summary, the actual sequence of events in nature is that the muscle excitations are the inputs, and all the other variables can be considered the outputs (Fig. 6.2A). By definition, all the outputs can be calculated if the inputs are known. So given an initial value for all independent outputs (e.g., muscle forces and the joint angles and velocities), the trajectory of muscle forces and body motion is known if the EMG trajectory is given.

Estimating Net Muscle Torques from Inverse Dynamics

Inverse dynamics can be employed to estimate the net torque that muscles must have produced to have generated the movement [17, 22]. Thus, the natural, direct multijoint dynamics are inverted and used to compute the n net muscle torques ($T_1^{mus} \dots T_n^{mus}$), given that the kinematics ($\phi_1 \dots \phi_n$; $\dot{\phi}_1 \dots \dot{\phi}_n$; $\ddot{\phi}_1 \dots \ddot{\phi}_n$) are known (Fig. 6.2, compare the direct multijoint dynamics with the inverse multijoint dynamics). Thus recorded data of joint angles versus time can be differentiated once to yield velocities versus time, or a second time to yield joint accelerations versus time. Once joint angular positions, velocities, and accelerations are calculated, then net torques can be computed.

However, reaction force data are often used to compute the net muscle torques [17, 20, 34, 81, 101, 120, 121, 128]. External reaction forces serve as a measure of the inertia forces of the body segments, which obviates modeling of the whole body [17, 34, 81]. In gait, for example, net ankle muscle torques can be computed from ground reaction forces, from a model of the foot, and from estimates of the acceleration of the foot [17, 101, 120]. Thus, neither the mass distribution of all segments proximal to the ankle, nor their accelerations, would have to be measured or computed. And acceleration data are especially noisy because position data are differentiated twice [20, 120]. Thus, the inertia force of all the segments proximal to the ankle, including the large inertia force of the trunk [68], does not have to be estimated from kinematic data.

This variation of the inverse dynamics approach to compute net muscle torques continues to be widely used in studies of many tasks, for example, in studies of human and animal gait [2, 20, 29, 101, 104, 105,

120, 121], jumping [1, 13, 76], sitting [113], and pedaling [46, 66, 91]. The reason for calculating net muscle torques, rather than examining recordings of body motion alone, is that finding net muscle torques makes it easier to infer which muscles need to generate force, and how much force. Knowledge of net muscle torques needed at the hip, knee, and ankle during walking, for example, can be helpful in the quantitative assessment of amputee and pathological gait, and in the design of subsequent rehabilitation strategies [22, 29, 85, 119–121].

Estimating Force Using Static Optimization

Once net muscle torques are computed, determining individual muscle forces is often not straightforward because the number of muscles m may exceed the number of joints n . In this case, there is not enough information to uniquely determine the individual muscle forces, and many solutions are possible (e.g., $R^{-1}(\phi)$ does not exist even though $R(\phi)$ does; see Fig. 6.2A, B). It's much like being asked to solve for x and y given only that $x + y = 6$. A solution can be found, however, if enough muscles are observed or assumed to be inactive so that the number of active muscles does not exceed the number of joints.

Static optimization can also be employed to find the muscles forces from the net muscle torques [27, 100, 103, 110]. With this approach, a mathematical description of what the body is trying to optimize at a specific instant, the cost function, is defined. Using relatively simple algorithms, a solution which minimizes the cost function is found, and the individual muscle forces are determined. Then the algorithm is repeated to find a solution corresponding to another instant of the motor task. A commonly used cost function is the sum of muscle stresses, where each stress is weighted by an exponent that is the same for each muscle [27, 28]. (Muscle stress is the force in muscle divided by its physiological cross-sectional area.)

The choice of a suitable cost function is critical to this approach, and the cost function determines what kinds of results are possible. For example, a minimal stress cost function cannot result in the coactivation of antagonist muscles, a strategy that might be useful for improving measures of performance unrelated to simple energetic efficiency.

Nevertheless, because static optimization is based on well-developed theories [79], computational algorithms are commercially available to readily solve such problems. Static optimization has been used to find the distribution of forces of limb muscles during walking, jumping, and cartwheeling [28, 29, 32, 47, 98, 102, 110, 116], of elbow and knee muscles during isometric and isokinetic tasks [4, 27, 33, 54, 103], of trunk muscles during posture [12], of facial muscles during biting [95], and of finger muscles during flexion [19]. Limitations of the approach include the somewhat arbitrary choice of a cost function [28, 33, 54,

100]; its sensitivity to parameters, such as muscle cross-sectional area [16]; and the resolution of the kinematic recordings or torque computations [98].

Estimating Force from Direct Dynamics

If recordings of EMG signals are available, a model of the system dynamics can be used to generate muscle forces and body motion (Fig. 6.2A). A simulation of human walking, partially based on the “central pattern generator” theory of locomotion [105], has employed this approach, though the effort to date seems, understandably, to have been in constructing and validating the simulation rather than in analyzing the simulation to gain fundamental understanding of human gait.

We have empathy because our past experience in using eight muscles to drive a four-segment model of the body to jump has indicated how critical muscle timing is [62]. We found it virtually impossible to find EMG inputs by trial and error that produce physiological trajectories, even when we based the simulation EMG inputs on measured EMG signals. The reason is, of course, that control of a four-segment, inverted pendulum is difficult (e.g., balancing a one-segment broom on one's hand is hard enough). In fact, we have recently found that a change by 5 milliseconds in the excitation of vastus lateralis from its optimal pattern, as determined by dynamic optimization, causes the propulsion phase of the jump to be very uncoordinated [97]. Nevertheless, having used the trial-and-error approach, we had gained insight into muscle, tendon, and body-segmental energetics [62]. Still, we feel that the application of dynamic optimization provides much greater insight and should therefore be used when possible.

Estimating Force Using Dynamic Optimization

The application of dynamic optimization to models of the system dynamics (Fig. 6.2A) is potentially the most powerful method for studying muscle forces during human movement. The power of the method derives from the scope of the modeling: Not only is there a model of the system dynamics, but also a model of what the motor task is attempting to achieve (the performance criterion) [9, 52, 127]. As with the cost function associated with static optimization, the performance criterion is often ambiguous, though reasonable conjectures are at times possible (e.g., maximum-height vertical jumping) [76, 127, 129].

A dynamic optimization algorithm uses a model of the system dynamics to find the inputs (e.g., EMGs), and all the outputs (e.g., forces), that maximize the performance of the whole task as defined by some single criterion. For some movements, alternative performance criteria may lead to similar neural control strategies (inputs) [90, 114]. One difference between static and dynamic optimization is that static optimization cal-

culates the inputs at only a single instant of time, independent of the past or the future. And the algorithm is run repeatedly to find the inputs at other instants. In contrast, a dynamic optimization algorithm is run once and computes the inputs (and outputs) for all times during the motor task.

Once a solution to a dynamic optimization problem is found, a wealth of information is available to compare with experimental data. Because there is a model of the system dynamics, the complete history of muscle excitations, muscle forces, external reaction forces, joint angles, joint velocities, and joint accelerations can be calculated. Thus the model can be used to understand all aspects of muscle and multijoint coordination. Not only would all muscle forces and their contributions to movement be known, but the neural control strategy for the coordination of the motor task would be known as well.

Dynamic optimization can thus provide a deep understanding of the sensitivity of the solution to assumptions made in formulating the model [76, 97, 123, 127], including assumptions of neural connectivity. For instance, the structure of the central nervous system (CNS) through which control must pass could be assumed to be a one-to-one mapping between the higher CNS structures and the muscles (motoneuronal pools). In this case, the model is permitting the independent excitation of muscles [127]. Or the CNS could be assumed to be constrained by some CNS network, which has fewer inputs than there are motoneuronal pools. In this case, the muscle excitation pattern found by the optimization would be different than the one found in the absence of the network. Also, performance of the motor task would be less when the network exists because performance is always greater in the absence of constraints [9].

Should the optimization results provide a good match with experimentally observed trajectories, then both the motion itself and the control strategy being used would appear to have been well modeled. In this case, one might as well “pack one’s bags and go home,” because the intellectual part of the job would seem to be done. Of course computer simulation of movement is never perfect. So, the challenge is to identify, perhaps through additional experiments, the significant inaccuracies of the current modeling data. Development of a next-generation, more complex model would then be justified to better comprehend the motor task, and to eliminate known inadequacies of the current-generation model.

Though dynamic optimization has been used to study jumping [53, 76, 97, 123], walking [23, 30], kicking [50], posture [24], and arm movements [38, 92], its full potential has not yet been realized. This is because solving dynamic optimization problems can be extremely difficult, especially if many body segments and muscles are modeled. In fact, the

techniques needed to implement suitable dynamic optimization algorithms are at the forefront of control system theory [76, 97].

A MOTOR TASK'S DYNAMIC EQUATIONS OF MOTION

Though muscle force can be found using any of the approaches reviewed in the previous section, an important question still remains: What is the contribution of a muscle's force to the body motion, particularly the angular accelerations of the joints? The answer can be found by applying mechanics to derive the direct multijoint dynamics (Fig. 6.2A). However, except for studies that apply dynamic optimization, the governing dynamical equations are usually not formulated to answer this question. And even when formulated, the dynamical equations are rarely analyzed in depth to ascertain the action of muscle on body motion.

The analysis of muscle action using the dynamical equations is worthwhile because the equations may differ among motor tasks, even though the mechanical properties of the body segments and joints are usually assumed to remain the same. Muscle action is therefore task dependent. The equations may differ because the body segments and the environment interact, just as the body segments interact among themselves, and all these interactions determine the dynamical equations [67, 72].

Though the subject of dynamics is beyond the scope of this view, this section reviews the dynamics of several motor tasks to convey the essence of the principles involved. Also discussed are the assumptions about body-segmental interactions made in studying multiarticular movement and muscle coordination. Using these same assumptions in subsequent sections, we present the dynamical equations for the simplest example of a multiarticular motor task, the two-joint system. These equations are then discussed to illustrate how multijoint movement differs from single-joint movement and to show how muscle action, as described in textbooks, can lead to misinterpretations of muscle action during multiarticular movement.

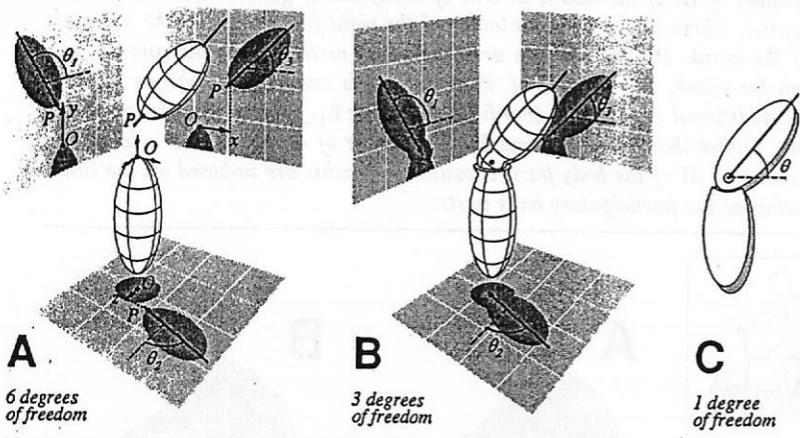
Modeling Body Segments and Joints

Coordination studies almost always assume that the body segments are rigid. Although rarely stated explicitly, the reason is that intersegmental motion is believed to be dominated by the relative movement among the body segments rather than by movement of parts within a body segment. We believe the “rigid-body” assumption is currently appropriate, except perhaps when the body impacts rigid objects, as evident by the subsequent shock-wave propagation in the body [31].

The least restrictive representation of the motion of one body segment relative to an adjacent segment allows both rotation and translation (Fig.

FIGURE 6.3

Number of degrees of freedom (df) of a joint. A, The maximum number of df of a joint is 6. θ_1 , θ_2 , and θ_3 specify the orientation of the upper body segment relative to the lower segment. x, y, and z specify the translation of the upper segment with respect to the lower segment (i.e., the position of P relative to O). B, A ball-and-socket joint has only 3 df (i.e., θ_1 , θ_2 , and θ_3) since P and O coincide. C, A pin joint has 1 df (i.e., θ) since the upper segment rotates in a plane, and about a fixed axis.



6.3A). Thus, the number of degrees of freedom (df) of each joint is at most 6: 3 associated with the orientation of one segment relative to the other (θ_1 , θ_2 , θ_3 in Fig. 6.3A), and 3 with the position of a point in one segment relative to the other segment (x, y, z in Fig. 6.3A). In studies of joint mechanics, where the emphasis is on stresses and strains in joint structures, it may be necessary to assume 6 df for each joint [3, 22, 74, 112]. Some studies that use static optimization to find muscle forces also assume 6 df joints [29, 100].

For sound practical reasons, models of joints are kept as simple as possible, and coordination studies almost always assume frictionless ball-and-socket joints or pin joints (Fig. 6.3B,C) [6, 51]. Translational motion within a joint, assumed to be small, is generally ignored. In studies of walking, the hip is often assumed to be a ball-and-socket joint (Fig. 6.3B), in which case the pelvis is assumed to rotate relative to the femur [80]. Thus, the relative location of the two segments can be described by three angles (Fig. 6.3B). In vertical jumping, for example, the hip may even be assumed to be a pin joint, which has only 1 df (θ in Fig. 6.3C), because sagittal plane motion is assumed to dominate performance of the motor

task [13, 123, 127]. A pin-joint hip has also been used to study coordination of walking, where the focus is on sagittal-plane motion [23, 30, 101, 120, 121]. When a joint is assumed to have less than 6 df, the implications are that joint structures such as bone, ligaments, and menisci, for example, constrain the joint to have less than 6 df and, except for these effects, affect intersegmental dynamics in no other way.^a

Because frictional losses in joints are assumed negligible, the joint reaction forces do not have to appear in the dynamical equations [72] (e.g., see equations in references 19, 30, 60, 97). Expressions for joint forces are often found, however, in the derivation of the dynamical equations [26, 75, 83, 94, 96]. If frictional losses in joints are high, such as in pathology, then joint reaction forces would have to appear explicitly in these equations. As a consequence, the net effect of muscle force on joint rotation would depend not only on the torque developed by the muscle, but also on its contribution to the magnitude and direction of the compressive forces produced between the articular surfaces of the joint (Appendix A) [51, 72].

Determining the Degrees of Freedom of a Motor Task and Choosing a Set of Generalized Coordinates

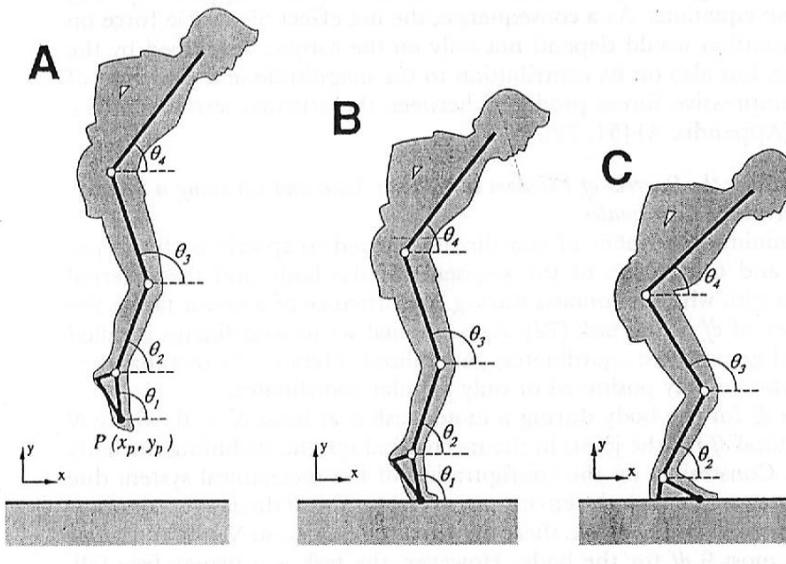
The minimum number of coordinates needed to specify both the position and orientation of the segments of the body and the external objects with which it contacts during performance of a motor task is the number of df of the task [72]. Any minimal set of coordinates is called a set of generalized coordinates; generalized refers to the fact that they need not be only positional or only angular coordinates.

The df for the body during a motor task is at most $N + 6$, where N is the total df for the joints in the mechanical system, including the body joints. Constraints on the configuration of the mechanical system due to its interaction with the environment reduce the df during motor tasks. For example, in Fig. 6.4A, there are three pin joints, so $N = 3$ and there are at most 9 df for the body. However, the task is a planar free fall, and remaining in the plane imposes three constraints. Thus there are 6 df during planar free fall; one possible set of generalized coordinates is shown, consisting of two Cartesian coordinates to specify the point P (i.e., x_p , y_p) and four angular coordinates to specify the orientation of the segments (i.e., θ_1 , θ_2 , θ_3 , θ_4). Other variables could be used for the generalized coordinates, such as the position of a point other than P, or intersegmental angles.

^aSometimes these structures are assumed to generate passive torques that act, additionally, to hinder joint rotation (e.g., to prevent joint hyperextension) (51, 52, 122).

FIGURE 6.4

Number of degrees of freedom (df) of a planar motor task (e.g., landing, jumping) whose parts form an open kinematic linkage. In A, B, and C, the body parts involved in the task are assumed to consist of three pin joints and four segments. The number of df of these body parts is four (i.e., $\theta_1, \theta_2, \theta_3, \theta_4$). A, The number of df of the task is 6; 4 to specify the orientation of the segments ($\theta_1, \theta_2, \theta_3, \theta_4$) and 2 (x_p, y_p) to specify the rectilinear position of the body with respect to the ground reference frame (x, y). B, The number of df of the task is 4 ($\theta_1, \theta_2, \theta_3, \theta_4$) since (x_p, y_p) are stationary. C, the number of df of the task is only 3 ($\theta_2, \theta_3, \theta_4$), since the foot is stationary. Notice that the number of df of the task can be greater than, equal to, or less than the number of df of the body parts (A, B, and C, respectively).

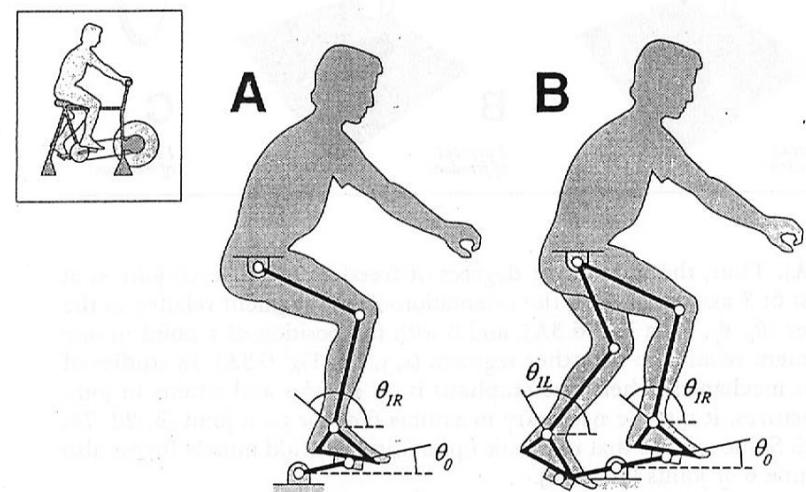


The df of a motor task may vary from start to finish, such as during falling and landing (Fig. 6.4A–C). The df decrease by 2 when the toes first make contact with the ground (Fig. 6.4B), and the number decreases by 1 more when the sole of the feet make contact (Fig. 6.4C). In vertical jumping, this sequence is reversed, and the df increase during the task.

The above are considered examples of an “open linkage” because one end of the “chain” of the rigid bodies is free to move. During some motor tasks, the body interacts with the environment to form a “closed linkage.” For example, while seated on a bicycle, the position of the hip may remain stationary, as does the axis of rotation of the crankshaft (Fig. 6.5). The

FIGURE 6.5

Number of degrees of freedom (df) of a planar motor task (pedaling, insert) whose parts form a closed kinematic linkage. In A and B the body parts involved in the task are assumed to consist of three pin joints (hip, knee, and ankle) and three segments (thigh, shank, and foot) of each leg. The number of df of the body parts participating in the task is thus 6, 3 for each leg. A, The two legs are assumed to pedal symmetrically and deliver power only during the downstroke. The task is thus specified by motion of either one of the two legs. The number of df of the task is 2. One of many sets of generalized coordinates is (θ_{IR}, θ_o), where θ_{IR} is the orientation of the right foot and θ_o is the orientation of the crank. B, The two legs are assumed to mechanically interact through the crank. The number of df of the task is one greater than in A, and the one additional generalized coordinate could be θ_{IL} , the orientation of the left foot. Notice that in both A and B, the number of df of the task is less than the number of df of the body parts because constraints are imposed on the configuration of the participatory body parts.



effect of a closed linkage is that there are additional constraints on the configuration, and thus the df are further reduced. One possible model of interlimb coordination of pedaling a stationary ergometer might assume that the hips are stationary, that the limbs are symmetrically controlled except for being 180° out of phase, and that each limb delivers power to the crank only during the power stroke (Fig. 6.5A). Thus the df of this model of pedaling is 2, and a set of generalized coordinates

could be the crank and right pedal (foot) angles (i.e., θ_0 and θ_{1R} , Fig. 6.5A). If mechanical coupling of the limbs is to be analyzed (Fig. 6.5B), then the *df* of the task increases by 1, and the left pedal angle (θ_{1L}) could be added as a generalized coordinate. An alternative set of generalized coordinates could be the knee and hip angles of the right side, and the knee (or hip) angle of the left leg, since orientation of all other joint angles and the crank would then be specified.

For some multiarticular, single *df* tasks, the generalized coordinate need not be a joint angle. The bilaterally symmetrical leg press, where the hips are assumed stationary, and the feet flat and fixed to a foot plate, has 1 *df*, and thus only one generalized coordinate (Fig. 6.6). The orientation of the foot plate (θ) is an appropriate generalized coordinate since all other parts that can move (the foot, the shank, and the thigh) are uniquely located by this coordinate. The orientation of the link at the other end (i.e., the thigh) is also an appropriate choice, as are either the knee or ankle angle.

Finding the Dynamic Equations of Motion

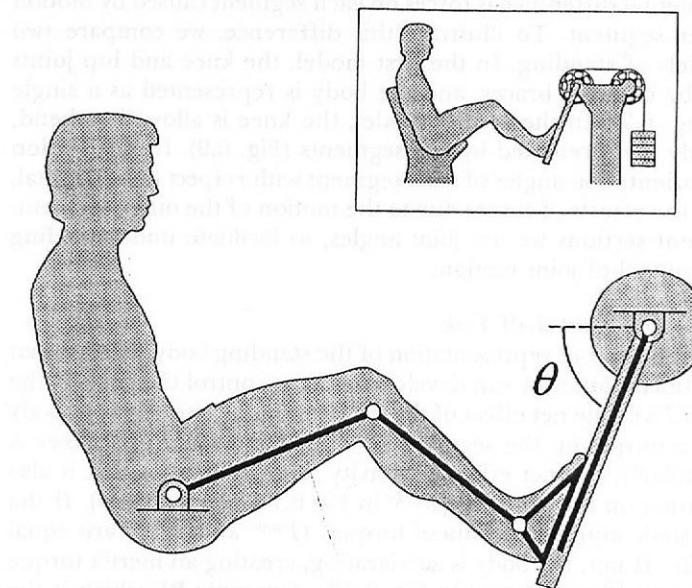
Once the linkage system among the body segments and external objects is defined and a set of generalized coordinates specified for the motor task, the governing dynamic equations of motion for the motor task can be written [72]. Once written, muscular control and motion of the motor task can be analyzed quantitatively. Since a set of generalized coordinates is nonunique, there are other ways of expressing the dynamic equations. However, the motion described by any set of equations is independent of the choice of generalized coordinates (i.e., the body and objects move the same way no matter how the motion is described).

For each generalized coordinate, there is a corresponding equation of motion [72]. Therefore, if the *df* change during a motor task, the dynamic equations also change. Since deriving just one set of equations is time consuming, especially if three-dimension motion is to be studied, often the coordination of only one part of a motor task is studied, for example the propulsion [76, 97] or the airborne [53] phase of jumping.

Different methods are available to derive the dynamic equations of motion for a motor task. In the commonly used Newton-Euler method [26, 75, 83, 94, 96], the multijoint system is "broken apart," and free-body diagrams are constructed to show the external forces and torques acting on the each rigid-body segment. Equations for the linear accelerations of the segments are written using Newton's second law, and expressions for the angular accelerations are written using Euler's equations. One disadvantage of this method is that many extra equations must be written and then, often with considerable effort, combined to form the governing dynamic equations. An advantage, however, is that

FIGURE 6.6

1-df, multiarticular motor task (leg press, insert). Though the hip, knee, and ankle can move, the configuration constraint imposed on the task reduces the task to 1 df. One possible generalized coordinate is θ , which although not a coordinate of the body, when known specifies all joint angles of the participatory body parts as well as the height of the weight.



sometimes these extraneous equations provide useful information (e.g., joint reaction forces). Although equations are not derived in this chapter, our discussions make use of D'Alembert's principle. Rather than thinking of a force causing an acceleration ($F = ma$), the use of D'Alembert's principle converts dynamics problems into statics problems by defining an inertia force ($F^* = -ma$) that places a rigid body into "equilibrium" ($F + F^* = 0$).

The Lagrangian method and Kane's method [72], also referred to as Lagrange's form of D'Alembert's principle [67, 70], are both based on the generalized coordinate concept. Kane's method is particularly advantageous when deriving equations for a task with many *df*, or if forces which are noncontributory to the motion, such as the equal and opposite forces acting at the joints and at the ground, are of no interest.

HOW THE DYNAMICS OF SINGLE- AND MULTIJOINT TASKS DIFFER

Our knowledge of how forces control the motion of one segment (e.g., an inverted pendulum) is often used to infer mechanisms of control of multiarticular motor tasks. However, there are basic differences between the dynamics of multi- and single-*df* movement that must be recognized. The fundamental difference is forces on each segment caused by motion of the other segment. To illustrate this difference, we compare two simple models of standing. In the first model, the knee and hip joints are locked by external braces, and the body is represented as a single segment (Fig. 6.7). In the second model, the knee is allowed to bend, and the body is represented by two segments (Fig. 6.9). In this section we use the orientation angles of each segment with respect to horizontal, to focus on the effects of forces due to the motion of the other segment. In subsequent sections we use joint angles, to facilitate understanding of muscle control of joint motion.

Dynamics of a One-Joint, 1-df Task

For a one-segment 1-*df* representation of the standing body, assume that soleus and tibialis anterior can develop forces to control the sway of the body (Fig. 6.7A). The net effect of these forces on the one-segment body is to exert a torque on the segment (T^{mus} in Fig 6.7B; Appendixes A and C). Similarly, the net effect of gravity (body weight) is that it also exerts a torque on the segment (T^{grav} in Fig 6.7B; Appendix B). If the body is in static equilibrium, these torques (T^{mus} and T^{grav}) are equal and opposite. If not, the body is accelerating, creating an inertia torque on the segment $\bar{I}\ddot{\theta}$ (not shown in Fig. 6.7B; Appendix B), which is the net difference of T^{mus} and T^{grav} (i.e., $T^{\text{net}} = T^{\text{mus}} - T^{\text{grav}}$). The one dynamic equation of motion can be expressed as a "torque balance equation" (Appendix C):

$$(\bar{I})\ddot{\theta} = T^{\text{mus}} - T^{\text{grav}} = T^{\text{net}}, \quad (1)$$

where θ is the one generalized coordinate.

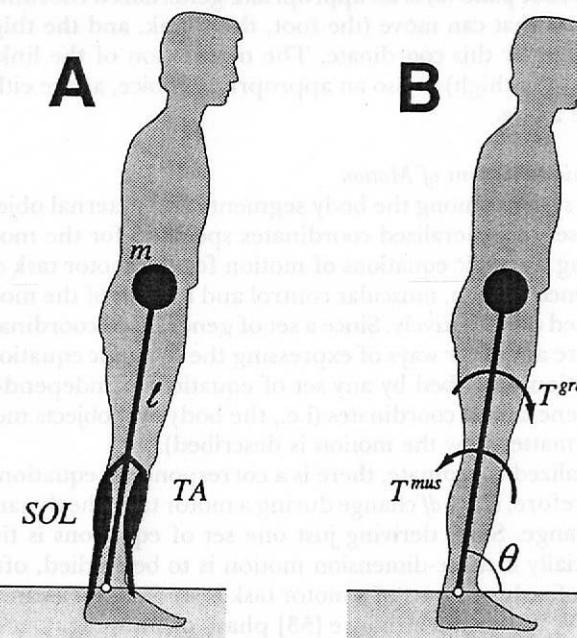
Notice from Equation 1 that the net torque on the shank (T^{net}) is proportional to the angular acceleration of the shank ($\ddot{\theta}$) and the constant of proportionality is \bar{I} . The angular acceleration of the body is therefore

$$\ddot{\theta} = \left(\frac{1}{\bar{I}} \right) T^{\text{mus}} - \left(\frac{1}{\bar{I}} \right) T^{\text{grav}} = \left(\frac{1}{\bar{I}} \right) T^{\text{net}}. \quad (2)$$

Just as the net torque equals the difference of the muscle and gravity torques, the (net) acceleration is the difference of the acceleration due to the muscle torque, and the acceleration due to the gravity torque.

FIGURE 6.7

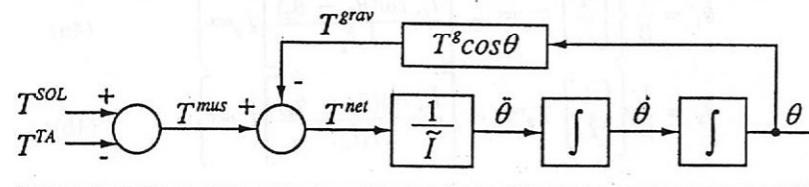
One-joint, 1-df sagittal-plane model of standing. A, Feet are assumed to always be flat on the ground, and knees and hips "locked." The distributed mass of the body is m , whose mass center is assumed to be located a distance of l from the pin-joint ankle and to have a moment of inertia about the mass center of I (not shown). Only muscles crossing the ankle (e.g., soleus [SOL] and tibialis anterior [TA]) contribute to body motion. B, The net effect of each ankle muscle is to generate a torque on the segment (Appendix A). The summed torques from all ankle muscles is T^{mus} (e.g., $T^{\text{mus}} = T^{\text{SOL}} - T^{\text{TA}}$). Gravity also develops a torque (T^{grav}), which opposes T^{mus} when the body lean forwards. θ is chosen as the generalized coordinate.



Therefore, there is a linear relationship between the net acceleration and the muscle and gravity torques (Fig. 6.8). Notice that soleus (SOL) and tibialis anterior (TA) act in opposition. The torque produced from soleus (T^{SOL}) acts instantaneously to accelerate the one body segment posteriorly (i.e., $+T^{\text{SOL}} > 0$, and thus causes $\ddot{\theta}$ to increase), and tibialis anterior torque (T^{TA}) acts instantaneously to accelerate the segment anteriorly (i.e., $-T^{\text{TA}} < 0$, and thus acts to decrease $\ddot{\theta}$, or to accelerate the segment anteriorly, or equivalently, to decelerate the segment). Thus,

FIGURE 6.8

Block diagram of the 1-df model of standing. Torques developed from each ankle muscle sum to produce the net muscle torque ($T^{\text{mus}} = T^{\text{SOL}} - T^{\text{TA}}$). The net muscle torque sums with the gravity torque (T^{grav}) to produce the net torque (T^{net}) acting on the segment. T^{net} , including each component of T^{net} , produces an instantaneous angular acceleration of the body ($\ddot{\theta}$). The body's angular velocity ($\dot{\theta}$) and orientation (θ) depends on the trajectory of $\dot{\theta}$ and $\ddot{\theta}$, respectively (i.e., refer to integrals). The mechanical system is nonlinear because gravity affects torque generation nonlinearly ($T^{\text{grav}} = T^g \cos \theta$).



torques from muscles sum to produce the net muscle torque (T^{mus}) acting on the segment. Torque from gravity acts instantaneously to hinder posterior acceleration when the body leans forward (i.e., $\cos\theta > 0$) and acts to assist posterior acceleration when the body leans backward (i.e., $\cos\theta < 0$).

Although there is a linear relationship between the angular acceleration and the torques, this is *not* a linear system [71]. The nonlinearity is due to the gravity torque, which depends on the cosine of the segment angle. Therefore, no simple relationship between the torques and the velocity or position of the segment exists. In a linear system, the motion resulting from each input acting alone can be summed to find the motion when all the inputs act together. This is known as superposition.

Suppose this one segment model was being used to understand the action of soleus during standing posture, and the body motion and muscle torques for an experimental trial were determined. It would be natural to wonder how the motion would differ if, for example, soleus had developed 20% more torque throughout the trial. If the dynamical equation were linear, it would be straightforward to answer this question using superposition.

Although the dynamics are not linear, one might assume the system to be linear if the range of motion during the trial was sufficiently small. However, unless the assumption of linearity can be justified, it is necessary to simulate the trial, using the dynamical equation, to determine the change in body motion resulting from a 20% increase in soleus torque.

When gravity does not affect the mechanics of a one-segment, 1-df motor task (e.g., during spaceflight, or during horizontal movement of the forearm), the mechanical system is linear, and segment positions, velocities, and accelerations are all affected proportionally by changes in muscle torque. However, in multiarticular tasks, the mechanical system is nonlinear, even when the task is conducted in a gravity-free environment (see below). Thus, in more complex tasks, only the accelerations, and not the positions or velocities, remain proportional to the muscle torques.

Dynamics of a Two-Joint, 2-df Task

For the two-segment, 2-df model of the standing body (Fig. 6.9A), muscles crossing the ankle add to the net torque acting on the shank, and muscles crossing the knee add to the net torques acting on both segments (Appendix A). In Fig. 6.9B, T_1^{mus} is therefore the net muscle torque acting on the shank, and T_2^{mus} is the net muscle torque acting on the thigh. When in static equilibrium, these net muscle torques balance the torques from gravity (Fig. 6.9B; $T_1^{\text{grav}} = T_1^{\text{mus}}$, and $T_2^{\text{grav}} = T_2^{\text{mus}}$). Otherwise the body is accelerating, and inertia torques and torques induced from motion of the other segment are generated (not shown in Fig. 6.9B). The dynamic equations of motion can be expressed as a “torque balance equation” for segment 1 (Equation 3a) and a “torque balance equation” for segment 2 (Equation 3b) with (θ_1, θ_2) as the two generalized coordinates (Appendix D):

$$[\tilde{I}_1] \ddot{\theta}_1 + [\tilde{I}_{cs} \cos(\theta_1 - \theta_2)] \ddot{\theta}_2 = T_1^{\text{net}} \quad (3a)$$

$$[\tilde{I}_2] \ddot{\theta}_2 + [\tilde{I}_{cs} \cos(\theta_1 - \theta_2)] \ddot{\theta}_1 = T_2^{\text{net}} \quad (3b)$$

where

$$T_1^{\text{net}} = T_1^{\text{mus}} - T_1^{\text{grav}} - [\tilde{I}_{cs} \sin(\theta_1 - \theta_2)] \dot{\theta}_2^2$$

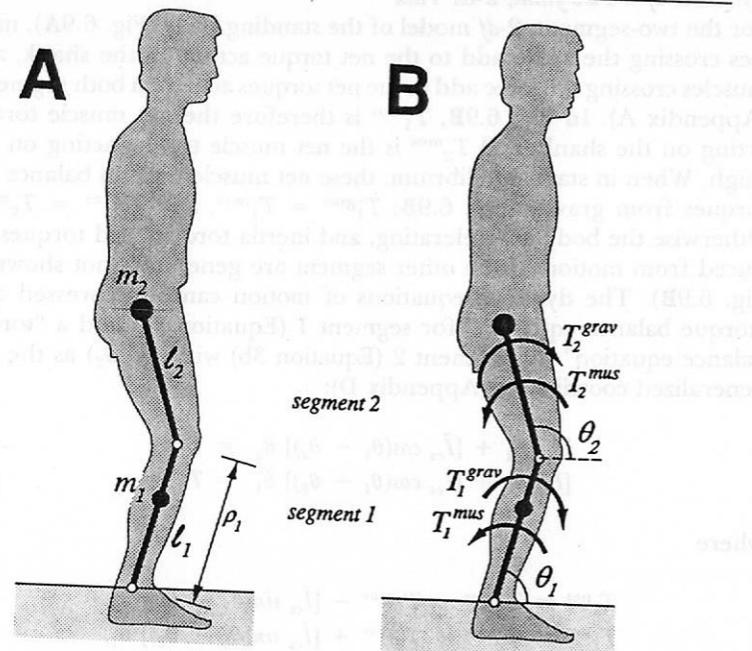
$$T_2^{\text{net}} = T_2^{\text{mus}} - T_2^{\text{grav}} + [\tilde{I}_{cs} \sin(\theta_1 - \theta_2)] \dot{\theta}_1^2,$$

and T_1^{net} and T_2^{net} are the net torques accelerating the segments. Notice that the accelerations are proportional to the net torques ($T_1^{\text{net}}, T_2^{\text{net}}$).

A fundamental difference of the 2-df and 1-df systems is the presence of torques from the motion of the other segment (Appendix E). For the one-segment, 1-df system, the net torque is the difference of the gravity and muscle torques acting on the segment (see Equation 1). For the 2-df system, the net torque (T_1^{net}), for example, contains an additional term due to the angular velocity of segment 2 ($\dot{\theta}_2$); similarly, the net torque (T_2^{net}) contains a term due to the angular velocity of segment 1 ($\dot{\theta}_1$). For the 1-df task, the inertia torque of the segment equals the net torque acting on the segment. For the 2-df task, however, this is not the

FIGURE 6.9

Two-joint, 2-df model of standing. A, The body is modeled by a shank segment (mass m_1 , length ρ_1 , and mass center location from ankle l_1) and an upper segment (mass m_2 and c.m. location from knee l_2). B, Ankle and knee muscles develop torque on the shank. Their summed torque is T_1^{mus} . For the sign convention assumed, extensor muscles of the ankle and knee develop a positive torque on the shank. Knee muscles also develop torque on the upper segment; extensor muscles contributing negatively to T_2^{mus} . Similarly, gravity produces a torque on both segments, T_1^{grav} and T_2^{grav} . The orientations of the two segments (θ_1, θ_2) are chosen to be the generalized coordinates.



case. Notice that the left-hand side of Equations 3a and 3b contains both the angular acceleration of segment 1 ($\ddot{\theta}_1$), and the angular acceleration of segment 2 ($\ddot{\theta}_2$).

The torques exerted on one segment due to the motion of the other segment depend not only on the angular acceleration and velocity of the other segment, but also on the relative orientation of the two segments ($\theta_1 - \theta_2$). Notice from Equations 3a and 3b that the torque from angular acceleration of the other segment is maximum when the seg-

ments are colinear (i.e., $\cos(\theta_1 - \theta_2) = 1$), and zero when perpendicular (i.e., $\cos(\theta_1 - \theta_2) = 0$). The opposite is true for the torque from the angular velocity of the other segment (i.e., it is maximum when the segments are perpendicular, when $\sin(\theta_1 - \theta_2) = 1$, and zero when colinear, when $\sin(\theta_1 - \theta_2) = 0$).

Using Equations 3a and 3b the angular acceleration of each segment caused by the net torques can be found:

$$\ddot{\theta}_1 = \frac{1}{\beta} \left\{ \left[\frac{1}{\tilde{I}_1} \right] T_1^{\text{net}} - \left[\frac{\tilde{I}_{cs} \cos(\theta_1 - \theta_2)}{\tilde{I}_1 \tilde{I}_2} \right] T_2^{\text{net}} \right\} \quad (4a)$$

$$\ddot{\theta}_2 = \frac{1}{\beta} \left\{ \left[\frac{1}{\tilde{I}_2} \right] T_2^{\text{net}} - \left[\frac{\tilde{I}_{cs} \cos(\theta_1 - \theta_2)}{\tilde{I}_1 \tilde{I}_2} \right] T_1^{\text{net}} \right\} \quad (4b)$$

where

$$\beta = [\tilde{I}_1 \tilde{I}_2 - \tilde{I}_{cs}^2 \cos^2(\theta_1 - \theta_2)] / (\tilde{I}_1 \tilde{I}_2) > 0 \text{ for all } (\theta_1 - \theta_2).$$

Notice that the angular accelerations of both segments depend on T_1^{net} and T_2^{net} . Consequently, muscles such as soleus, which contribute only to T_1^{net} , act to accelerate both segments 2 and 1. In fact, the effect on the other segment can be large. For example, in upright posture, T_{SOL} (or for that matter any term contributing to T_1^{net}) can act to accelerate the thigh (segment 2) about the same as it acts to accelerate the shank (segment 1) [42].

Even with just one extra segment, there is a large increase in the complexity of the dynamics (Fig. 6.10). The ways that one segment affects the motion of the other are indicated by heavy lines. Notice that in the absence of the dynamic coupling, the flow of events in the two segments would be independent. Therefore, segment 1 could be controlled by exciting muscles crossing the ankle (to control T_1^{net}), and segment 2 could be controlled by exciting muscles crossing the hip (to control T_2^{net}). However, dynamical coupling must always exist since $\cos(\theta_1 - \theta_2)$ and $\sin(\theta_1 - \theta_2)$ cannot both be zero.^b

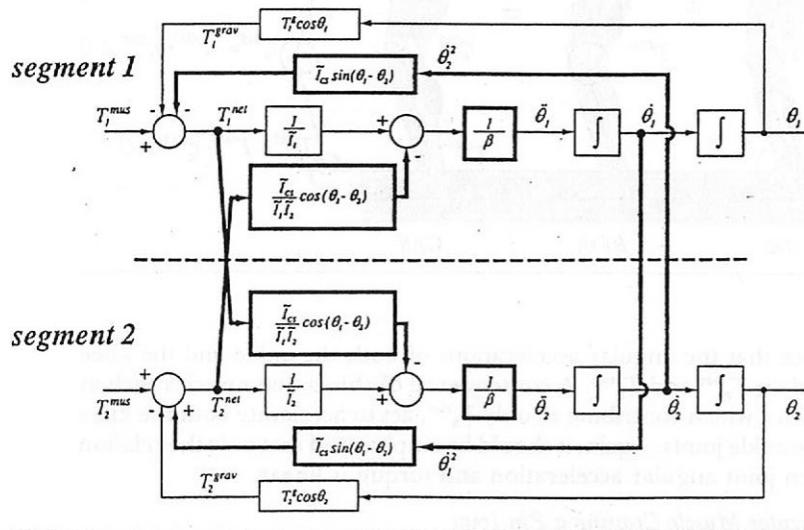
For a two-joint, as for a one-joint, system, there is a linear relationship between torque and acceleration. Should the torque from any one muscle change, the angular accelerations of all the segments would change proportionally.

However, there is no linear relationship between the segment's angular velocities and the torques, or between the segment's orientations and

^bActually, β also depends on $\cos(\theta_1 - \theta_2)$ (see Fig. 6.10 and Equations 4a and 4b).

FIGURE 6.10

Block diagram of the 2-df model of standing. Notice how much more complex the dynamics are when the body is assumed to be articulated (compare with Fig. 6.8). If the segments were to move independently of one another, the heavily outlined boxes would be absent (i.e., no dynamic coupling would exist). The block diagram of the two-segment dynamics would then be the sum of two, one-segment block diagrams (thin lines; compare with Fig. 6.8). Dynamic coupling always exists, however, and is significant (see text). Notice that the torques acting on one segment affect the angular acceleration of both segments viz the feed-forward $\cos(\theta_1 - \theta_2)$ terms, that the angular velocities of the segments generate torques on the segments viz the feedback $\sin(\theta_1 - \theta_2)$ terms, and that the segmental orientations affect the feed-forward gain because β depends on θ_1 and θ_2 (see Equations 4a and 4b). Observe that the dynamics are nonlinear even in the absence of gravity (i.e., when $T_1^g = T_2^g = 0$).



the torques. The system dynamics contain several nonlinearities (Fig. 6.10, sines and cosines). Therefore, superposition does not apply. So, a computer simulation of the task is required to determine how force (or torque) developed by a muscle affects the angular velocities and orientations of the segments. Similarly, forces and joint angular velocities and angles are nonlinearly related.

In other multijoint systems, it can be shown that dynamic coupling exists across all segments [26, 42, 67, 72, 96, 97, 124]. For example, in a planar analysis of standing or walking, the trunk exhibits coupling to

both the thigh and the shank, and the shank and thigh to the trunk. Again, torques arising from angular acceleration of another segment are maximum when the two segments are colinear, and angular-velocity induced-torques are maximum when the two segments are perpendicular. For example, during standing, soleus will act to accelerate the pelvis the most when the pelvis's center of mass is colinear with the shank's. The old saying, the "shank bone is connected to the thigh bone, the thigh bone is connected to the hip bone, the hip bone etc." shows perhaps that intersegmental coupling is generally recognized, though we believe its effect on how muscles act to accelerate the segments is less appreciated (see section entitled "Muscle Action on Joint Motion").

Are the Dynamical Interactions Among Segments Significant?

The answer is yes. First, we mentioned briefly above that soleus can act to angularly accelerate the thigh by the same amount that it acts to accelerate the shank near upright posture when humans stand flat-footed. The consequence of this is that soleus acts two times more powerfully to accelerate the knee into extension than it does the ankle (see next section). This analysis is based on a two-segment model of a person. Fortunately, this result appears robust, being insensitive to both typical anthropometrical variations found among children and adults, and to the number of segments used to model the head–arms–trunk segment [42, 73]. Other results to be presented in the next section are likewise robust, unless otherwise stated.

Analyzing a variety of movements in animals and humans has indicated that intersegmental coupling is indeed significant. For example, analysis of arm movements, based on a two-segment model, has shown that angular velocity of one joint induces a significant torque at the other joint, even when the reaching task is conducted slowly [60]. Similarly, the torque induced from angular acceleration of the other joint was found to be significant during these forearm and upper arm movements. Intersegmental (or joint) motion has also been found to affect the torques that need to be developed by ankle and knee muscles in the cat's swing leg during gait [63]. Dynamic analysis of cat paw shaking has revealed that proximal muscles have to counter the torques induced by motion of the distal segments [64]; these findings have led to hypotheses about neural control of knee and ankle muscles during the paw shake [65]. Finally, in human gait studies, intersegmental dynamic interactions are deemed, *prima facie*, significant [17, 20, 22, 101, 120, 121].

MUSCLE ACTION ON JOINT MOTION

In this section, we review the relation between muscle torques and the joint angular accelerations they cause. It is important to recognize that

muscle acts to accelerate all the joints of the body, and the acceleration of an unspanned joint can greatly exceed that of a spanned joint. Further, multiarticular muscles may accelerate a spanned joint in the direction opposite to that of the torque it applies to the joint. Because muscle action is task dependent, it is necessary to use the task-specific dynamic equations to find muscle's action on the body. Therefore, as an example, the two-segment model of standing posture is used again, although the equations are expressed in joint rather than segmental coordinates. With this model, soleus acts to accelerate the knee (the unspanned joint) into extension twice as much as it acts to accelerate the ankle (the spanned joint) into extension. Although gastrocnemius exerts a knee flexor torque, it can act to accelerate the knee into extension.

Two-Joint Dynamic Equations

Consider again the two-segment model of standing posture, with the equations expressed using the joint angles (ϕ_a and ϕ_k , Fig. 6.11A). The dynamic equations in joint space (see Equations 8a and 8b in Appendix F) become on rearrangement:

$$\ddot{\phi}_a = \frac{1}{\gamma} \left\{ \left[\frac{1}{\tilde{I}_a} \right] T_a^{\text{net}} - \left[\frac{\tilde{I}_{\text{cj}}}{\tilde{I}_a \tilde{I}_k} \right] T_k^{\text{net}} \right\} \quad (5a)$$

$$\ddot{\phi}_k = \frac{1}{\gamma} \left\{ \left[\frac{1}{\tilde{I}_k} \right] T_k^{\text{net}} - \left[\frac{\tilde{I}_{\text{cj}}}{\tilde{I}_a \tilde{I}_k} \right] T_a^{\text{net}} \right\} \quad (5b)$$

where

$$T_a^{\text{net}} = T_a^{\text{mus}} - T_a^{\text{grav}} + T_a(\dot{\phi}_a, \phi_k, \dot{\phi}_k)$$

$$T_k^{\text{net}} = T_k^{\text{mus}} - T_k^{\text{grav}} + T_k(\dot{\phi}_a, \phi_k)$$

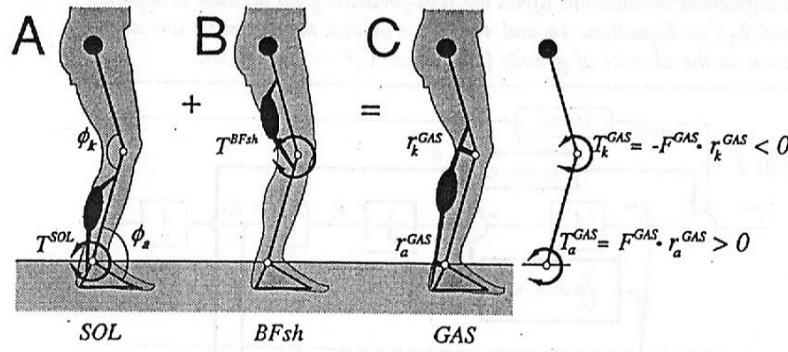
and

$$\gamma = [\tilde{I}_a \tilde{I}_k - I_{\text{cj}}^2] / [\tilde{I}_a \tilde{I}_k] > 0 \text{ for all } \phi_k.$$

The structure of these equations, expressed in joint coordinates, is similar to the structure of Equations 4a and 4b, which are the equations expressed in segmental coordinates. Again, torques from muscle, gravity, and velocity sum to produce the net torques (T_a^{net} , T_k^{net}). Torques from muscles crossing the ankle sum to produce T_a^{mus} , the total muscle torque at the ankle. Similarly, torques from muscles crossing the knee sum to produce T_k^{mus} , the total muscle torque at the knee. The sign convention is that torques in the direction of joint extension are positive; thus soleus torque (T^{SOL} , Fig. 6.11A) is positive, and biceps femoris (BFsh) torque (T^{BFsh} , Fig. 6.11B) is negative.

FIGURE 6.11

Relationships among torques generated by uni- and biarticular muscles. A, ϕ_k and ϕ_a define the knee and ankle joint angles. Soleus (SOL) produces an ankle-extensor torque ($T^{\text{SOL}} > 0$). B, Biceps-femoris, short-head (BFsh) generates a knee-flexor torque ($T^{\text{BFsh}} < 0$). C, Gastrocnemius (GAS) develops both an ankle-extensor torque ($T_a^{\text{GAS}} > 0$) and a knee-flexor torque ($T_k^{\text{GAS}} < 0$). Gastrocnemius can be considered to be the sum of two uniarticular muscles, an ankle-extensor muscle like soleus and a knee-flexor muscle like biceps femoris. The ratio of the magnitude of gastrocnemius's torque at the knee to the ankle is equal to the ratio of gastrocnemius's moment arm at the knee to the ankle (i.e., $r_k^{\text{GAS}} : r_a^{\text{GAS}}$). F^{GAS} is the force developed by gastrocnemius.



Notice that the angular accelerations of both the ankle and the knee depend on T_a^{net} and T_k^{net} . A consequence of this is that muscles such as the soleus, which contribute to only T_a^{net} , act to accelerate both the knee and the ankle joints. Again, it should be emphasized that only the relation between joint angular acceleration and torque is linear.

Uniarticular Muscle Crossing a Pin Joint

A uniarticular muscle crossing a pin joint will always act to accelerate the joint in the direction of the muscle's torque. Thus a muscle like the soleus, which develops an extensor torque at the ankle, will accelerate the ankle into extension. The certainty of the conclusion follows from energetic arguments [7, 8, 73]. That is, the soleus, acting alone to generate force on a body initially at rest, must initially shorten to perform work to initiate movement of the body. In order to shorten, the ankle must move initially into extension. However, all the energy developed by the soleus, even initially, does not have to be spent on initiating movement of the ankle; some energy, and perhaps a lot, can be expended on initiating movement of other joints.

Analysis of the dynamical equations leads to the same conclusion [42, 124]. For example, since the soleus contributes only to T_a^{net} , the ankle angular acceleration caused by the soleus can be found from the coefficient of T_a^{net} in Equation 5a. Since $\bar{I}_a > 0$ and $\gamma > 0$, and $T^{\text{SOL}} > 0$, the soleus accelerates the ankle into extension. Similarly, the uniarticular biceps femoris, which produces a negative (flexor) torque at the knee and thus contributes to T_k^{net} , accelerates the knee into flexion (since $\bar{I}_k > 0$ and $T^{\text{BFSH}} < 0$). More generally, it can be shown from analysis of the structure of the dynamic equations that a uniarticular muscle crossing a pin joint will always accelerate the joint in the direction of the muscle torque, regardless of the task [42, 124].

The dynamic equations can also be analyzed to study the action of uniarticular muscles on unspanned joints [41, 42, 73, 124, 126]. For example, the sign of the knee angular acceleration caused by the soleus for the two-joint model of standing depends on the sign of \bar{I}_{g} in Equation 5b, which can be positive, negative, or zero, depending on ϕ_k and the inertial parameters of the two segments.

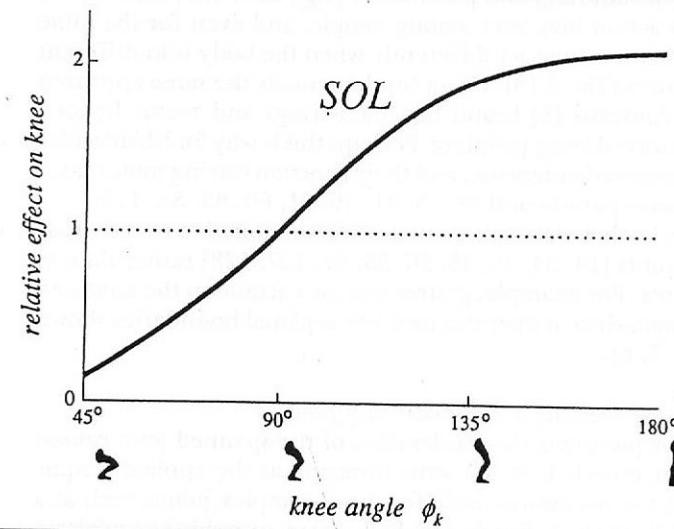
The action on unspanned joints can be powerful [42, 124]. For example, for positions near upright posture, the soleus accelerates the knee into extension twice as much as the ankle (Fig. 6.12). Notice that with the knee more flexed, the soleus's action on the knee lessens, and when flexed beyond 90° ($\phi_k < 90^\circ$), the soleus's action at the ankle dominates. Similarly, using Equation 5a, the biceps femoris accelerates the ankle into flexion (dorsiflexion), though its action at the ankle is less than its action at the knee, and the ratio of knee and ankle accelerations varies little with knee angle. The qualitative nature of these actions is insensitive to the inertial parameters of the body and the number of segments used to represent flat-footed standing posture [42].

Biarticular Muscle Crossing Two Pin Joints

A biarticular muscle crossing two pin joints accelerates at most one spanned joint at a time in a direction opposite to its torque at that joint [7, 8, 41, 42, 124, 126]. Energetic arguments can be invoked to show that muscle can shorten in three possible ways when acting alone on the body [7, 8]. For example, gastrocnemius can accelerate either (a) the knee into flexion and the ankle into extension (plantarflexion), (b) the knee into extension and the ankle into extension, or (c) the knee into flexion and the ankle into flexion (dorsiflexion). These three possibilities are also consistent with analysis of the dynamical equations [41, 42, 124, 126]. For example, gastrocnemius behaves as soleus and biceps femoris acting together since gastrocnemius develops a flexor knee torque (T_k^{GAS}) and an extensor ankle torque (T_a^{GAS}) (Fig. 6.11). Since soleus accelerates both joints into extension, and biceps femoris accelerates both joints into flexion, the net action of gastrocnemius on either one (but

FIGURE 6.12

The effect of soleus (SOL) on angular acceleration of the knee relative to the ankle. Soleus's effect on knee and ankle angular acceleration depends on the knee angle (ϕ_k), and thus so does the ratio of the two accelerations (see text). Notice that when the knee is flexed less than 90° (i.e., $\phi_k > 90^\circ$), SOL accelerates the knee (into extension) more than it accelerates the ankle (into extension) (bold line is above the dotted line).



not both) of the joints can be opposite to the direction of its torque at that joint.

The dynamic equations of motion can be analyzed in detail to ascertain which of the three possible ways a biarticular muscle will act if, in addition, the muscle's moment arms at the spanned joints are given [42, 124]. For example, the action of gastrocnemius on the ankle is given by the sum of the accelerations caused by gastrocnemius's ankle torque (T_a^{GAS}) and its knee torque (T_k^{GAS}), and can be computed from Equation 5a by setting $T_a^{\text{net}} = T_a^{\text{GAS}}$ and $T_k^{\text{net}} = T_k^{\text{GAS}}$. Thus, the ratio of knee to ankle torque developed by gastrocnemius ($T_k^{\text{GAS}} : T_a^{\text{GAS}}$), or, equivalently, the ratio of its moment arm at the knee to ankle ($r_k^{\text{GAS}} : r_a^{\text{GAS}}$) (Fig. 6.11C), influences the net action of gastrocnemius on the ankle, as well as on the knee (see Equation 5b). Therefore, for a specific body configuration (knee angle ϕ_k in this example), the action of gastrocnemius to accelerate the knee and ankle can be found if its moment-arm ratio ($r_k^{\text{GAS}} : r_a^{\text{GAS}}$) is also specified.

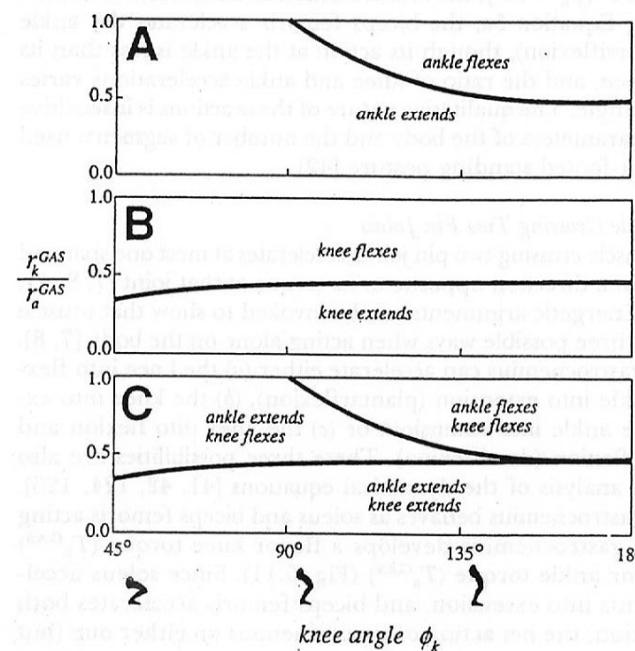
Results from this model of standing and others (42) suggest that gastrocnemius may indeed act, at times, either to accelerate the ankle into flexion (dorsiflexion) or to accelerate the knee into extension (Fig. 6.13). Notice that all three possible combinations of ankle and knee action can occur. Assuming the knee-to-ankle moment-arm ratio is about 0.5 in the human (M. G. Hoy, personal communication, 1988), gastrocnemius's action will be sensitive to model structure (e.g., a two- versus three-segment model of standing) and parameters (e.g., moment arms). Thus, gastrocnemius's action may vary among people, and even for the same person, gastrocnemius may act differently when the body is in different flat-footed postures (Fig. 6.13). Using fundamentally the same approach outlined here, Andrews [8] found that hamstrings and rectus femoris also act in three ways during pedaling. Perhaps this is why limb biarticular muscles are considered enigmatic, and their function during motor tasks sometimes appears paradoxical [8, 15, 21, 46, 61, 69, 82, 86, 115].

In fact, it may be that biarticular muscles are well suited to transferring power among joints [14, 34, 35, 45, 56, 58, 87, 107, 128] rather than to accelerating joints. For example, gastrocnemius's action on the knee and ankle will be small, since it operates near the regional boundaries shown in Figure 6.13 [7, 8].

Uniarticular Muscle Crossing a Ball-and-Socket Joint

Even though for pin joints the acceleration of the spanned joint caused by a uniarticular muscle is in the same direction as the applied torque, this result does not necessarily hold for more complex joints, such as a ball-and-socket joint (B. J. Fregly and F. E. Zajac, unpublished analysis, 1988). For example, gluteus maximus, which crosses the ball-and-socket hip joint and generates both an extensor and an abductor torque [80], will not necessarily accelerate the hip simultaneously into extension and abduction. Energetic arguments require only that the muscle shorten, and there are many possible joint accelerations consistent with such a shortening. For instance, its extensor torque might accelerate the hip into adduction which, when added to the hypothetically small abductor acceleration caused by the muscle's abductor torque, may produce a net acceleration of adduction. The reason torque can cause angular accelerations around axes different from the axis around which torque is generated is the same as the reason why a biarticular muscle can accelerate a spanned joint in the direction opposite to its applied torque: It is a result of dynamic coupling. To ascertain even a qualitative action for gluteus maximus, or for other uniarticular muscles crossing complex joints, demands deriving the dynamic equations for the specific motor task under study.

FIGURE 6.13 Directionality of gastrocnemius's (GAS's) action on the angular acceleration of the ankle and the knee as a function of GAS's moment-arm ratio ($r_k^{\text{GAS}}:r_a^{\text{GAS}}$) and knee angle (ϕ_k). **A**, GAS's action on ankle angular acceleration. The bold line separates the regions where GAS accelerates the ankle into extension (i.e., ankle extends) and into flexion (i.e., ankle flexes). Between $135^\circ < \phi_k < 180^\circ$, and for ratios less than 0.5, GAS accelerates the ankle into extension because its ankle-extensor torque dominates the action at the ankle. For ratios greater than 0.5, GAS accelerates the ankle into flexion because the ankle extensor action produced from its ankle-extensor torque is less than the ankle flexor action produced from GAS's knee-flexer torque (see text). **B**, GAS's action on knee angular acceleration. For moment-arm ratios greater than about 0.5, GAS accelerates the knee into flexion (i.e., knee flexes), regardless of how much the knee is flexed, because its knee-flexer torque dominates. For moment-arm ratios less than 0.5, GAS accelerates the knee into extension (i.e., knee extends) because the knee flexor effect caused by its knee-flexer torque is less than the knee extensor action generated by its ankle-extensor torque. **C**, Regions showing the three possible actions of GAS. Notice that between $135^\circ < \phi_k < 180^\circ$, slight variations of GAS's moment-arm ratio around the nominal 0.5 may cause GAS to have a different action on either ankle or knee rotation, or on both.



Anatomical Classification and Its Connotation for Movement: Reasons to Be Cautious

Though textbooks [5, 25, 108] use anatomy to correctly infer the directionality of torque developed by a muscle, their stated inference of muscle action may be erroneous. Using an anatomical classification, a muscle is called either a flexor or an extensor muscle, depending only on its torque. The interpretation is that muscle y defined anatomically as a *flexor muscle* of joint z , will act *to flex* joint z . Thus, “*to flex*” needs to be defined. A common interpretation is that if a muscle were acting alone from all other muscles, then it would, on development of force, move the joint into flexion. And if the muscle should develop more force, the joint would move faster and further into flexion and, as long as no other forces act on the body, the joint would in no way move into extension. When do we know with certainty that this interpretation is correct?

This classical interpretation is correct when the body is constrained so that only one joint, the joint under consideration, can move, and in only one direction. Only muscles crossing that joint will act to “move” the joint. If there are no other forces acting on the body, the mechanical system is linear. Thus the force developed by an anatomically defined flexor muscle will indeed accelerate the joint from rest into flexion and, at a later time, the joint will be flexing (i.e., moving with a certain velocity into flexion) and will have moved into flexion. If the muscle develops more force, the joint will accelerate into flexion faster, flex faster, and move to a flexed position faster. Even for biarticular muscles, no ambiguity arises since the other spanned joint will have been constrained from moving. And no ambiguity occurs for uniarticular muscles crossing a ball-and-socket joint either, because the joint is constrained from moving in the other directions and is only allowed to move in one direction (e.g., flex/extend).

In multiarticular motor tasks, however, the classical interpretation should be subjected to analysis. As we have repeatedly emphasized, there is no simple relationship among joint angle, joint velocity, and an individual torque acting alone because the mechanical system is nonlinear and superposition does not apply. However, the joint acceleration and the individual torque are linearly related. Thus, statements like “muscle y flexes joint z ” or “the action of muscle y is to flex joint z ” should be interpreted as “muscle y acts to *accelerate* joint z into flexion.” Previously, we avoided using “*to flex*” for fear of causing ambiguity.

We have also emphasized that the way in which a muscle acts to accelerate a joint cannot necessarily be inferred from anatomy. For example, gastrocnemius can at times extend the knee, and at other times dorsiflex the ankle, even though it develops always a knee flexor torque and an ankle extensor torque. And a uniarticular muscle, like gluteus maximus, may flex the ball-and-socket hip at times, or adduct the hip

at other times, even though it develops extension torque and abduction torque.

Finally, muscle action, as inferred by anatomical classification, conveys the notion that muscles do not instantaneously affect unspanned joints. Muscles do indeed act to accelerate all joints; at times, such as in standing, muscles crossing the ankle can extend or flex the knee much more than they flex or extend the ankle. Clearly, muscle action on body motion is a topic worthy of more careful scrutiny than appears in textbooks.

MUSCLE AGONIST–ANTAGONIST GROUPS AND SYNERGIES

Muscle function on the body during a motor task can be analyzed to determine if a muscle assists or hinders the desired task, and to determine if a muscle functions identically or oppositely to other muscles. When torque is used to define muscle function, a muscle is called an *agonist* if its torque is in the same direction as the net muscle torque, or two muscles are called an *agonist–antagonist pair* if their torques are in opposition signs [6–8, 80]. Alternatively, the joint accelerations caused by muscle may be used to determine agonist/antagonist relationships [7, 8, 124, 126]. We believe that using acceleration to classify agonist/antagonist function rather than torque will lead to a better understanding of muscle coordination [42, 124]. However, because muscles that work together need not function identically, it may be best to study how muscles work *synergistically* [41, 48, 84, 88] rather than *agonistically*, in accomplishing the motor task.

For single-joint motor tasks, the use of torque to define muscle function is consistent with the use of acceleration, and thus “*agonist–antagonist*” is unambiguous. In the control of elbow flexion/extension with the arm horizontal, for example, only the elbow is free to move. Thus whether the muscles are uni- or biarticular is immaterial, since each muscle develops either a flexor or an extensor elbow torque and thus acts to accelerate the elbow either into flexion or extension, respectively. Thus the relation between agonists, antagonists and task performance is easily related to whether the muscle is a flexor or extensor muscle, as defined by anatomy. In such highly constrained tasks, therefore, classifying muscles into agonist–antagonist groups is a convenient and meaningful way to view muscles and has led to the elucidation of neural control mechanisms [39, 40, 44, 111], though such mechanisms may not necessarily operate during multiarticular tasks [49, 78].

Uniarticular muscles crossing a pin joint can also be consistently classified as agonists or antagonists. For example, if the ankle is assumed to be a pin joint, then soleus and tibialis anterior are opposites in the anatomical sense because the former exerts an ankle extensor torque, and the latter exerts an ankle flexor torque. It is simple to show that

when they exert torques of equal magnitudes, all the joint accelerations caused by tibialis anterior are exactly opposite to all the joint accelerations caused by soleus. Thus soleus and tibialis anterior function oppositely when function is defined either anatomically (by torque), or dynamically (by acceleration). Thus, when EMG activity is recorded in such muscles, it may well be because the task requires joint stiffening [59].

With the exception of uniarticular muscles crossing a pin joint, the use of torque to define muscle function in tasks with many degrees of freedom can lead to different agonist/antagonist relationships than the use of acceleration [7, 8, 41, 42, 124, 126]. For example, consider hamstrings and rectus femoris. Anatomically they are antagonists because their torques at both the hip and the knee are opposite. However, they would not necessarily act to accelerate the joints oppositely should the ratio of their hip-to-knee torques be unequal. Thus although their torques have opposite signs, hamstrings and rectus femoris may both act, for example, in flat-footed postures, to extend the knee [42]. Similarly, the action of a uniarticular muscle crossing a ball-and-socket joint depends on the ratio of its three torque components. Thus uniarticular muscles which are anatomical opposites may not act to accelerate the body oppositely.

While we feel that muscle coordination is best understood when dynamic equations are used to analyze accelerations caused by muscle, it can be useful to compare a muscle's torque with the net muscle torque required to perform the task. For example, if a net ankle extensor torque is required, tibialis anterior would be an antagonist to the task, since it exerts an ankle flexor torque. If both an ankle-extensor and a knee-flexor torque is required, gastrocnemius would be an agonist at both joints because it contributes positively to both muscle torques. Indeed, such analyses have fostered insight into why some muscles are excited and others not during task performance [8, 34, 35, 69, 70, 73, 80, 120, 121].

The limitation of comparing the directions of torques developed by muscles to the directions required of muscles in the task is that coordination of muscles is discussed in the context of their ability to develop the required torques. However, such discussions do not provide insight into the action that these torques have on accelerating the joints. For example, soleus may be an agonist to fulfill the requirement of the task that ankle extensor torque be generated, but that does not necessarily mean that soleus (or any other muscle generating ankle extensor torque) is acting primarily to extend the ankle, even if the ankle should be accelerating into extension. Soleus may instead be acting mostly to accelerate the knee into extension (Fig. 6.12). And furthermore, though the net torque at the ankle is an extensor torque, the net acceleration of the ankle, resulting from torques at all the joints, need not be in extension.

If agonist–antagonist comparisons based on directionality of the torques needed and produced among muscles at joints can lead to misconceptions of coordination of joint movement, do agonist–antagonist comparisons based on the directionality of muscle action to accelerate joints have utility? For highly constrained tasks, it may be useful to use the dynamical equations of motion to find groups of muscles that accelerate the joints identically or oppositely. Obviously, single-joint, 1-df tasks fulfill this requirement. But some multiarticular tasks do also. For example, in the leg press (Fig. 6.6), there is only 1 df, so all lower limb muscles either accelerate the weight up ($\ddot{\theta} > 0$) or down ($\ddot{\theta} < 0$), and knowledge of a muscle's action on the weight specifies exactly how it will accelerate each of the three joints.

For tasks with many degrees of freedom, it may not be useful to identify agonist/antagonist groups because muscles may work together, yet not act identically. For example, in standing posture, muscles must actually work together even though their action on joints oppose one another [41, 42, 84, 89]. That is, a muscle must be excited even though its action at some of the joints is in opposition to task requirements. To counter the muscle's undesired action, a synergistic muscle must be excited. Since muscles need not act identically to work together, it may be best to study how muscles work as synergists rather than as agonists.

CONCLUSIONS

1. Dynamic optimization is the most powerful method for determining muscle forces and how they coordinate movement, because both the dynamics and what the task is attempting to achieve must be modeled.
2. A muscle acts to accelerate not only the joints it spans, but the unspanned joints as well. The angular acceleration of unspanned joints can exceed that of spanned joints.
3. Muscle's action is *task dependent* because the dynamic equations describing the relation among forces and motion depend on the interactions among the body segments and the environment.
4. Anatomy determines muscle-torque directionality, which is *task independent*. Therefore, extreme caution must be exercised when anatomy is used to infer muscle action during movement.
5. Since muscles need not function identically as agonists/antagonists to work together, it may be best to study how muscles work as synergists rather than as agonists.

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APPENDIX A: MUSCLE FORCE, MOMENT, COUPLE AND TORQUE

Although muscle develops force (see Muscle Force During Movement section), often other terms are used in the biomechanics and motor control literature, for example, “moment of muscle force” about a joint, a muscle’s “force couple,” a muscle’s “joint reaction force,” the “torque of the force couple” of a muscle, the “joint torque” developed by a muscle, and the muscle’s “torque at the joint.” The relation among these terms [6, 34–37, 72, 81, 99] is reviewed here for uniarcticular muscles, though similar relations apply to bi- and multiarticular muscles.

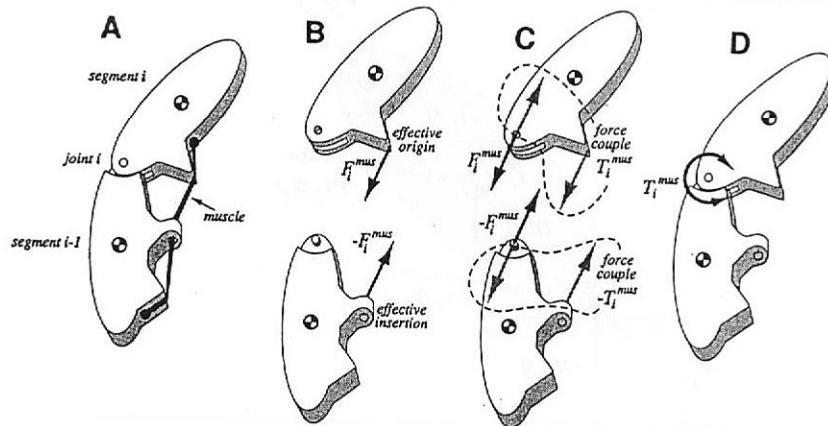
Assuming uniform tension along a straight line path from muscle’s “effective” origin to its “effective” insertion (Fig. 6.14A), the muscle exerts a force F_i^{mus} acting on segment i at the effective origin, and an opposite force $-F_i^{\text{mus}}$ on segment $i - 1$ at the effective insertion (Fig. 6.14B).

The set of forces shown in Figure 6.14B is equivalent to the same forces acting at the joint, together with a force couple of torque T_i^{mus} acting on segment i , and a force couple of torque $-T_i^{\text{mus}}$ acting on segment $i - 1$ (Fig. 6.14C). A force couple is a pair of equal and opposite forces. A special property of a force couple is that its moment about any point is the same, and is called the *torque* of the force couple. Because the moment of a force about its point of application is zero, T_i^{mus} is equal in magnitude and direction to the moment of muscle force F_i^{mus} about the joint center. And $-T_i^{\text{mus}}$ is equal to the moment of muscle force $-F_i^{\text{mus}}$ about the joint center.

If the joint allows for translation of the two adjacent segments, or if frictional forces are significant, both the muscle’s force and the torque acting on each segment must be included in a dynamic analysis. Otherwise, the muscle forces sum to zero at the joint, and only the two segment torques contribute to the motion. Because the torque on segment $i - 1$ is exactly opposite to the torque on segment i , the muscle acts as a mechanical torque actuator would, such as those used in robotics [26, 60]. Therefore, the combination of these two segment torques is called

FIGURE 6.14

Relations among a uniarcticular muscle (A), the force it exerts on each of the two adjacent segments (B), the equivalent joint reaction forces and torques of the force couples acting on the two segments (C), and the equivalent joint-torque (D).



the muscle’s *joint-torque* (Fig. 6.14D), or the *torque of the muscle at the joint*; these terms are often used in coordination studies [10, 30, 47, 60, 98, 109]. For brevity, we refer simply to the muscle’s *torque* in this chapter.

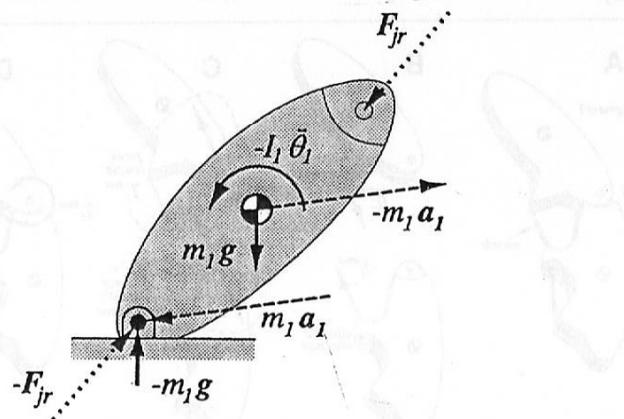
Similarly, one can show that a biarticular muscle acts as two uniarcticular muscles. So, for frictionless pin- and ball-and-socket joints, a biarticular muscle acts as two “torque actuators,” one at each of the spanned joints.

APPENDIX B: GRAVITY AND INERTIA FORCES, COUPLES AND TORQUES

With at least one point of one of the body segments stationary, every force acting on a segment is accompanied by an equal and opposite reaction force that also acts on the segment. For example, consider a segment connected to another segment by a joint (the upper joint in Fig. 6.15), and to the ground by another joint (the lower joint). Suppose only two external forces act on this segment: the reaction force F_{jr} at the upper joint, and the force from gravity m_1g (Fig. 6.15). In addition, the internal inertia force $-m_1a_1$ of the segment acts on itself, as does its inertia torque $-I_1\ddot{\theta}_1$ (Fig. 6.15). Notice that three equal and opposite

FIGURE 6.15

For each force acting on a segment, there is an equal and opposite reaction force that also acts on the segment. The force and its reaction force are a force couple. The moment of the force couple about any point is the same, and is called the torque of the couple.



reaction forces appear at the lower joint: two in reaction to the external forces (i.e., $-F_{jr}$ and $m_1 g$), and one in reaction to the inertia force (i.e., $m_1 a_1$).

Thus, because of the reaction forces, both external and inertia forces occur in pairs. Each pair constitutes a force couple which has a torque associated with it (see Appendix A). For example, the torque of the gravity force couple acting on this segment equals the moment of the gravity force about the lower joint. So, for frictionless pin- and ball-and-socket joints, and with at least one point of the body stationary, the net effect of gravity and inertia on the motion of the segments is given by their torques.

APPENDIX C: DYNAMIC EQUATION OF A ROTATING PLANAR SEGMENT

The dynamic equation of the rotating planar segment shown in Figure 6.7 is as follows:

$$(I)\ddot{\theta} = T^{\text{mus}} - T^{\text{grav}} = T^{\text{net}}. \quad (1)$$

The segment has a mass m , and its mass center is located a distance l from the joint, and the segment is oriented at an angle θ with respect to horizontal. Muscle torques T^{SOL} and T^{TA} act in opposition; therefore,

$$T^{\text{mus}} = T^{\text{SOL}} - T^{\text{TA}}.$$

The net and gravity torques are, respectively,

$$T^{\text{net}} = (\bar{I})\ddot{\theta} = (I + ml^2)\ddot{\theta} \quad \text{and} \quad T^{\text{grav}} = mgl \cos\theta = T^g \cos\theta.$$

APPENDIX D: DYNAMIC EQUATIONS FOR TWO ROTATING PLANAR SEGMENTS

The dynamic equation for the two-segment system shown in Figure 6.9 can be derived from the recursive equations in reference 75. The equations are as follows:

$$(\bar{I}_1)\ddot{\theta}_1 + [\bar{I}_{cs} \cos(\theta_1 - \theta_2)]\ddot{\theta}_2 = T_1^{\text{mus}} - T_1^{\text{grav}} - [\bar{I}_{cs} \sin(\theta_1 - \theta_2)]\dot{\theta}_2^2. \quad (3a)$$

$$(\bar{I}_2)\ddot{\theta}_2 + [\bar{I}_{cs} \cos(\theta_1 - \theta_2)]\ddot{\theta}_1 = T_2^{\text{mus}} - T_2^{\text{grav}} + [\bar{I}_{cs} \sin(\theta_1 - \theta_2)]\dot{\theta}_1^2. \quad (3b)$$

Our notation is m_i for the mass of segment i , ρ_i for the length of segment 1, l_i for the distance from the joint nearest to the ground to the mass center, I_i for the moment of inertia about the mass center, and θ_i for the angle of the segment with respect to horizontal. Thus,

$$\bar{I}_1 = I_1 + m_1 l_1^2, \quad \bar{I}_2 = I_2 + m_2 l_2^2, \quad \bar{I}_{cs} = m_2 l_2 \rho_1,$$

$$T_1^{\text{grav}} = T_1^g \cos\theta_1 = g(m_1 l_1 + m_2 \rho_1) \cos(\theta_1),$$

$$T_2^{\text{grav}} = T_2^g \cos\theta_2 = g(m_2 l_2) \cos(\theta_2).$$

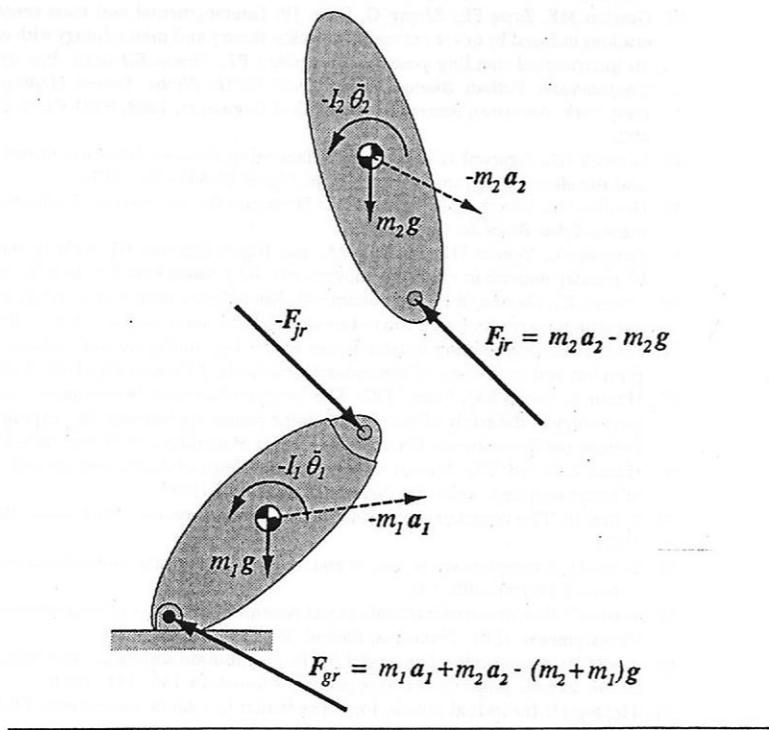
APPENDIX E: ANALYSIS OF DYNAMIC COUPLING

The forces acting on segment 2 are the gravity force $m_2 g$, and the joint reaction force F_{jr} , as well as the inertia force $-m_2 a_2$ (Fig. 6.16). The coupling arises on segment 2 because its own acceleration, a_2 , is a function of θ_1 , $\dot{\theta}_1$, and $\ddot{\theta}_1$. Therefore, the coupling on segment 2 occurs for purely kinematic reasons; that is, the inertia force of segment 2 depends on the kinematics of segment 1.

In contrast, coupling from segment 2 on segment 1 occurs because of joint reaction forces. The forces acting on segment 1 are the gravity force $m_1 g$ and the reaction force at each of the two joints ($-F_{jr}$, F_{gr}). Therefore, since the acceleration of segment 1 does not depend on θ_2 or its time derivatives, the coupling on segment 1 occurs because both the inertia force of segment 2 ($-m_2 a_2$) and its gravity force ($m_2 g$) have reactionary forces at the joints.

FIGURE 6.16

Dynamic coupling arises because inertia and gravity forces may be transmitted via the joints to the other segments, and because the inertia force of one segment may depend on the kinematics of other segments.



APPENDIX F: DYNAMIC EQUATIONS OF TWO SEGMENTS IN JOINT SPACE

Although the equations of motion can be derived directly in joint space [26], the equations are easily converted from segment to joint coordinates. The segment and joint coordinates, and torques are related by:

$$\begin{aligned}\theta_1 &= \phi_a, & T_1^{\text{net}} &= T_a^{\text{net}} + T_k^{\text{net}}, \\ \theta_2 &= \phi_a - \phi_k + \pi, & T_2^{\text{net}} &= -T_k^{\text{net}}.\end{aligned}\quad (6)$$

Therefore,

$$\begin{aligned}\ddot{\theta}_1 &= \ddot{\phi}_a, & \dot{\theta}_1 &= \dot{\phi}_a, & \ddot{\theta}_2 &= \ddot{\phi}_a - \ddot{\phi}_k, & \dot{\theta}_2 &= \dot{\phi}_a - \dot{\phi}_k, \\ \cos(\theta_1 - \theta_2) &= -\cos(\phi_k), & \sin(\theta_1 - \theta_2) &= -\sin(\phi_k).\end{aligned}\quad (7)$$

Substituting Equations 6 and 7 into Equations 3a and 3b, the dynamic equations in joint coordinates are as follows:

$$\begin{aligned}(\tilde{I}_a) \ddot{\phi}_a + (\tilde{I}_{cj}) \ddot{\phi}_k &= T_a^{\text{net}} \\ (\tilde{I}_k) \ddot{\phi}_k + (\tilde{I}_{cj}) \ddot{\phi}_a &= T_k^{\text{net}}\end{aligned}\quad (8a)\quad (8b)$$

where

$$\tilde{I}_a = \tilde{I}_1 + \tilde{I}_2 - 2\tilde{I}_{cs} \cos\phi_k, \quad \tilde{I}_k = \tilde{I}_2, \quad \tilde{I}_{cj} = \tilde{I}_{cs} \cos\phi_k - \tilde{I}_2,$$

and

$$\begin{aligned}T_a^{\text{net}} &= T_a^{\text{mus}} - T_a^{\text{grav}} + T_a(\dot{\phi}_a, \phi_k, \dot{\phi}_k), \\ T_k^{\text{net}} &= T_k^{\text{mus}} - T_k^{\text{grav}} + T_k(\dot{\phi}_a^2, \phi_k), \\ T_a^{\text{mus}} &= T_1^{\text{mus}} + T_2^{\text{mus}}, \\ T_k^{\text{mus}} &= -T_2^{\text{mus}}, \\ T_a^{\text{grav}} &= T_1^{\text{grav}} + T_2^{\text{grav}}, \\ T_k^{\text{grav}} &= -T_2^{\text{grav}}, \\ T_a(\dot{\phi}_a, \phi_k, \dot{\phi}_k) &= \tilde{I}_{cs} \sin(\phi_k) (\dot{\phi}_k^2 - 2\dot{\phi}_a \dot{\phi}_k), \\ T_k(\dot{\phi}_a^2, \phi_k) &= \tilde{I}_{cs} \sin(\phi_k) \dot{\phi}_a^2.\end{aligned}$$

REFERENCES

- Alexander RMcN: The mechanics of jumping by a dog (*Canis familiaris*). *J Zool (Lond)* 173:549–573, 1974.
- Alexander RMcN, Vernon A: The mechanics of hopping by kangaroos (*Macropodidae*). *J Zool (Lond)* 177:265–303, 1975.
- An KN, Chao EY: Kinematic analysis of human movement. *Ann Biomed Eng* 12:585–597, 1984.
- An KN, Kwak BM, Chao EY, Morrey BF: Determination of muscle and joint forces: a new technique to solve the indeterminate problem. *J Biomech Eng* 106:364–367, 1984.
- Anderson JE: *Grant's Atlas of Anatomy*, ed 8. Baltimore, Williams & Wilkins, 1983.
- Andrews JG: On the relationship between resultant joint torques and muscular activity. *Med Sci Sports Exerc* 14:361–367, 1982.
- Andrews JG: A general method for determining the functional role of a muscle. *J Biomech Eng* 107:348–353, 1985.
- Andrews JG: The functional role of the hamstrings and quadriceps during cycling: Lombard's paradox revisited. *J Biomech* 20:565–575, 1987.
- Athans M, Fall PL: *Optimal Control: An Introduction to the Theory and Its Application*. New York, McGraw-Hill, 1966, p 879.
- Atkeson CG, Hollerbach JM: Kinematic features of unrestrained vertical arm movements. *J Neurosci* 5:2318–2330, 1985.

11. Barry D, Ahmed AM: Design and performance of a modified buckle transducer for the measurement of ligament tension. *J Biomech Eng* 108:149–152, 1986.
12. Bean JC, Chaffin DB, Schultz AB: Biomechanical model calculation of muscle contraction forces: a double linear programming method. *J Biomech* 21: 59–66, 1988.
13. Bobbert MF, Huijing PA, van Ingen Schenau, GJ: A model of the human triceps surae muscle-tendon complex applied to jumping. *J Biomech* 19: 887–898, 1986.
14. Bobbert MF, Huijing PA, van Ingen Schenau GJ: An estimation of power output and work done by the human triceps surae muscle-tendon complex in jumping. *J Biomech* 19:899–906, 1986.
15. Bock WJ: Mechanics of one- and two-joint muscles. *Am Mus Novit* 2319:1–45, 1968.
16. Brand RA, Pedersen DR, Friederich JA: The sensitivity of muscle force predictions to changes in physiologic cross-sectional area. *J Biomech* 19:589–596, 1986.
17. Bresler B, Frankel JP: The forces and the moments in the leg during level walking. *Trans ASME (Am Soc Mech Eng) J Biomech Eng* 72:27–35, 1950.
18. Brown TD, Sigal L, Njus GO, Njus NM, Singerman RJ, Brand RA: Dynamic performance characteristics of the liquid metal strain gauge. *J Biomech* 19:165–173, 1986.
19. Buchner HJ, Hines MJ, Hemami HA: Dynamic model for finger interphalangeal coordination. *J Biomech* 21:459–468, 1988.
20. Cappozzo A, Leo T, Pedotti A: A general computing method for the analysis of human locomotion. *J Biomech* 8:307–320, 1975.
21. Carlsoo S, Molbeck S: The functions of certain two-joint muscles in a closed muscular chain. *Acta Morphol Neerl Scand* 6:377–386, 1966.
22. Chao EYS: Biomechanics of the human gait. In Schmid-Schonbein GW, Woo SLY, Zweifach BW (eds): *Frontiers in Biomechanics*. New York, Springer-Verlag, 1986, pp 225–244.
23. Chow CK, Jacobson DH: Studies of human locomotion via optimal programming. *Math Biosci* 10:239–306, 1971.
24. Chow CK, Jacobson DH: Further studies of human locomotion: postural stability and control. *Math Biosci* 15:93–108, 1972.
25. Cooper JM, Adrian M, Glassow RB: *Kinesiology*, ed 5. St. Louis, Mosby, 1982, p 452.
26. Craig JJ: *Introduction to Robotics: Mechanics and Controls*. Reading, MA, Addison-Wesley, 1986, p 303.
27. Crowninshield D: Use of optimization techniques to predict muscle forces. *J Biomech Eng* 100:88–92, 1978.
28. Crowninshield RD, Brand RA: A physiologically based criterion of muscle force prediction in locomotion. *J Biomech* 14:793–801, 1981.
29. Crowninshield RD, Johnston RC, Andrews JG, Brand RA: A biomechanical investigation of the human hip. *J Biomech* 11:75–85, 1978.
30. Davy DT, Audu ML: A dynamic optimization technique for predicting muscle forces in the swing phase of gait. *J Biomech* 20:187–201, 1987.
31. Dickinson JA, Cook SD, Leinhardt TM: The measurement of shock waves following heel strike when running. *J Biomech* 18:415–422, 1985.
32. Dul J, Johnson GE, Shiavi R, Townsend MA: Muscular synergism-II. A minimum-fatigue criterion for load sharing between synergistic muscles. *J Biomech* 17:675–684, 1984.
33. Dul J, Townsend MA, Shiavi R, Johnson GE: Muscular synergism—I. On criteria for load sharing between synergistic muscles. *J Biomech* 17:663–673, 1984.
34. Elftman H: Forces and energy changes in the leg during walking. *Am J Physiol* 125:339–356, 1939.
35. Elftman H: The function of muscles in locomotion. *Am J Physiol* 125:357–366, 1939.
36. Elftman H: The action of muscles in the body. *Biol Symp* 3:191–209, 1941.
37. Elftman H: Biomechanics of muscle. *J Bone Joint Surg* 48A:363–377, 1966.
38. Flash T, Hogan N: The coordination of arm movements: An experimentally confirmed mathematical model. *J Neurosci* 5:1688–1703, 1985.
39. Ghez C, Gordon J: Trajectory control in targeted force impulses: I. Role of opposing muscles. *Exp Brain Res* 67:225–240, 1987.
40. Ghez C, Martin JH: The control of rapid limb movement in the cat: III. Agonist–antagonist coupling. *Exp Brain Res* 45:115–125, 1982.
41. Gordon ME, Zajac FE, Hoy MG: Postural synergies dictated by segmental accelerations from muscles and physical constraints. *Soc Neurosci Abstr* 12:1425, 1986.
42. Gordon ME, Zajac FE, Khang G, Loan JP: Intersegmental and mass center accelerations induced by lower extremity muscles: theory and methodology with emphasis on quasivertical standing postures. In Spilker RL, Simon BR (eds): *Proc Symposium Computational Methods Bioengineering: 1988 ASME Winter Annual Meeting/Chicago*. New York, American Society of Mechanical Engineers, 1988, BED-Vol 9, pp. 481–492.
43. Gottlieb GL, Agarwal GC: Dynamic relationship between isometric muscle tension and the electromyogram in man. *J Appl Physiol* 30:345–351, 1971.
44. Gottlieb GL, Corcos DM, Agarwal GC: Strategies for the control of voluntary movements. *Behav Brain Sci* (in press).
45. Gregoire L, Veeger HE, Huijing PA, van Ingen Schenau CJ: Role of mono- and biarticular muscles in explosive movements. *Int J Sports Med* 5:301–305, 1984.
46. Gregor RJ, Cavanagh PR, LaFortune M: Knee flexor moments during propulsion in cycling—a creative solution to Lombard's Paradox. *J Biomech* 18:307–316, 1985.
47. Hardt DE: Determining muscle forces in the leg during normal walking—an application and evaluation of optimization methods. *J Biomech Eng* 100:72–78, 1978.
48. Hasan Z, Enoka RM, Stuart DG: The interface between biomechanics and neurophysiology in the study of movement: some recent approaches. In Terjung RL (ed): *Exercise and Sport Sciences Reviews*. New York: Macmillan, 1985, pp. 169–234.
49. Hasan Z, Stuart DG: Animal solutions to problems of movement control: The role of proprioceptors. *Annu Rev Neurosci* 11:199–223, 1988.
50. Hatze H: The complete optimization of a human motion. *Math Biosci* 28:99–135, 1976.
51. Hatze H: A complete set of control equations for the human musculo-skeletal system. *J Biomech* 10:799–805, 1977.
52. Hatze H: Neuromusculoskeletal control systems modeling—a critical survey of recent developments. *IEEE Trans Auto Control AC-25:375–385*, 1980.
53. Hatze H: A comprehensive model for human motion simulation and its application to the take-off phase of the long jump. *J Biomech* 14:135–142, 1981.
54. Herzog H: Individual muscle force prediction in athletic movements. Ph.D. dissertation, University of Iowa, Ames, 1985.
55. Hill AV: The heat of shortening and the dynamic constants of muscle. *Proc R Soc Lond (Biol)* 126:136–195, 1938.
56. Hof AL, Geelen BA, Van Den Berg JW: Calf muscle moment, work and efficiency in level walking: role of series elasticity. *J Biomech* 16:523–537, 1983.
57. Hof AL, Pronk CNA, van Best JA: Comparison between EMG to force processing and kinetic analysis for the calf muscle moment in walking and stepping. *J Biomech* 20:167–178, 1987.
58. Hof AL, Van Den Berg JW: EMG to force processing I: an electrical analogue of the Hill muscle model. *J Biomech* 14:747–758, 1981.
59. Hogan N: The mechanics of multi-joint posture and movement control. *Biol Cybern* 52:315–331, 1985.
60. Hollerbach JM, Flash T: Dynamic interactions between limb segments during planar arm movement. *Biol Cybern* 44:67–77, 1982.
61. Houtz SJ, Fischer FJ: An analysis of muscle action and joint excursion during exercise on a stationary bicycle. *J Bone Joint Surg* 41-A:123–131, 1959.
62. Hoy MG, Zajac FE, Topp EL, Cady CT, Levine WS: Synergistic control of uniarticular and biarticular muscles in human jumping: a computer simulation study. *Soc Neurosci Abstr* 12:1425, 1986.

63. Hoy MG, Zernicke RF: Modulation of limb dynamics in the swing phase of locomotion. *J Biomech* 18:49–60, 1985.
64. Hoy MG, Zernicke RF: The role of intersegmental dynamics during rapid limb oscillations. *J Biomech* 19:867–877, 1986.
65. Hoy MG, Zernicke RF, Smith JL: Contrasting roles of inertial and muscle moments at knee and ankle during paw-shake response. *J Neurophysiol* 54:1282–1294, 1985.
66. Hull M, Davis RR: Measurement of pedal loading in bicycling: I. Instrumentation. *J Biomech* 14:843–856, 1981.
67. Huston RL, Passerello CE, Harlow MW: Dynamics of multirigid-body systems. *J Appl Mech* 45:889–894, 1978.
68. Inman VT, Ralston HJ, Todd F: *Human Walking*. Baltimore, Williams & Wilkins, 1981, p 154.
69. Jorge M, Hull L: Analysis of EMG measurements during bicycle pedaling. *J Biomech* 19:683–694, 1986.
70. Ju MS, Mansour JM: Simulation of the double limb support phase of human gait. *J Biomech Eng* 110:223–229, 1988.
71. Kailath T: *Linear Systems*. Englewood Cliffs, Prentice-Hall, 1980, p 682.
72. Kane TR, Levinson DA: *Dynamics: Theory and Applications*. New York, McGraw-Hill, 1985, p 379.
73. Khang G: Paraplegic standing controlled by functional neuromuscular stimulation: computer model, control-system design, and simulation studies. Ph.D dissertation, Stanford University, Stanford, CA, 1988, p 115.
74. Kinzel GL, Gutkowski LJ: Joint models, degrees of freedom, and anatomical motion measurement. *J Biomech Eng* 105:55–62, 1983.
75. Koozekanani SH, Barin K, McGhee RB, Chang HT: A recursive free-body approach to computer simulation of human postural dynamics. *IEEE Trans Biomed Eng* 30:787–792, 1983.
76. Levine WS, Zajac FE, Belzer MR, Zomlefer MR: Ankle controls that produce a maximal vertical jump when other joints are locked. *IEEE Trans Auto Control AC*-28:1008–1016, 1983.
77. Lippold OCJ: The relation between integrated action potentials in a human muscle and its isometric tension. *J Physiol* 117:492–499, 1952.
78. Loeb GE: Hard lessons in motor control from the mammalian spinal cord. *Trends Neurosci* 10:108–113, 1987.
79. Luenberger DG: *Linear and Nonlinear Programming*. Reading, Addison-Wesley, 1984, p 491.
80. Mansour JM, Pereira JM: Quantitative functional anatomy of the lower limb with application to human gait. *J Biomech* 20:51–58, 1987.
81. Manter JT: The dynamics of quadrupedal walking. *J Exp Biol* 15:522–540, 1938.
82. Markee JE, Logue JT, Williams M, Stanton WB, Wrenn RN, Walker LB: Two-joint muscles of the thigh. *J Bone Joint Surg* 37-A:125–142, 1955.
83. Marshall RN, Jensen RK, Wood GA: A general Newtonian simulation of an n -segment open chain model. *J Biomech* 18:359–367, 1985.
84. McCollum G, Nashner M: Mechanics of stance and locomotion in physiological coordinates: a biomechanical model taking into account the physiology of muscle contraction and the activation patterns of muscle synergies. *Soc Neurosci Abstr* 8:284, 1982.
85. Miller D: Resultant lower extremity joint moments in below-knee amputees during running stance. *J Biomech* 20:529–541, 1987.
86. Morrison JB: The mechanics of the knee joint in relation to normal walking. *J Biomech* 3:51–61, 1970.
87. Morrison JB: The mechanics of muscle function in locomotion. *J Biomech* 3:431–451, 1970.
88. Nashner LM: Fixed patterns of rapid postural responses among leg muscles during stance. *Exp Brain Res* 30:13–24, 1977.

89. Nashner LM, McCollum G: The organization of human postural movements: a formal basis and experimental synthesis. *Behav Brain Sci* 8:135–172, 1985.
90. Nelson WL: Physical principles for economies of skilled movements. *Biol Cybern* 46: 135–147, 1983.
91. Newmiller J, Hull ML, Zajac FE: A mechanically decoupled two force component bicycle pedal dynamometer. *J Biomech* 21:375–386, 1988.
92. OGUZTÖRELİ MN, Stein RB: Optimal control of antagonistic muscles. *Biol Cybern* 48:91–99, 1983.
93. Olney SJ, Winter DA: Prediction of knee and ankle moments of force in walking from EMG and kinematic data. *J Biomech* 18:9–20, 1985.
94. Orin DF, McGhee RB, Vukobratovic M, Hartoch G: Kinematic and kinetic analysis of open-chain linkages utilizing Newton-Euler methods. *Math Biosci* 43:107–130, 1979.
95. Osborn JW, Baragar FA: Predicted pattern of human muscle activity during clenching derived from a computer assisted model: symmetric vertical bite forces. *J Biomech* 18:599–612, 1985.
96. Pandy MG, Berme N: A numerical method for simulating the dynamics of human walking. *J Biomech* (in press).
97. Pandy MG, Zajac FE, Hoy MG, Topp ER, Tashman S, Stevenson PJ, Cady C, Sim E, Levine WS: Sub-optimal control of a maximum-height, countermovement jump. In Stein JL (ed): *Proceedings Symposium Modeling and Control Issues in Biomechanical Systems: 1988 ASME Winter Annual Meeting/Chicago*. New York, American Society of Mechanical Engineers, 1988, DSC-Vol 12, pp 27–44.
98. Patriarco AG, Mann RW, Simon SR, Mansour JM: An evaluation of the approaches of optimization models in the prediction of muscle forces during human gait. *J Biomech* 14:513–525, 1981.
99. Paul JP: Letter to the editor: torques produce tension. *J Biomech* 11:87, 1978.
100. Pedersen DR, Brand RA, Cheng C, Arora JS: Direct comparison of muscle force predictions using linear and nonlinear programming. *J Biomech Eng* 109:192–199, 1987.
101. Pedotti A: A study of motor coordination and neuromuscular activities in human locomotion. *Biol Cybern* 26:53–62, 1977.
102. Pedotti A, Krishnan VV, Stark L: Optimization of muscle-force sequencing in human locomotion. *Math Biosci* 38:57–76, 1978.
103. Penrod DD, Davy DT, Singh DP: An optimization approach to tendon force analysis. *J Biomech* 7:123–129, 1974.
104. Pierrynowski MR, Morrison JB: Estimating the muscle forces generated in the human lower extremity when walking: a physiological solution. *Math Biosci* 75:43–68, 1985.
105. Pierrynowski MR, Morrison JB: A physiological model for the evaluation of muscular forces in human locomotion: theoretical aspects. *Math Biosci* 75:69–102, 1985.
106. Riemersma DJ, Lammertink Jos LMA: Calibration of the mercury-in-silastic strain gauge in tendon load experiments. *J Biomech* 21:469–476, 1988.
107. Robertson DGE, Winter DA: Mechanical energy generation, absorption and transfer amongst segments during walking. *J Biomech* 13:845–854, 1980.
108. Romanes GJ (ed): *Cunningham's Textbook of Anatomy*, ed 12. Oxford, Oxford University Press, 1981, p 1078.
109. Romick-Allen R, Schultz AB: Biomechanics of reactions to impending falls. *J Biomech* 21:591–600, 1988.
110. Scireg A, Arvikar RJ: The prediction of muscular load sharing and joint forces in the lower extremities during walking. *J Biomech* 8:89–102, 1975.
111. Sherrington CS: Flexion-reflex of the limb, crossed extension-reflexes, and reflex stepping and standing. *J Physiol* 40:28–121, 1910.
112. Shiavi R, Limbird T, Frazer M, Stivers K, Strauss A, Abramovitz J: Helical motion

- analysis of the knee—II. Kinematics of uninjured and injured knees during walking and pivoting. *J Biomech* 20:653–665, 1987.
113. Son K, Miller JAA, Schultz AB: The mechanical role of the trunk and lower extremities in a seated weight-moving task in the sagittal plane. *J Biomech Eng* 110:97–103, 1988.
 114. Stein RB, O'Guztöreli MN, Capaday C: What is optimized in muscular movements? In Jones NL, McCartney N, McComas AJ (eds): *Human Muscle Power*. Champaign, IL, Human Kinetics, 1986, pp 131–150.
 115. Suzuki S, Watanabe S, Homma S: EMG activity and kinematics of human cycling movements at different constant velocities. *Brain Res* 240:245–258, 1982.
 116. Vaughan CL, Hay JG, Andrews JG: Closed loop problems in biomechanics. Part II—An optimization approach. *J Biomech* 15:201–210, 1982.
 117. Walmsley B, Hodgson JA, Burke RE: Forces produced by medial gastrocnemius and soleus muscles during locomotion in freely moving cats. *J Neurophysiol* 41:1203–1216, 1978.
 118. Whiting WC, Gregor RJ, Roy RR, Edgerton VR: A technique for estimating mechanical work of individual muscles in the cat during treadmill locomotion. *J Biomech* 17:685–694, 1984.
 119. Winter DA: The locomotion laboratory as a clinical assessment system. *Med Prog Technol* 4:95–106, 1976.
 120. Winter DA: *Biomechanics of Human Movement*. New York, Wiley, 1979, p 202.
 121. Winter DA: *The Biomechanics and Motor Control of Human Gait*. Waterloo, University of Waterloo Press, 1987, p 72.
 122. Yoon YS, Mansour JM: The passive elastic moment at the hip. *J Biomech* 15:905–910, 1982.
 123. Zajac FE: Thigh muscle activity in cats during maximal height jumps. *J Neurophysiol* 53:979–993, 1985.
 124. Zajac FE: Dynamics of limb movement: interpretation of EMG signals and the effects of musculotendon forces on body acceleration. In Butler DL, Torzilli PA (eds): *ASME 1987 Biomechanics Symposium in Cincinnati*. New York: The American Society of Mechanical Engineers, 1987, vol. AMD-84:391–394.
 125. Zajac FE: Muscle and tendon properties: models, scaling, and application to biomechanics and motor control. *CRC Crit Rev Biomed Eng*, in press.
 126. Zajac FE, Gordon ME, Hoy MG: Physiological classification of muscles into agonist–antagonist muscle action groups: theory and methodology based on mechanics. *Soc Neurosci Abstr* 12:1424, 1986.
 127. Zajac FE, Levine WS: Novel experimental and theoretical approaches to study the neural control of locomotion and jumping. In Talbott R, Humphrey D (eds): *Posture and Movement: Perspective for Integrating Sensory and Motor Research on the Mammalian Nervous System*. New York, Raven Press, 1979, pp 259–279.
 128. Zarrugh MY: Kinematic prediction of intersegment loads and power at the joints of the leg in walking. *J Biomech* 14:713–725, 1981.
 129. Zomlefer MR, Ho R, Levine WS, Zajac FE: The use of optimal control in the study of a normal physiological movement. *Proceedings of the 1975 IEEE Conference on Decision and Control in Houston*. New York: The Institute of Electrical and Electronic Engineers, 1975, pp 402–407.

7

Impedance Cardiography: Noninvasive Assessment of Human Central Hemodynamics at Rest and During Exercise

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Few of us are partial to any invasive procedure to assess our cardiac status, and fewer still have the fortitude to catheterize their own heart as Forssman did in 1929 [55]. Yet, there is no question that assessment of the mechanical pumping activity of the heart provides information of critical importance for evaluation of the circulatory system. The use of electrical impedance to assess the cardiac status has developed into a relatively simple, repeatable, atraumatic, cost-effective, and, most important, noninvasive procedure to assess the cardiac pump on a beat-by-beat basis. This review summarizes selected literature, primarily of the 1980s, to provide a perspective on the use of electrical impedance to assess human cardiac performance in basic and applied physiology, as well as clinical practice.

Historical Perspective

Electrical impedance has been used to evaluate heart function and a host of other divergent physiological events since the early 1900s [55]. Reviews on the detection of physiological events by evaluation of impedance changes have been presented by Geddes and Baker [25], Miller and Horvath [72], Baker [3], Mohapatra [78], Lamberts et al. [55], Penney [88], and Schuster and Schuster [97]. This review differs from previously published material in that it summarizes the 1980s work with special consideration given to our current understanding of the impedance waveform genesis, advances in waveform conditioning, and applications at rest and during exercise which provide relevant information concerning myocardial function.

The earlier measurements of the changes in electrical impedance resulting from the activity of the heart have been labeled dielectrography, radiocardiography, rheocardiography, and cardiographie à haute fréquence. Nyboer [82] was a pioneer in suggesting the use of a thoracic tetrapolar electrode arrangement and should be credited with proposing the term electrical impedance plethysmography. In the simplest terms,