Direct State funding of Chilean universities

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1 Calculation

1.1 Yearly evaluation

1.1.1 Determination

Art. 2 of Decree with Force of Law 4 of 1980, with modifications from Art. 1 of Ministry of Education Decree 116 of 2002, indicates that 5% part of the total funding of year $n+1$ is distributed to University i according to its metrics reported after year n[1]. They involve

- $U_{i,n}$, the number of undergraduate students ("estudiantes de pregrado");
- $M_{i,n}$, the number of majors ("carreras");
- $S_{i,n}$, the number of equivalent full-time scholars ("académicos"), i.e. professors and researchers;
- $P_{i,n}$, the number of equivalent full-time scholars with a post-graduate title such as master or a PhD;
- $G_{i,n}$, the number of research grants ("proyectos");
- $P^{\mathrm{I}}_{i,n}$, the number of Web of Science publications (WoS)[2];
- and $P^{\mathrm{S}}_{i,n}$, the number of non-WoS publications indexed by the Scientific Electronic Library Online (Scielo) Chile.

The metrics, defined in the aformentioned decrees, are ratios meant to measure an output v. staff efficiency[3]

$$x_{i,n,1} = U_{i,n}/M_{i,n}, \tag{1a}$$
$$x_{i,n,2} = U_{i,n}/S_{i,n}, \tag{1b}$$
$$x_{i,n,3} = P_{i,n}/S_{i,n}, \tag{1c}$$
$$x_{i,n,4} = G_{i,n}/S_{i,n}, \tag{1d}$$
$$x_{i,n,5} = (P^{\mathrm{I}}_{i,n} + \frac{33}{100} P^{\mathrm{S}}_{i,n})/S_{i,n} \tag{1e}$$

Table 1: Coefficients used for university evaluation since 1998.

	ratio	value
c_1	students-to-majors	0.01
c_2	students-to-staff	0.14
c_3	postgrad staff-to-staff	0.24
c_4	grants-to-staff	0.25
c_5	papers-to-staff	0.35

According to Art. 3 of Ministry of Education Decree 128 of 1991, the evaluation formula renormalises the aforemention ratios in this way[4]:

$$\mu_{n,k} = \frac{1}{N} \sum_j x_{j,n,k} \qquad \text{(mean)} \tag{2a}$$
$$\sigma_{n,k} = \sqrt{\frac{1}{N}\Big(\sum_j x^2_{j,n,k}\Big) - N\mu^2_{n,k}} \qquad \text{(std. dev.)} \tag{2b}$$
$$\xi_{i,n,k} = \frac{x_{i,n,k} - \mu_{n,k}}{\sigma_{n,k}} \qquad \text{(reduced coeff.)} \tag{2c}$$
$$y_{i,n,k} = \exp\left[-\frac{7}{5} + \frac{\xi_{i,n,k}}{4}\right]^3 \tag{2d}$$

where N is the total number of universities. The transform in Eq. (2c) ensures that Universities are compared by how much they deviate from the mean. The exponential in Eq. (2d) is supposed to simulate a biological growth. Figure 2 displays the exponential nature of the rating.

Art. 2 of Decree with Force of Law 4 of 1980 indicates that 5% of the funding is indexed on a weighted average of the metrics $y_{i,n,k}$ (k in $1 \cdots 5$) (see Sect. 1.1.1). The weights c_k may vary from year to year, but have been constant since 1998 (see Table 1). University i is thus assigned a score

$$y_{i,n} = \sum_k c_k y_{i,n,k}. \tag{3a}$$

and, using the total score

$$y_n = \sum_i y_{i,n}, \tag{3b}$$

[1] Some are from year n and others from year $n-1$.

[2] At the time of the Decree 116 it was known as ISI

[3] While the number of publications is defined by the number of WoS plublications plus one third of Scielo one by Ministry of Education Decree 116 of 2002, the Ministry has consistently used factor 0.33 instead of 1/3 for the calculation.

[4] Although not specified by the decree, the Ministry has consistently used the population variance, not the sample variance, for the calculation.

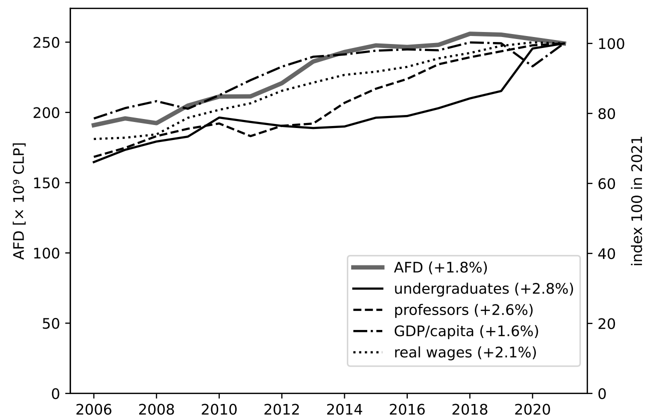

Figure 1: Evolution of total direct State funding to traditional Chilean Universities, in 2020 pesos (inflation-corrected) compared to the evolution of the GDP per capita (World Bank data), the mean real wage (Instituto Nacional de Estadísticas data), the number of undergraduate students and the number of professors.

a funding share

$$f_{i,n} = \frac{y_{i,n}}{y_n} \tag{3c}$$

1.1.2 Marginal earnings

In this section, I focus on a yearly snapshot and drop the n index in the formulae. I examine the case where university i decides to increase one of its ratios number k by a small number of standard deviations $\Delta\xi_{i,k}$, so that the ratio $x_{i,k}$ improves by $\Delta x_{i,k} = \sigma_k \Delta\xi_{i,k}$.

For any university j, the new value of the score $y_{j,k}$ is usually modified because the mean and standard deviation are changed via $x_{i,k}$. The difference $\Delta y_{j,k}$ is given by differentiating Eq. (2d), and in turn Eqs. (2a–2c) on which it depends. The calculation, detailed in in Appendix A.1, yields

$$\frac{\Delta y_{j,k}}{y_{j,k}} = \frac{3}{4}\left(\frac{\xi_{j,k}}{4} - \frac{7}{5}\right)^2 \left(\delta_{ij} - \frac{1}{N} - \frac{\xi_{i,k}\xi_{j,k}}{N}\right)\Delta\xi_{i,k}, \tag{4}$$

where $\delta_{ij} = 1$ if universities i and j are the same and zero otherwise.

The meaning of Eq. (4) is the following:

1. $(\xi_{j,k}/4 - 7/5)^2$ *factor*: The relative improvement depends on the relative standing of the University in the ranking. A university lagging behind by 2 standards deviations gets a relative improvement 4,5 times higher than a university standing out by 2 standard deviations.

2. δ_{ij} *term:* University i generally benefits from an increase of its own ratio $\Delta x_{i,k}$: the δ_{ij} ($= 1$ for $i = j$) term in the equation is the only one that is not in $1/N$ and thus dominates. However, if $|\xi_{i,k}| > \sqrt{N-1} \approx 5$ standard deviations, University would lose from improving. Nevertheless, the data of the Ministery from 2006 to 2020 can be used to show that the highest deviation any of the ratio has ever reached is 3.7.

3. University j may benefit from, or be harmed by, the improvement of University i. There are two effects at play.

   (a) $1/N$ *term*: The increase of the mean, would on its own hurt all other universities as their position relative to the mean $\xi_{j,k}$ would drop (see the $1/N$ term in the equation).

   (b) $\xi_{i,j}\xi_{j,k}/N$ *term*: However, the modification of the standard deviation works both ways. Intuitively, if a university with a high $\xi_{i,k} > 0$ ($x_{i,k} > \mu_k$) improves, it will increase the standard deviation, so that all universities deviate less from the mean: other universities with $\xi_{j,k} > 0$ will lose some of their good standing and lower tier ones with $\xi_{j,k} < 0$ will decrease their lag . Conversely, on can see that the improvement of a University with a lower rank $\xi_{j,k} < 0$, by decreasing the standard deviation of the sample when it goes closer to the mean, will help those with good standing to stand out more and harm other lower tier ones.

   For both effects combined, University j benefits if $\xi_{i,k}\xi_{j,k} < -1$ and is harmed otherwise.

To determine the additional funding fraction Δf_j, let's propagate Eq. (4) into Eqs. (3a–3c):

$$\frac{\Delta f_j}{f_j} = c_k \left[\left(1 - \frac{y_j}{y}\right)\frac{\Delta y_{j,k}}{y_j} - \sum_{l \neq j} \frac{y_l}{y}\frac{\Delta y_{l,k}}{y_l}\right]. \tag{5}$$

The first term in the square brackets has the same sign as $\Delta y_{j,k}$ because, by definition, $y_j < y$. In most cases, the second term is smaller than the first one because the $\Delta y_{l,k}$ partially cancel out (some positives and some negatives) and $y_l < y$. It means that the funding received by a university that has an improved rating normally receives additional funding. It is possible, though, that a university with a very small $\Delta y_{j,k}$ (e.g. $\xi_{j,k} < -2$) will be harmed by increasing its score, because the other, larger, $\Delta y_{l,k}$ coould lead to a second term larger than the first term under these circumstances. In years 2006–2019 it has ocurred once, very marginally, in 2015, for Universidad de Talca. It would have received 1,000 CLP less had it substituted four regular professors with ones owning a postgraduate degree (improvement of $y_{\text{U. Talca},2015,3}$). It happened on that year that Universidad de Talca had the highest negative standard deviation observed for any metrics in the period 2006–2019 ($\xi \approx -2.8$).

1.2 Time evolution

Total funding. The total funding in year n, F_n, is a slowly increasing series (see Fig. 1). In half of the years it approximately follows the consumer price index, but it has received a modest boost in other years. The average inflation-corrected increase has been 1.8% per year in period 2006–2021. This increases matches the increase in undergraduate students (+2.8% in 2004–2019), professors (+2.6% in 2005–2020), real wages (+2.1% in 2006–2021), and GDP per capita (+1.6% in 2006–2021). Table 3 in Appendix A.2 gives an overview of the main macroeconomic quantities in this period. Note that the pre-pandemic trends for macroeconomic data and AFD evolution were about 0.2% higher.

Increase in standard of living and student population are long-term trends that I would expect to hold for at least the next decade, so I assume that University funding by the State will still follow this trend. For predictions, beyond 2021, I will therefore take

$$F_{n+1} = (1+q)F_n, \tag{6}$$

where $q = 2\%$.

University funding. Let $F_{i,n}$ be the funding received by university i at year n. Art. 2 of Decree with Force of Law 4 of 1980 indicates that 5% of the funding is indexed on metrics $y_{i,n,k}$ and 95% of the funding is related to the previous year's share of the total funding. So,

$$F_{i,n+1} = \frac{19F_{i,n}}{20F_n}\left(F_{n+1} - F^{\mathrm{new}}_{n+1}\right) + \frac{y_{i,n}}{20y_n}F_{n+1}. \tag{7}$$

where F^{new}_{n+1} is the part of the 95% apportionned to newly admitted traditional universities. In the period 2006–2021, it has been nonzero in 2018 when State Universities of O'Higgins and Aysén where included (Decree 228 of 2017).

Funding by metric. For an analysis of the strengths and weaknesses of each Universities, it is useful to determine what part of the total AFD funding originates from "historic" funding at year n_0 and which parts originate from each of the metrics computed for the funding of years n_0+1 to n. Let $H_{i,n}$ be the share of "historic" funding of university i in year n and $F_{i,n,k}$ that of the metric number k. They can be determined via the initial conditions and recurrence relations:

$$H_{i,n_0} = F_{i,n_0}, \tag{8}$$

$$H_{i,n+1} = \frac{19H_{i,n}}{20F_n}\left(F_{n+1} - F^{\mathrm{new}}_{n+1}\right), \tag{9}$$

$$F_{i,n_0,k} = 0, \tag{10}$$

$$F_{i,n+1,k} = \frac{19F_{i,n,k}}{20F_n}\left(F_{n+1} - F^{\mathrm{new}}_{n+1}\right) + \frac{c_k y_{i,n,k}}{20y_n}F_{n+1} \quad \text{for } 1 \le k \le 5. \tag{11}$$

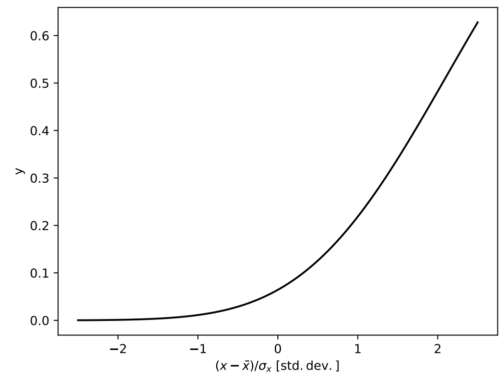

Figure 2: Transformation of the metric $x_{i,n,k}$ into $y_{i,n,k}$, before a weighted sum $\sum_k c_{i,n,k}y_{i,n,k}$ is performed to determine the rating of university i in year n.

It is straighforward to check that $F_{i,n} = H_{i,n} + \sum_k F_{i,n,k}$ since the six recurrence equations (Eqs. 9 & 11) sum to Eq. (7).

In steady state—ignoring the small non-zero values for F^{new} in 2018—we expect that the historic to total funding ratio decreases by 5% each year from a value of 1 in year n_0. I define the surging index as

$$\mathcal{S}_{i,n} = \left(\frac{19}{20}\right)^{n-n_0} - \frac{H_{i,n}}{F_{i,n}} \tag{12}$$

with positive values for surging Universities boasting better metrics in the recent years than in the past.

I also define the research index as the ratio of funding obtained for scientific activities (metrics 4 & 5: number of grants and papers) to the funding received for all metrics.

$$\mathcal{R}_{i,n} = \frac{F_{i,n,4} + F_{i,n,5}}{F_{i,n} - H_{i,n}}. \tag{13}$$

A value of 1 denotes a research institution while a value of 0 a university only dedicated to teaching.

1.3 Checks

Yearly evaluation I have checked the calculations of the 5% using open data from the Eduction Ministry for years 2006 to 2021. For each year since 2011 and 2007–2009, the percentages I derived match within numerical rounding errors (8 digits) with those of the Ministry. The subsidies I predict for each university differ by at most CLP 1,000 ($\approx$ 1.20 USD as of November 2021) with the official ones due to rounding errors, as the accounting unit used in the official documents is 1,000 Chilean pesos. In 2010, the Ministry used the 2009 calculation with 2008 metrics, instead of 2009 ones, leading to large differences if the 2009 metrics given in the Ministry's spreadsheet is used. Difference are zero within rounding errors using 2008 data instead. In 2006, there is an unexplained 0.01% discrepancy between my determination and the Ministry's. The official spreadsheet file from the ministry, with additional sheets showing my calculations, is available from github project `https://github.com/loqueelvientoajuarez/afd`.[5].

Time-evolution I have checked the induction formula Eq. (7) using the total amount given for each year F_n. The 95% funding is well predicted from year to year, except again for 2010, where I had to substitute 2008 funding percentages to the expected 2009 ones. Because of rounding errors cumulating from year to year, the amounts I predict differ by up to 5,000 pesos ($\approx$ 6 USD as of November 2021) with those of the Ministry.

Marginal earnings Marginal earnings have been determined by two methods. The first one, using differential calculus in Sect. 1.1.2, and the second one, by doing the full calculation with Eq. (7) using the new values of the coefficients. I have checked that both method agree within a few significant digits as long as the variations remain small.

2 Incentives & desincentives

2.1 Marginal earnings

If an additional paper has been published by a researcher of University i in year $n-1$, it will reflect in the 5% funding of year n. Let us call $\Delta F_{i,n}$ the additional earnings of the university in that year. In the subsequent years, it will reflect via the 95% (first term of the right handside of Eq. (7)) in this way:

$$\begin{aligned}\Delta F_{i,n+k} &= \frac{19}{20}\Delta F_{n+k-1}\frac{F_{n+k}}{F_{n+k-1}},\\ &= \Delta F_{n+k-1}\frac{19(1+q)}{20},\\ &= \Delta F_n\left(\frac{19(1+q)}{20}\right)^k. \end{aligned}\tag{14}$$

In order to obtain the present value corresponding to these future earnings, we take a yearly depreciation of $d = 5\%$ in real terms (i.e. $\approx 8\%$ in pesos).

$$\Delta F^{\mathrm{PV}}_{i,n+k} = \Delta F_n\left(\frac{19(1+q)}{20(1+d)}\right)^k \tag{15}$$

The additional funding obtained by the university in all years is therefore

$$\begin{aligned}\Delta F^{\mathrm{PV}}_i &= \sum_{k=0}^{+\infty}\Delta F_{i,n+k},\\ &= \sum_{k=0}^{+\infty}\left(\frac{19(1+q)}{20(1+d)}\right)^k\Delta F_{i,n}\\ &= \frac{20(1+d)}{1+20d-19q}\Delta F_{i,n},\\ &\approx 13\Delta F_{i,n}.\end{aligned}\tag{16}$$

The determination of $\Delta F_{i,n}$ is straightforward. The calculations in Eqs. (1a–7) are done with the metrics provided by the Ministry (see Sect. 1.1.1) and for the same ones with an additional publication. The difference in funding is $\Delta F_{i,n+1}$.

Table 2.1 gives the 2021 funding a University would have received, had an additional 2020 paper been published, an additional one-year science staff (professor) been contracted, or an additional grant been open (postdoct staff or other project). I have made the hypothesis that no other Traditional University has co-authored the paper, in which case the amount may vary.

My figures differ from those derived by Ramírez and Alfaro [2012]. The reasons are that they

1. only consider the first five years after the paper is published though their dampening law of $95\%/(1+8\%)$ still yields about 50% of the inital sum at that point, meaning that they underestimate the total revenue obtained with a paper by a factor of ≈ 2;
2. do not take into account that the total funding increases by IPC + 2% $\approx$ 5% per year, meaning that they further underestimate the present value of a paper by a factor of ≈ 1.5;
3. publish at a time of significantly larger incentives, by a factor of ≈ 2 for well established universities; and
4. seem to use different values for the coefficients than those published by the Ministry and actually used for the calculations. Their Fig. 4 doesn't match the corrected coefficients I derive for 2009, 2010, or 2011. For instance, both my data and the Ministry's official figures for years 2006 to 2012 show a systematic discrepancy between U. de Chile and P. U. Católica de Chile between 31 and 60% in the weighted sum of corrected coefficients and share of the 5%, while their Figure shows a discrepancy of only about 10%.

[5] The original ministry file can be obtained from `http://dfi.mineduc.cl/usuarios/MECESUP/File/2021/AFD/AFD_2006_al_2021_MontosVariables5xc.xlsx`.

Table 2: Additional earnings in 2021 Chilean pesos for thes marginal improvement of 2020 metrics: an additional one-year full-time contract of a postgraduate professor, an additional research grant, and an additional Web of Science (ex-ISI) publication with no collaborators from other Traditional Universities. Assumptions for the all years funding is: State contribution continues to grow by 2% a year real terms, a yearly depreciation of 5% in real terms to future earnings is applied.

university	postgraduate staff		research grant		WoS publication	
	2021	all years [CLP]	2021	all years [CLP]	2021	all years [CLP]
U. de Chile	317,000	4,109,259	1,601,000	20,753,704	467,000	6,053,704
P. U. Católica de Chile	366,000	4,744,444	1,649,000	21,375,926	449,000	5,820,370
U. de Concepción	1,376,000	17,837,037	2,009,000	26,042,593	586,000	7,596,296
U. Católica de Valparaíso	3,595,000	46,601,852	5,665,000	73,435,185	1,557,000	20,183,333
U. Téc. Federico Sta. María	1,740,000	22,555,556	5,306,000	68,781,481	1,248,000	16,177,778
U. de Santiago	177,000	2,294,444	1,704,000	22,088,889	407,000	5,275,926
U. Austral	2,113,000	27,390,741	2,481,000	32,161,111	767,000	9,942,593
U. Católica del Norte	1,451,000	18,809,259	1,709,000	22,153,704	1,391,000	18,031,481
U. de Valparaíso	1,216,000	15,762,963	1,475,000	19,120,370	294,000	3,811,111
U. de Antofagasta	1,584,000	20,533,333	891,000	11,550,000	1,939,000	25,135,185
U. de la Serena	485,000	6,287,037	1,835,000	23,787,037	1,865,000	24,175,926
U. de Bío-bío	5,056,000	65,540,741	2,960,000	38,370,370	805,000	10,435,185
U. de la Frontera	2,186,000	28,337,037	6,905,000	89,509,259	2,124,000	27,533,333
U. de Magallanes	216,000	2,800,000	2,678,000	34,714,815	645,000	8,361,111
U. de Talca	4,061,000	52,642,593	4,764,000	61,755,556	657,000	8,516,667
U. de Atacama	78,000	1,011,111	607,000	7,868,519	339,000	4,394,444
U. de Tarapacá	7,353,000	95,316,667	5,562,000	72,100,000	2,847,000	36,905,556
U. Arturo Prat	595,000	7,712,963	505,000	6,546,296	85,000	1,101,852
U. Metropolitana	5,426,000	70,337,037	545,000	7,064,815	90,000	1,166,667
U. de Playa Ancha	4,838,000	62,714,815	1,675,000	21,712,963	105,000	1,361,111
U. Tecnológica Metropolitana	1,621,000	21,012,963	914,000	11,848,148	161,000	2,087,037
U. de Los Lagos	2,279,000	29,542,593	1,139,000	14,764,815	206,000	2,670,370
U. Católica de Maule	548,000	7,103,704	1,203,000	15,594,444	552,000	7,155,556
U. Católica de Temuco	1,148,000	14,881,481	1,113,000	14,427,778	140,000	1,814,815
U. C. de la Sant. Concepción	315,000	4,083,333	1,108,000	14,362,963	375,000	4,861,111
U. de O'Higgins	8,124,000	105,311,111	19,909,000	258,079,630	4,234,000	54,885,185
U. de Aysén	16,241,000	210,531,481	11,825,000	153,287,037	4,095,000	53,083,333

2.2 Value of collaborations

A Appendix

A.1 Derivation of equation

The variation in $\Delta y_{j,k}$ is linked to $\Delta\xi_{i,k}$ via the derivative:

$$\Delta y_{j,k} \approx \frac{\partial\, y_{j,k}}{\partial x_{i,k}}\,\Delta x_{i,k}, \tag{17}$$

$$\approx \frac{\mathrm{d}\, y_{j,k}}{\mathrm{d}\xi_{i,k}}\,\frac{\partial\, \xi_{i,k}}{\partial x_{i,k}}\,\sigma_k \Delta\xi_{i,k}, \tag{18}$$

so, substituting Eq. (2c), for the second factor

$$\approx \frac{\mathrm{d}\, y_{j,k}}{\mathrm{d}\xi_{i,k}}\left[\frac{\partial\, x_{j,k}}{\partial x_{i,k}} - \frac{\partial\, \mu_k}{\partial x_{i,k}} - \frac{x_{j,k}-\mu_k}{\sigma_k}\frac{\partial\, \sigma_k}{\partial x_{i,k}}\right]\Delta\xi_{i,k} \tag{19}$$

and, backsubstituting Eq. (2c),

$$\approx \frac{\mathrm{d}\, y_{j,k}}{\mathrm{d}\xi_{i,k}}\left[\frac{\partial\, x_{j,k}}{\partial x_{i,k}} - \frac{\partial\, \mu_k}{\partial x_{i,k}} - \xi_{j,k}\frac{\partial\, \sigma_k}{\partial x_{i,k}}\right]\Delta\xi_{i,k}. \tag{20}$$

The first factor is the derivative of the function in the right handside of Eq. (2d). It is:

$$\frac{\mathrm{d}\, y_{i,k}}{\mathrm{d}\xi_{i,k}} = \frac{3}{4}\left[-\frac{7}{5} + \frac{\xi_{i,k}}{4}\right] y_{i,k}. \tag{21}$$

In the second factor, the first term is one if $i = j$, $x_{j,k}$ and $x_{i,k}$ being then the same variable, and zero otherwise. The second term is the variation of the mean when one of the term varies, it is therefore $1/N$ the variation of the individual term. So,

$$\frac{\partial\, x_{j,k}}{\partial x_{i,k}} = \delta_{ij}, \tag{22}$$

$$\frac{\partial\, \mu_k}{\partial x_{i,k}} = \frac{1}{N}\sum_j \frac{\partial\, x_{j,k}}{\partial x_{i,k}} = \frac{1}{N}\sum_j \delta_{ij} = \frac{1}{N}. \tag{23}$$

Table 3: Macroeconomic data for Chile 2006 to 2020: UF, a price-indexed fiscal currency unit, the growth of the GDP per capita, and the growth of the mean wage.

year	UF[1] [CLP]	GDP per capita[2] [growth in %]	Mean wage[3] [growth in %][4]
2006	18,152.61	5.20	3.0
2007	18,627.89	3.79	0.5
2008	20,260.76	2.426	1.3
2009	20,930.92	−2.601	6.4
2010	21,204.99	4.75	2.9
2011	21,892.81	5.058	2.3
2012	22,627.36	4.309	4.3
2013	22,852.67	3.032	2.7
2014	24,026.01	0.693	2.5
2015	24,984.62	1.106	1.0
2016	26,053.81	0.372	1.5
2017	26,665.98	−0.247	2.6
2018	27,161.48	2.281	1.6
2019	27,908.86	−0.245	1.4
2020	28,695.46	−6.581	1.0
2021	29,712.80	7[6]	−0.3[7]

[1] Source: SII. UF (price indexed) on July 1st
[2] Sourec: World Bank
[3] Source: Instituto Nacional de Estadísticas
[4] From January to January
[5] Up to September
[6] Forecast
[7] January-September

The last term requires some more calculation. Let's use Eq. (2b):

$$\frac{\partial\,\sigma_k}{\partial x_{i,k}} = \frac{\partial}{\partial x_{i,k}}\sqrt{\frac{1}{N}\Big(\sum_j x_{j,n,k}^2\Big) - \mu_{n,k}^2}, \tag{24}$$

$$= \frac{1}{2\sigma_k}\frac{\partial}{\partial x_{i,k}}\left[\frac{1}{N}\Big(\sum_j x_{j,k}^2\Big) - \mu_k^2\right], \tag{25}$$

$$= \frac{1}{2\sigma_k}\left[\frac{2}{N}\sum_k x_{j,k}\frac{\partial\,x_{j,k}}{\partial x_{i,k}} - 2\mu_k\frac{\partial\,\mu_k}{\partial x_{i,k}}\right], \tag{26}$$

so, using Eq. (22) and Eq. (23),

$$= \frac{1}{2\sigma_k}\left[\frac{2\xi_{i,k}}{N} - \frac{2\mu_k}{N}\right] \tag{27}$$

and, finally, with Eq. (2c),

$$= \frac{\xi_{i,k}}{N}. \tag{28}$$

A.2 Macroeconomic quantities

The Chilean consumer price index narrowly followed by the unidad de fomento (UF), a fiscal currency unit with daily conversions published by the inland revenue service known as Servicio de Impuestos Internos (SII). The GDP growth per capita has been taken from the World Bank online database and the growth of the mean wage from the Chilean institute for Statistics (Instituto Nacional de Estadística). They are summed up in Table 3.

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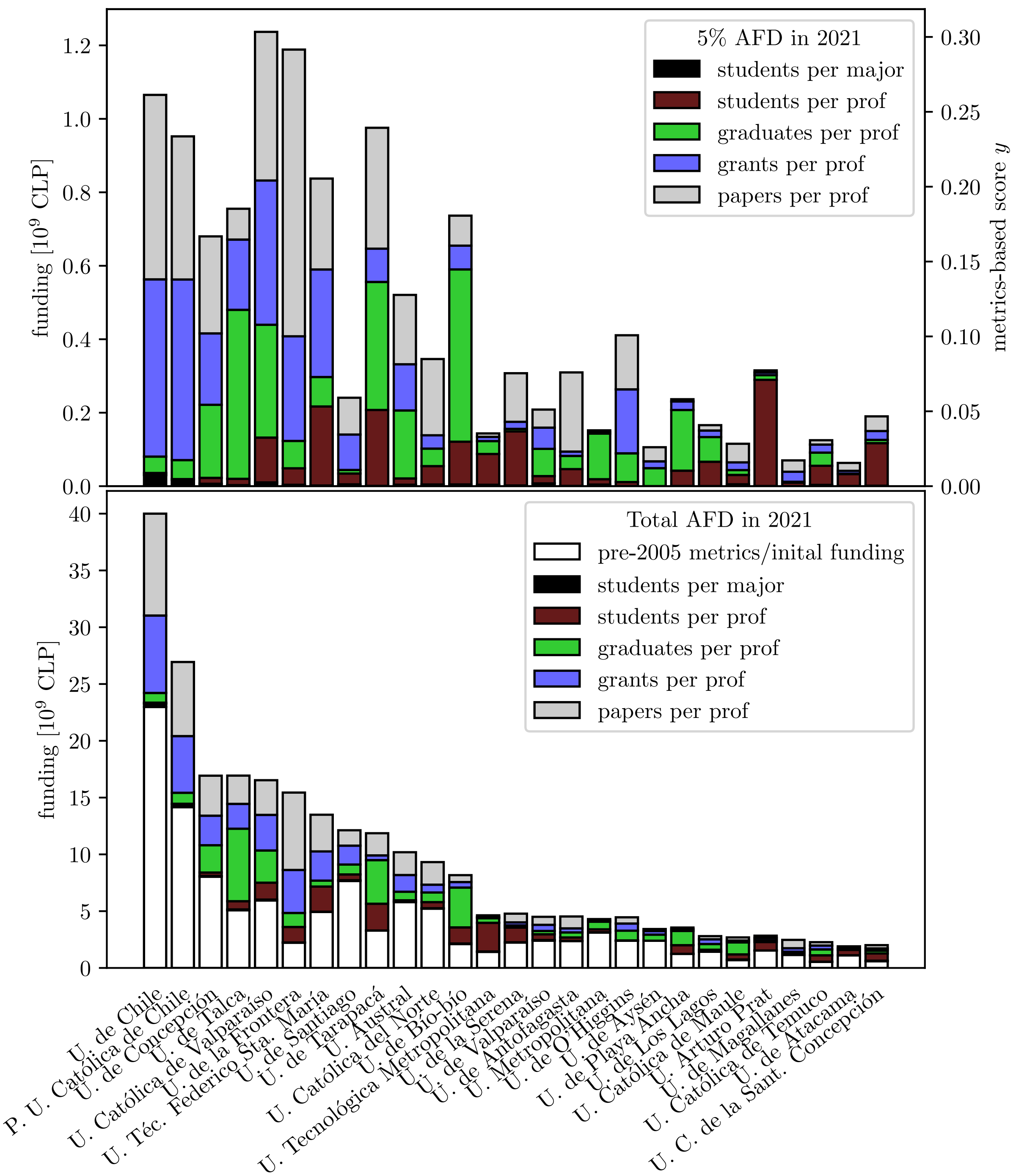

Figure 3: Contributions of each metric to the AFD funding of traditional Chilean universities in 2021. *Top panel:* contributions of 2020 metrics to the 5%. *Bottom panel:* contributions of the recent metrics and historic pre-2005 (or pre-creation for the most recent universities) ones to the total funding. *Left axis:* thousand million Chilean pesos, of the order of million US dollars. *Right axis:* university scores $c_k y_k$ and its total $y = \sum c_k y_k$.

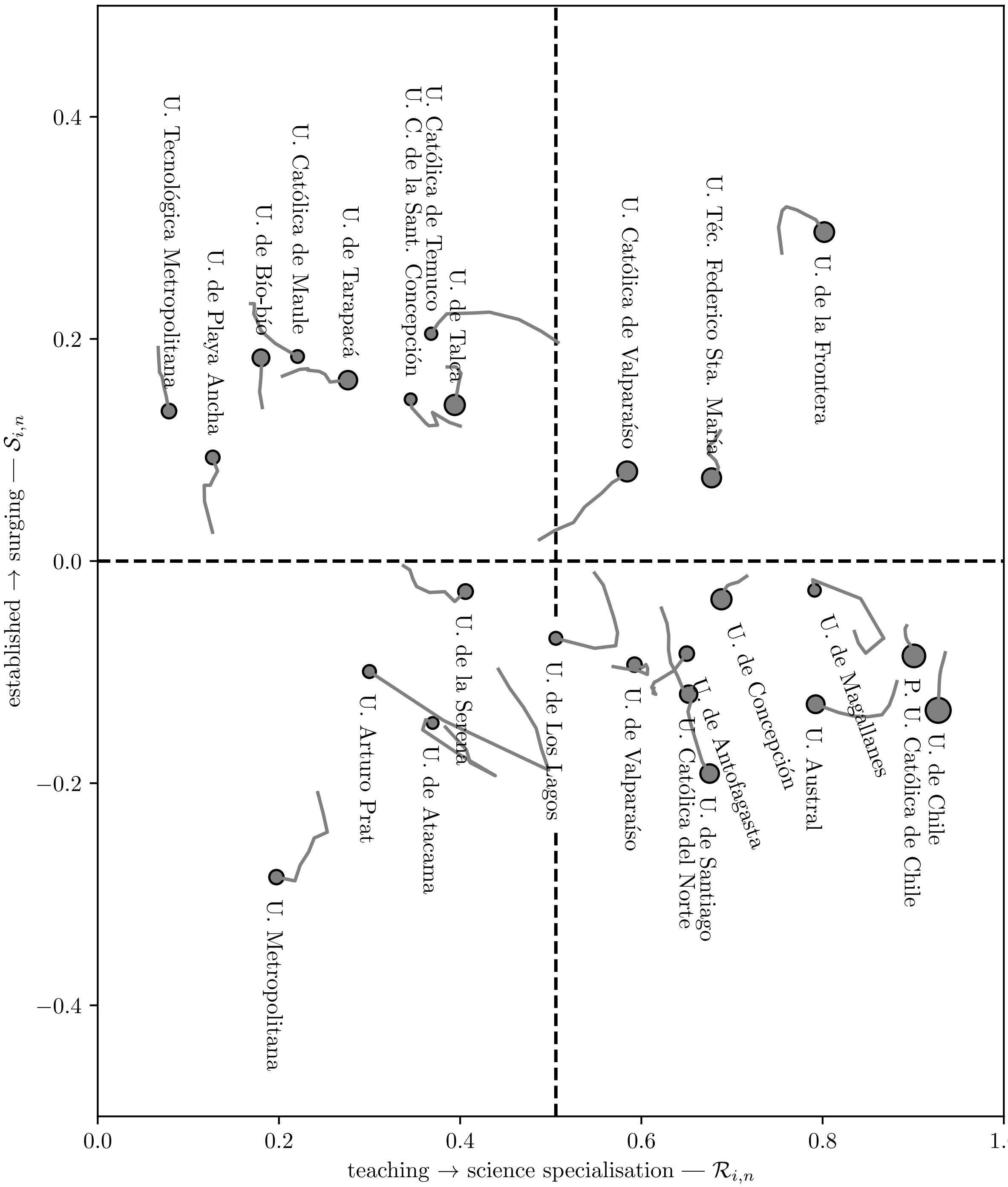

Figure 4: Classification of traditional universities according to their teaching v. science specialisation and their established v. surging status. The two newly created institutions have not been included. *Circles:* 2021 standing, area is proportional to the total AFD funding received in 2021. *Gray tracks:* evolution of the universities through years 2014–2021. *Horizontal axis:* research index, i.e. ratio of funding originating from the 2005–2020 science metrics (number of grants and papers) to the one from all 2005–2020 metrics. The vertical line indicates the median value. *Vertical axis:* surging index, i.e. ratio of funding obtained from recent metrics to total funding minus its expected steady state value.