STEADY-STATE SIMULATION MODEL

This document accompanies the function "Simulation_Model_WDN.m" written in Matlab which executes the steady-state simulation of a water distribution network (WDN). The function performs Demand-Driven Analysis (DDA) or Pressure-Driven Analysis (PDA) allowing the prediction of the background leakages by means of Germanopoulos' model (1985). The function is extracted from WDNetXL system (www.hydroinformatics.it) and it is intended for research purpose only in order to explain the modelling strategy reported in Giustolisi et al. (2008). In fact, it is limited to background leakage and/or hydraulic pressure deficient conditions modelling and does not implement devices as for example flow-pressure control valves; check valves, etc..

Steady-State Simulation of Water Distribution Networks

A steady-state water distribution network (WDN) simulation model run is a snapshot representing a real hydraulic system in a time interval ΔT. The hydraulic system is then supposed to be in a steady-state condition although that condition is an abstraction which holds under some assumptions that will be discussed. The mathematical formulation of a WDN model, for a given hydraulic system composed of n_p pipes, n_n nodes and n_0 reservoirs, is based on $(n_p + n_n)$ energy and mass balance equations,

$$\begin{cases} \mathbf{A}_{pp}\mathbf{Q}_p + \mathbf{A}_{pn}\mathbf{H}_n = -\mathbf{A}_{p0}\mathbf{H}_0 + \mathbf{H}_p^{pump} \\ \begin{bmatrix} \mathbf{A}_{np}\mathbf{Q}_p = \dfrac{\mathbf{V}_n}{\Delta T} & \text{demand-driven analysis} \\ \mathbf{A}_{np}\mathbf{Q}_p - \dfrac{\mathbf{V}_n(\mathbf{H}_n)}{\Delta T} = \mathbf{0}_n & \text{pressure-driven analysis} \end{bmatrix} \end{cases} \quad (1)$$

where:
$\mathbf{Q}_p = [n_p,1]$ column vector of unknown pipe flow rates;
$\mathbf{H}_n = [n_n,1]$ column vector of unknown nodal heads;
$\mathbf{H}_0 = [n_0,1]$ column vector of known nodal heads of reservoirs;
$\mathbf{H}_p^{pump} = [n_p,1]$ column vector of static heads of pump systems installed along pipes (if any);
$\mathbf{V}_n = [n_n,1]$ column vector of volumes of water withdrawals in the nodes;
ΔT = time interval of the real hydraulic system snapshot;
$\mathbf{A}_{pn}=\mathbf{A}^T_{np}$ and $\mathbf{A}_{p0}$ = topological incidence sub-matrices of size $[n_p,n_n]$ and $[n_p,n_0]$, respectively, derived from the general topological matrix $\bar{\mathbf{A}}_{pn}=[\mathbf{A}_{pn} \mid \mathbf{A}_{p0}]$ of size $[n_p,n_n+n_0]$;
$\mathbf{A}_{pp}\mathbf{Q}_p = [n_p,1]$ column vector of pipe head losses containing the terms related to internal head losses of pump systems, minor head losses and evenly distributed head losses.
Afterward, a steady-state modeling for a given time interval ΔT and volume $\mathbf{V}_n$ of nodal water withdrawals, provides $\mathbf{Q}_p$ and $\mathbf{H}_n$.
The curve of a pump installed on the kth pipe has equation:

$$\Delta H = \pm\omega_k^2\left(H_k^{pump} - r_k\left(\frac{Q_k}{\omega_k}\right)^{c_k}\right) \quad (2)$$

where:
ΔH = dynamic head;
ω_k = speed factor;
H_k^{pump} = static head;
r_k and c_k = parameters of internal head loss.

The sign in front of ω_k depends on the installation direction of the pump with respect to the positive sign of the flow rate in the kth pipe, then the non-null elements of $\mathbf{H}_p^{pump}$ are $\pm\omega_k^2 H_k^{pump}$. The classical formulation of the system in (1) reports $\mathbf{d}_n = \mathbf{V}_n/\Delta T$ = column vector of demands lumped in the nodes, whilst the use of $\mathbf{V}_n$ and ΔT instead of $\mathbf{d}_n$ is more effective to discuss model assumptions from a technical standpoint.

Demand-driven analysis (DDA) and pressure-driven analysis (PDA)

In the past, the steady-state modeling was generally performed using the approach of fixed demands, i.e. fixed volumes. As a consequence, the first form of the two mass balance equations in (1) was used. As a result, the demands drive the solution ($\mathbf{Q}_p$; $\mathbf{H}_n$) of the system in (1) and the analysis was named demand-driven. DDA generally applies when the nodal volumes can be fixed (fixed demands per ΔT) because known *a priori* or when they are prior assumed. For example, in classical pipe sizing the volumes required by customers at the pick hour are assumed and the background leakages are fixed as a percentage of those volumes. Recently, the steady-state modeling is performed using the most realistic assumption of a demand dependency on pressure through the nodal heads ($\mathbf{H}_n$), i.e. the volume of a nodal outflow depends on pressure. As a consequence, the second form of the mass balances in (1) is used. As a result, the pressures drive the solution ($\mathbf{Q}_p$; $\mathbf{H}_n$) of the system in (1) and the analysis is named pressure-driven.

Steady-state assumption and WDN model volumes in ΔT

The steady-state analysis of a real hydraulic system, as a snapshot of its steady-state condition in a time interval ΔT, assumes a slow time-varying boundary conditions such as demands, water tank levels, working condition and state of control valves, etc.. Under these assumptions the inertial and dynamic effects are considered negligible and are not reflected on the model. Nevertheless, at smaller time and spatial scales some demands are actually pulses having a volume and a stochastic behavior (e.g. due to customers' use of the faucets) while others are unit volumes on time from continuous flows (e.g. background leakage outflows). The model volumes are actually a summation of components related to the water withdrawals from different types of orifices which can be controlled or not.
The orifices can be classified as:

- controlled by customers. Their use generates pulses having a random intensity, duration and volume;
- controlled (on/off status) by filling levels of fixed volumes. Their use generates pulses having a random intensity and duration but a fixed volume;
- uncontrolled or free. They are orifices (for example of sprinkler systems) which continuously work for a given time interval depending on the system working status;
- uncontrolled of diffuse background leakages or single bursts. They are orifices whose shape and size depend on pipe material and pressure.

In the case of pulses, their spatial and temporal aggregations on ΔT provide the WDN model nodal volumes (average demands per ΔT) related to the controlled orifices. Similarly, spatial and temporal aggregations on ΔT of the continuous flows through the uncontrolled orifices (e.g. sprinkler systems and leakages) provide the WDN model nodal volumes (average demands per ΔT). The four volumes figures are the components of $\mathbf{V}_n(\mathbf{H}_n)$.

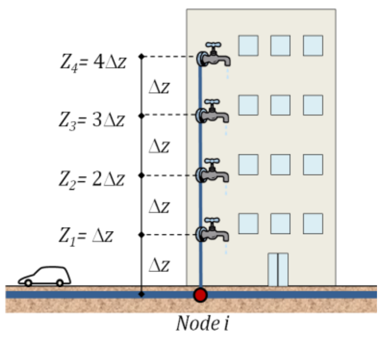
Pressure-demand relationships for V_n

The mathematical formulations of pressure-demand models implemented in part in function "Simulation_Model_WDN.m" and generally in WDNetXL vr. 3.0 are here reported. They are based on pressure variables instead of on head variables. The use of head variables $\mathbf{H}_n$ to represent $\mathbf{V}_n$ (or $\mathbf{d}_n$) dependence is more consistent in a model formulation which is based on the state variables $\mathbf{Q}_p$ and $\mathbf{H}_n$. The use of pressures, which are the actual drivers of the outflows, is more consistent to present and discuss the demand models. Those models are classically represented as pressure-demand relationships, whilst the simple scaling of the demand by ΔT allows perceiving the relationships as volume-pressure.

Orifices controlled by costumers (human/customer-based demands)
(The function "Simulation_Model_WDN.m" allows a Single Level of Orifices)

$$d_i^{hum}\left(P_i - Z_j\right) = \sum_{j=1}^{m} d_j^{hum}\left(P_i - Z_j\right) = \sum_{j=1}^{m} \begin{cases} d_j^{req,hum} & P_i \geq P_j^{ser} \\ a_j^{hum}\sqrt{P_i - Z_j} & Z_j < P_i < P_j^{ser} \\ 0 & P_i \leq Z_j \end{cases} \qquad (3)$$

$$a_j^{hum} = \frac{d_j^{req,hum}}{\sqrt{P_j^{ser} - Z_j}} = \frac{d_j^{req,hum}}{\sqrt{P_j^{ser,res}}}$$



where:

i = subscript of the ith node;
j = subscript of the jth level of orifices (e.g. in multi-floor buildings);
P_i = model pressure at the ith node;
Z_j = elevations of the jth level of orifices;
d_i^{hum} = actually supplied demand at the ith node as sum of those at each level of orifices;
d_j^{hum} = actually supplied demand at the jth level of orifices depending on residual pressure $P_i - Z_j$ at that jth level;
$d_j^{req,hum}$ = required customer demand at the jth level of orifices;
P_j^{ser} = pressure for a correct service of each jth level of orifices;
a_j^{hum} = model coefficient representing the sum of the outflow coefficients of the statistically working orifices delivering the required customer demand $d_j^{req,hum}$ at the residual pressure for a correct service $P_j^{ser,res}$ (= $P_j^{ser} - Z_j$).

The actually supplied demand (d_i^{hum}) can be simply transformed in the volume V_i^{hum} (= $\Delta T d_i^{hum}$) multiplying the Eqs. (3) by ΔT. This way the technical soundness of an actually supplied volume computed by the model snapshot is preserved.
The model in (3) states that the volume V_i^{hum} actually supplied by the hydraulic system in a time interval ΔT equals the summation of required volume $V_j^{hum,req}$ (= $\Delta T d_j^{hum,req}$) at each level of the orifices supplied by the ith node. This fact occurs when the pressure P_i determines at each jth level a pressure P_j which is greater than equal to a residual pressure for a correct service $P_j^{ser,res}$ (e.g. 5 m). This means that the pressure P_i must be $P_j^{ser,res}$ (e.g. 5 m) greater than the elevation Z_m of the last level of orifices. In this circumstance, the hydraulic system capacity is not exceeded by each volume request $V_j^{hum,req}$ and the customers actually control the flow intensity at each orifice. This working condition of the system is named normal and DDA is performed under this assumption with respect to the volumes (demands) required by customers.
When the hydraulic system capacity is exceeded to supply each volume request $V_j^{hum,req}$, that is the pressure P_j is not sufficient at some level, a pressure-deficient working condition occurs at an orifice level. In this circumstance, the customers do not control the flow intensity at orifices with low pressure. They are opened at their maximum degree in order to get the maximum volume allowed by pressure. The pressure-demand relationship is then consistent with Torricelli law and, as in Eqs. (3), the volume (demand) is proportional to the square root of the residual pressure $P_i - Z_j$.
Consequently, PDA is mandatory to compute the actually supplied volume in the ith node which is lower than the required one. In order to account for several levels of orifices, WDNetXL allows to set up an initial level Z_1, a constant ΔZ between levels and the number of levels (i.e. it is considered the most common situation of a multi-floor building). Then, the required demand $d_j^{hum,req}$ is the total required demand at the ith node divided by the number of floors. The total required demand at the ith node is retrieved from the table of nodes and computed from the pipe level total demands by concentrating them in the pipe terminal nodes. Finally P_j^{ser} are computed starting from that retrieved from the table of the nodes, which refer to the first level, by adding a ΔZ increasing the levels. This corresponds to set up a constant $P_j^{ser,res}$ through the levels which is equal to $P_1^{ser,res}$ (= $P_1^{ser} - Z_j$).

Uncontrolled or free orifices (uncontrolled or free orifice-based demands)
(The function "Simulation_Model_WDN.m" does not allow Free Orifices)

The formulation of the uncontrolled orifice-based demands is similar to that of customer-based demands, but the upper bound of the Eqs. (3) does not exist as the customer control does not exist even in a normal working condition of the hydraulic system. Hence,

$$d_i^{unc}\left(P_i - Z_j\right) = \sum_{j=1}^{m} d_j^{unc}\left(P_i - Z_j\right) = \sum_{j=1}^{m} \begin{cases} a_j^{unc}\sqrt{P_i - Z_j} & P_i > Z_j \\ 0 & P_i \le Z_j \end{cases} \tag{4}$$

where:
i = subscript of the ith node;
j = subscript of the jth level of orifices (e.g. in multi-floor buildings);
P_i = model pressure at the ith node;
Z_j = elevations of the jth level of orifices;
d_i^{unc} = actually supplied demand at the ith node as sum of those at each level of orifices;
d_j^{hum} = actually supplied demand at the jth level of orifices depending on residual pressure $P_i - Z_j$ at that jth level;
a_j^{unc} = sum of the outflow coefficients of the working orifices at the jth assumed the same residual pressure (= $P_i - Z_j$).

It is noteworthy that the actually supplied demand (d_i^{unc}) can be simply transformed in the volume V_i^{unc} (= $\Delta T d_i^{unc}$) multiplying the Eqs. (4) by ΔT. This way the technical soundness of an actually supplied volume computed by the model snapshot is preserved.

The model in (4) states that the actually supplied volume V_i^{unc} by the hydraulic system in a time interval ΔT equals the summation of the volumes V_j^{unc} (= $\Delta T d_j^{unc}$) at each level. The volumes V_j^{unc} are different as the residual pressures (= $P_i - Z_j$) are not the same. In this circumstance V_i^{unc} strictly depends on the actual pressure of the hydraulic system and the concepts of normal and pressure-deficient working conditions do not apply. PDA is mandatory to compute the actually supplied volume V_i^{unc} in the ith node as it depends on pressure. In order to account for several level of orifices WDNetXL allows to set up an initial level Z_1, a constant ΔZ between levels and the number of levels (i.e. it is considered the most common situation of a multi-floor building). Then, the table of nodes reports a_i^{unc}, while the outflow coefficients a_j^{unc} are obtained by dividing a_i^{unc} by the corresponding number of floors. The exponent in the table of nodes for this kind of demands should be equal to 0.5.

Uncontrolled or free orifices with varying size and shape (Leakage-based demands)
(The function "Simulation_Model_WDN.m" allows Background Leakages)

The formulation of the leakage based-demands is similar to that of uncontrolled orifice-based demands, but the outflow coefficients are now function of the pressure as the size and shape of the leaks depend on it. In addition, there are two kind of leakage-based demands: (1) the background leakages which refer to diffuse leaks along the mains and (2) the bursts which are single outflows of relevant flow rates due to pipe breaks. The first are always present in the hydraulic system and runs continuously, the second are accidental occurrences whose modeling is generally useful for operational purposes. In addition, as the outflows are at the pipe level, m = 1 and Z_1 = 0 can be generally assumed, i.e. the concept of multi-level orifices at a different residual pressure does not apply. Hence,

$$d_i^{leaks}(P_i) = \begin{cases} a_i^{leaks}(P_i)\sqrt{P_i} & P_i > 0 \\ 0 & P_i \le 0 \end{cases} \tag{5}$$

where:
i = subscript of the ith node;
P_i = model pressure at the ith node;
d_i^{leaks} = leakage outflow at the ith node;
a_i^{leaks} = outflow coefficient depending on pressure.

In the case of **background leakages** they are computed at pipe level as a summation of small outflows due to cracks, small water losses from fitting, etc.. The pressure is the mean pressure along the pipe ($P_i = P_{k,mean}$) and the model formulation becomes:

$$d_k^{leaks}(P_{k,mean}) = \begin{cases} a_k^{leaks}(P_{k,mean})\sqrt{P_{k,mean}} = \beta_k L_k P_{k,mean}^{\alpha_k} & P_{k,mean} > 0 \\ 0 & P_{k,mean} \le 0 \end{cases}$$
$$a_k^{leaks}(P_{k,mean}) = \beta_k L_k P_{k,mean}^{(\alpha_k - 0.5)} \tag{6}$$

where:
k = subscript of the kth pipe;
$P_{k,mean}$ = model mean pressure along the kth pipe;
d_k^{leaks} = background leakages outflow along the kth pipe;
β_k and α_k= model parameters to be set in table of pipes;
L_k = length of the kth pipe.

During a simulation model run the pipe outflows in (6) are lumped in the nodes using the equation,

$$\mathbf{d}_n^{leaks} = \frac{1}{2}\left|\mathbf{A}_{np}\right|\mathbf{d}_p^{leaks} = \frac{1}{2}\left|\mathbf{A}_{np}\right|\begin{bmatrix} d_1^{leaks} \\ \dots \\ d_k^{leaks} \\ \dots \\ d_{n_p}^{leaks} \end{bmatrix} \tag{7}$$

In the case of a single leak in the *i*th node due to a burst the model formulation becomes:

$$d_i^{leaks}\left(P_i\right) = \begin{cases} a_i^{leaks}\left(P_i\right)\sqrt{P_i} = \beta_i P_i^{\alpha_i} & P_i > 0 \\ 0 & P_i \le 0 \end{cases}$$
$$a_i^{leaks}\left(P_i\right) = \beta_i P_i^{\left(\alpha_i - 0.5\right)} \tag{8}$$

where:
i = subscript of the *i*th node;
P_i = model mean pressure at the *i*th node;
d_i^{leaks} = outflow at the *i*th node due to a burst;
β_i and α_i = model parameters to be set in table of nodes;

It is worth noting that the model in (8) is the model in (4) assuming one orifice on a pipe whose outflow coefficient is a monomial function of pressure.
The leakages outflows (d_i^{leaks} and d_k^{leaks}) can be simply transformed in the volumes V_i^{leaks} (= $\Delta T d_i^{leaks}$) and V_k^{leaks} (= $\Delta T d_k^{leaks}$) by multiplying Eqs. (6) and (8) by ΔT. This way the technical soundness of a volume of water losses computed by the model snapshot is preserved. The models in (6) and (8) state that the actual water volumes lost from a single burst V_i^{leaks} and from background leakages V_k^{leaks} in a time interval ΔT strictly depend on the pressure status of the hydraulic system. Consequently, PDA is mandatory to compute the actual water losses in the *i*th node and along the *k*th pipe and the concepts of normal and pressure-deficient working conditions do not apply. In order to account for a single burst, WDNetXL allows setting up β_i and α_i in the table of nodes. In this case the level Z_1 and ΔZ should to be set to null values.
Similarly, the table of pipes reports β_k and α_k for the background leakages along the pipes. The simulation model computes the pipe level outflows and concentrates them in the nodes.

Finally, it is important to remark that, given a pipe k whose end nodes are i and j, the model for background leakages in Eq. (6) cannot be confused with the model for bursts in Eq. (8). In fact, the model in Eq. (6) states that the background leakages for pipe k are:

$$d_k^{leaks}\left(P_{k,mean}\right) = \beta_k L_k \left(\frac{P_i + P_j}{2}\right)^{\alpha} \tag{10}$$

and they are concentrated, for modelling purpose, as two water withdrawals at the end nodes using the coefficient ½.

$$d_i^{leaks}\left(\frac{P_i + P_j}{2}\right) = d_j^{leaks}\left(\frac{P_i + P_j}{2}\right) = \frac{\beta_k L_k}{2}\left(\frac{P_i + P_j}{2}\right)^{\alpha} \tag{11}$$

Concentrating the pipe level outflow at the end nodes preserves the mass balance while causes an error in the energy balance equation. The magnitude of the error can be evaluated as in Giustolisi and Todini (2009) or Giustolisi (2010).
In any case, in order to concentrate in advance the background leakages in the end nodes of pipes, Eq. (10) cannot be substituted using

$$d_i^{leaks}\left(P_i\right) = \frac{\beta_k L_k}{2}\left(P_i\right)^{\alpha};\; d_j^{leaks}\left(P_j\right) = \frac{\beta_k L_k}{2}\left(P_j\right)^{\alpha} \;\;\Rightarrow\;\; d_k^{leaks}\left(P_i, P_j\right) = \beta_k L_k \frac{\left(P_i\right)^{\alpha} + \left(P_j\right)^{\alpha}}{2} \tag{12}$$

The difference between d_k of Eq. (12) and Eq. (10) is function of asset and hydraulic parameters,

$$\beta_k L_k \left(\frac{P_i + P_j}{2}\right)^{\alpha} - \beta_k L_k \frac{\left(P_i\right)^{\alpha} + \left(P_j\right)^{\alpha}}{2} = f\left(\alpha, \beta_k, L_k, \left|P_i - P_j\right|\right) \tag{13}$$

Note that even if α=1 concentrating in advance the background leakages in the end nodes as in Eq. (12), results into a different distribution of nodal outflows, see Eq. (11) and, thus, into a different distribution of pressures through the network which change the background leakage outflows. In other words, for any $\alpha \neq 1$ the background leakages prediction looking at the single pipe is already different, while for $\alpha=1$ the prediction becomes different because of the dissimilar demand and pressure distribution in the network.

DDA vs. PDA

We are here considering the comparison between DDA and PDA to discuss the approximations of DDA. As for example, assuming customer/human- and leakage-based demands, PDA formulation of the volumes in model (1) is

$$\mathbf{V}_n(\mathbf{H}_n) = \mathbf{V}_n^{human}(\mathbf{H}_n) + \mathbf{V}_n^{leaks}(\mathbf{H}_n) \tag{14}$$

where $\mathbf{V}_n{}^{hum}$ and $\mathbf{V}_n{}^{leaks}$ are based on the pressure-demand relationships in (3) and (6-7 or 8), respectively. In the case of $\mathbf{V}_n{}^{hum}$, it is assumed that a required volume of 10 L/h ($V^{req,hum} = \Delta T d^{req,hum}$) is supplied if the pressure for a correct service (P^{ser}) is greater than equal to 10 m. A single level of orifices with a null elevation is assumed. For $\mathbf{V}_n{}^{leaks}$ curve, β is estimated by considering a volume of water losses equal to 25% of the required volume $V^{req,hum}$ and α equal to 1.2.
In the case of DDA, both the volumes are assumed constant ($\mathbf{V}_n{}^{hum}$ = 10 L/h for and ($\mathbf{V}_n{}^{leaks}$ = 2.5 L/h = 25% of $\mathbf{V}_n{}^{hum}$) as required by model assumption. Consequently, DDA does not model the pressure-deficient condition and the volume actually supplied to customers is not computed; moreover, the actual volume of water losses is not computed in both normal and pressure-deficient working conditions. Figure 1 reports the six curves related to pressure-volume (the volume is simply the demand scaled by ΔT) of the previous two components (as defined in DDA and PDA) and their summation in the model. As clear from Figure 1, in nodes experiencing a pressure-deficient condition, DDA overestimates the water volume actually supplied to customers and that lost from background leakages. In the other nodes, DDA underestimates the water losses.
In addition, the assumption of fixed volumes in DDA causes water flowing through the network and head losses independent on head status. In a pressure-deficient condition, this fact causes an important difference in the nodal heads computed using PDA with respect to those using DDA. In fact, DDA overestimates the head losses (more water flowing in the network) and the nodal heads result underestimated. As a result, large negative nodal heads computed by model are usually experienced in DDA.

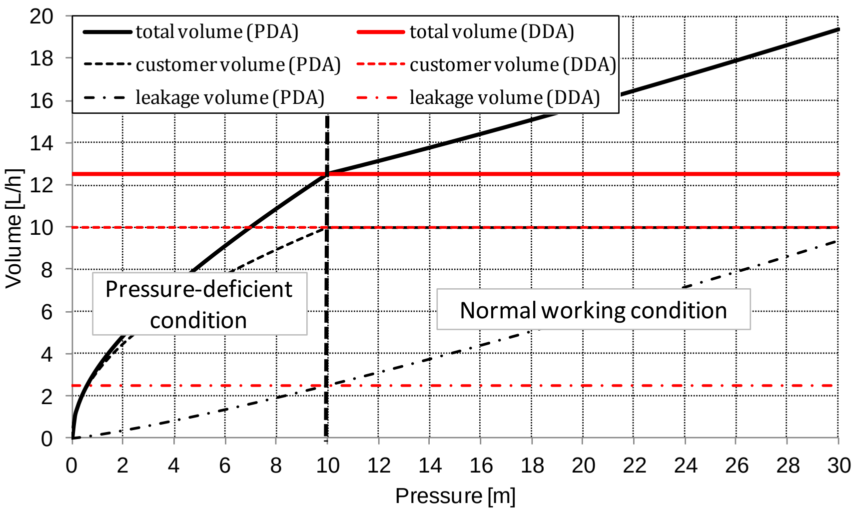


Figure 1 - Pressure-Volume curves in DDA and PDA

Acronyms

DDA	=	Demand-Driven Analysis.
GGA	=	Global Gradient Algorithm.
PDA	=	Pressure-Driven Analysis.
WDN	=	Water Distribution Network.

Suggested References

Germanopoulos, G. 1985 A technical note on the inclusion of pressure dependent demand and leakage terms in water supply network models. *Civil Eng. Syst.*, 2(3), 171–179.
Giustolisi, O. & Walski, T.M. 2012 A Demand Components in Water Distribution Network Analysis. *J. Water Resour. Plan. Manage.*, 138(4), 356 -367.
Giustolisi, O. & Todini, E. 2009 Pipe hydraulic resistance correction in WDN analysis. *Urban Water J.*, 6(1), 39–52.
Giustolisi, O., Savic, D.A. & Kapelan, Z. 2008 Pressure-driven demand and leakage simulation for water distribution networks. *J. Hydr. Eng.*, 134(5), 626–635.

Other references of further modelling implementations

Giustolisi, O., Berardi, L., Laucelli, D. 2014 Modeling local water storages delivering customer-demands in WDN models, *J. Hydr. Eng.*. *J. Hydr. Eng.*, 140(1), 1–16.
Giustolisi, O., Berardi, L., & Laucelli, D. 2012 “Accounting for directional devices in WDN modeling.. *J. Hydr. Eng*., 138(10), 858-869.
Giustolisi, O., Berardi, L., Laucelli, D. & Savic, D.A. 2012 A Computationally efficient modeling method for large size water network analysis. *J. Hydr. Eng*., 138(4), 313-326.
Giustolisi, O., Berardi, L. & Laucelli, D. 2012 Generalizing WDN simulation models to variable tank levels. *J. Hydroinf.*, 12(3) 562–573.
Giustolisi, O. & Laucelli, D. 2011 Water distribution networks pressure-driven analysis using EGGA. *J. Water Resour. Plan. Manage.*, 137(6), 117-127.
Giustolisi, O. 2010 Considering actual pipe connections in WDN analysis. *J. Hydr. Eng.* 136(11), 889–900.