已知：直线 $AB \parallel CD$，点 P 为平面上一点，连接 AP 与 CP。

(1) 如 1，点 P 在直线 AB、CD 之间，当 $\angle BAP = 60^\circ$，$\angle DCP = 20^\circ$ 时，求 $\angle APC$。

(2) 如 2，点 P 在直线 AB、CD 之间，当 $\angle BAP$ 与 $\angle DCP$ 的角平分线相交于 K，写出 $\angle AKC$ 与 $\angle APC$ 之间的数量关系，并说明理由。

(3) 如 3，点 P 落在 CD 之外，$\angle BAP$ 与 $\angle DCP$ 的角平分线相交于 K，$\angle AKC$ 与 $\angle APC$ 有何数量关系？并说明理由。

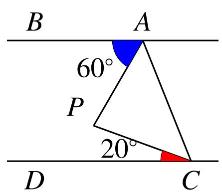


图 1

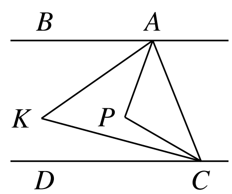


图 2

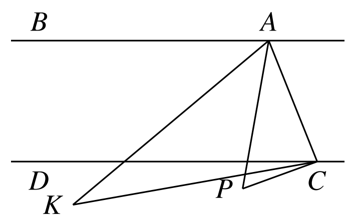


图 3

(1) **解.** $\because AB \parallel CD$

$\therefore \angle BAC + \angle DCA = 180^\circ$

又 $\because \angle BAC = \angle BAP + \angle PAC$

$\angle DCA = \angle ACP + \angle DCP$

$\therefore \angle BAP + \angle PAC + \angle ACP + \angle DCP = 180^\circ$

又 $\because$ 三角形内角和 $= 180^\circ$

$\therefore \angle APC + \angle PAC + \angle ACP = 180^\circ$

从而可得：

$$\begin{aligned}\angle APC &= \angle BAP + \angle DCP \\ &= 60^\circ + 20^\circ \\ &= 80^\circ\end{aligned}$$

□

(2) **解.** $\because AB \parallel CD$

$\therefore \angle BAC + \angle DCA = 180^\circ$

又 $\because \angle BAC = \angle BAP + \angle PAC$

$\angle DCA = \angle ACP + \angle DCP$

$\therefore \angle BAP + \angle PAC + \angle ACP + \angle DCP = 180°$

又 $\because$ 三角形内角和 $= 180°$

$\therefore \angle APC + \angle PAC + \angle ACP = 180°$

$\therefore \angle APC = \angle BAP + \angle DCP$

同理可得：$\therefore \angle AKC = \angle BAK + \angle DCK$

又 $\because K$ 是 $\angle BAP$ 与 $\angle DCP$ 的角平分线的交点

从而可得：

$$\begin{aligned}\angle AKC &= \frac{1}{2}\angle BAP + \frac{1}{2}\angle DCP \\ &= \frac{1}{2}(\angle BAP + \angle DCP) \\ &= \frac{1}{2}\angle APC\end{aligned}$$

□

(3) **解**. $\because AB \parallel CD$

$\therefore \angle BAC + \angle DCA = 180°$

又 $\because \angle BAC = \angle BAP + \angle PAC$

$\angle DCA = \angle ACP - \angle DCP$

$\therefore \angle BAP + \angle PAC + \angle ACP - \angle DCP = 180°$

又 $\because$ 三角形内角和 $= 180°$

$\therefore \angle APC + \angle PAC + \angle ACP = 180°$

$\therefore \angle APC = \angle BAP - \angle DCP$

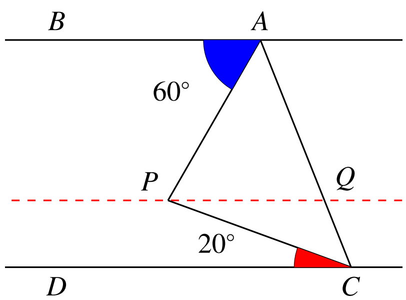
同理可得：$\therefore \angle AKC = \angle BAK - \angle DCK$

又 $\because K$ 是 $\angle BAP$ 与 $\angle DCP$ 的角平分线的交点

从而可得：

$$\begin{aligned}\angle AKC &= \frac{1}{2}\angle BAP - \frac{1}{2}\angle DCP \\ &= \frac{1}{2}(\angle BAP - \angle DCP) \\ &= \frac{1}{2}\angle APC\end{aligned}$$

□



(1) **解.** 过 P 点作辅助线 PQ，并且使 $PQ \parallel CD$

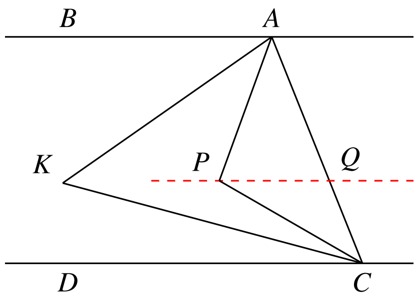
$\because AB \parallel CD$

$\therefore \angle APQ = \angle BAP$，并且 $\angle CPQ = \angle DCP$

从而可得：

$$\begin{aligned}\angle APC &= \angle APQ + \angle CPQ \\ &= \angle BAP + \angle DCP \\ &= 60^\circ + 20^\circ \\ &= 80^\circ\end{aligned}$$

□



(2) **解**. 过 P 点作辅助线 PQ，并且使 $PQ \parallel CD$

$\because AB \parallel CD$

$\therefore \angle APQ = \angle BAP$，并且 $\angle CPQ = \angle DCP$

$\therefore \angle APC = \angle BAP + \angle DCP$

同理可得：$\angle AKC = \angle BAK + \angle DCK$

又 $\because K$ 是 $\angle BAP$ 与 $\angle DCP$ 的角平分线的交点

从而可得：

$$\begin{aligned}\angle AKC &= \frac{1}{2}\angle BAP + \frac{1}{2}\angle DCP \\ &= \frac{1}{2}(\angle BAP + \angle DCP) \\ &= \frac{1}{2}\angle APC\end{aligned}$$

$\square$